Chapter 8: Exponential and Logarithmic Functions

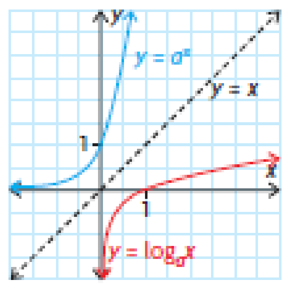
Skills and Concepts you need: 1, 2, 3

Exploring the Logarithmic Function

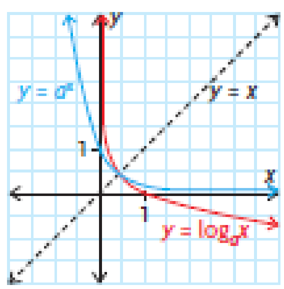
The **logarithmic function** is the inverse of the exponential function $y = a^x$, ie. $x = \log_a y$. As with the exponential function, a>0, a!=1. This can be read as "the logarithm of y to the base of x", and understood as "the exponent that must be applied be applied to base a to get the value x is y".

- The general shape of the graph of the logarithmic function depends or value of the base.

When $a > 1$, the exponential function is an increasing function, and the logarithmic function is also an increasing function.



When $0 < a < 1$, the exponential function is a decreasing function and the logarithmic function is also a decreasing function.



- The y-axis is the vertical asymptote for the logarithmic function. The x-axis is the horizontal asymptote for the exponential function.
- The x-intercept of the logarithmic function is 1, while the y-intercept of the exponential function is 1.
- The domain of the logarithmic function is $\{x \in \mathbf{R} \mid x > 0\}$, since the range of the exponential function is $\{y \in \mathbf{R} \mid y > 0\}$.
- The range of the logarithmic function is $\{y \in \mathbf{R}\}$, since the domain of the exponential function is $\{x \in \mathbf{R}\}$.

FYU 1,2,3 a,c – Class discussion

FYU 1,2,3 c, 7ac – Group discussion

Potential Assignment Questions: 1/2/3 d, 5, 7bd, 9, 10

8.2 Transformations of Logarithmic Functions

Logarithms of base 10 are considered the default, and are called a common logarithm. They may be written as $y = \log x$.

Group discussion – Graph $y = 10^x, y = \log_{10} x$

These functions can be transformed in the usual way as $f(x) = \log_{10} x \rightarrow a\log_{10}(k(x-d)) + c$, where :

- |a| gives the vertical stretch or compression with a<0 indicating a reflection in the x axis.
- 1/|k| gives the horizontal stretch or compression, with k<0 indicating a reflection in the y-axis.
- d gives the horizontal translation, and can affect the domain of the function as it may move the location of the vertical asymptote. If the function is now to the left of the asymptote x=d, the domain is x<d. If the function is to the right of the asymptote x=d, the domain is x>d. The range of a logarithmic function is always $\{y \in \mathbb{R}\}$.
- c gives the vertical translation.

Ex1 – Class discussion

Ex2 – Group discussion

CYU 1,3 – Take home

Potential Assignment questions: 4, 5, 7, 8, 9

8.3 Evaluating Logarithms

Ex1 - Class discussion

Ex2 – Class discussion a,c – Group discussion b,d

Exponential equations can be solved by:

- expressing both sides as powers with a common base and then equating the exponents.
- graphing both sides of the equation and determine the intersection point.
- Rewriting the equation in logarithmic form and simplifying.

In general, it's important to keep in mind that logarithms of negative numbers don't exist, as a negative number cannot be written as a power of a positive base. Any logarithm can be estimated with a calculator and/or graphing technology.

Ex 5 – Class discussion

Additionally:

- $\log_a 1 = 0$
- $\log_a a^x = x$
- $a^{\log_a x} = x$

Ex 3,4 – Group discussion

Potential Assignment Problems: 1-6,11-17

8.4 Laws of Logarithms

Remember the laws for exponents:

Products: $a^x a^y = a^{x+y}$

Quotients: $\frac{a^x}{a^y} = a^{x-y}$

Powers: $(a^x)^y = a^{xy}$

Ex1,2,3 – Class discussion

The laws of logarithms are then:

Products:
$\log_a xy = \log_a x + \log_a y$

Quotients:
$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Powers: $\log_a x^r = r log_a x$

Ex 4 a,b – Class discussion

Ex 4c, Ex6 – Group discussion

Potential Assignment Questions: 1-4, 6-12,

8.5 Solving Exponential Equations

Two exponential expressions with the same base are equal if their exponents are equal. If two expressions are equal, then taking the logarithms of both sides maintains their equality.

If it is not possible to set both sides of an exponential equation to have the same base, solve using logarithms by taking the common logarithm of both sides and using the rules of logarithms to simplify and solve. Additionally, exponential equations can be solved graphically.

Ex1 – Class discussion

Ex2 – Group discussion

Ex3 – Class discussion

CYU1,3 – Group discussion + take home

Potential Assignment questions: 4-8, 10-13

8.6 Solving Logarithmic Equations

A logarithmic equation can be solved by simplifying it using the laws of logarithms and/or expressing it in exponential form. Be sure to check for inadmissible solutions (ie. solutions that lie outside of the domain or an undefined value in the original equation).

Ex1, Ex2 a – Class discussion

Ex2b, Ex3,Ex4 – Group discussion + take home

Potential Assignment questions: 4-10, 13-17

8.7 Solving Problems with Exponential and Logarithmic Functions

When a range of values varies greatly, using a logarithmic scale with powers of 10 makes comparisons more manageable. In particular, growth and decay situations can be modelled by exponential functions of the form $f(x) = ab^x$, where f is the final amount or number, a is the initial amount or number, b = 1+ growth rate or b=1-decay rate, and x is the number of growth or decay periods.

Ex1,2 – Class discussion

Ex3,4 – Group discussion + take home

Potential assignment questions: 4-15