MHF4U Assignment # 7 Solutions.

1. [2K, 2T, 2C]

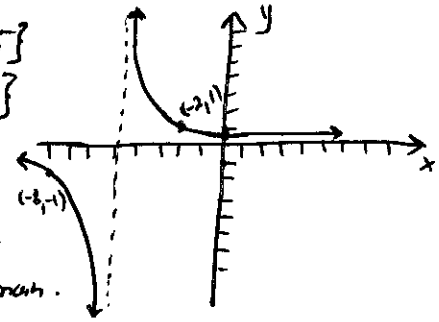
a) $f(x) = \dfrac{3}{x+5}$

V.A. at $x = -5$
H.A. at $y = 0$
No holes.

$D: \{x \in \mathbb{R} \mid x \neq -5\}$
$R: \{y \in \mathbb{R} \mid y \neq 0\}$
x int: none.
y int: $3/5$

+ve intervals: $x > -5$
−ve intervals: $x < -5$

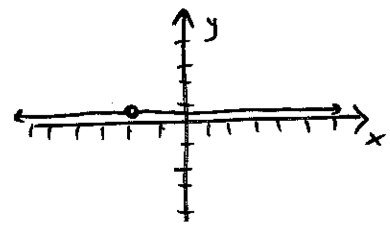
increasing: never.
decreasing: all domain.



b) $f(x) = \dfrac{x+2}{5x+10} = \dfrac{x+2}{5(x+2)}$

No V.A.
Hole at $x = -2$.
H.A. at $y = 1/5$
$D: \{x \in \mathbb{R} \mid x \neq -2\}$
$R: \{y = 1/5\}$

+ve interval: all
−ve interval: none.
No intervals of increasing or decreasing.

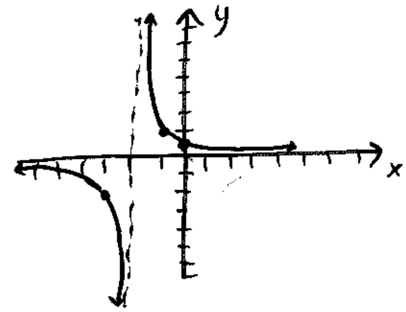


2. [2K, 2T, 2A]

a) V.A. at $x = -2 \rightarrow (x+2)$ in denominator
H.A. at $y = 0 \rightarrow a = 0$.

$\left.\begin{array}{l} \text{+ve } x \in (-\infty, -2) \\ \text{+ve } x \in (-2, \infty) \end{array}\right\} b > 0 \Rightarrow f(x) = \dfrac{1}{x+2}.$

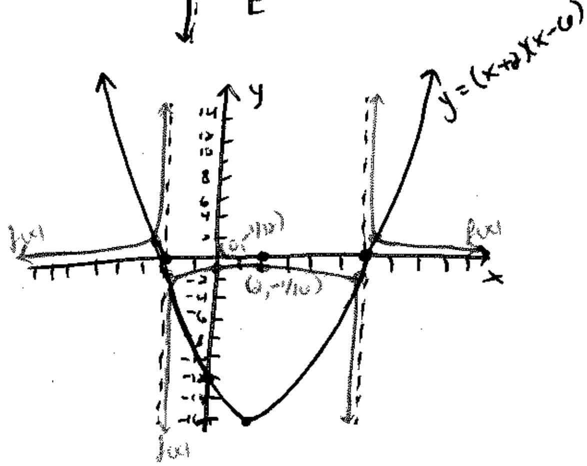
always decreasing.



b) V.A. at $x = -2, 6 \rightarrow (x+2)(x-6)$ in denominator.

H.A at $y = 0 \rightarrow a = 0$

$\left.\begin{array}{l} \text{+ve } x \in (-\infty, -2), (6, \infty) \\ \text{−ve } x \in (-2, 6) \\ \text{increasing } x \in (-\infty, 2) \\ \text{decreasing } x \in (2, \infty) \end{array}\right\} b > 0 \Rightarrow f(x) = \dfrac{1}{(x+2)(x-6)}$



3. [1K, 1T, 1C, 2A]

$c(t) = \dfrac{2t}{2+t} \rightarrow$ V.A at $t = -2$ (outside domain)
H.A at $y = 2$.
$D: \{t \in \mathbb{R} \mid 0 \le t < 24\}$, $R: \{c(t) \in \mathbb{R} \mid 0 \le c < 2\}$.

At $t = 0$, $c(0) = \underline{0}$
$c(t) > 0$ for all t.
$c(24) = \dfrac{48}{26} \cong 1.85$

$c(t)$ is always increasing, more quickly at early times and more slowly at late times.

4. [4K, 4T].

a) $\frac{2}{x} + \frac{5}{3} = \frac{7}{x}$

$\Rightarrow \frac{6}{3x} + \frac{5x}{3x} = \frac{21}{3x}$

$\Rightarrow 6 + 5x = 21$

$\Rightarrow 5x = 15$

$\Rightarrow \underline{x = 3}$

b) $\frac{2x}{x-3} = 1 - \frac{6}{x-3}$

$\Rightarrow \frac{2x}{x-3} = \frac{x-3}{x-3} - \frac{6}{x-3}$

$\Rightarrow 2x = x - 3 - 6$

$\Rightarrow \underline{x = -9}$

c) $\frac{3}{x} + \frac{4}{x+1} = 2$

$\Rightarrow \frac{3(x+1)}{x(x+1)} + \frac{4x}{x(x+1)} = \frac{2x(x+1)}{x(x+1)}$

$\Rightarrow 3x + 3 + 4x = 2x^2 + 2x$

$\Rightarrow 0 = 2x^2 - 5x - 3$

$\Rightarrow 0 = (2x+1)(x-3)$

$\underline{x = -0.5, 3}$

d) $\frac{1}{x} - \frac{1}{45} = \frac{1}{2x-3} \Rightarrow \frac{45(2x-3)}{45x(2x-3)} - \frac{x(2x-3)}{45x(2x-3)} = \frac{45x}{(2x-3)45x}$

$\Rightarrow 90x - 135 - 2x^2 + 3x = 45x \Rightarrow 0 = 2x^2 - 48x + 135$

$x = \frac{48 \pm \sqrt{2304 - 1080}}{4} \cong 20.75, 3.25 \quad \therefore \underline{x = 3.25, 20.75}$

5. [1T, 1A, 1C]

Machine A: s minutes/case

Machine B: s + 10 minutes/case.

Together: 15 minutes/case:

Case/length of time:

$\underset{A}{\overset{\uparrow}{\frac{1}{s}}} + \underset{B}{\overset{\uparrow}{\frac{1}{s+10}}} = \underset{A+B}{\overset{\uparrow}{\frac{1}{15}}}$.

$\Rightarrow \frac{15(s+10)}{15s(s+10)} + \frac{15s}{15s(s+10)} = \frac{s(s+10)}{15(s)(s+10)}$

$\Rightarrow 15s + 150 + 15s = s^2 + 10s$

$\Rightarrow 0 = s^2 - 20s - 150$

$s = \frac{20 \pm \sqrt{400 + 600}}{2} = 25.8$ or $\underbrace{-5.8}_{\text{inadmissable!}}$

* Shouldn't have any -ve s values.

$\therefore s = 25.8$ minutes.

Machine A takes 25.8 minutes, machine B takes 35.8 minutes to fill a case.

6. [1T, 2A, 2C]

Whole box: \$300.

Whole box − 15 sold for \$330

⇒ \$330 is a profit of \$1.50 on each.

• Say x comic books in the box originally.

∴ x comic books for \$300

x − 15 for \$330 with a profit of 1.50 each.

Buying price per comic: $\frac{300}{x}$

Selling price per comic: $\frac{330}{x-15}$

If there's a profit of \$1.50 per comic book:

$\Rightarrow -\frac{300}{x} + \frac{330}{x-15} = 1.50$

$\Rightarrow \frac{(x-15)(-300)}{x(x-15)} + \frac{330x}{x(x-15)} = \frac{1.50\,x(x-15)}{x(x-15)}$

$\Rightarrow -300x + 4500 + 330x = 1.5x^2 - 7.5x$

$\Rightarrow 0 = 1.5x^2 - 37.5x - 4500$

$\Rightarrow 0 = x^2 - 25x - 3000$

$x = \frac{25 \pm \sqrt{625 + 12000}}{2}$

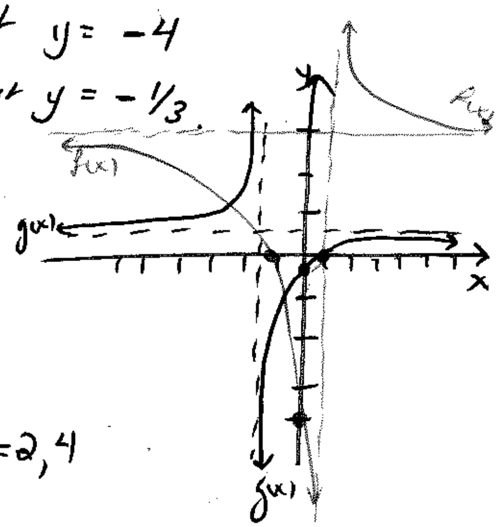
$\cong 68.7, \underbrace{-43.7}_{\text{inadmissable!}}$

∴ There were ~68 comics in the box, each one cost $\frac{\$300}{68} \cong \$4.41$ each originally.

Speed Round

1. [1K, 1T, 1C] $f(x) = \frac{3x+4}{x-1}$, $g(x) = \frac{x-1}{2x+3}$.

$f(x)$ has a V.A. at $x = 1$ and a H.A. at $y = 3$.
$g(x)$ has a V.A at $x = -3/2$ and a H.A. at $y = 1/2$.
$f(x)$ is +ve for $x > 1$ and $x < -4/3$, -ve for $-4/3 < x < 1$.
$g(x)$ is +ve for $x > 1$ and $x < -3/2$, -ve for $-3/2 < x < 1$.
$f(x)$ has a x-int at $x = -4/3$ and a y-int at $y = -4$
$g(x)$ has a x-int at $x = 1$ and a y-int at $y = -1/3$.
$f(x)$ is increasing never, and always decreasing.
$g(x)$ is increasing always and ~~always~~ never decreasing.



2. [1K, 1T, 2C]

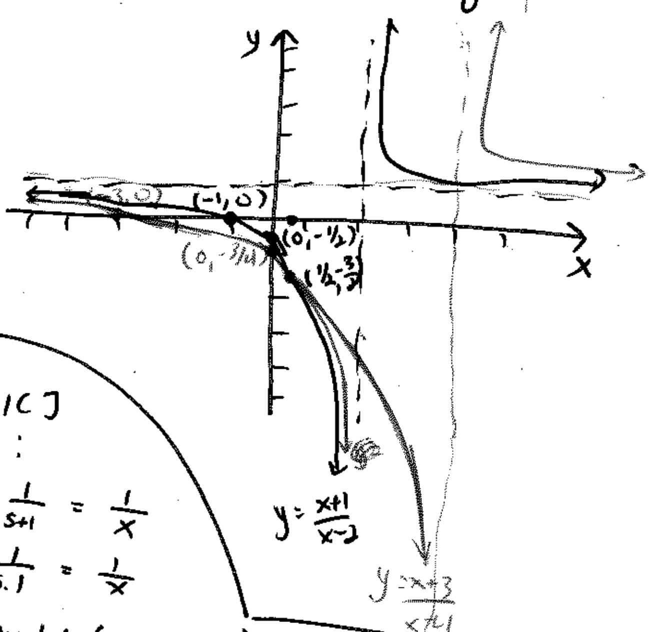
$$\frac{x+1}{x-2} = \frac{x+3}{x-4} \Rightarrow \frac{(x+1)(x-4)}{(x-2)(x-4)} = \frac{(x+3)(x-2)}{(x-2)(x-4)} \Big\} x \neq 2, 4$$

$\Rightarrow (x+1)(x-4) = (x+3)(x-2)$

$\Rightarrow x^2 - 3x - 4 = x^2 + x - 6$

$\Rightarrow 0 = 4x - 2$

$\Rightarrow 2 = 4x \Rightarrow \underline{x = 1/2}$



3. a) [1K, 2T, 3A, 1C]

① $\frac{1}{s} + \frac{1}{s-2} = \frac{1}{1.33}$ } min/order for Tom + Paco

② $\frac{1}{s-2} + \frac{1}{s+1} = \frac{1}{1.5}$ } min/order for Paco + Carl.

b) [1T, 1C]

Together:

$\frac{1}{s} + \frac{1}{s-2} + \frac{1}{s+1} = \frac{1}{x}$

$\frac{1}{4.1} + \frac{1}{2.1} + \frac{1}{5.1} = \frac{1}{x}$

$x \approx 0.92$ minutes (~55 seconds) All together.

→①: $1.3(s-2) + 1.3s = s(s-2)$
$1.3s - 2.6 + 1.3s = s^2 - 2s$
$\Rightarrow 0 = s^2 - 4.6s + 2.6$
$s = \frac{4.6 \pm \sqrt{21.16 - 10.4}}{2}$
$\approx 0.66, 3.99$

②: $1.5(s+1) + 1.5(s-2) = (s-2)(s+1)$
$1.5s + 1.5 + 1.5s - 3 = s^2 - s - 2$
$\Rightarrow 0 = s^2 - 4s - 0.5$
$s = \frac{4 \pm \sqrt{16+2}}{2} \approx 4.12, -0.1$

-0.1 and 0.66 are inadmissable.
∴ S is something like 3.99 – 4.12, average to $\underline{S \sim 4.1 \text{ minutes}}$ ∴ Tom takes 4.1 min, Paco takes 2.1 min, Carl takes 5.1 min.