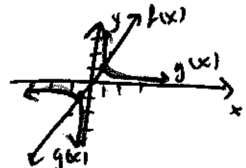
MHF4U Assignment #6. Solutions

1 [3K, 3T, 3C].

a). $f(x) = 2x$.

$$g(x) = \frac{1}{f(x)} = \frac{1}{2x}.$$

$g(x)$ has a vertical asymptote at $x=0$.

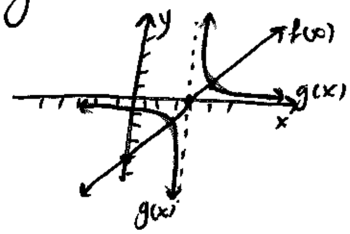


* $g(x)$ intersects $f(x)$ at $f(x) = \pm 1$.
* $g(x)$ is always decreasing.
* horizontal asymptote at $y=0$.

b) $f(x) = x-4$

$$g(x) = \frac{1}{f(x)} = \frac{1}{x-4}$$

$g(x)$ has a vertical asymptote at $x=4$.

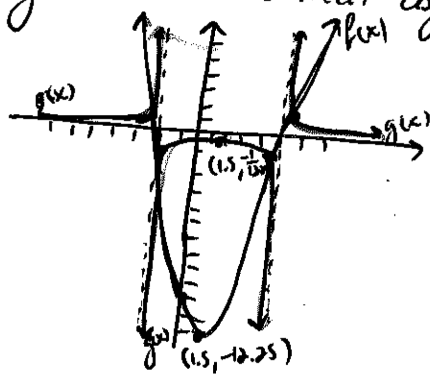


* $g(x)$ intersects $f(x)$ at $f(x) = \pm 1$
* $g(x)$ is always decreasing
* horizontal asymptote at $y=0$.

c) $f(x) = x^2 - 3x - 10$

$$g(x) = \frac{1}{f(x)} = \frac{1}{x^2-3x-10}$$
$$= \frac{1}{(x-5)(x+2)}$$

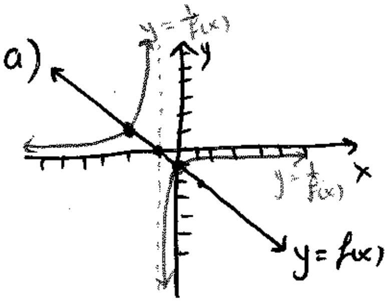
$g(x)$ has a vertical asymptote at $x=-2$ and $x=5$.



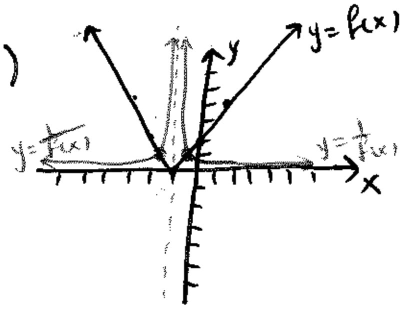
* $g(x)$ intersects $f(x)$ 4 times, when $f(x) = \pm 1$.
* $g(x)$ is increasing until $x=1.5$, then is decreasing
* $g(x)$ has a max where $f(x)$ has a min, at $x=1.5$.
* horizontal asymptote at $y=0$.

2. [2K, 2T, 2A).

a)



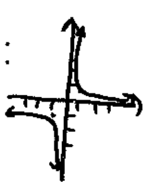
b)



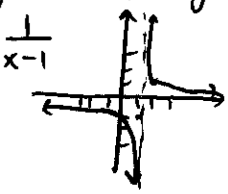
3. [2K, 1T, 1A, 1C]

For $g(x) = \frac{1}{x+n}$, the functions are always decreasing; have a horizontal asymptote at $y=0$; and have a vertical asymptote at $x=-n$. They are identical to $m(x) = \frac{1}{x}$ but are translated by $|n|$ to the right (if $n<0$) or $|n|$ to the left (if $n>0$). They will always hit the points $(-n+1, 1)$ and $(-n-1, -1)$.

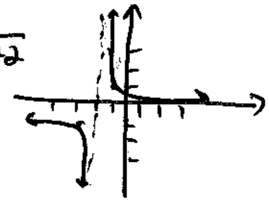
ex. $\frac{1}{x}$:



$\frac{1}{x-1}$



$\frac{1}{x+2}$



(3 cont'd)

a) Domain: $\{x \in \mathbb{R} \mid x \neq -n\}$, Range: $\{y \in \mathbb{R} \mid y \neq 0\}$.

b) For $f(x) = x + n$, the y-intercept will be n. As n changes, the function is translated by n up (if $n > 0$), or down (if $n < 0$). This affects $g(x)$ as the y-intercept moving also moves the x-intercept, which defines where $g(x)$ should have a vertical asymptote.

c) $f(x)$ and $g(x)$ ~~would~~ intersect where $f(x) = g(x) = \pm 1$.

4. [4K, 4T].

a) $y = \frac{1}{2x+3}$: VA at $x = -3/2$, no holes. Horizontal asymptote.

b) $y = \frac{x^2-9}{x+3} = \frac{(x-3)(x+3)}{x+3}$: Hole at $x = -3$, no V.A. Oblique asymptote.

c) $y = \frac{x+4}{x^2-16} = \frac{x+4}{(x+4)(x-4)}$: Hole at $x = -4$, V.A. at $x = 4$. Horizontal asymptote.

d) $y = \frac{x}{5x-3}$: V.A. at $x = 3/5$, no holes. Horizontal asymptote.

5. [2T, 2A].

a) horizontal asymptote along the x-axis $\Rightarrow y = 0$ so need a numerator of 1. A vertical asymptote anywhere implies a zero in the denominator anywhere. Can interpret in 2 ways:

$$f(x) = \frac{1}{x+n}, n \in \mathbb{R}. \quad \underline{\text{or}} \quad f(x) = \frac{1}{x-x}$$

b) Hole at $x = -2 \Rightarrow$ Both numerator and denominator have a zero at $x = -2$, and so must both have a factor $(x+2)$.
Vertical asymptote at $x = 1 \Rightarrow$ only denominator has a zero at $x = 1$, and so must have a factor $(x-1)$.

$$\Rightarrow f(x) = \frac{x+2}{(x-1)(x+2)} = \frac{x+2}{x^2+x-2}$$ (one option!).

Speed Round

1. [1K, 1T, 1C]

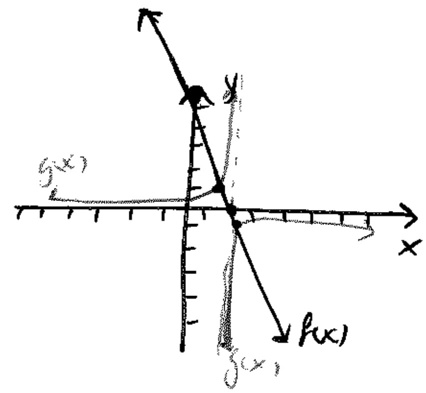
$f(x) = -3x + 6$

$g(x) = \dfrac{1}{-3x+6}$.

g(x) has a V.A at $x = 2$

Domain: $\{x \in \mathbb{R} \mid x \neq 2\}$

Range: $\{y \in \mathbb{R} \mid y \neq 0\}$.



2. [2T, 2C, 2A]. $b(t) = \dfrac{10000}{t}$.

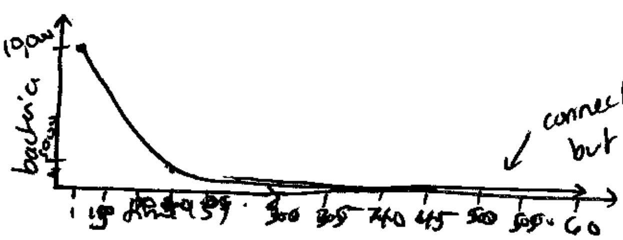
a) $b(20) = \dfrac{10000}{20} = \underline{500}$ bacteria.

b) $5000 = \dfrac{10000}{t} \Rightarrow t = \underline{2s}$; after $t = 2s$ 5000 bacteria will be left.

$1 = \dfrac{10000}{t} \Rightarrow \underline{t = 10{,}000s}$; after $t = 10{,}000s$, 1 bacteria will be left.

c) Model assumes range (b(t)) is real numbers, can't have partial bacteria. At $t = 0$, have an <u>infinite</u> amount of bacteria, which is not reasonable. Most reasonable to really implement solution at a value close to $t = 0$ that gives a finite and whole # of bacteria $\Rightarrow \underline{\underline{t = 1s}}$.

d)



connecting line, but really discrete points!

Domain: $\{t \in \mathbb{R} \mid t \geq 1\}$

Range: $\{B \in \mathbb{N}\}$.

3. [1T, 1A, 1C].

$f(x) = \dfrac{p(x)}{q(x)} = \dfrac{x+2}{1} = x + 2$.

The function <u>itself</u> is the oblique asymptote!