Assignment # 5 solutions

1. a) $x^3 - 6x^2 - x + 30 = 0$

[4K]

* can't use grouping, use factor theorem method.

→ possible factors: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$f(-2) = (-2)^3 - 6(-2)^2 - (-2) + 30 = -8 - 24 + 2 + 30 \stackrel{?}{=} 0$ ∴ $x+2$ is a factor.

$$\begin{array}{r|rrrr} -2 & 1 & -6 & -x & 30 \\ & & -2 & 16 & -30 \\ \hline & 1 & -8 & 15 & \underline{0} \end{array} \Rightarrow (x+2)(x^2 - 8 + 15) = (x+2)(x-5)(x-3)$$

∴ $x^3 - 6x^2 - x + 30 = 0$ when $x = -2, 3, 5$.

b) $x^4 - 6x^3 + 10x^2 - 2x = x^2 - 2x$

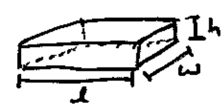
* move everything over to LHS

$x^4 - 6x^3 + 10x^2 - x^2 - 2x + 2x = x^4 - 6x^3 + 9x^2 = 0$

$\Rightarrow x^2(x^2 - 6x + 9) = 0$

$x^2(x-3)^2 = 0$ ∴ $x^4 - 6x^3 + 10x^2 - 2x = x^2 - 2x$ when $x = 0, 3$

2. Volume of a rectangular prism box: $V = l \times h \times w$.



[3T, 2A] From the diagram: $\underbrace{h = x, \quad l = 30 - 2x, \quad w = 20 - 2x}_{\text{all in cm!}}$

Solve for dimension of squares ⇒ find value of x!

$V = 1008\,cm^3 = (x\,cm)(30 - 2x)cm\,(20 - 2x)cm$ [drop units for algebraic manipulation].

$= x(4x^2 - 100x + 600)$

$= 4x^3 - 100x^2 + 600x$

$\Rightarrow 4x^3 - 100x^2 + 600x = 1008$

$4x^3 - 100x^2 + 600x - 1008 = 0.$

$4\underbrace{(x^3 - 25x^2 + 150x - 252)}_{\text{can't use grouping, use factor theorem.}} = 0$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 7, \pm 9, \pm 12, \pm 14, \pm 18, \pm 21, \pm 28, \pm 36, \pm 42, \pm 63, \pm 84, \pm 126, \pm 252.$

$f(3) = 3^3 - 25(3)^2 + 150(3) - 252 = \underline{0}$ ∴ $x - 3$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & -25 & 150 & -252 \\ & & 3 & -66 & 252 \\ \hline & 1 & -22 & 84 & \underline{0} \end{array}$$

$\Rightarrow 4(x-3)\underbrace{(x^2 - 22x + 84)}_{\text{doesn't nicely factor.}} = 0$

$x = \dfrac{22 \pm \sqrt{484 - 4(1)(84)}}{2}$

$= \dfrac{22 \pm \sqrt{148}}{2} = \dfrac{22 \pm 2\sqrt{37}}{2}$

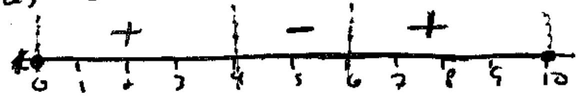
$= 11 \pm \sqrt{37}$ ⇒ both +ve.

∴ x could be $4 \times 3 = \underline{12cm}$, $4(11 + \sqrt{37}) = (44 + 4\sqrt{37})cm$, or $4(11 - \sqrt{37}) = (44 - 4\sqrt{37})cm$.

3. $S(x) = x(x-4)(x-6)$ for $x < 10$, $x \in W$.

[3A, 2C] a) we have 3 x-int: $x = 0, 4, 6$. We know since $n = 3$ (degree), and $a_n = 1 > 0$ that the end behaviours are $x \to -\infty, y \to -\infty, x \to \infty, y \to \infty$ and so Maya's score never remains at 0. We can also assume that at game 0, everyone's scores are 0 (they just started!) $\therefore$ after game 4 and game 6, Maya's score is 0.

b) It doesn't make sense for games to count in -ve numbers, so we can make the number line:



Between $x = 0$ and $x = 4$, the graph is +ve (ex. $S(1) = (1)(1-4)(1-6) = (1)(-3)(-5) = \underline{15} > 0$)

Between $x = 4$ and $x = 6$, the graph is $\underline{-ve}$ (ex. $S(5) = (5)(5-4)(5-6) = (5)(1)(-1) = -5 < 0$)

Between $x = 6$ and $x = 10$, the graph is +ve (ex. $S(7) = 7(7-4)(7-6) = 7(3)(1) = 21 > 0$)

We would look between $x = 4$, $x = 6$ at whole numbers, and have already found that Maya's score is -5 after game 5.

c) There are 2 possible sections of the domain where Maya's score could be 16. Solve directly:

$$16 = x(x-4)(x-6) = x(x^2 - 10x + 24) = x^3 - 10x^2 + 24x$$

$$\Rightarrow 0 = \underbrace{x^3 - 10x^2 + 24x - 16}_{\text{cannot use grouping, use factor theorem.}} \qquad \pm 1, \pm 2, \pm 4, \pm 8, \pm 16.$$

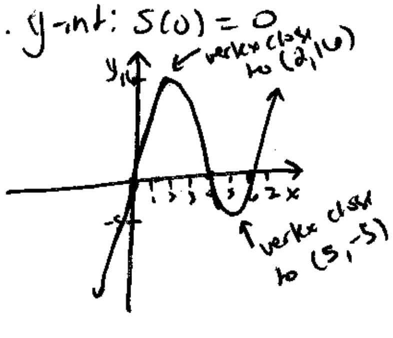
$$(2)^3 - 10(2)^2 + 24(2) - 16 = 8 - 40 + 48 - 16 \stackrel{!}{=} 0 \quad \therefore x - 2 \text{ is a factor.}$$

$$\begin{array}{r|rrrr} 2 & 1 & -10 & 24 & -16 \\ & & 2 & -16 & 16 \\ \hline & 1 & -8 & 8 & \underline{0} \end{array} \qquad \therefore x^3 - 10x^2 + 24x - 16 = (x-2)\underbrace{(x^2 - 8x + 8)}_{\text{use quadratic formula.}}$$

$$x = \frac{8 \pm \sqrt{64 - 4(1)(8)}}{2} = \frac{8 \pm \sqrt{32}}{2} = \frac{8 \pm 4\sqrt{2}}{2} = \underbrace{4 \pm 2\sqrt{2}}_{\text{always +ve.}}$$

$\therefore 0 = x^3 - 10x^2 + 24x - 16$ when $x = 2, 4 - 2\sqrt{2}, 4 + 2\sqrt{2}$. However, we can only count games in whole numbers, so Maya's score is 16 after game $\underline{2}$.

d). y-int: $S(0) = 0$



This is not a good model because:

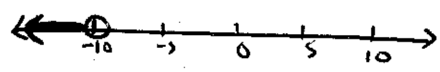
- $x < 0$ doesn't makes sense, can't have -ve games
- $x > 10$ doesn't apply to the problem.
- $x \notin \mathbb{N}$ doesn't make sense in terms of games played

4. a) $2x-8>4x+12 \Rightarrow 2x-4x-8-12>0$
[4K, 4T, 4C]

$-2x-20>0$

$-2(x+10)>0$ ← does not include x-int.

1 x-int at $x=-10$



try to the left and right of $x=-10$:

$-2(-11+10) = -2(-1) = 2 \overset{?}{>} 0$

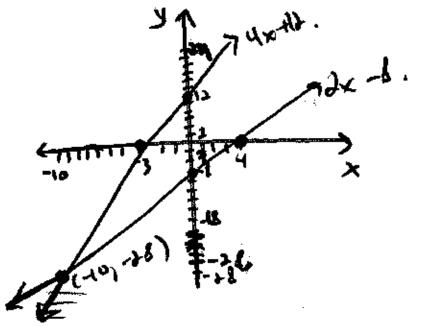
to the right of $x=10$:

$-2(9+10) = -2(19) = -38 \underline{\underline{<0}}$

} ∴ only values <-10 satisfy the inequality.

(or. $2x-8>4x+12 \Rightarrow 2x-4x>12+8 \Rightarrow -2x>20 \Rightarrow \underline{x<-10}$)

0 is <u>not</u> in the solution set.



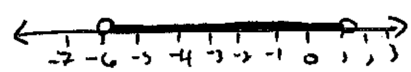
$2x-8$ has y-int -8, x-int 4

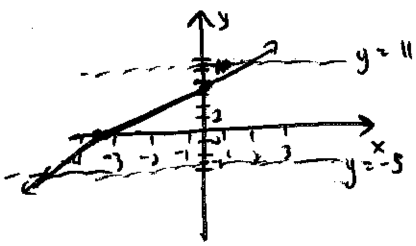
$4x+12$ has y-int 12, x-int -3

The graph of $2x-8$ goes below the graph of $4x+12$ at $(-10,-28)$ ie. for $x<-10$.

b) $-5<2x+7<11 \Rightarrow -12<2x<4 \Rightarrow \underline{-6<x<2}$

0 <u>is</u> in the solution set.



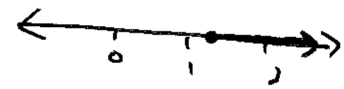


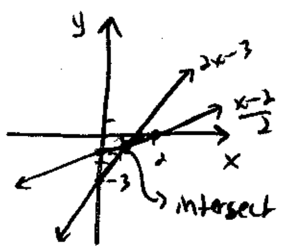
$2x+7$ has a y-int of 7, x-int of $-7/2$.

Only the parts of the graph <u>between</u> -5, 11 are part of the solution. $2(-6)+7 = -12+7 \overset{?}{=} -5$, $2(2)+7 = 4+7 \overset{?}{=} 11$.

c) $\frac{x-2}{2} \le 2x-3 \Rightarrow x-2 \le 4x-6 \Rightarrow -2+6 \le 4x-x \Rightarrow +4 \le 3x \Rightarrow \underline{\frac{+4}{3} \le x}$

0 <u>is not</u> in the solution set.





$\frac{x-2}{2}$ has x-int of 2, y-int of -1

$2x-3$ has x-int of $3/2$, y-int of -3.

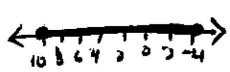
} $\frac{x-2}{2}$ is <u>below</u> $2x-3$ on the graph only for $x \ge 4/3$.

d) $-1 \le -x+9 \le 13$

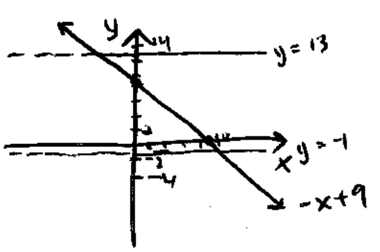
$\Rightarrow -1-9 \le -x \le 13-9$

$\Rightarrow -10 \le -x \le 4$

$\Rightarrow \underline{10 \ge x \ge -4}$

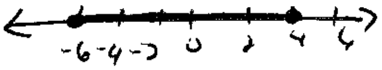


0 <u>is</u> in the solution set.



$-x+9$ has y-int 9, x-int 9.

$-x+9$ is only between 13 and -1 for $x=-4$ $(-(-4)+9 \overset{?}{=} 13)$ and $x=10$ $(-(10)+9 \overset{?}{=} -1)$.

5. 

[2K, 2T] a) $-6 \le x \le 4 \longrightarrow \{x \in \mathbb{R} \mid -6 \le x \le 4\}$.

b) (There are many potential solutions!!).

$-2 \le \frac{1}{2}x + 1 \le 3$.

6. $C = \frac{5}{9}(F-32)$, C should be between 18°C , 22°C.

[1K, 1T, 3A]. a) $\underbrace{18^\circ C < C < 22^\circ C}_{\text{can interpret as } < \text{ or } \le .} \Rightarrow 18 < \frac{5}{9}(F-32) < 22.$

b) $18 < \frac{5}{9}(F-32) < 22$

$\Rightarrow \frac{9}{5}(18) < F - 32 < \frac{9}{5}(22)$

$\Rightarrow \frac{162}{5} < F - 32 < \frac{198}{5}$

$\Rightarrow \frac{162}{5} + 32 < F < \frac{198}{5} + 32.$

$\Rightarrow \underline{64.4 < F < 71.6}$

Check!

$F = \frac{9}{5}C + 32.$

$\Rightarrow \frac{9}{5}(18) + 32 \stackrel{!}{=} 64.4$

$\Rightarrow \frac{9}{5}(22) + 32 \stackrel{!}{=} 71.6.$