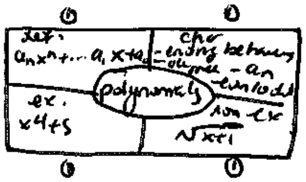
Assignment # 3 (Ch. 3.1 – 3.3)

1. a) $f(x) = 2x^3 + x^2 - 5$: yes①, it is a polynomial b/c it follows the general form of polynomials: ①
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$.

b) $y = \sqrt{x+1}$: No①, not a polynomial because① degree would be $1/2 \notin \mathbb{N}$.

2. linear : $y = 2x + 5$①, quadratic: $y = 2x^2 + 2x + 5$①, cubic: $y = x^3 + 5$①, quartic: $x^4 + 5$①

3. 

(free-form response).

4. a) $f(x) = 2x^2 - 3x + 5$ degree $= 2$① $\Rightarrow$ $x \to -\infty$, $y \to \infty$①
$a_n = +2$ $x \to +\infty$, $y \to \infty$

b) $f(x) = -3x^2 - 2x + 6$ degree $= 2$① $\Rightarrow$ $x \to -\infty$, $y \to -\infty$①
$a_n = -3$ $x \to +\infty$, $y \to -\infty$

5. Odd degree polynomial functions have opposite end behaviours (eg. $x \to -\infty, y \to -\infty$ $x \to +\infty, y \to +\infty$ or vice versa), and so have an unbounded range. When the domain is restricted, they can of course have local max/min. Even degree polynomials have like end behaviours (eg. $x \to -\infty, y \to -\infty$ $x \to +\infty$ $y \to -\infty$ or vice versa) and so always have either an upper <u>or</u> lower bound, implying a global/absolute max/min respectively.

6. $y = -0.1x^4 + 0.5x^3 + 0.4x^2 + 10x + 7$. , x = # of years since 1900.

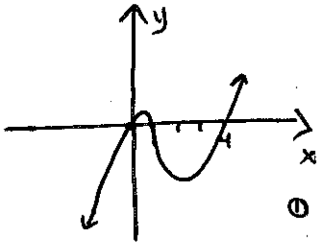
a) $y = 0 + 7 = \underline{\underline{7}}$① (number of years since 1900 <u>in</u> 1900 $= 0$)

b) degree : 4 $\Rightarrow$ $x \to -\infty, y \to -\infty$ Of course, $y \underset{①}{\geq} 0$ (can't have a negative population)!
$a_n = -0.1$ $x \to +\infty, y \to -\infty$①

$\therefore$ Brighton's population collapsed overtime①, with $\underset{①}{\underline{2}}$ instances (degree $= 4$) of population increases before total collapse.

7. a) $y = x(x-4)(x-1)$

x-int: $x=0$ y-int: 0
$x=4$
$x=1$ ①



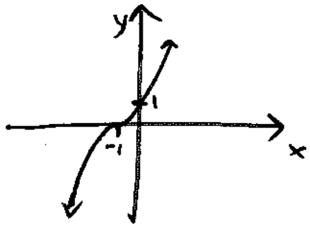
all order 1 factors!
∴ linear x-axis crossings.

degree: 3, $a_n = +1$

b) $y = (x+1)^3$ ①

x-int: $x=-1$, y-int: 1

degree: 3, $a_n = +1$



Only factor is of order 3!
∴ saddle point.

8. $f(x) = kx^3 - 8x^2 - x + 3k + 1$, zero at $x = 2$.

$0 = k(2)^3 - 8(2)^2 - (2) + 3k + 1$

$\Rightarrow 0 = 8k - 32 - 2 + 3k + 1$

$\Rightarrow 0 = 11k - 33$

$\Rightarrow \underline{\underline{k=3}}$ ①

$\Rightarrow f(x) = 3x^3 - 8x^2 - x$ ~~[illegible]~~ $+10$

one factor is $(x-2)$, need to find the other factors (can also read off x-int from Desmos!).

↳ we don't know how to factor cubics yet, but we can work backwards! We know we'll have something of the form: ①

$f(x) = (x-2)(ax^2 + bx + c)$ since $(x-2)$ is a confirmed factor.

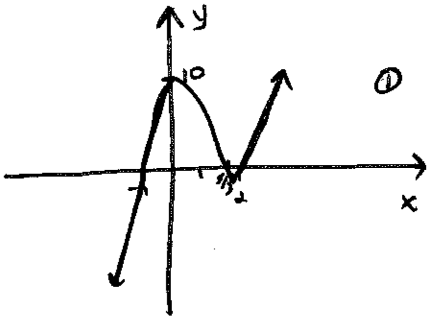
$a = 3$ to get the $3x^3$ term, $c = -5$ to get the $+10$ term. Then: $f(x) = (x-2)(3x^2 + bx - 5)$. multiplying $3x^2$ and -5 by the other term in the factor, we get $-6x^2$ and $-5x$ when everything is multiplied out. To match the $-8x^2$, $-x$ terms in $f(x)$, we need b to give us $-2x^2$, $+4x \Rightarrow b = -2$.

$\therefore f(x) = (x-2)(3x^2 + 2x - 5)$, factor the quadratic term to get:

$= 3(x-2)(x+1)(x - 5/3)$ ①

$3(x^2 - 2/3\,x - 5/3) = 3(x+1)(x-5/3)$

∴ zeros: $x = -1, 2, 5/3$. ①



x-int: $-1, 2, 5/3$
y-int: 10

9\. a) $y = a(x-2)^2(x-4)^2$.

~~Quadratic~~ degree = 4 · ending behaviours are the same ①

unknown a_n. · $\underline{2}$ parabolic x-intercepts at $x=2, 4$, so the function never crosses the x-int. ①

· y-int = $64a$ · "W" or "M" shape.

b) $y = a(x+4)(x-3)^2$

degree = 3 · ending behaviours are opposites. ①

unknown a_n. · 1 linear x-int at $x=-4$, 1 parabolic x-int at $x = 3$. ① .