Nonlinear growth of streaming instability in Shocks

$$\frac{\partial F}{\partial t} - S_0(z,p,t) = \frac{\partial}{\partial z}\left[K_1 \frac{\partial F}{\partial z}\right] - v\frac{\partial F}{\partial z} + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 \Gamma \frac{\partial F}{\partial p}\right]$$ — Fermi II

$$+ \frac{p}{3}\frac{\partial}{\partial z}\left[U + \frac{1}{4p^2}\frac{\partial}{\partial p}(p^2 v A_1)\right]\frac{\partial F}{\partial p}$$ — Fermi I

→ adiabatic deceleration

$$A_1 = \int_{-1}^{1} d\mu \, (1-\mu^2) \frac{D_{\mu p}(\mu)}{D_{\mu\mu}(\mu)}$$

Nonlinear growth rate of beaming

$$\Gamma(k) \simeq \frac{a k \xi U^3}{c V_A \left[\gamma_{max}^{a} - (1+a)^{-1}\right] (k r_{g0})^a (1+A_{tot}^2)^{(1+a)/2}}$$

$$\Rightarrow \gamma_{max} \simeq \left[\frac{a \xi (L e B_0)^{0.5} U^3}{\sqrt{m} V^2 c^2}\right]^{1/(0.5+a)}$$

ξ is the ratio of pressure of CRs at the shock and the upstream momentum flux entering the shock front, $a-4$ is the spectrum index of CRs at the shock front. $A_{tot} = \left(\frac{\delta B}{B_0}\right)^2$

△ Original picture: (Cesarsky 1980)

the instability could provide confinement for CRs up to 100 GeV in partially ionized gas, for all CRs in fully ionized gas.

△ Modern picture: (Yan & Lazarian 2002, 2004; Farmer & Goldreich 2004)

the instability is limited by turbulence in fully ionized medium

damping by fast modes, $\Gamma_{fast} \simeq \sqrt{\frac{k}{L}} \frac{V^2}{V_{ph}^2}$

by $\Gamma_{Alf} = \sqrt{\frac{k}{L}} V_A$

If adopting the number density of CRs near the Sun.

$N(\geq E) = 2\times10^{-10} (E/GeV)^{-1.6} cm^{-3} sr^{-1}$

$\Rightarrow \gamma_{max} \simeq 1.5\times10^{-9} [n_p^{-1} (V_{ph}/V) (L v \Omega_0 / V^2)^{0.5}]^{1/1.1}$

The change of magnetic field is $\phi \sim \delta B/B$, the scattering that is a random walk requires $N \sim 1/\phi^2$ interaction,

this
$$\lambda \sim N r_L \sim \frac{r_L}{\phi^2} \sim \frac{r_L B^2}{\delta B^2}$$

The timescale for scattering through 90°:
$$\tau_s \sim \frac{\lambda}{v} = \left(\frac{r_L}{v}\right)\left(\frac{V_A^2 N_p m_p}{U_{wave}}\right), \quad U_{wave} = \frac{B_1^2}{8\pi}$$

To seek for the rate of momentum transfer to the waves,
$$\frac{dP_{wave}}{dt} = \frac{d}{dt}\left(\frac{U_{wave}}{V_A}\right),$$

this has to be equal to the momentum supplied by CRs over time scale τ_s

$$\frac{1}{v_A}\frac{dU_{wave}}{dt} = \frac{E v N(E)}{\tau_s c^2} = \frac{E N(E) v^2}{r_L V_A^2 N_p m_p c^2} U_{wave}$$

$$\Rightarrow \Gamma = \frac{E N(E) v^2}{r_L V_A N_p m_p c^2},$$

$$= \Omega_0 \frac{N(N \geq E)}{N_p}\left(\frac{|v|}{V_A} - 1\right)$$, turns off at $|v| \sim V_A$

Beaming instability

Energy of CRs go into waves, the higher disturbance the more particle-wave interaction.

$$\begin{pmatrix}\text{increase}\\ \text{of momentum}\\ \text{of the wave}\end{pmatrix} = \begin{pmatrix}\text{decrease}\\ \text{of momentum}\\ \text{of the particles}\end{pmatrix}$$

Instability developes until the streaming velocity of high energy particles is reduced to the Alfvén speed

$$V_A = \frac{B}{\sqrt{4\pi\rho}}$$

If neutrals are present, Alfvén waves are damped, particles & waves get decoupled.

ionized gas	naive picture of the Galaxy, Kulsrud & Pearce (1969)
neutral + ionized gas	
ionized gas	

Leaky box picture $N_{int} \cdot C = N_{ext} V_A$ residence time

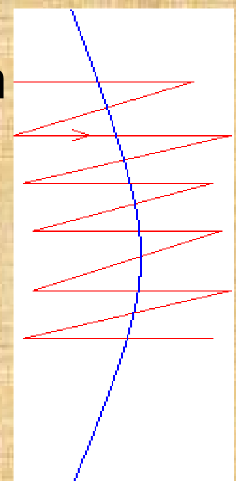
$$\frac{L}{c} \times \frac{c}{V_A} \sim 10^7 \text{ years}$$

Streaming instability of CR

Acceleration in shocks requires scattering of particles back from the upstream region.

Downstream

Upstream



Turbulence generated by shock

Turbulence generated by streaming

Streaming cosmic rays result in formation of perturbation that scatters cosmic rays back and increases perturbation.
This is streaming instability that can return cosmic rays back to shock and may prevent their fast leak out of the Galaxy.

Streaming instability of CR (Cont.)

1. MHD turbulence can suppress streaming instability (*Yan & Lazarian 2002*).

2. Calculations for weak case ($\delta B < B$):

With background compressible turbulence (*Yan & Lazarian 2004*):

$$E_{max} \approx 1.5 \times 10^{-9} \left[n_p^{-1} (V_A/V)^{0.5} (Lc\Omega_0/V^2)^{0.5}\right]^{1/1.1} E_0$$

This gives $E_{max} \approx 20$GeV for HIM.

This is similar to the estimate obtained with background Alfvenic turbulence (*Farmer & Goldreich 2004*).

Streaming instability of CR (Cont.)

3. Strong case (e.g. shocks):

Magnetic field itself can be amplified through inverse cascade.

As a result, $\delta B > B_0$, the growth rate becomes higher in this case.

And the streaming instability operates till higher energies (Yan & Lazarian 2004):

$$\gamma_{max} \approx (a\varepsilon(LeB_0)^{0.5}U^3/(m^{0.5}V^2c^2))^{1/(0.5+a)},$$

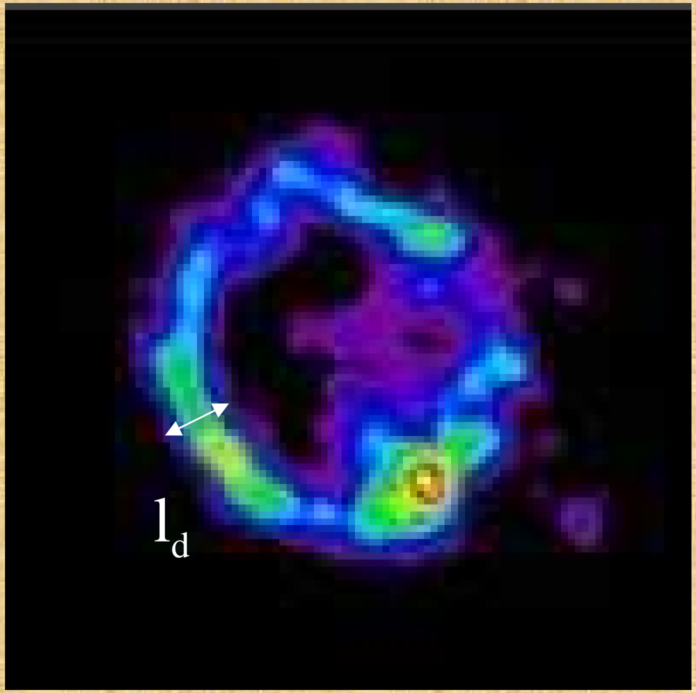
where ε is the ratio of the pressure of CRs at the shock and the upstream momentum flux entering the shock front,

U is the shock front speed, a-4 is the spectrum index of CRs at the shock front. This gives $\gamma_{max} \approx 2 \times 10^7 (t/kyr)^{-9/4}$ for HIM.

<u>Shock acceleration should be revised.</u>

<u>Cosmic Ray confinement in galaxies should be revised.</u>

Nonthermal X-ray filaments in young SNRs



$E_{loss}/(dE/dt) \sim R/U_{ds}$

$dE/dt \propto B_0^2$

→ much stronger B than typical

Even for $B=100\ \mu G$, $l_d > 10^{17}$ cm

Importance and overview