Exercises for
Cosmology (WS2013/14)

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Exercise sheet 8

Due: Dec. 10, 2013 16:00

To be handed in before the exercise class or emailed as a pdf or scanned hand-written document (via e-mail to walther@mpia.de). Remember to put your names on the document; groups of ≤ 3 are allowed.

1. **The Mass Function** (10 points)

   Masses and length scales are related by the mean background density

   $$M = \frac{4\pi}{3}\bar{\rho}R^3. \tag{1}$$

   The mass M_* corresponding to the scale R_* on which the variance becomes unity

   $$\sigma_*^2 = 4\pi \int_0^{k_*} \frac{k^2 \mathrm{d}k}{(2\pi)^3} P_\delta(k) = 1 \tag{2}$$

   is called the *nonlinear mass*.

   (a) Show that, for a power-law power spectrum with index n, $P_\delta(k) = Ak^n$, the variance can be written in the form

   $$\sigma^2 = \left(\frac{M_*}{M}\right)^{1+n/3}. \tag{3}$$

   (b) Discuss the special cases $n = 1$ and $n = -3$ and draw qualitatively σ^2 as a function of M by interpolating the asymptotic cases of the linear power spectrum.

   (c) Use this result to bring the Press-Schechter mass function into the form

   $$N(M,a)\mathrm{d}M = \sqrt{\frac{2}{\pi}} \frac{\alpha\bar{\rho}\delta_\mathrm{c}}{M_* D_+} m^{\alpha-2} \exp\left(-\frac{\delta_\mathrm{c}^2}{2D_+^2} m^{2\alpha}\right) \mathrm{d}m, \tag{4}$$

   where $m = M/M_*$ and $\alpha = 1/2 + n/6$.

2. **The Nonlinear Mass** (10 points)

   (a) Using *Planck* data, the latest cosmic microwave background measurements yield $\sigma_8 \approx 0.8$ (i.e., σ on the scale of $8\,h^{-1}$ Mpc). Assume that $n = -1$ and estimate the nonlinear mass today.

   (b) How does the nonlinear mass evolve with time?

   (c) Calculate the present abundance of objects with mass M_* according to the Press-Schechter mass function (use $\Omega_{\mathrm{m},0} = 0.3$). Assuming, for simplicity, that these halos are randomly distributed through space, estimate the mean separation between these objects. Compare your estimate with the actual distance of the Milky Way to the Virgo cluster.

3. **Mass-to-Light Ratio** (10 points)

   According to the current standard paradigm, galaxies live in extended cold dark matter halos that are distributed according to the Press-Schechter mass function (or more elaborate cousins). The goal of this problem is to understand the connection of galaxies to their parent halos. The distribution of galaxies with luminosity L is conveniently parametrized with a Schechter function,

$$\phi(x)\mathrm{d}x = \phi_* x^{-\alpha_L} \exp(-x)\mathrm{d}x, \tag{5}$$

   with $x = L/L_*$. At $z = 0$, typical values for the faint-end slope α_L are $\alpha_L \approx 1$ and $\phi_* = 1.5 \times 10^{-2} (h^{-1}\,\mathrm{Mpc})^{-3}$ is the number density.

   (a) First, we look at the faint-end of the distribution function. Assuming a constant mass-to-light ratio, what would be the power-law index n of the mass function that is needed to reconcile the galaxy luminosity and halo mass functions? Argue why this can be ruled out.

   (b) Hence, the only alternative appears to be a halo-mass dependent luminosity-to-mass ratio, L/M. Compute the slope of L/M at small masses for a realistic power spectrum slope of $n = -1$, $\alpha_L = 1$, and a power-law relation between galaxy luminosity and host-halo mass, $L \propto M^\beta$.

   (c) Galaxies with characteristic luminosity L_* are hosted by halos of $M \simeq 3 \times 10^{11} h^{-1}\mathrm{M}_\odot$ and are able to turn approximately 20% of baryons into stars. Plot the ratio of baryons that locked up in luminous stars to the total mass, M_L/M (normalized to the cosmic baryon-to-total mass ratio) as a function of halo mass (in units of solar masses). Finding explanations for the deviations of M_L/M at small and large halo masses from the cosmic mean are among the most famous problems in galaxy formation.