Exercises for Cosmology (WS2013/14)

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Exercise sheet 5

Due: Nov. 19, 2013 16:00

To be handed in before the exercise class or emailed as a pdf or scanned hand-written document (via e-mail to walther@mpia.de). Remember to put your names on the document; groups of ≤ 3 are allowed.

1. **Epochs of Big Bang Nucleosynthesis and Recombination** (10 points)
   (a) Calculate how long after the Big Bang the epoch of $kT = 0.1$ MeV (primordial nucleosynthesis) and $T = 3600$ K (recombination) was reached. In doing so, please account for the contribution of photons and neutrinos to the radiation energy density and provide you answer in terms of kT. Please use the numerical values $T_{\mathrm{photo},0} = 2.726$ K and $\{\Omega_{\mathrm{m},0}, h\} = \{0.3, 0.7\}$.
   Hint: note that primordial nucleosynthesis falls into the 'radiation dominated' era, while recombination does not.

   (b) Compare the age of the Universe at recombination that you obtain from only using the matter-dominated analytic solution for $a(t)$ to the solution that accounts for the different early functional dependence of $a(t)$ in the radiation-dominated era.

   (c) How long was the period between $kT = 0.8$ MeV (until then protons and neutrons were in equilibrium) and $kT = 0.1$ MeV (when neutrons ended up in ^{2}H, ^{4}He, etc.)? To this end, you can assume that there are only photons and neutrinos contributing to the effective degeneracy factor. What fraction of neutrons decayed during that period, given their half-life time is 881 s? What is the resulting neutron-to-proton ratio that is entering the abundances for ^{2}H, ^{4}He, etc.?

2. **Big Bang Nucleosynthesis** (10 points)
   Explain *qualitatively* how the predictions of Big Bang Nucleosynthesis for the ^{4}He and ^{2}H abundances would be altered if the CMB temperature were lowered.

3. **Saha Equation** (10 points)
Here, we are going to calculate the redshift of the release of the CMB radiation. To express the (free) electron number density in terms of the baryon number density, we need an expression of the ionization degree x. According to Saha's equation it obeys

$$\frac{x^2}{1-x} = \frac{0.26}{\eta} \left(\frac{m_e c^2}{kT} \right)^{3/2} \mathrm{e}^{-\chi/kT}, \tag{1}$$

where $\eta = 6 \times 10^{-10}$ is the baryon-to-photon ratio of the Universe, m_e is the electron mass, c is the light speed, k is the Boltzmann constant, and χ is the ionization potential of a hydrogen atom.

Let us now estimate at which temperature T recombination sets in, $x = 0.95$, and at what temperature it is largely done, $x = 10^{-3}$.

(a) Argue why Saha's equation needs to be solved numerically and why it may be more appropriate to take the logarithm of this equation. Use a method of your choice to solve this equation and send your program code to Michael by email (walther@mpia.de). E.g., you can use the Newton-Raphson algorithm, which finds the roots of the equation $f(x) = 0$. Starting from an educated guess x_0, roots are refined by the iteration

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}. \tag{2}$$

Using a sketch, show how this iteration works (or the method of your choice). Discuss possible cases where such an algorithm may fail to converge.

(b) What is the final resulting temperature for the two values of $x = 0.95$ and $x = 10^{-3}$? What are the corresponding redshifts? How much time did reionization take (between these two redshifts)?

(c) Discuss (just a few sentences) why we only 'see' the CMB photons now that were released when the matter had become almost completely neutral ($x \ll 1$).