Controlling eccentricity in binary black hole simulations

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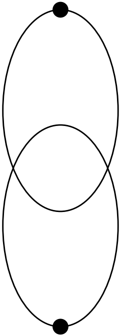
gr-qc/0702106; collaborators: D. Brown, L. Kidder, L. Lindblom, G. Lovelace, M. Scheel

Eccentricity

- Orbits circularize
- Peters (1965)

$e \propto a^{19/12}$



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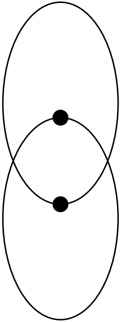

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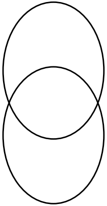

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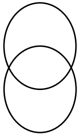

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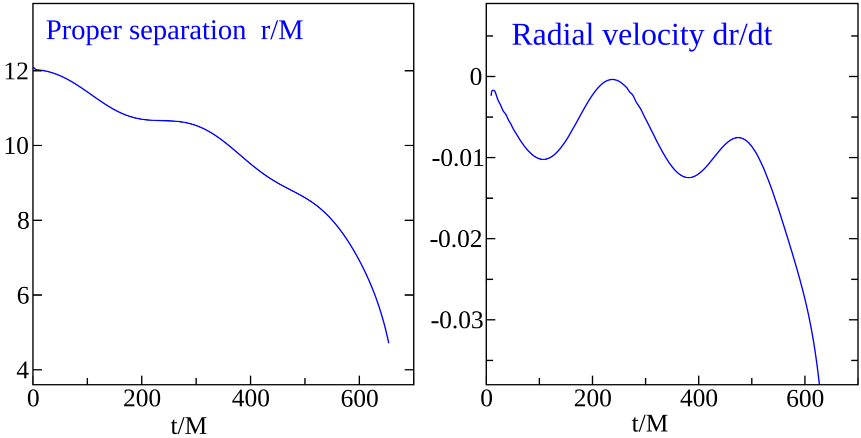
- Hulse-Taylor pulsar
  – today: $e_0 = 0.617$, $a_0 = 2 \times 10^9 m = 230.000 R_S$
  – Prior to merger: At $a \sim 10 R_s$: $e \sim 10^{-7}$

Quasi-circular binary black hole initial data is eccentric



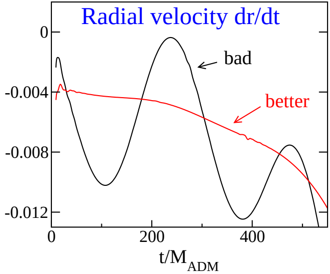
- Evolution of $v_r = 0$ initial data (Cook & Pfeiffer, 2004).
- Additional contributions from limited knowledge of orbital parameters and junk radiation.

Initial data with radial velocity

- Incorporate $\dot{r}_0$ into initial data (gr-qc/0702106)
  - Quasi-equilibrium, conformal thin sandwich
  - Add radial component to shift BC on horizon
  - Preserves quasi-equilibrium of horizon

- Two parameters $\Omega_O, \dot{r}_0$ for equal-mass non-spinning BBH initial data

- Idea: Choose $\Omega_0, \dot{r}_0$ to remove oscillations from inspiral trajectory



Parameter tuning

- Evolve ~ 1 orbits
- Fit $\dot{r}(t) = A_0 + A_1 t + B\cos(\omega t + \phi)$
  - $A_0 + A_1 t$ desired, smooth inspiral
  - $B\sin(\omega t + \phi)$ unwanted oscillations due to eccentricity

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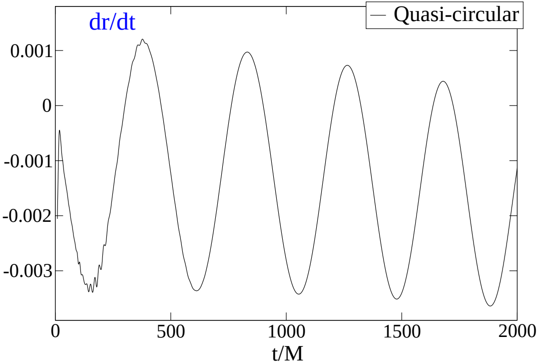
- Obtain e as byproduct:

$$\dot{r}_{\mathrm{N}} = e\,r_0\omega\cos(\omega t + \phi) \quad \Rightarrow \quad e = \frac{B}{r_0\omega}$$

Example: $\sim$ 15 orbit inspirals



Ω_0	$\dot{r}_0$	$B\cos(\omega t+\phi)$	e
0.0080108	0.00	$2.3\cdot 10^{-3}\cos(0.0136t+1.11)$	$1.0\cdot 10^{-2}$

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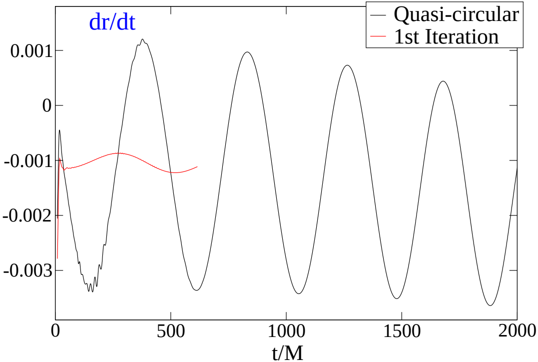

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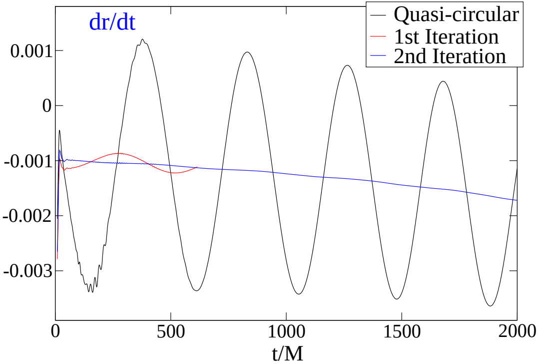

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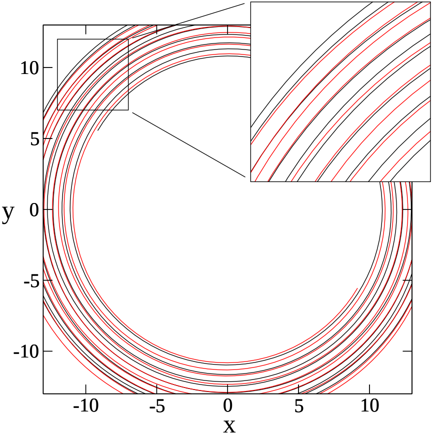
0.0080401	$-8.52\cdot 10^{-4}$	$1.0\cdot 10^{-5}\cos(0.0137t+0.42)$	$4.4\cdot 10^{-5}$

Orbital trajectory

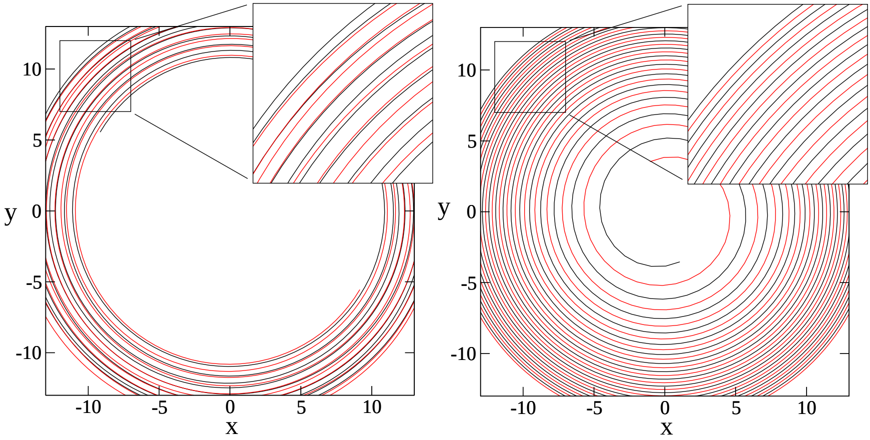
Quasi-circular



Orbital trajectory

Quasi-circular

Eccentricity reduced



Summary

- BBH initial data usually has some eccentricity due to
  – Incomplete knowledge of inspiral orbital parameters
  – Junk radiation
- Iterative parameter tuning removes this eccentricity.
- Low-eccentricity initial data forms starting point
  for other EGM10-talks (Abdul Mroue, Larry Kidder)