Bayesian CMB foreground separation with a correlated log-normal model

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in collaboration with:
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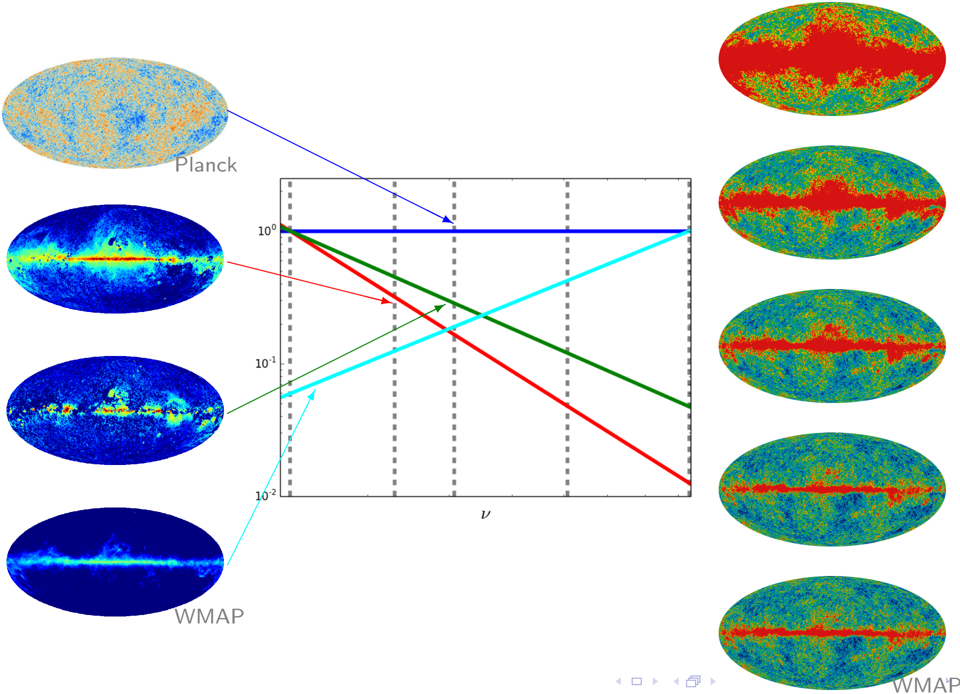
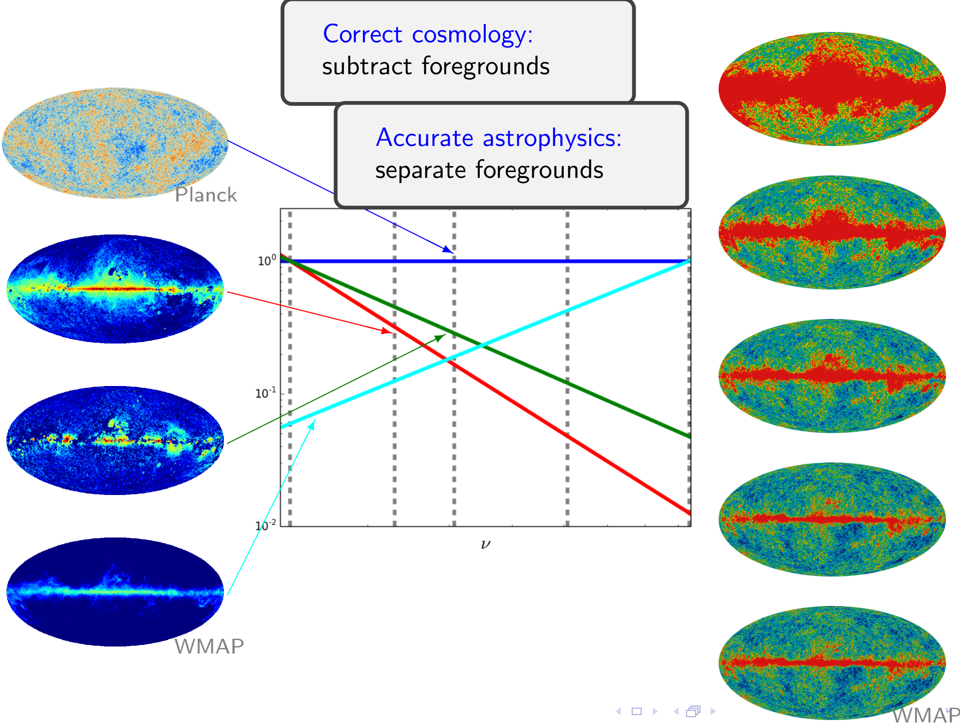
Accurate astrophysics. Correct cosmology. London, 2015-07-14


Planck
10^{0}
10^{-1}
10^{-2}
ν
WMAP
WMAP

Correct cosmology:
subtract foregrounds

Accurate astrophysics:
separate foregrounds



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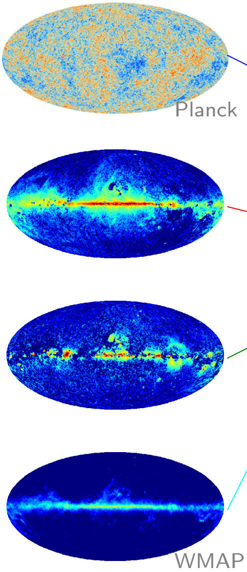

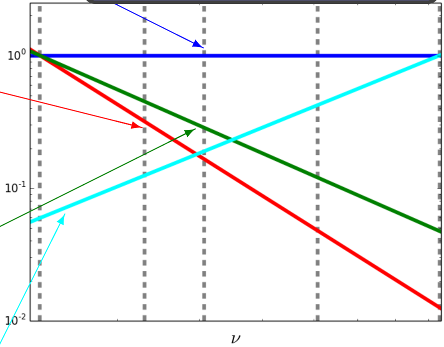

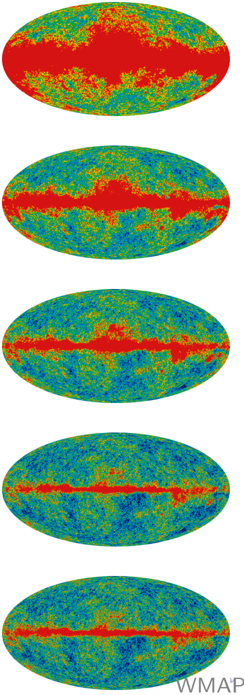

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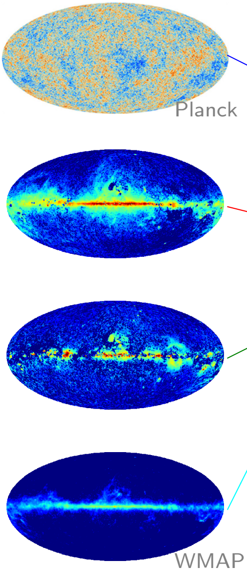
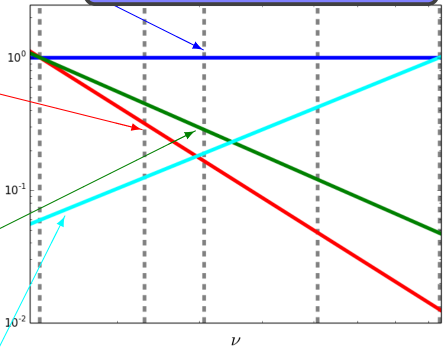
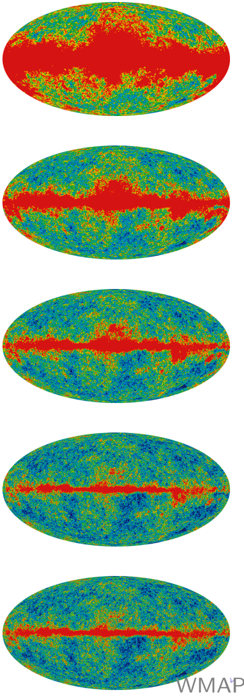




1) find frequency spectra

2) invert mixture



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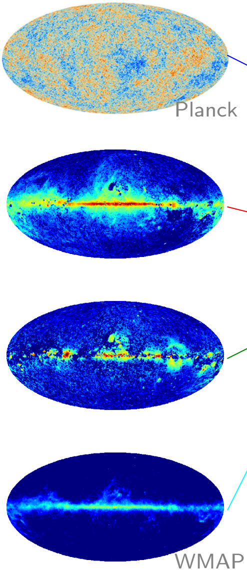
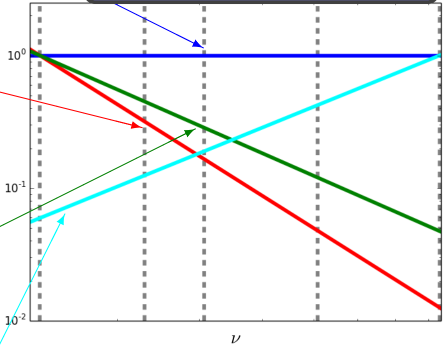
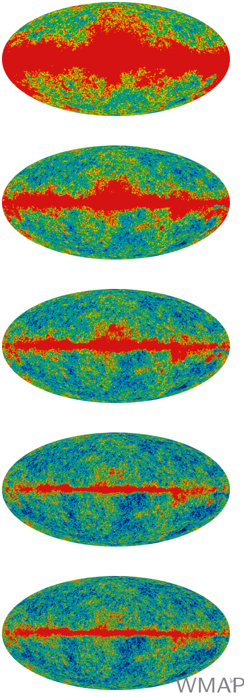


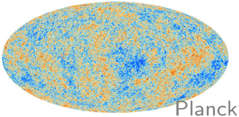




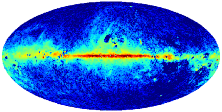
1) find frequency spectra

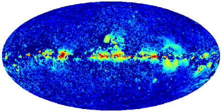
2) invert mixture

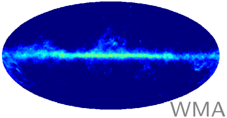
Idea



Planck

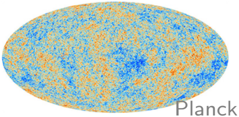






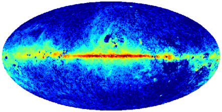
WMAP

Idea



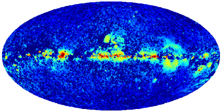
Planck

1) foregrounds (and CMB) spatially correlated



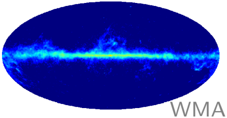
2) foregrounds cross-correlated

3) foregrounds non-Gaussian



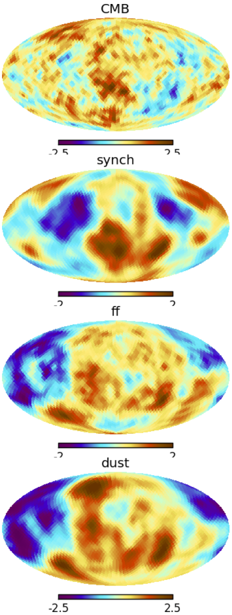
4) foreground-fluctuations vary over orders of magnitude

5) foregrounds strictly positive



WMAP

Model



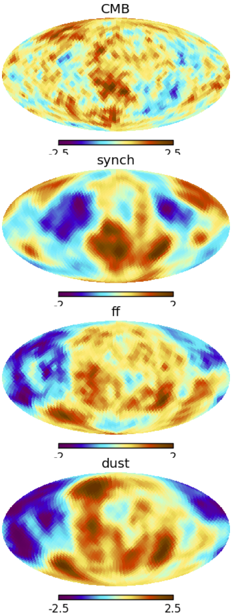
$$\text{data}_\nu = s^{(\text{CMB})}$$

$$+ f_\nu^{(\text{synch})} \, s^{(\text{synch})}$$

$$+ f_\nu^{(\text{ff})} \, s^{(\text{ff})}$$

$$+ f_\nu^{(\text{dust})} \, s^{(\text{dust})}$$

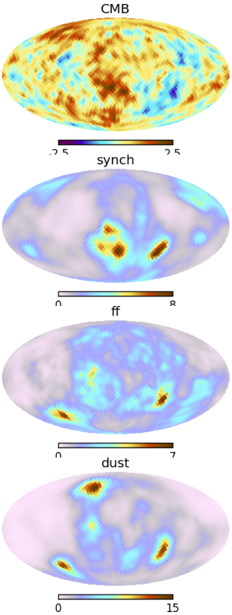
Model



$$\text{data}_\nu = s^{(\text{CMB})} + f_\nu^{(\text{synch})}\, s^{(\text{synch})} + f_\nu^{(\text{ff})}\, s^{(\text{ff})} + f_\nu^{(\text{dust})}\, s^{(\text{dust})}$$

Gaussian prior

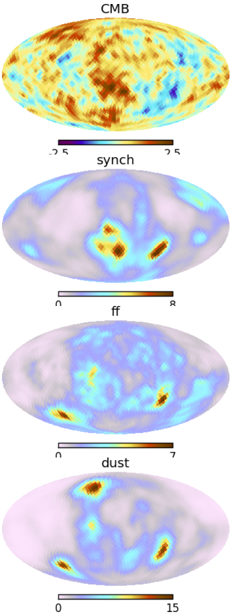
Model



$$\text{data}_\nu = s^{(\text{CMB})} + f_\nu^{(\text{synch})}\, e^{s^{(\text{synch})}} + f_\nu^{(\text{ff})}\, e^{s^{(\text{ff})}} + f_\nu^{(\text{dust})}\, e^{s^{(\text{dust})}}$$

Gaussian prior

Model



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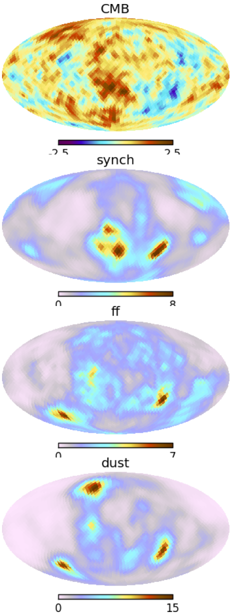
Gaussian prior

described by isotropic covariance matrix:

$$\left\langle s^{(\alpha)}_{\ell,m}\, \bar{s}^{(\beta)}_{\ell',m'} \right\rangle = \delta_{\ell,\ell'}\, \delta_{m,m'}\, C_\ell^{(\alpha,\beta)}$$

Model



$$\text{data}_\nu = s^{(\text{CMB})}$$
$$+ f_\nu^{(\text{synch})}\, e^{s^{(\text{synch})}}$$
$$+ f_\nu^{(\text{ff})}\, e^{s^{(\text{ff})}}$$
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Gaussian prior

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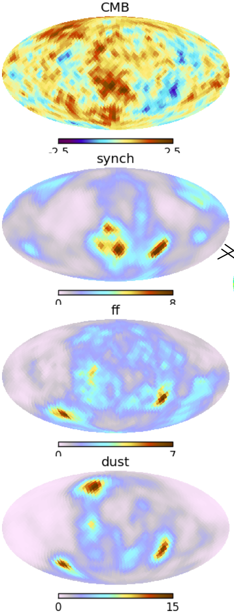
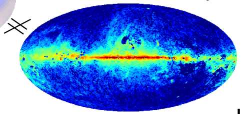
$$\left\langle s_{\ell,m}^{(\alpha)}\, \bar{s}_{\ell',m'}^{(\beta)} \right\rangle = \delta_{\ell,\ell'}\, \delta_{m,m'}\, C_\ell^{(\alpha,\beta)}$$

inverse-Wishart prior plus spectral smoothness prior

Model





$$\text{data}_\nu = s^{(\text{CMB})} + f_\nu^{(\text{synch})}\, e^{s^{(\text{synch})}} + f_\nu^{(\text{ff})}\, e^{s^{(\text{ff})}} + f_\nu^{(\text{dust})}\, e^{s^{(\text{dust})}}$$

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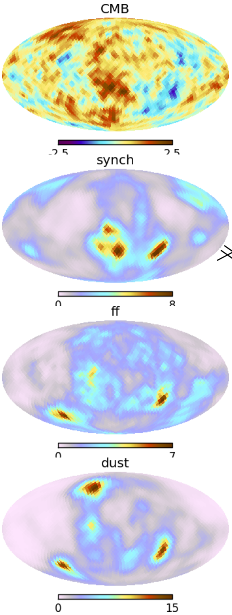

$$\text{data}_\nu = s^{(\text{CMB})}$$
$$+ f_\nu^{(\text{synch})}\, \mathrm{e}^{s^{(\text{synch})}}$$
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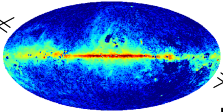
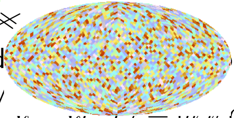
Gaussian prior





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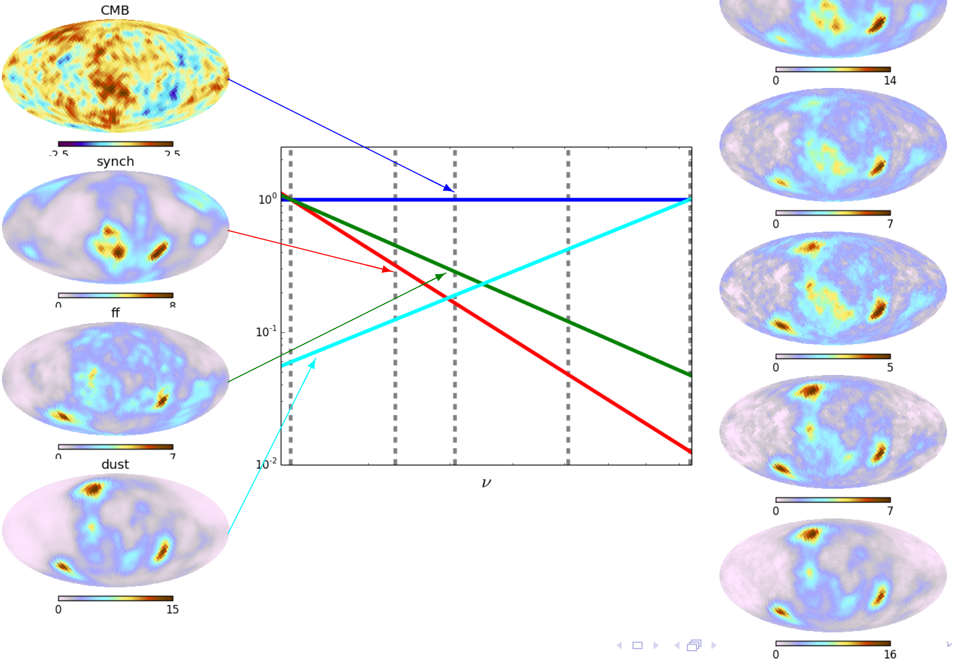
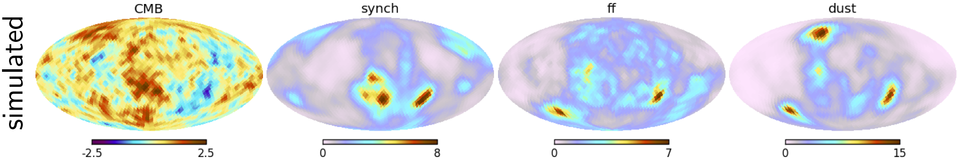
$$\left\langle \ldots_{\ell,m} \ldots_{\ell',m'} \right\rangle = \ldots_{\ell,\ell'}\, \delta_{m,m'}\, C_\ell^{(\alpha,\beta)}$$

inverse-Wishart prior plus spectral smoothness prior

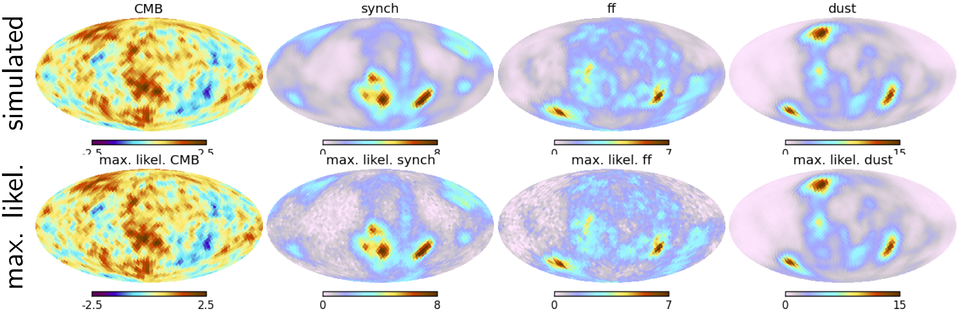
Test case



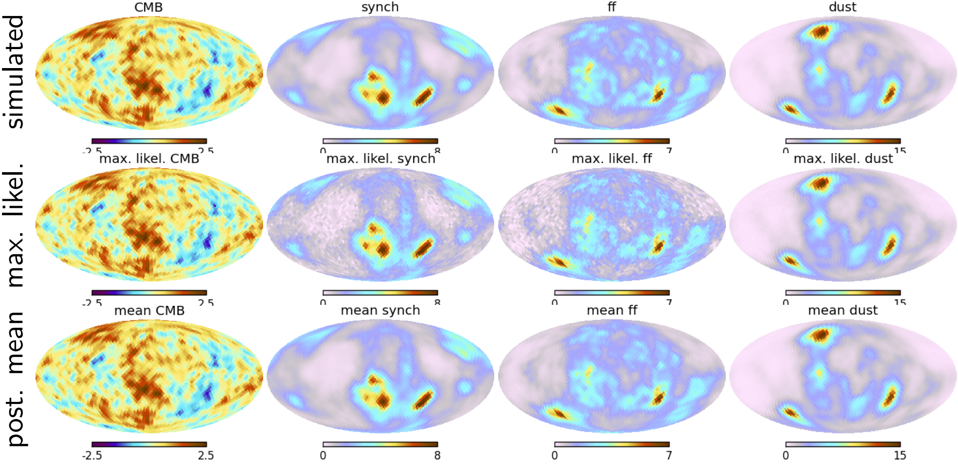
Results I



Results I

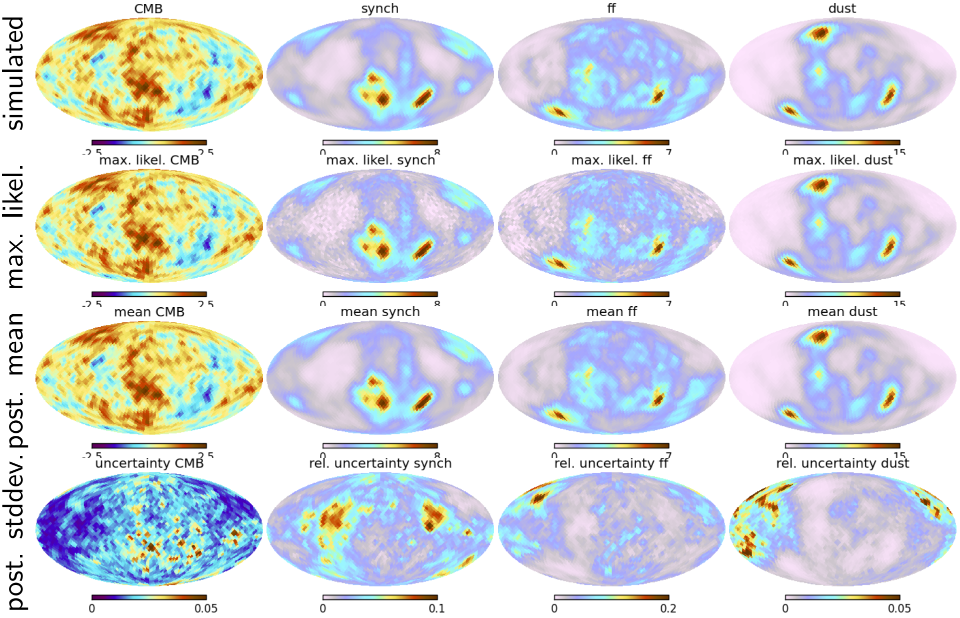

simulated
max. likel.
CMB
synch
ff
dust
-2.5
2.5
0
8
0
7
0
15
max. likel. CMB
max. likel. synch
max. likel. ff
max. likel. dust
-2.5
2.5
0
8
0
7
0
15

Results I

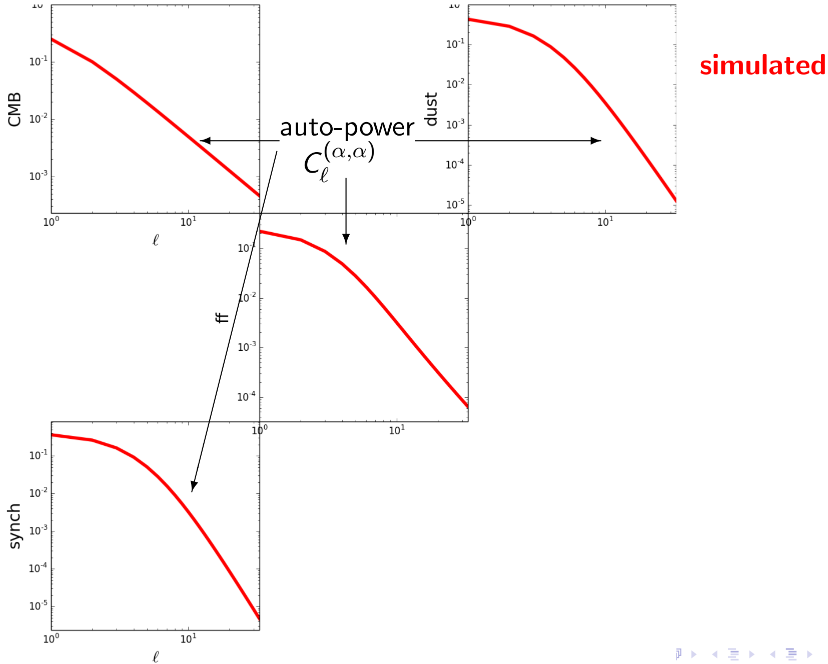


simulated
max. likel.
post. mean
CMB
synch
ff
dust
-2.5
2.5
0
8
0
7
0
15
max. likel. CMB
max. likel. synch
max. likel. ff
max. likel. dust
-2.5
2.5
0
8
0
7
0
15
mean CMB
mean synch
mean ff
mean dust
-2.5
2.5
0
8
0
7
0
15

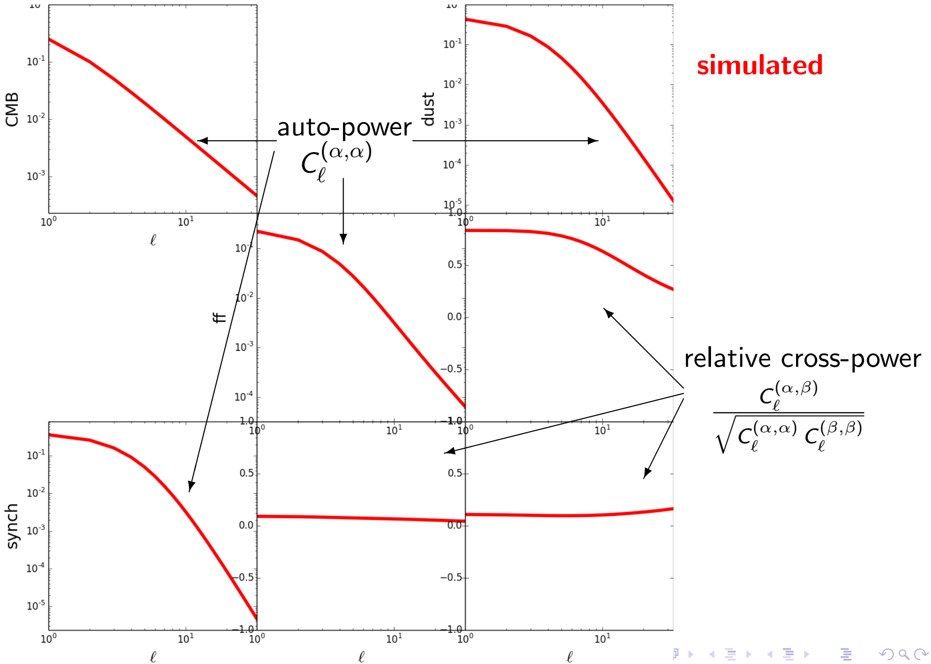
Results I


simulated
max. likel.
post. mean
post. stddev.
CMB
synch
ff
dust
max. likel. CMB
max. likel. synch
max. likel. ff
max. likel. dust
mean CMB
mean synch
mean ff
mean dust
uncertainty CMB
rel. uncertainty synch
rel. uncertainty ff
rel. uncertainty dust
-2.5
2.5
0
8
7
15
0.05
0.1
0.2

Results II

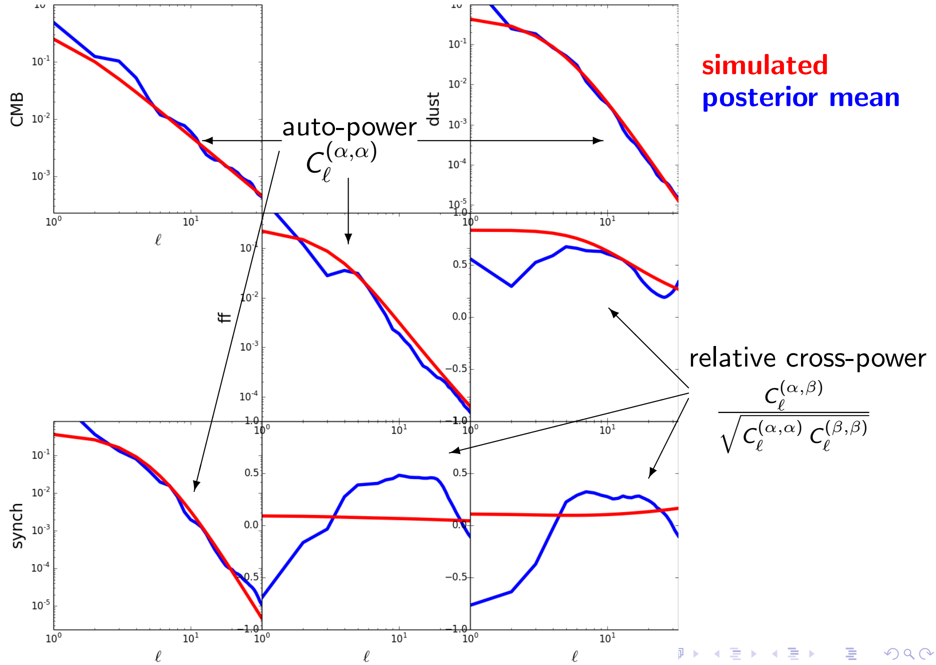

CMB
dust
ff
synch
auto-power
$C_\ell^{(\alpha,\alpha)}$
simulated
ℓ
ℓ

Results II

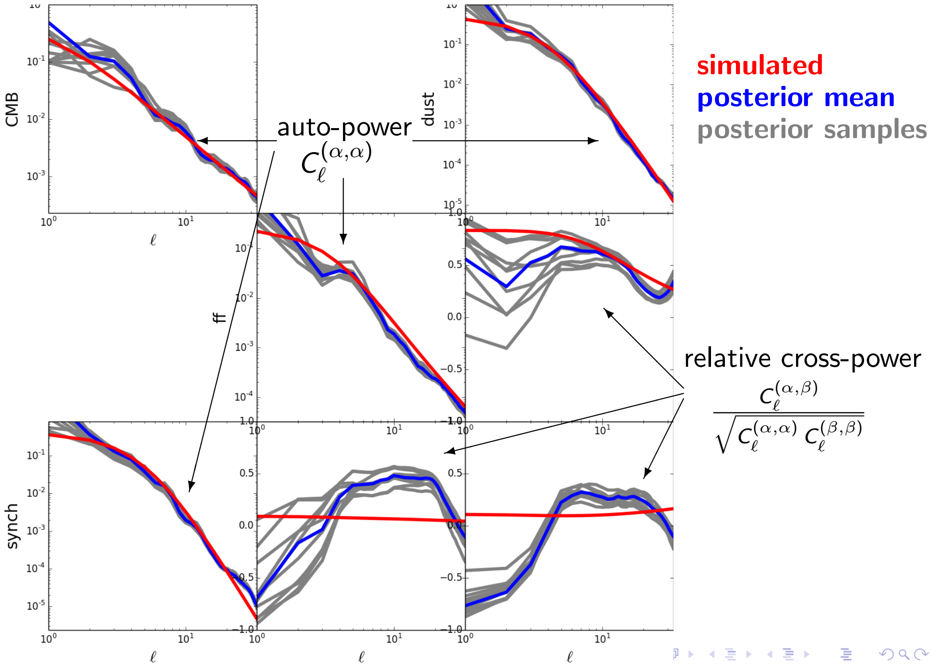


simulated
auto-power
$C_\ell^{(\alpha,\alpha)}$
relative cross-power
$\frac{C_\ell^{(\alpha,\beta)}}{\sqrt{C_\ell^{(\alpha,\alpha)}\, C_\ell^{(\beta,\beta)}}}$
CMB
dust
ff
synch
ℓ

Results II


CMB
dust
ff
synch
ℓ
simulated
posterior mean
auto-power
$C_\ell^{(\alpha,\alpha)}$
relative cross-power
$\frac{C_\ell^{(\alpha,\beta)}}{\sqrt{C_\ell^{(\alpha,\alpha)}\,C_\ell^{(\beta,\beta)}}}$

Results II


simulated
posterior mean
posterior samples
auto-power
$C_\ell^{(\alpha,\alpha)}$
relative cross-power
$\frac{C_\ell^{(\alpha,\beta)}}{\sqrt{C_\ell^{(\alpha,\alpha)}\, C_\ell^{(\beta,\beta)}}}$
CMB
dust
ff
synch
ℓ

Bottom line

"All models are wrong, but some are useful." (Box & Draper 1987)
Maybe being less wrong makes it more useful.

- spatial correlations
- cross-correlations between components

Applications:

- all-sky diffuse component separation (?)
- separation in targeted regions:
  - synchrotron due to different populations of electrons
  - thermal dust emission at different temperatures
  - spinning dust emission due to different species
- extend to polarization, 21cm, EoR,...
- ...