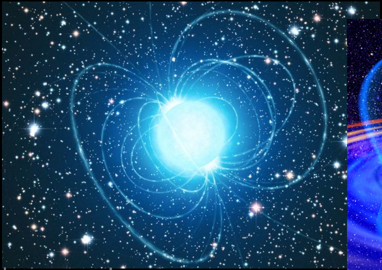
Magnetic fields seen through Faraday rotation – from the Milky Way to cosmic scales

Niels Oppermann

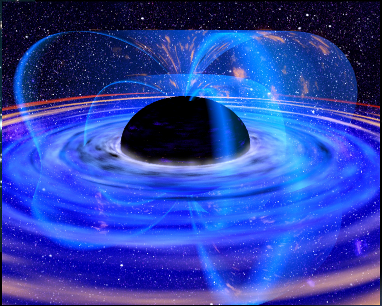
CITA ICAT
Canadian Institute for Theoretical Astrophysics
L'institut Canadien d'astrophysique théorique

with: Torsten Enßlin, Henrik Junklewitz, Valentina Vacca,
Mike Bell, Bryan Gaensler, Dominic Schnitzeler, Jeroen Stil,
...

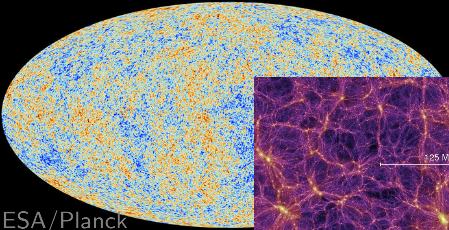
Astro lunch talk, University of Waterloo, 2014-10-15



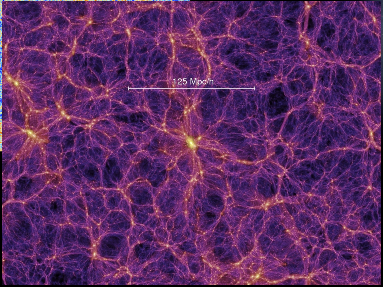
h+ magazine



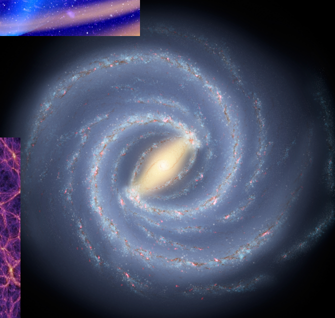
Dana Berry, NASA



ESA/Planck

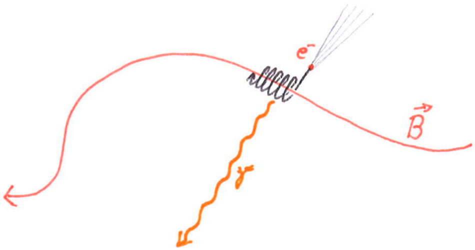


MPA

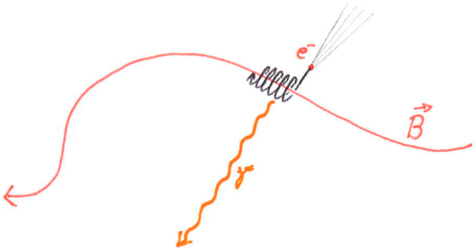


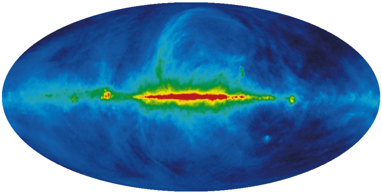
NASA/JPL-Caltech

Synchrotron

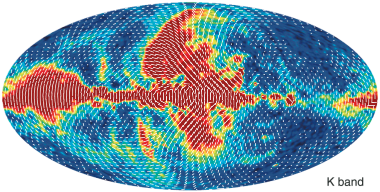


Synchrotron



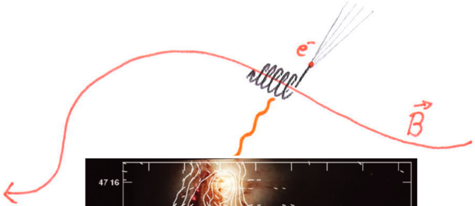


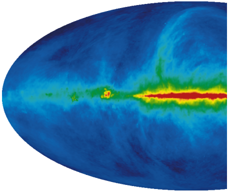
Haslam et al. (1981)



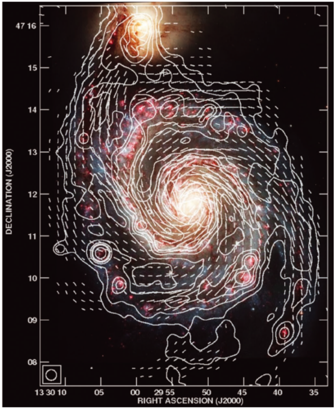
Hinshaw et al. (2009)

Synchrotron

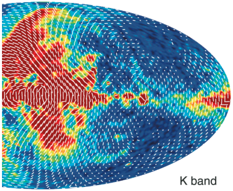




Haslan

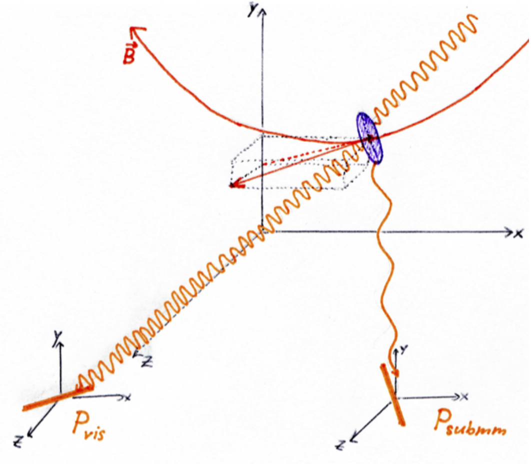


Fletcher et al. (2011)

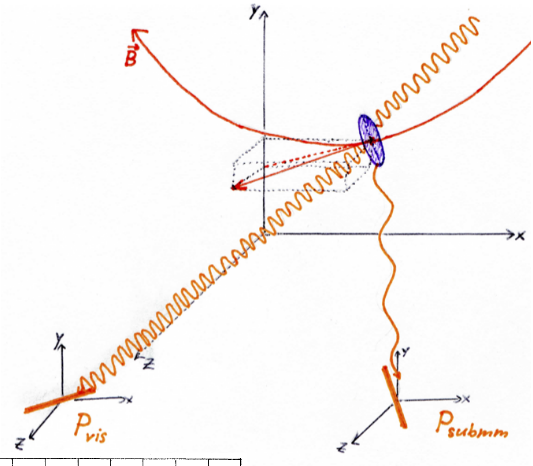


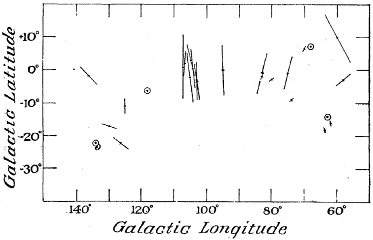
Hinshaw et al. (2009)

Dust

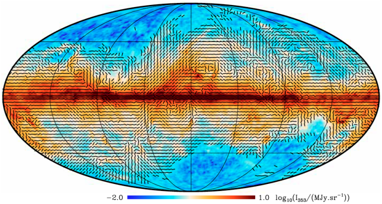


Dust



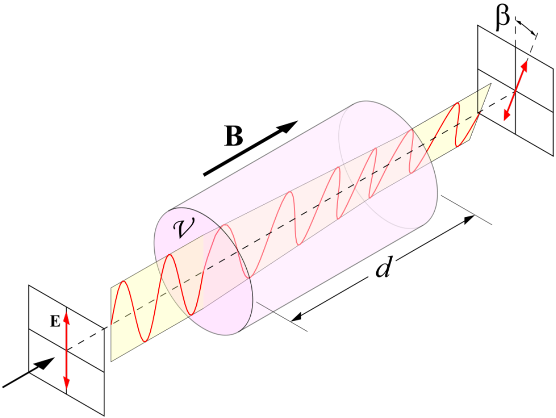


Hall (1949)



Planck Collaboration Int. XIX (2014)

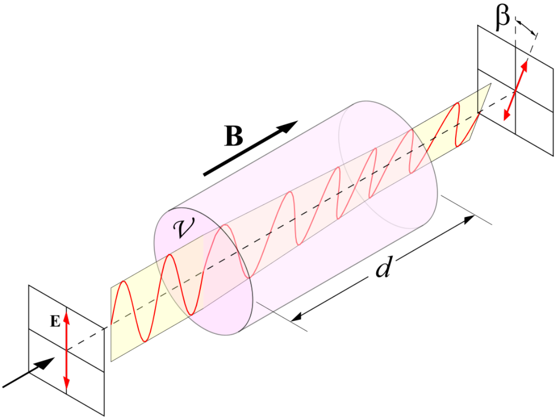
Faraday rotation



$$\mathrm{d}\beta \propto \lambda^2 n_{\mathrm{e}} B_r \, \mathrm{d}r$$

$$\Rightarrow \quad \beta \propto \lambda^2 \int_{r_{\mathrm{source}}}^{0} (1+z)^{-2} \; n_{\mathrm{e}} B_r \, \mathrm{d}r$$

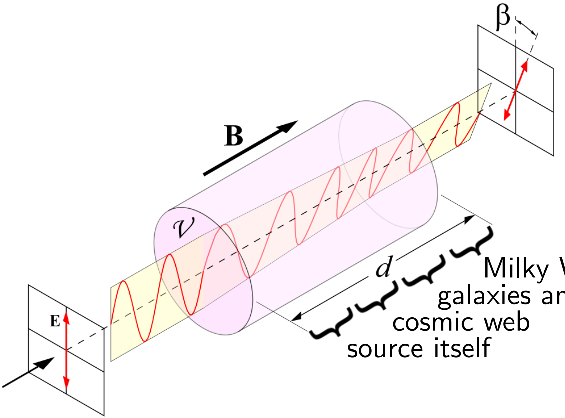
Faraday rotation

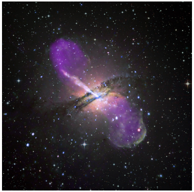


Faraday depth: $\phi \propto \int_{r_{\mathrm{source}}}^{0} (1+z)^{-2}\; n_{\mathrm{e}}\, B_r \,\mathrm{d}r$

$$\beta = \phi\lambda^2$$

Faraday rotation

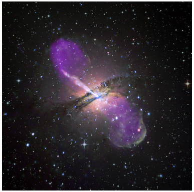


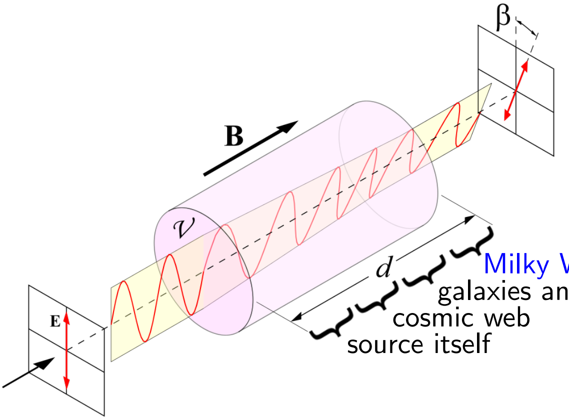


Faraday depth: $\phi \propto \int_{r_{\mathrm{source}}}^{0} (1+z)^{-2}\; n_{\mathrm{e}}\, B_r \,\mathrm{d}r$

$$\beta = \phi\lambda^2$$

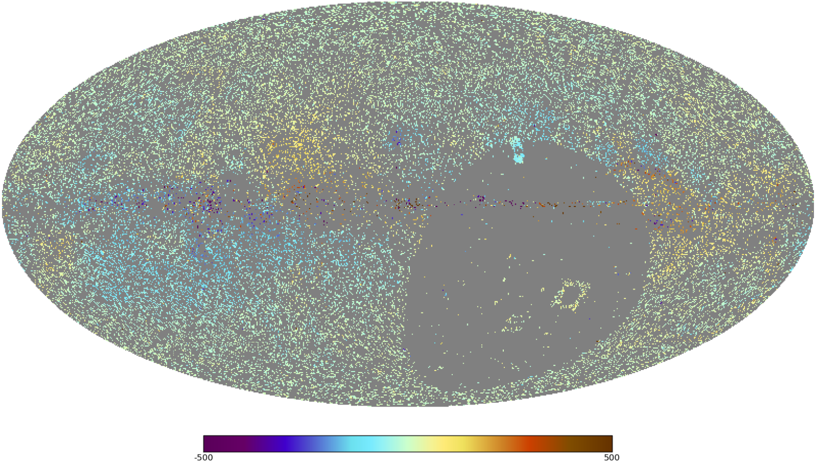
Extracting the Galactic contribution



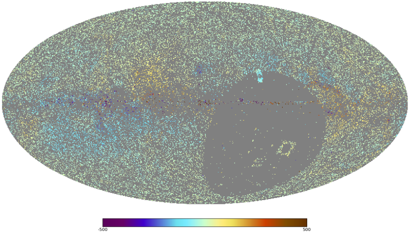


Galactic Faraday depth:

$$\phi_{g} \propto \int_{r_{\mathrm{MilkyWay}}}^{0} (1+z)^{-2}\, n_{e}\, B_{r}\, \mathrm{d}r$$



$\gtrsim 40\,000$ data points



Challenges

- Regions without data
- Extragalactic contributions unknown
- Uncertain error bars

$$d = \phi_{\mathrm{g}} + \phi_{\mathrm{e}} + n$$

Wiener filter:

$$\hat{\phi}_{\mathrm{g}} = G\,(G + E + N)^{-1}\,d$$

Covariance matrices:

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$
$$E_{ij} = \delta_{ij}\,\sigma_{\mathrm{e}}^2$$
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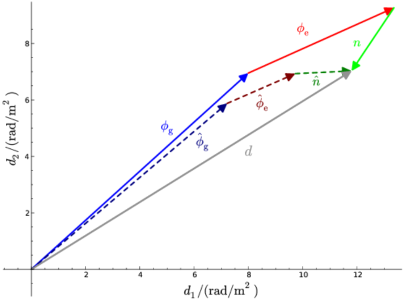

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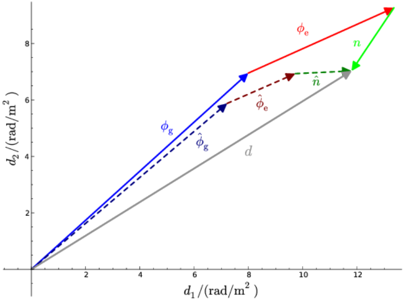

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$$E_{ij} = \delta_{ij}\,\sigma_{\mathrm{e}}^2$$
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Posterior uncertainty:

$$D_{\mathrm{g}} = \left(G^{-1} + (E+N)^{-1}\right)^{-1}$$
$$D_{\mathrm{e}} = \left(E^{-1} + (G+N)^{-1}\right)^{-1}$$
$$D_n = \left(N^{-1} + (G+E)^{-1}\right)^{-1}$$

$$d = \phi_\mathrm{g} + \phi_\mathrm{e} + n$$

Covariance matrices:

Wiener filter:

$$\hat{\phi}_\mathrm{g} = G\,(G + E + N)^{-1}\,d$$

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$

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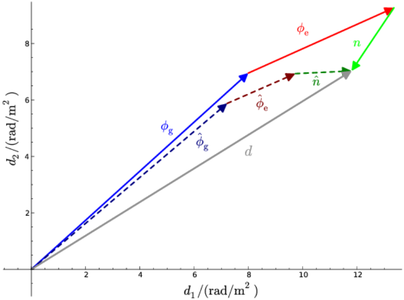

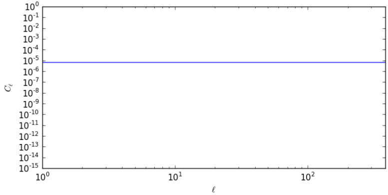

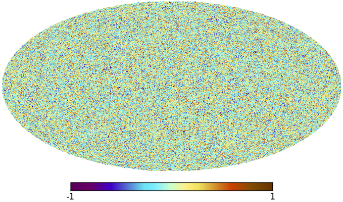

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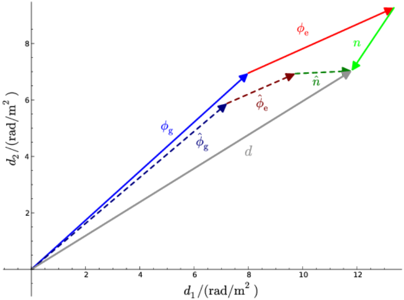

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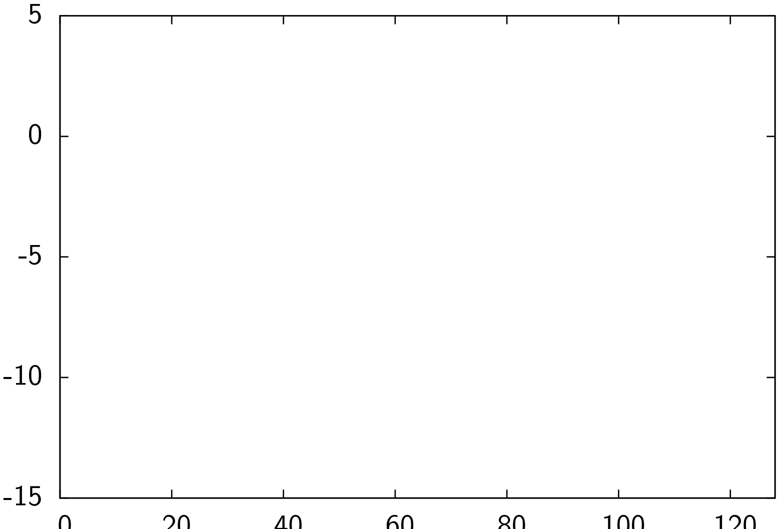
1D example

Assumptions:

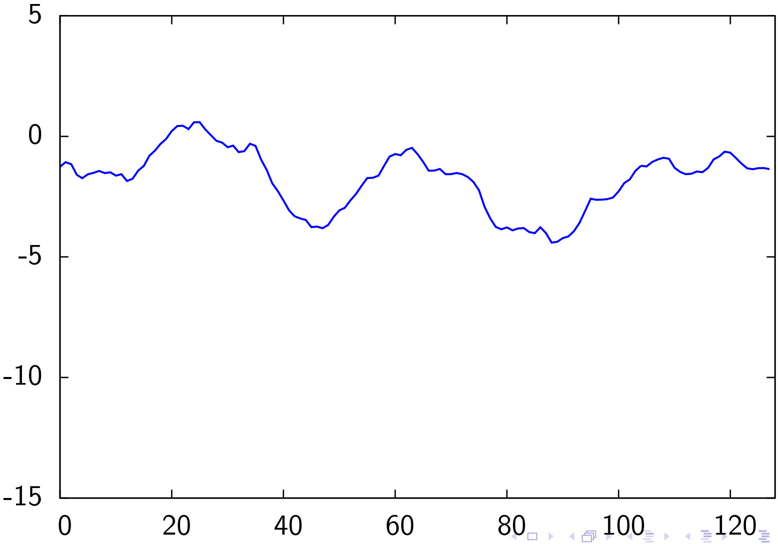
-
-



1D example

Assumptions:

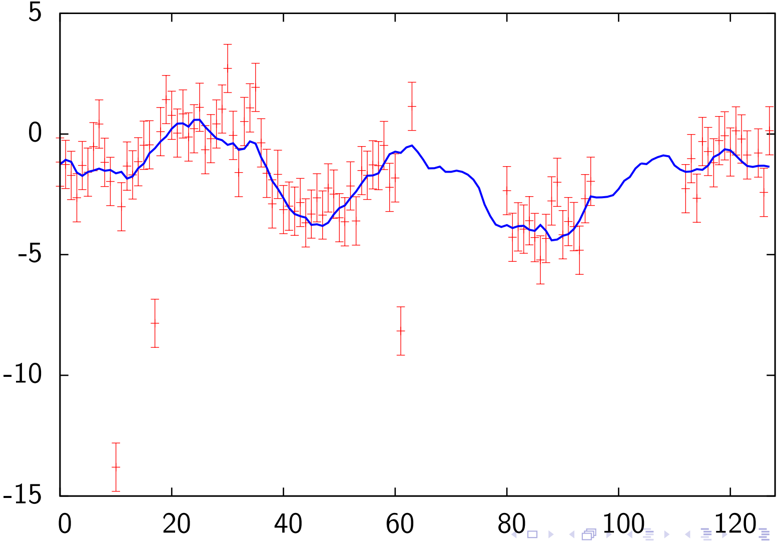
- signal field statistically homogeneous Gaussian random field
-



1D example

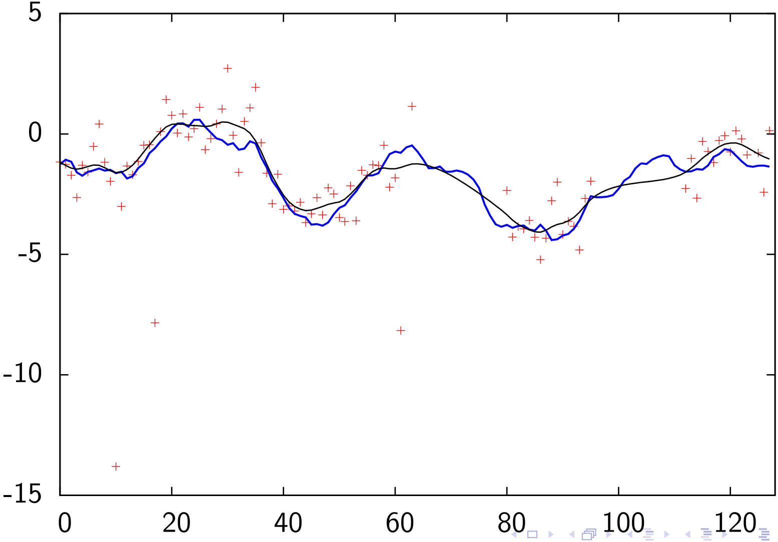
Assumptions:

- signal field statistically homogeneous Gaussian random field
- noise uncorrelated, Gaussian



1D example

- Reconstruct (iteratively):
  signal, power spectrum, noise variance



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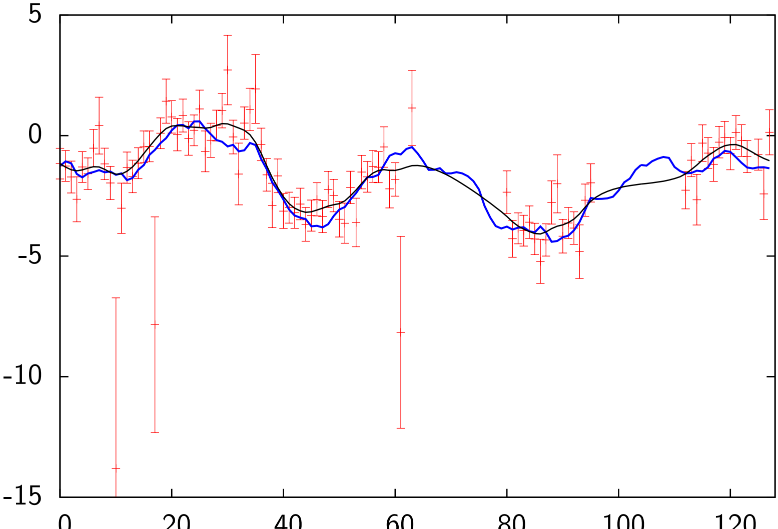

1D example

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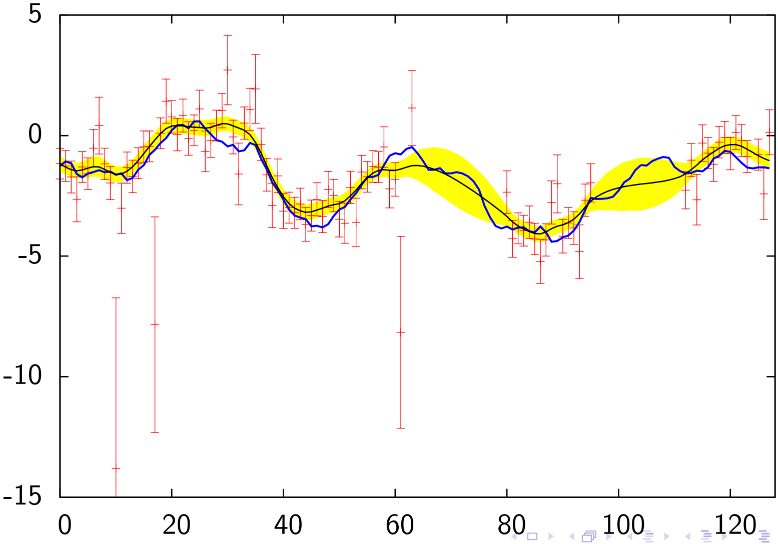
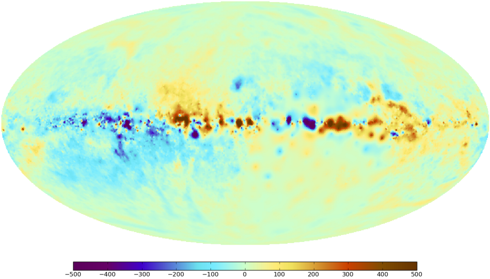
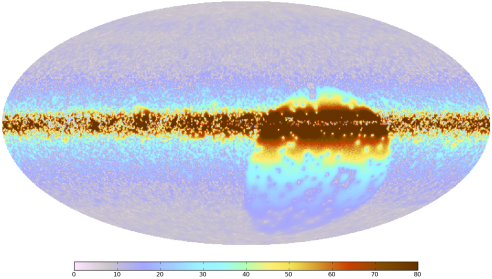


Galactic Faraday depth

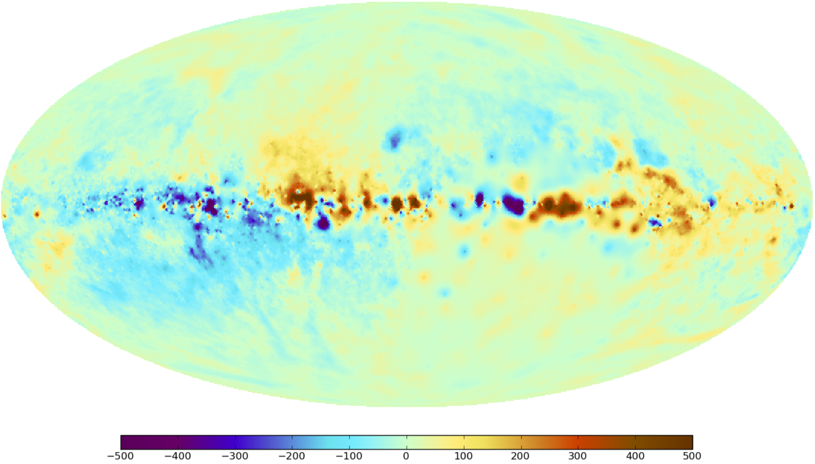


uncertainty



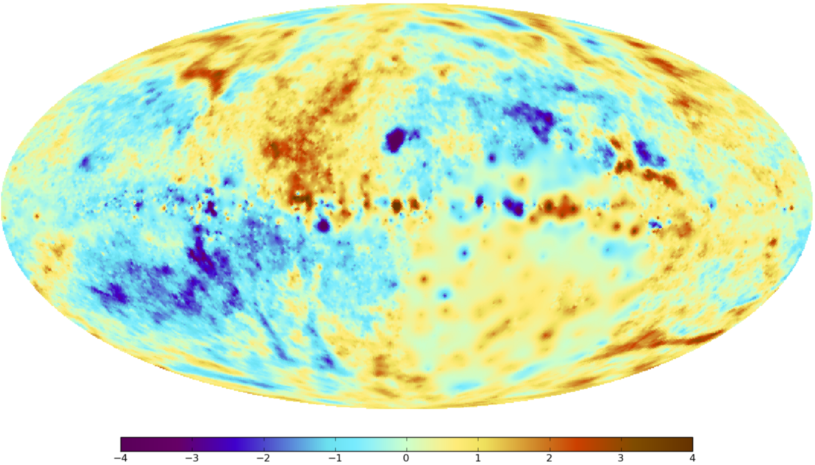
Oppermann et al. (2012/2014)

Galactic Faraday depth



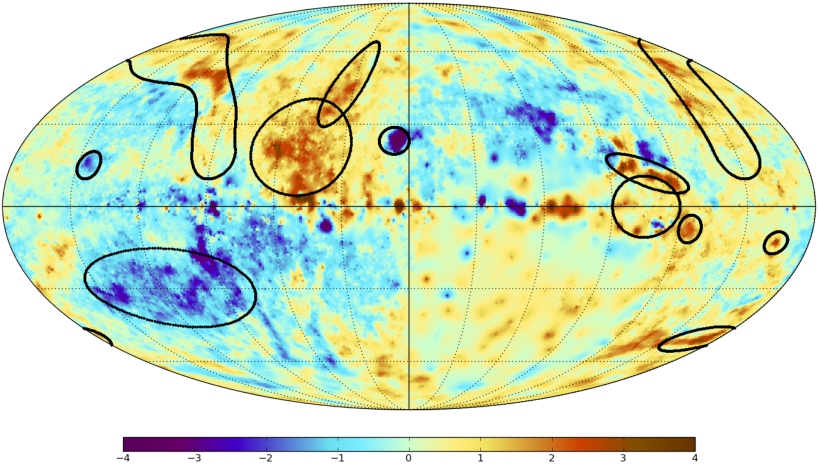
Oppermann et al. (2012/2014)

rescaled Galactic Faraday depth



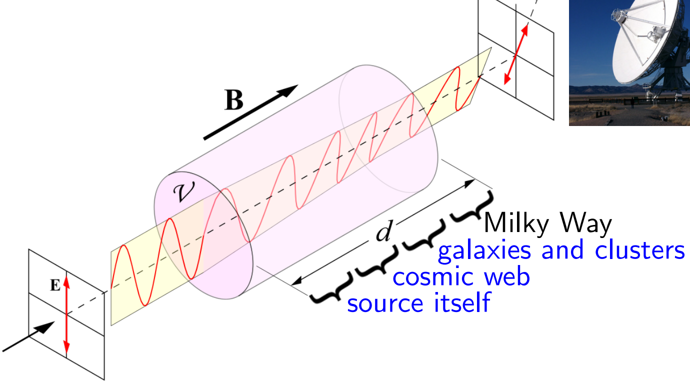
Oppermann et al. (2012/2014)

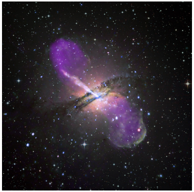
rescaled Galactic Faraday depth



Oppermann et al. (2012/2014)

Extracting the extragalactic contribution





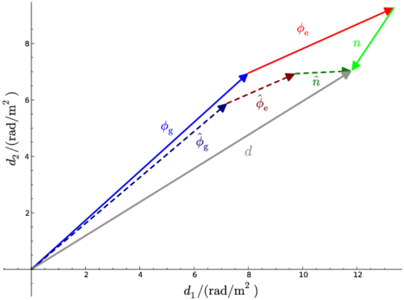
extragalactic Faraday depth:

$$\phi_{\mathrm{e}} \propto \int_{r_{\mathrm{source}}}^{r_{\mathrm{MilkyWay}}} (1+z)^{-2}\; n_{\mathrm{e}}\, B_r \,\mathrm{d}r$$

$$d = \phi_g + \phi_e + n$$

Wiener filter:

$$\hat{\phi}_g = G\,(G + E + N)^{-1}\,d$$



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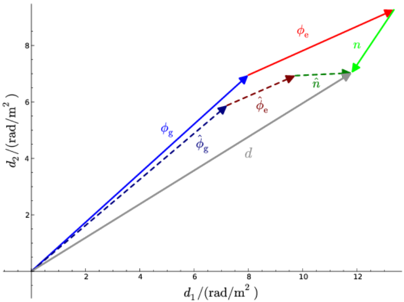

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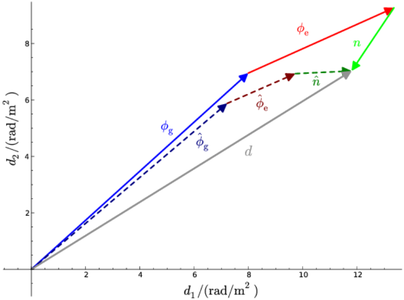

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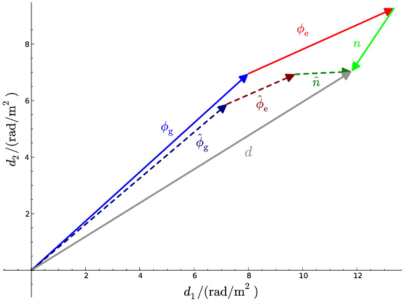
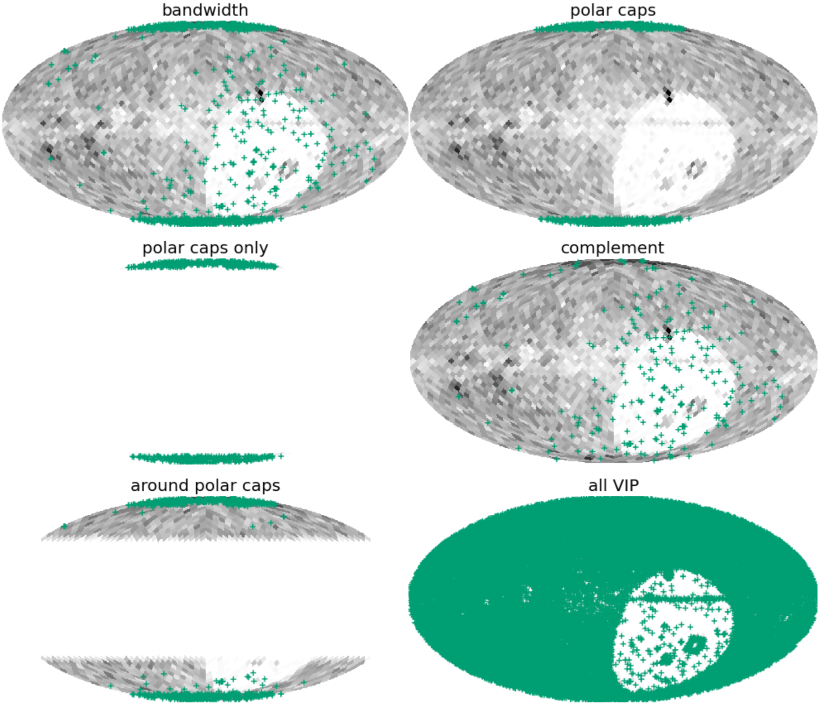
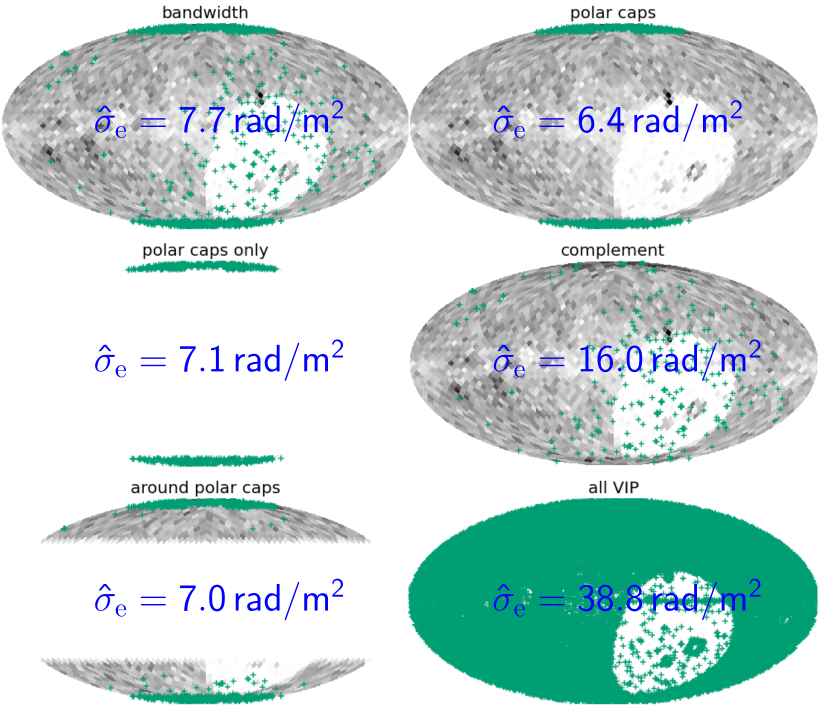
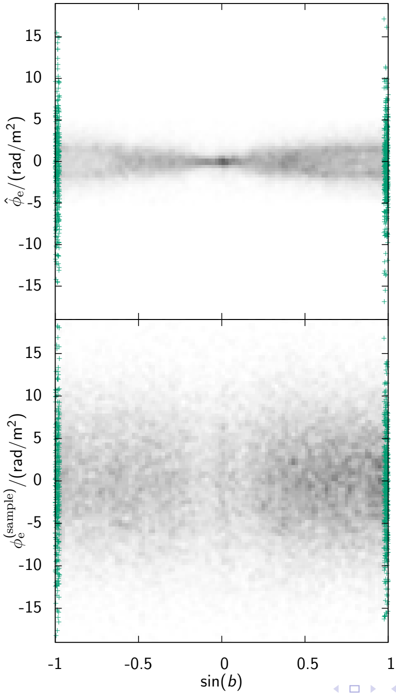


idea: find subset of data for which $\eta_i \equiv 1$

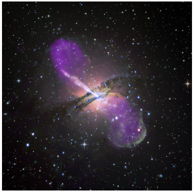

bandwidth
polar caps
polar caps only
complement
around polar caps
all VIP

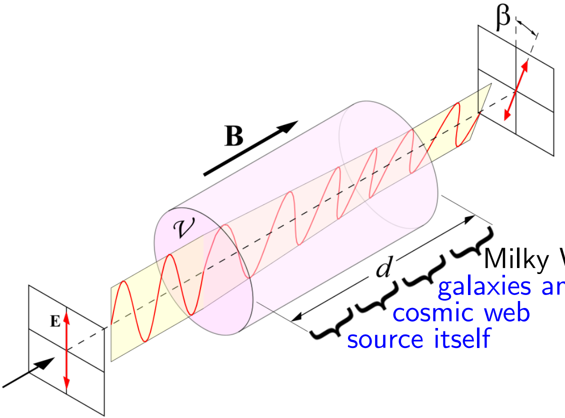

bandwidth
$\hat{\sigma}_{\rm e} = 7.7\,\text{rad}/\text{m}^2$
polar caps
$\hat{\sigma}_{\rm e} = 6.4\,\text{rad}/\text{m}^2$
polar caps only
$\hat{\sigma}_{\rm e} = 7.1\,\text{rad}/\text{m}^2$
complement
$\hat{\sigma}_{\rm e} = 16.0\,\text{rad}/\text{m}^2$
around polar caps
$\hat{\sigma}_{\rm e} = 7.0\,\text{rad}/\text{m}^2$
all VIP
$\hat{\sigma}_{\rm e} = 38.8\,\text{rad}/\text{m}^2$



15
10
5
0
-5
-10
-15
$\hat{\phi}_e/(\mathrm{rad/m^2})$
$\phi_e^{(\mathrm{sample})}/(\mathrm{rad/m^2})$
-1
-0.5
0
0.5
1
sin(b)

What is the extragalactic contribution?





extragalactic Faraday depth:

$$\phi_{\mathrm{e}} \propto \int_{r_{\mathrm{source}}}^{r_{\mathrm{MilkyWay}}} (1+z)^{-2}\, n_{\mathrm{e}}\, B_r \,\mathrm{d}r$$

$$d = \phi_{\mathrm{g}} + \phi_{\mathrm{e}} + n$$

Covariance matrices:

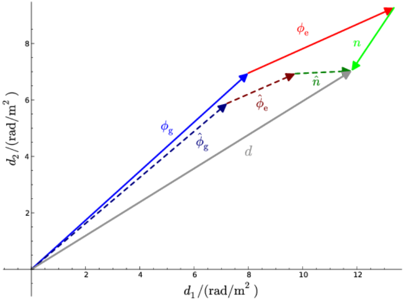
Wiener filter:

$$\hat{\phi}_{\mathrm{e}} = E\,(G + E + N)^{-1}\,d$$

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$

$$E_{ij} = \delta_{ij}\,\sigma_{\mathrm{e}}^2\,\eta_{\mathrm{e}}$$

$$N_{ij} = \delta_{ij}\,\sigma_i^2\,\eta_i$$



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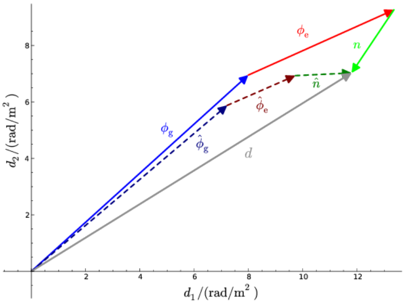

$$E_{ij} = \delta_{ij}\,\left(\sigma^{(\mathrm{source})2} + \sigma_i^{(\mathrm{cluster})2} + \sigma_i^{(\mathrm{filament})2} + \sigma_i^{(\mathrm{void})2}\right)$$

$$d = \phi_\mathrm{g} + \phi_\mathrm{e} + n$$

Covariance matrices:

Wiener filter:

$$\hat{\phi}_\mathrm{e} = E\,(G + E + N)^{-1}\,d$$

$$G_{(\ell,m),(\ell',m')} = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$

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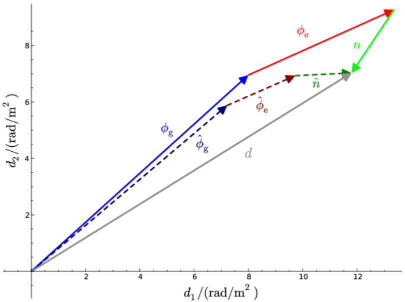
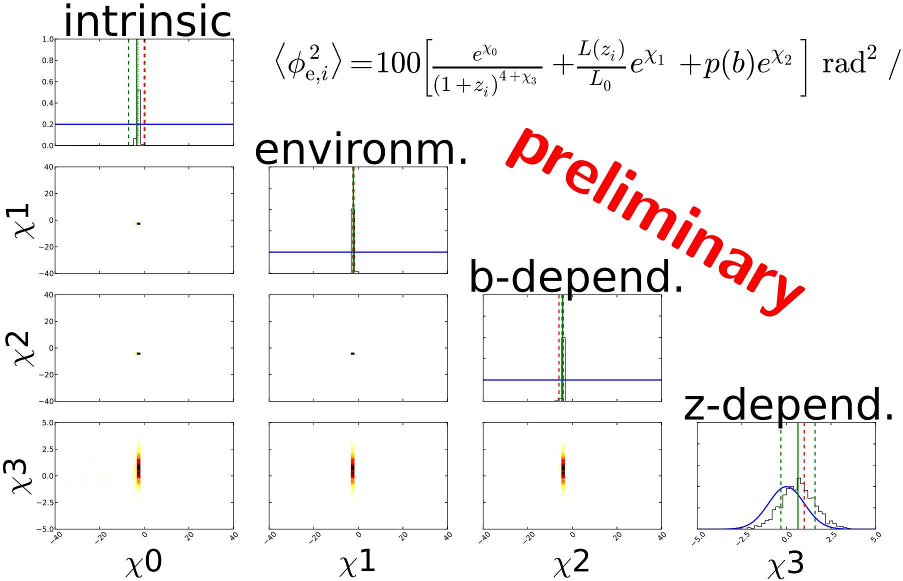


$$E_{ij} = \delta_{ij}\left(\frac{\mathrm{e}^{\chi_0}}{(1+z_i)^4} + \mathrm{e}^{\chi_1}\,L(z_i) + \ldots\right)$$

$$L(z_i) \propto \int_0^{r(z_i)} \frac{\mathrm{d}r}{(1+z(r))^4}$$

Simulation

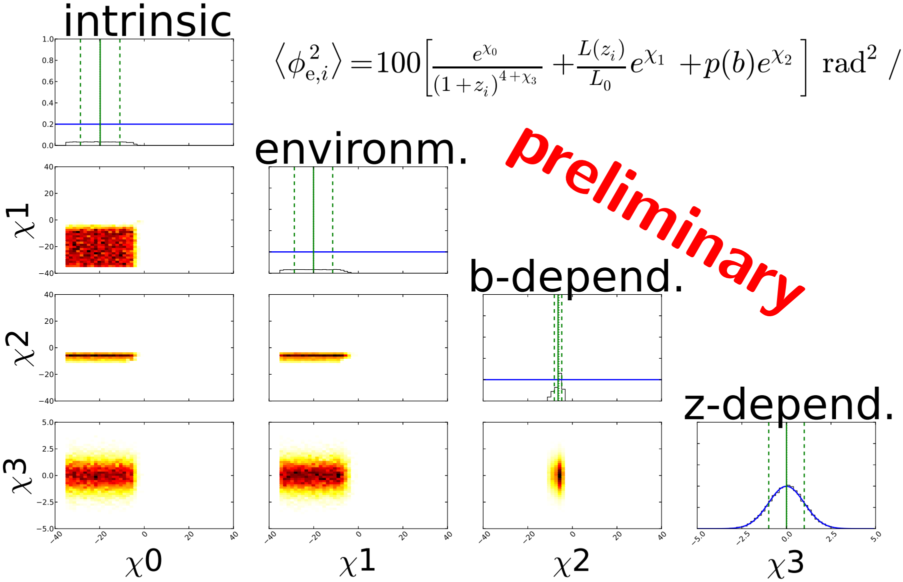
$$\left\langle \phi_{\mathrm{e},i}^{2} \right\rangle = 100 \left[\frac{e^{\chi_0}}{(1+z_i)^{4+\chi_3}} + \frac{L(z_i)}{L_0} e^{\chi_1} + p(b) e^{\chi_2} \right] \mathrm{rad}^2 / \mathrm{m}^4$$



plots courtesy of Valentina Vacca

Real data

$$\langle \phi_{e,i}^2 \rangle = 100\left[\frac{e^{\chi_0}}{(1+z_i)^{4+\chi_3}} + \frac{L(z_i)}{L_0}e^{\chi_1} + p(b)e^{\chi_2}\right] \text{ rad}^2/\text{m}^4$$



plots courtesy of Valentina Vacca

Summary

- Galactic contribution (correlated) can be separated from rest (uncorrelated)
- Rest can be separated statistically into extragalactic and noise
- Uncertainties are large and should not be ignored

All results at
http://www.mpa-garching.mpg.de/ift/faraday/