MAX-PLANCK-GESELLSCHAFT

Max Planck Institute
for Astrophysics

The diffuse Milky Way – Sharpening the picture with new inference techniques

Niels Oppermann

with
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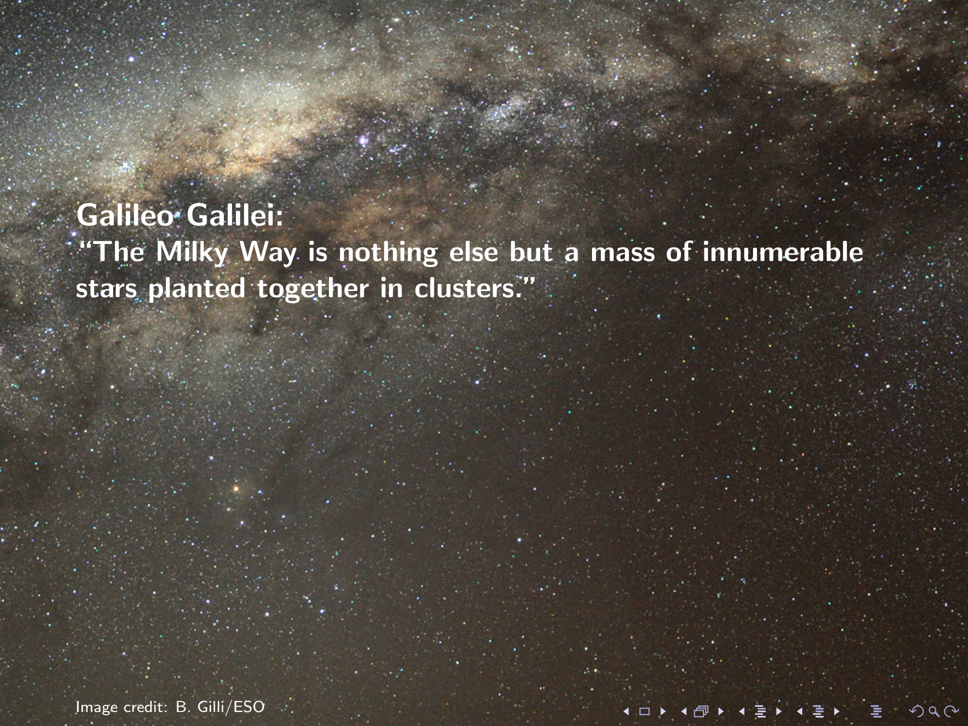
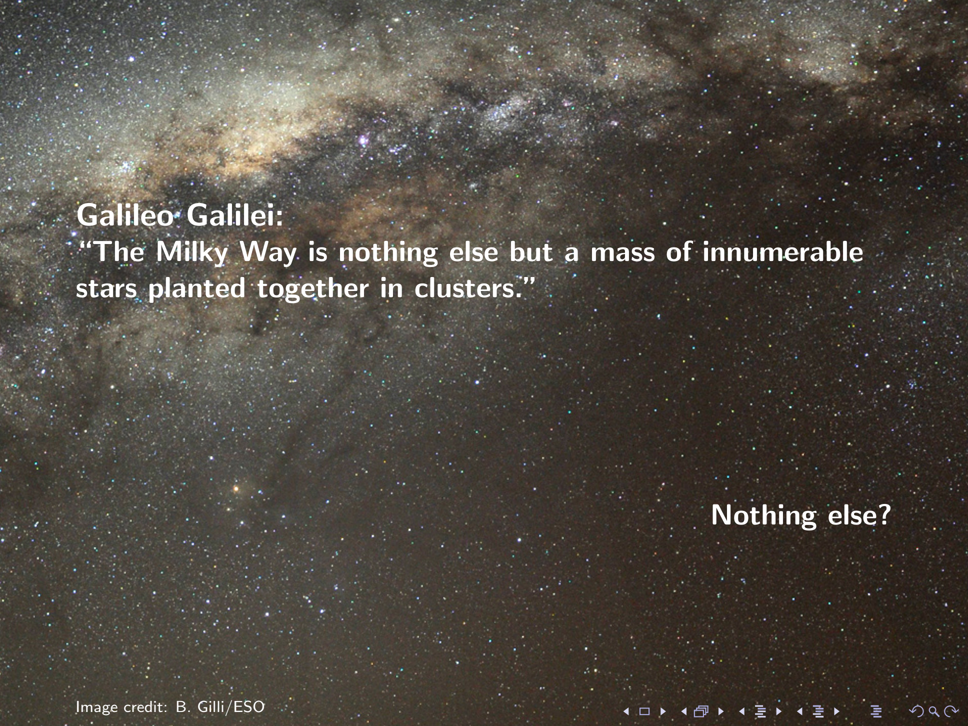
MPA seminar, Garching, 2013-07-01



Galileo Galilei:
"The Milky Way is nothing else but a mass of innumerable stars planted together in clusters."

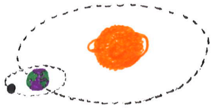
Image credit: B. Gilli/ESO

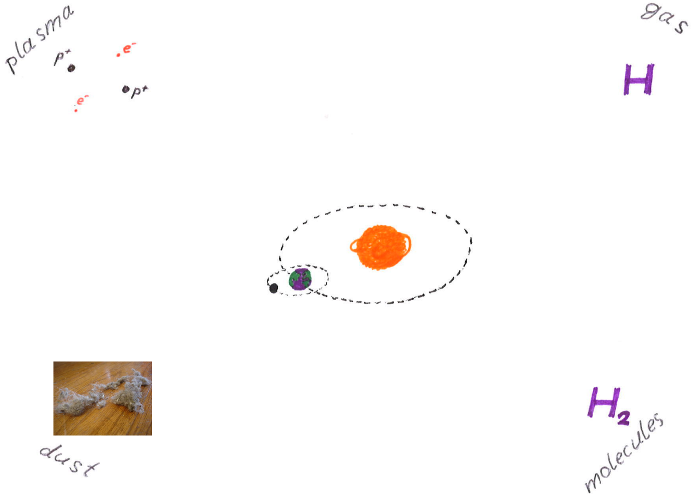


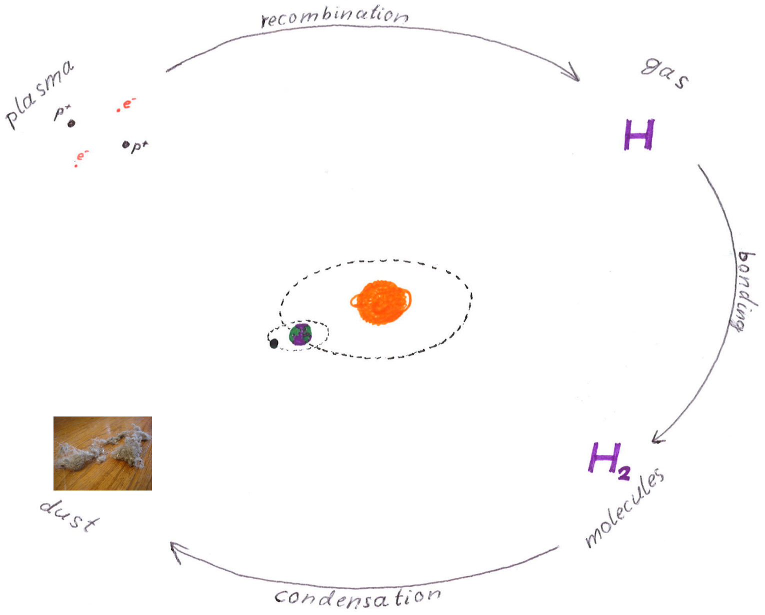
Galileo Galilei:
“The Milky Way is nothing else but a mass of innumerable stars planted together in clusters.”

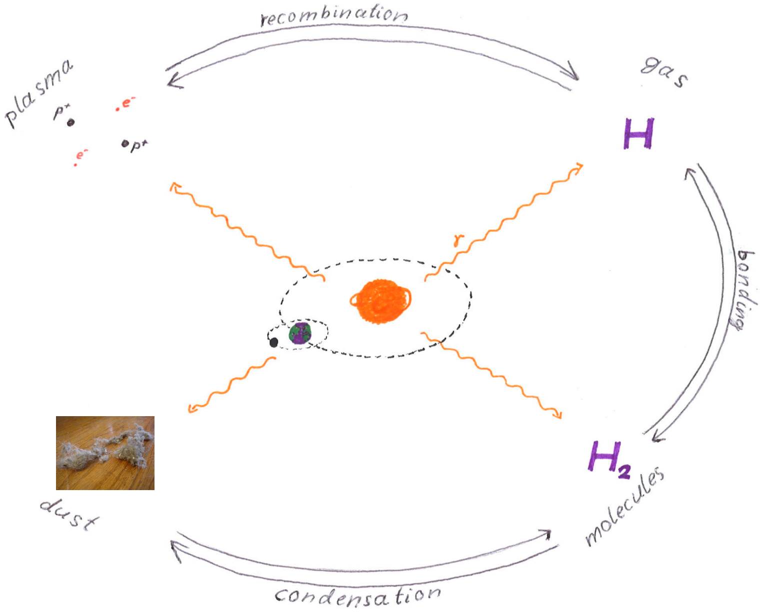
Nothing else?

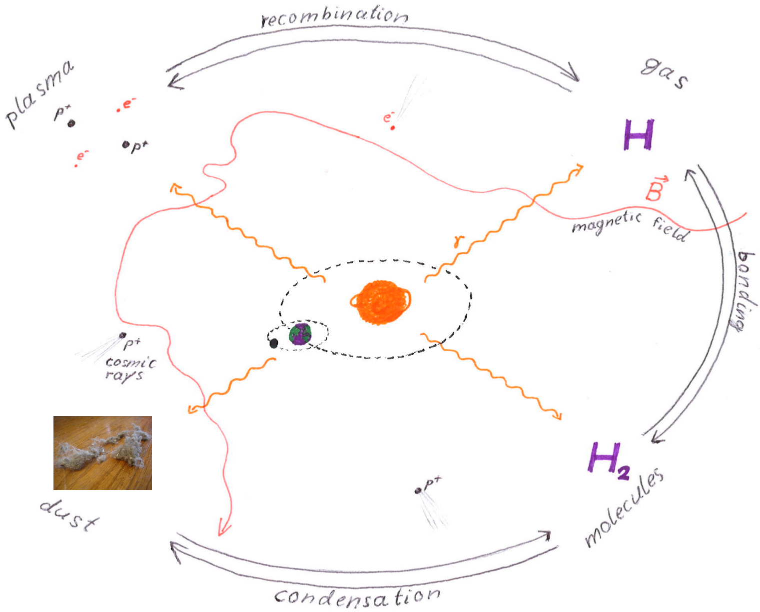
Image credit: B. Gilli/ESO



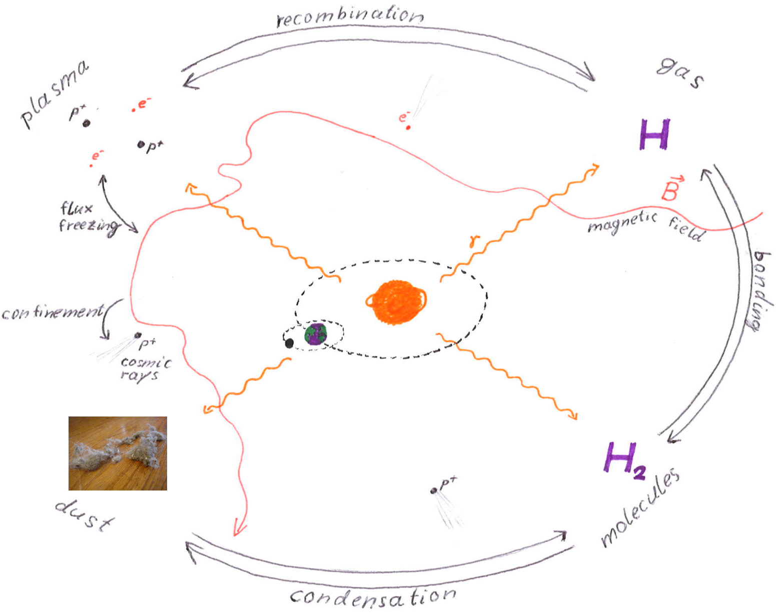

plasma
p+
e-
e-
p+
gas
H
dust
H2
molecules


recombination
gas
H
bonding
H2
molecules
condensation
dust
plasma
p+
e-
p+
e-


recombination
plasma
p^+
e^-
e^-
p^+
gas
H
γ
bonding
H_2
molecules
dust
condensation



recombination
plasma
p+
e-
e-
p+
e-
gas
H
$\vec{B}$
magnetic field
bonding
γ
p+
cosmic rays
H_2
molecules
p+
dust
condensation


recombination
plasma
gas
p^+
e^-
e^-
p^+
e^-
H
$\vec{B}$
flux freezing
magnetic field
γ
bonding
confinement
p^+
cosmic rays
H_2
molecules
p^+
dust
condensation

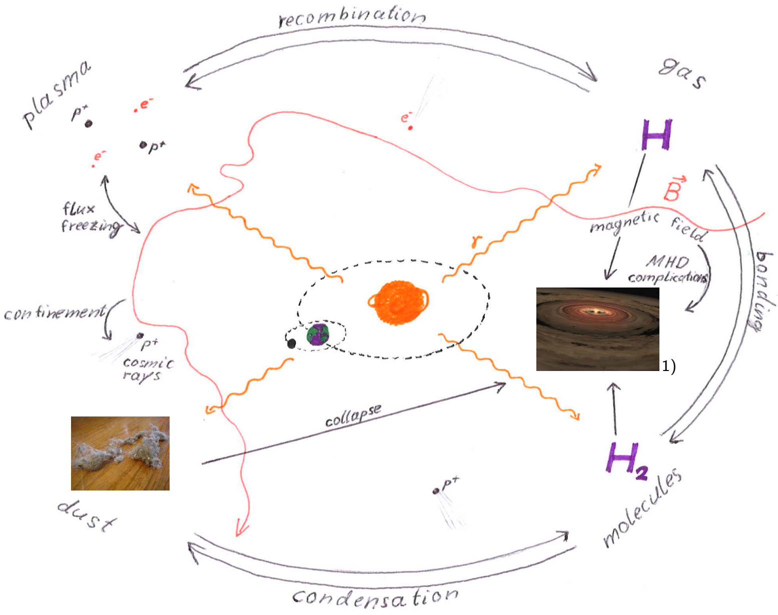


Image credits: 1) D. Darling

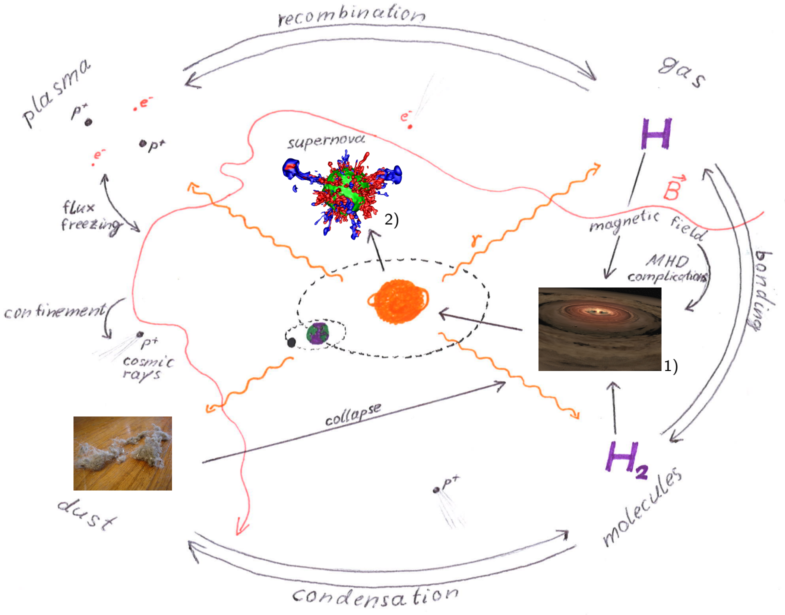
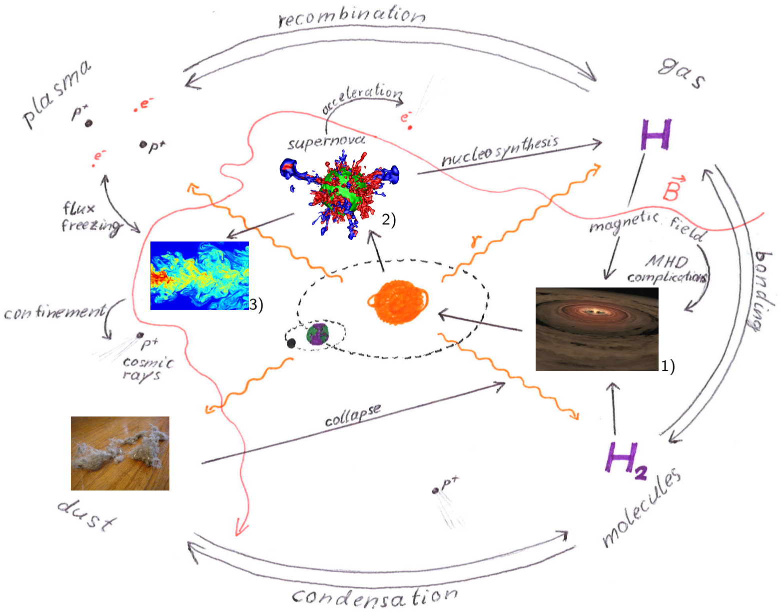


Image credits: 1) D. Darling; 2) N.J. Hammer/MPA



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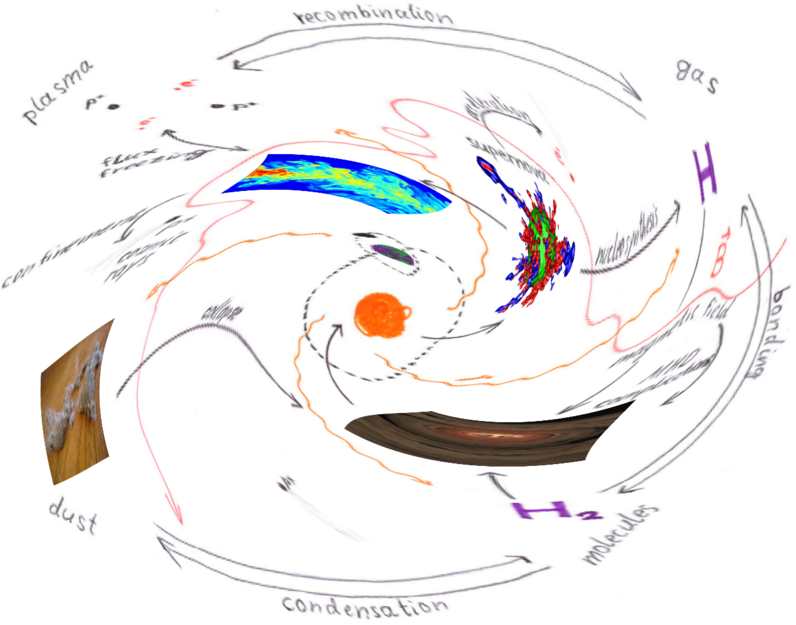

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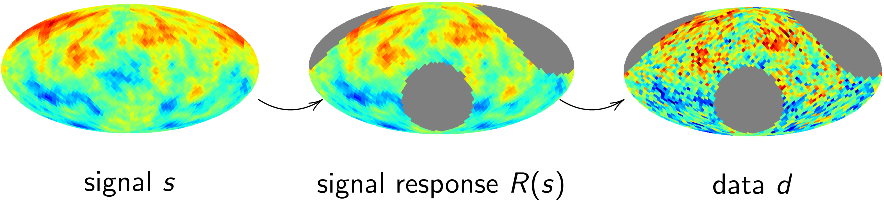
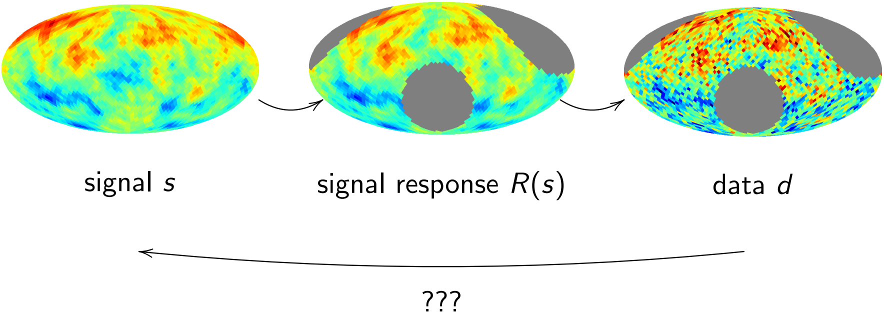
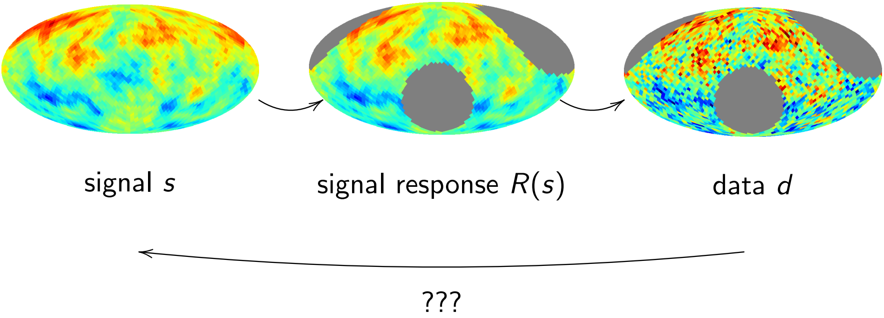
Signal inference



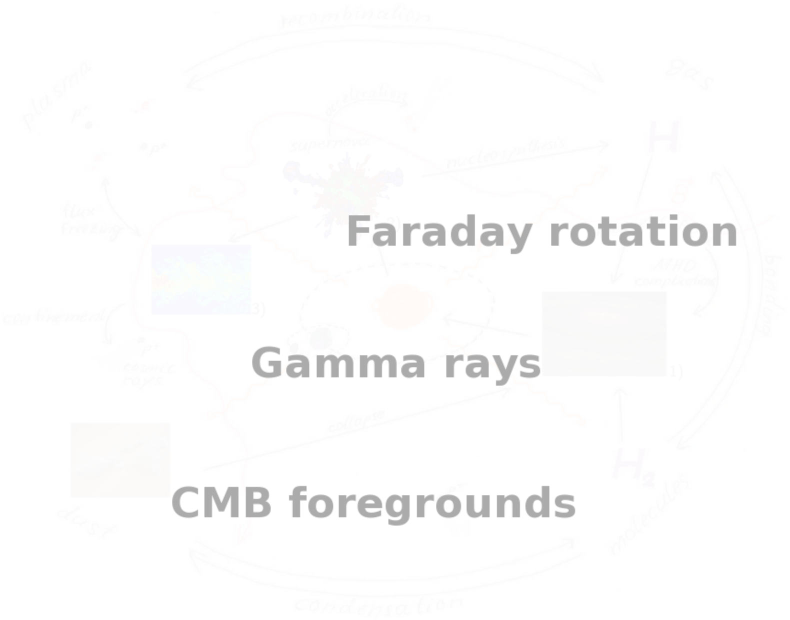
Signal inference



Signal inference



$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\,\mathcal{P}(s)}{\mathcal{P}(d)}$$



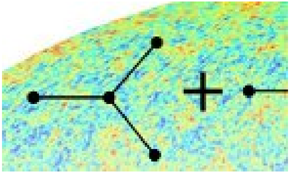
Faraday rotation

Gamma rays

CMB foregrounds

Image credits: 1) D. Darling; 2) N.J. Hammer/MPA; 3) C. Fukushima/TUDelft

Information Field Theory

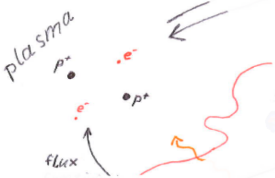


Numerical IFT for python

NIFTY

http://www.mpa-garching.mpg.de/ift/

http://www.mpa-garching.mpg.de/ift/nifty/



Faraday rotation

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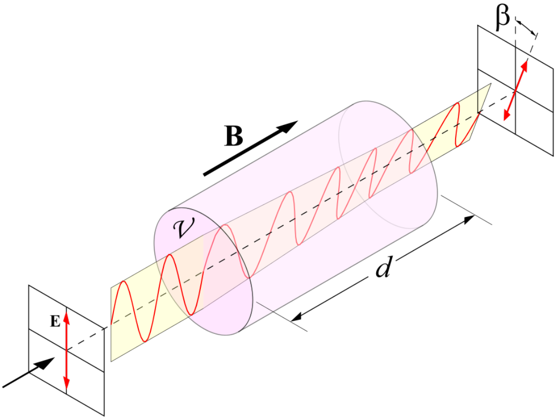
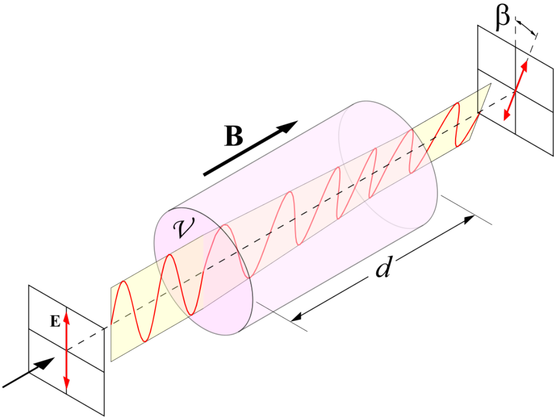


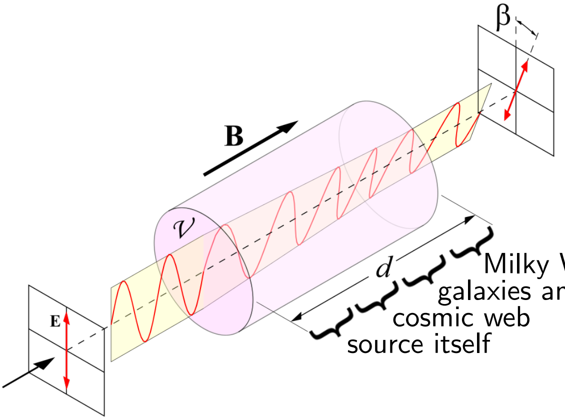
$$\mathrm{d}\beta \propto \lambda^2 n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

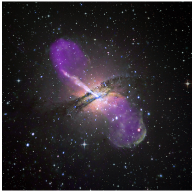
$$\Rightarrow \quad \beta \propto \lambda^2 \int_{r_\mathrm{source}}^{0} n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$



Faraday depth: $\phi \propto \int_{r_{\text{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$

$$\beta = \phi \lambda^2$$





$$\text{Faraday depth: } \phi \propto \int_{r_{\mathrm{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

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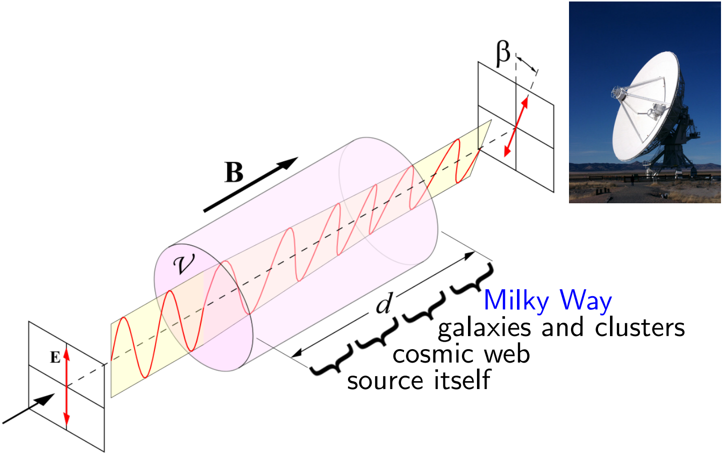
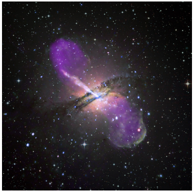
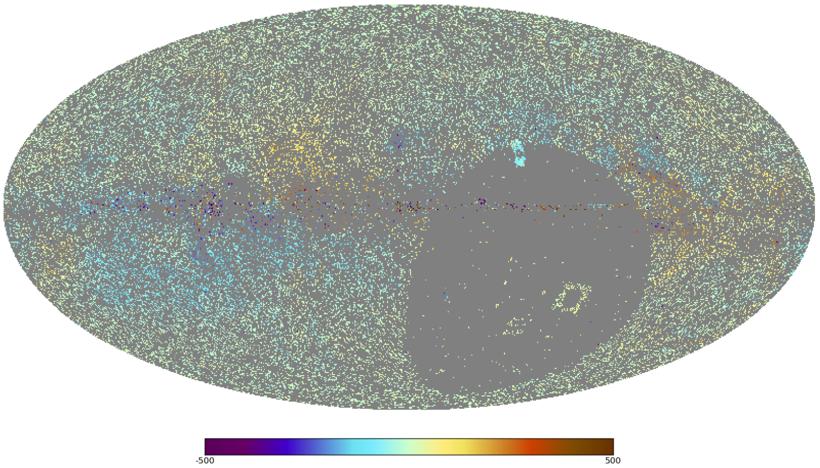


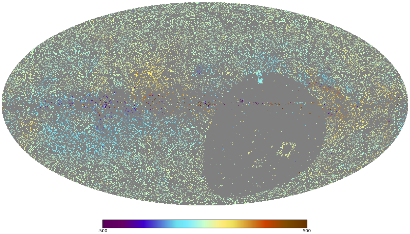


Galactic Faraday depth:

$$\phi \propto \int_{r_{\mathrm{MilkyWay}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

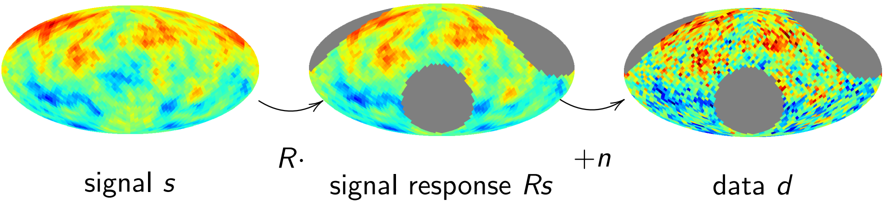


41 330 data points

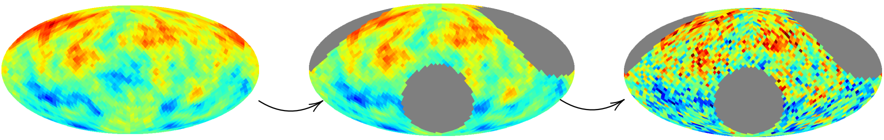


Challenges

- Regions without data
- Uncertain error bars:
  - complicated observations
  - $n\pi$-ambiguity
  - extragalactic contributions unknown



$$\boxed{d = Rs + n}$$



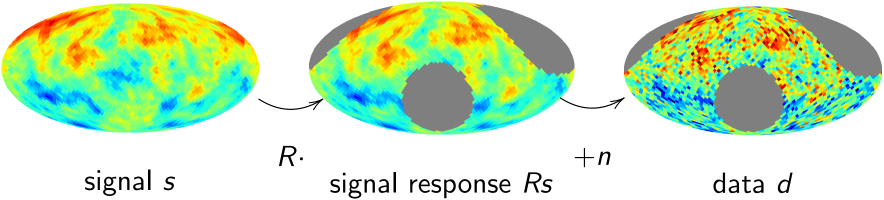
signal s $\quad R\cdot \quad$ signal response Rs $\quad +n \quad$ data d

$$\boxed{d = Rs + n}$$

$$\mathcal{P}(s) = \mathcal{G}(s, S)$$

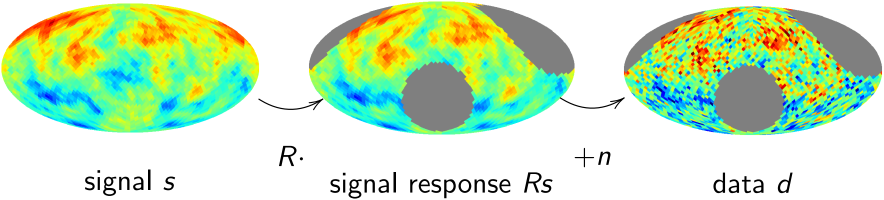
$$\mathcal{P}(n) = \mathcal{G}(n, N)$$

$$\mathcal{G}(s, S) = \frac{1}{\left|2\pi S\right|^{1/2}} \exp\left[\frac{1}{2} s^{\dagger} S^{-1} s\right]$$



$$d = Rs + n$$

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

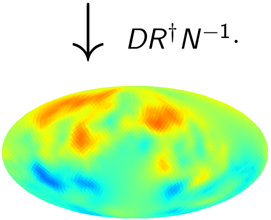


Wiener Filter

$$d = Rs + n$$

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$m = Dj, \text{ where } \quad \begin{aligned} j &= R^\dagger N^{-1} d \\ D &= \left(S^{-1} + R^\dagger N^{-1} R\right)^{-1} \end{aligned}$$

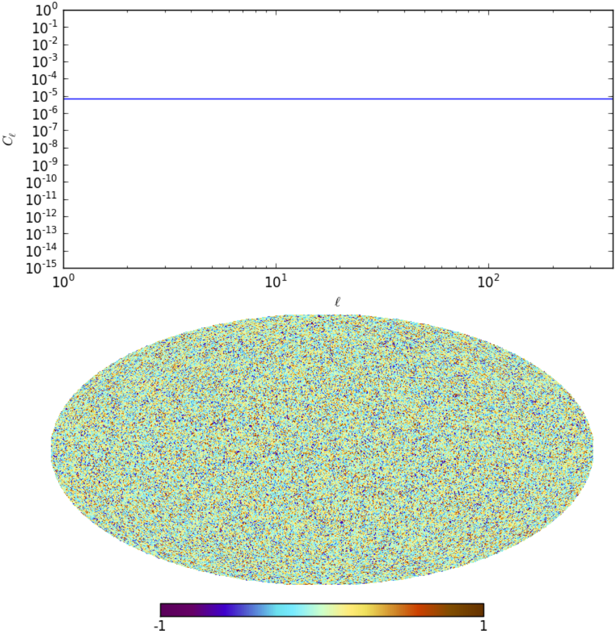


$$S(\hat{n}, \hat{n}') = \int \mathcal{D}s \; s(\hat{n}) s(\hat{n}') \mathcal{P}(s)$$

$$\Rightarrow S_{(\ell m),(\ell' m')} = \int \mathcal{D}s \; s_{\ell m} s^*_{\ell' m'} \mathcal{P}(s)$$

$$
\begin{aligned}
S(\hat{n}, \hat{n}') &= \int \mathcal{D}s\ s(\hat{n}) s(\hat{n}') \mathcal{P}(s) \\
&= S(\hat{n} \cdot \hat{n}') \\
\Rightarrow S_{(\ell m),(\ell' m')} &= \int \mathcal{D}s\ s_{\ell m} s^*_{\ell' m'} \mathcal{P}(s) \\
&= \delta_{\ell \ell'} \delta_{m m'} C_\ell
\end{aligned}
$$

$\hookrightarrow$ angular power spectrum



C_ℓ
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{0}
10^{1}
10^{2}
ℓ
-1
1

$$
\begin{aligned}
S(\hat{n}, \hat{n}') &= \int \mathcal{D}s \; s(\hat{n}) s(\hat{n}') \mathcal{P}(s) \\
&= S(\hat{n} \cdot \hat{n}') \\
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&= \delta_{\ell\ell'} \delta_{mm'} C_\ell
\end{aligned}
$$

$\hookrightarrow$ angular power spectrum

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\begin{aligned}
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&= \delta_{\ell \ell'} \delta_{m m'} C_\ell
\end{aligned}
$$

$\hookrightarrow$ angular power spectrum

$$
N_{ij} = \delta_{ij} \sigma_i^2 \eta_i
$$

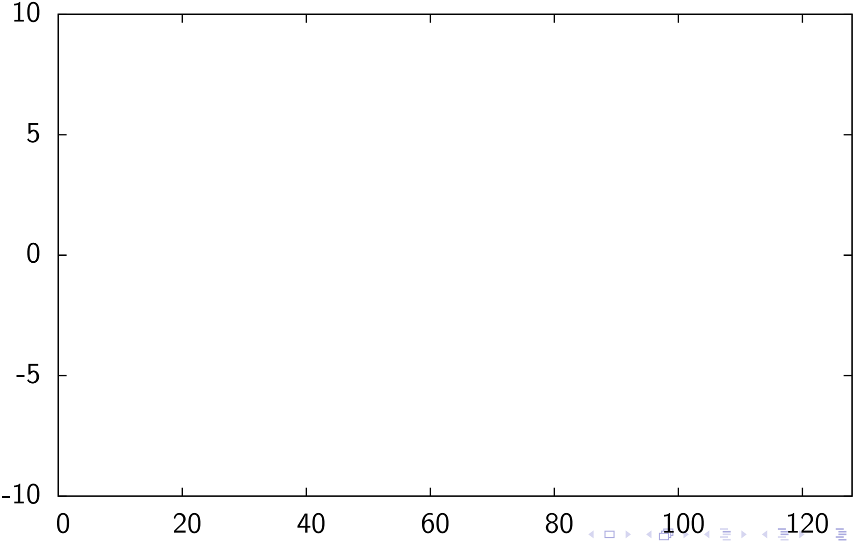
$\hookrightarrow$ error bar correction factors

(uncorrelated noise)

1D example

Assumptions:

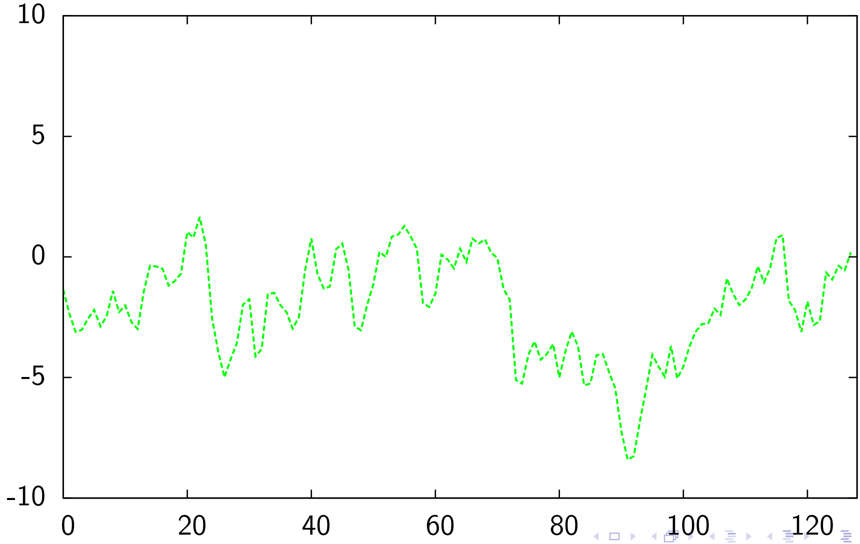
-
-



1D example

Assumptions:

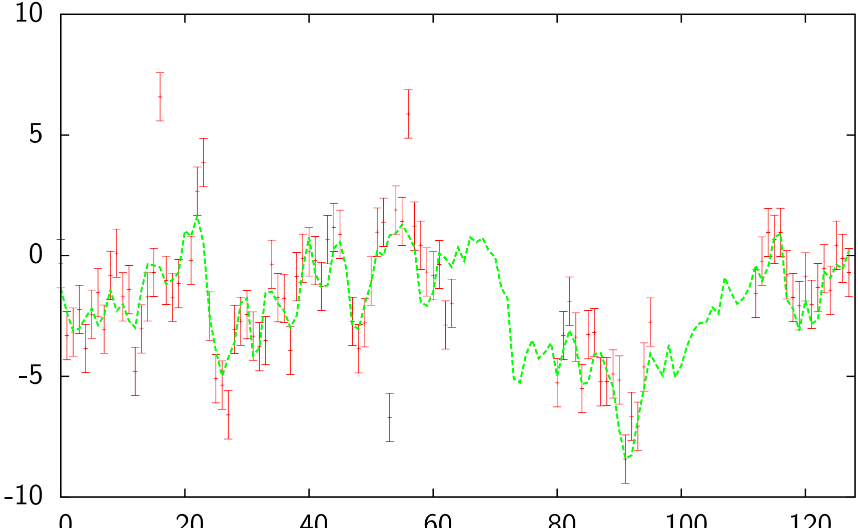
- signal field statistically homogeneous Gaussian random field
-



1D example

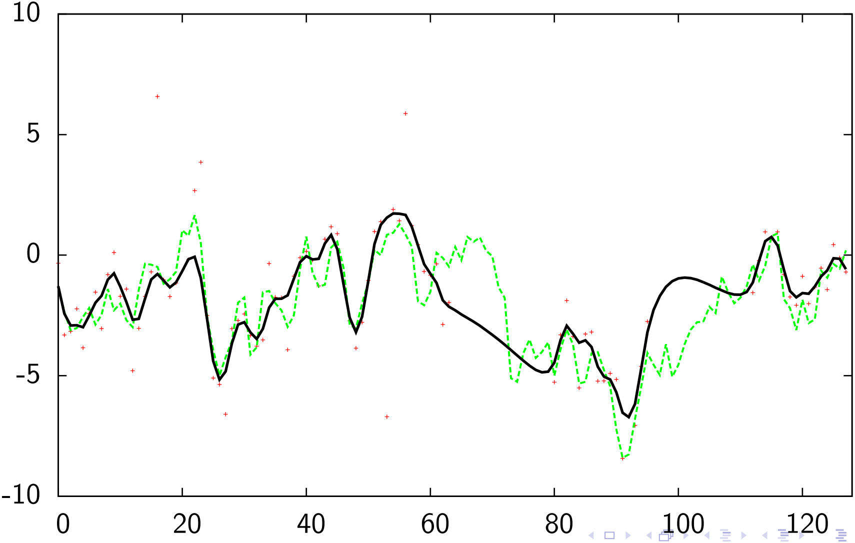
Assumptions:

- signal field statistically homogeneous Gaussian random field
- noise uncorrelated, Gaussian



1D example

- Reconstruct (iteratively):
  signal, power spectrum, noise variance



1D example

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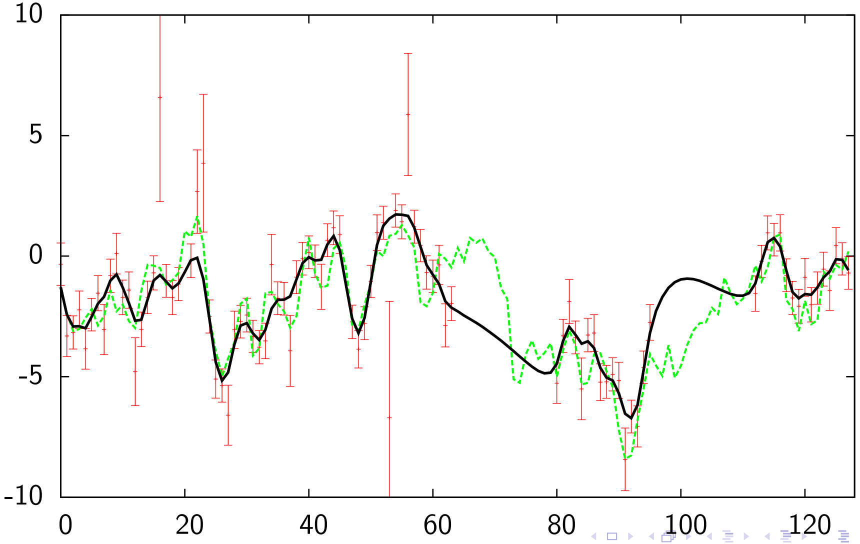
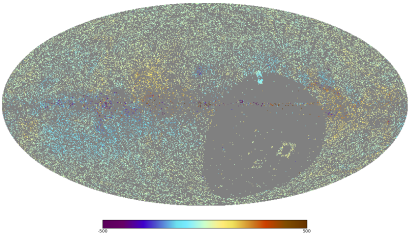
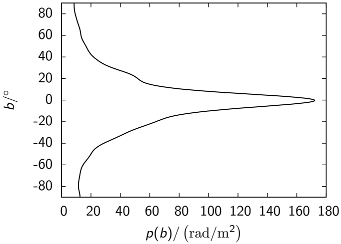
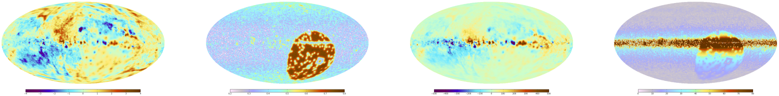


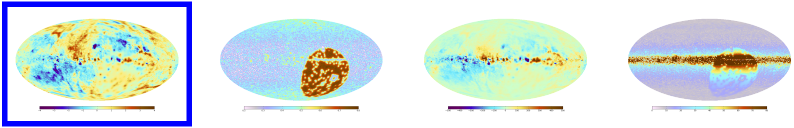




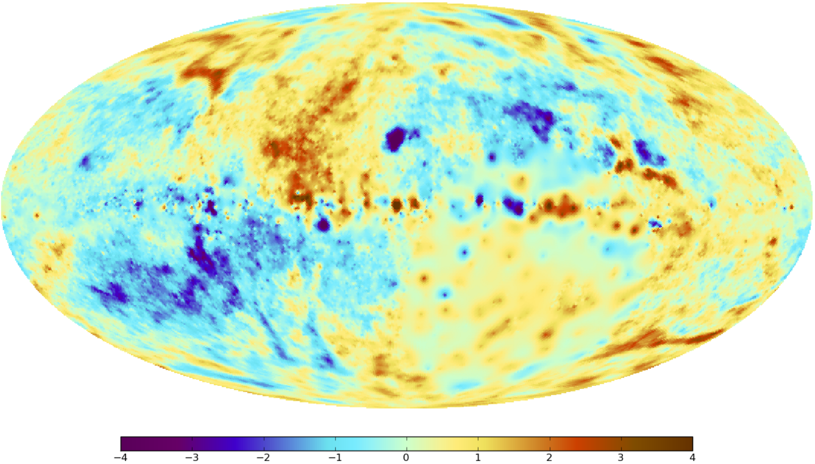
- Approximate $s(b,l) := \frac{\phi(b,l)}{p(b)}$ as a statistically isotropic Gaussian field
- R: multiplication with $p(b)$ and projection on directions of sources
- $N_{ij} = \delta_{ij}\eta_i\sigma_i^2$



Oppermann et al. (2012)

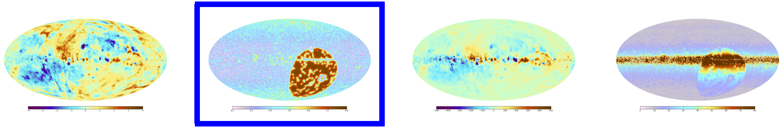


posterior mean of the signal

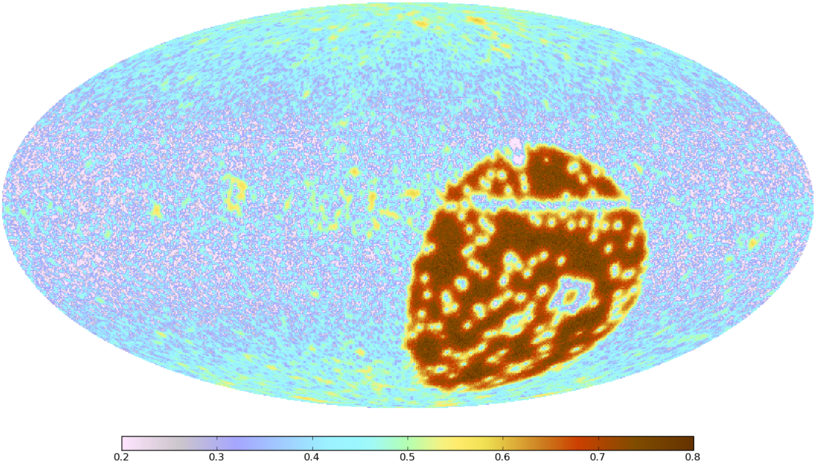


Oppermann et al. (2012)

m

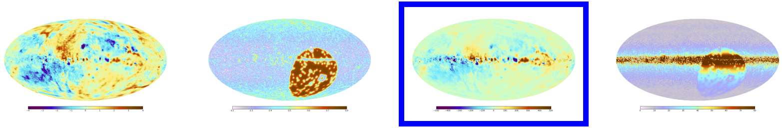


uncertainty of the signal map

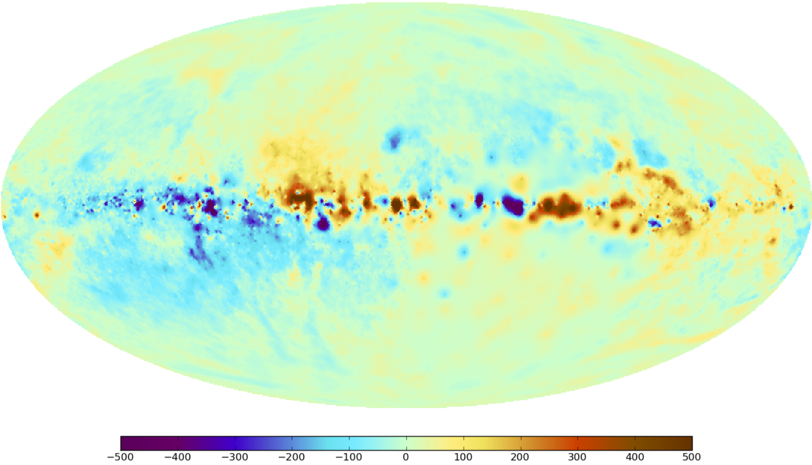


Oppermann et al. (2012)

$$\sqrt{\mathrm{diag}\,(D)}$$

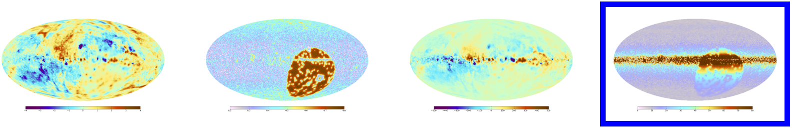


posterior mean of the Faraday depth

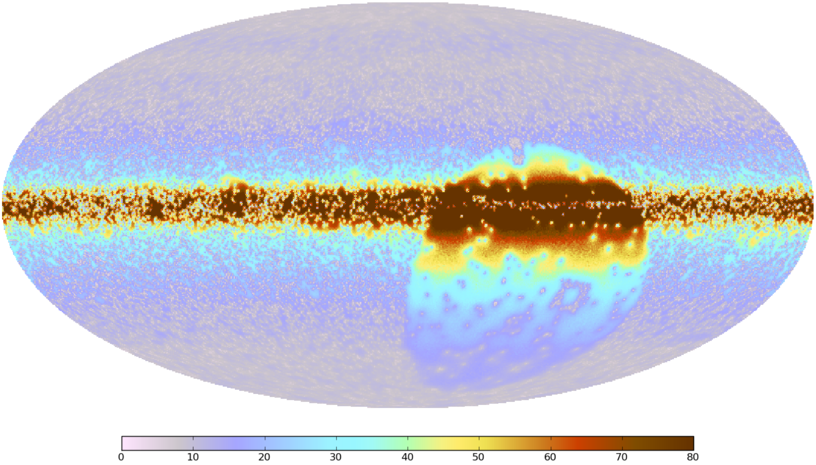


Oppermann et al. (2012)

pm

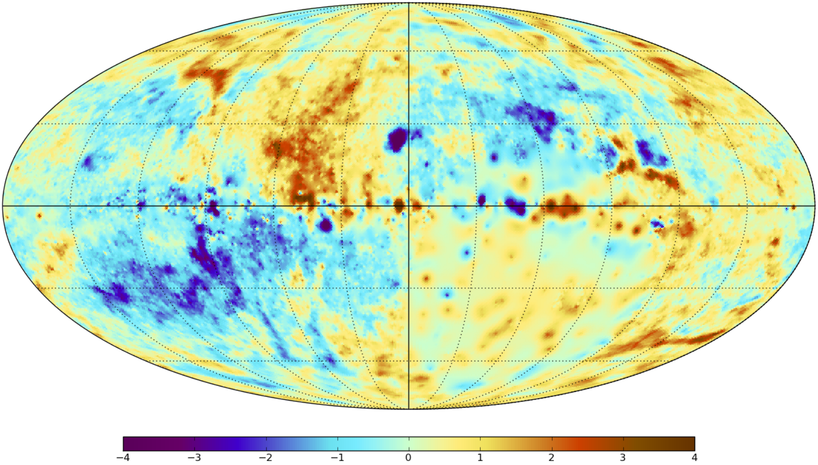


uncertainty of the Faraday depth

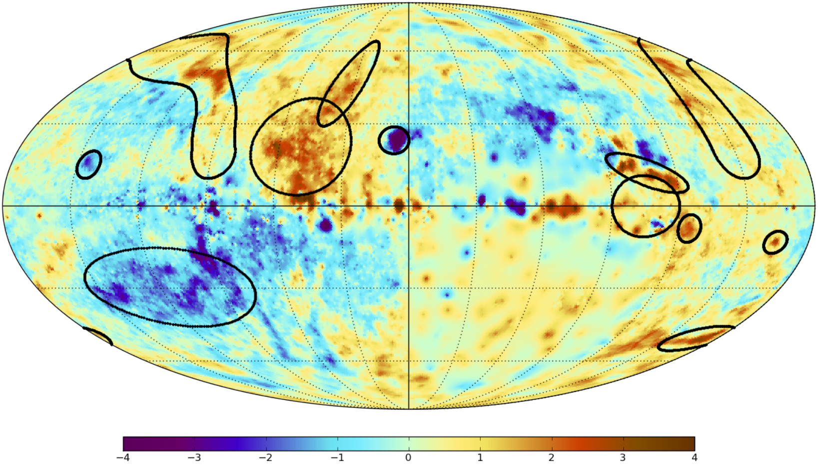


Oppermann et al. (2012)

$$p\sqrt{\mathrm{diag}\left(D\right)}$$



Oppermann et al. (2012)



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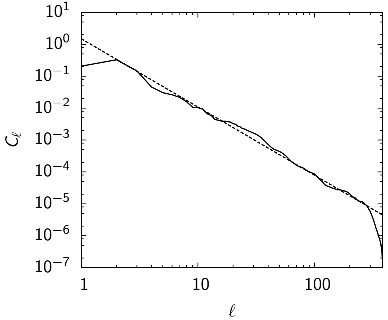
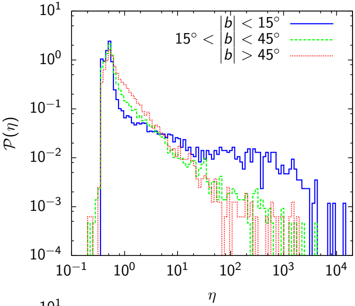
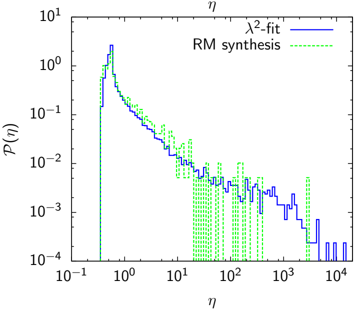


$$C_\ell \propto \ell^{-2.17}$$





$$N_{ij} = \langle n_i n_j \rangle = \delta_{ij} \eta_i \sigma_i^2$$

Oppermann et al. (2012)

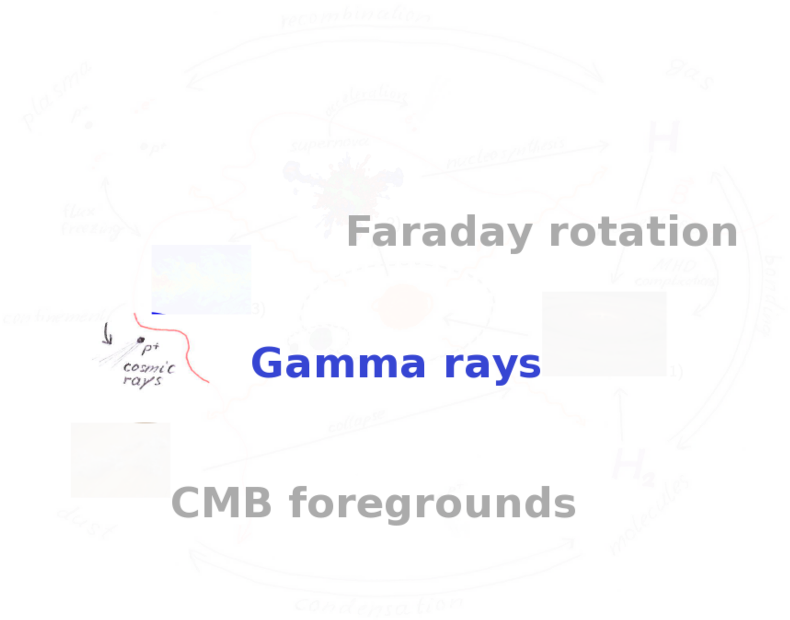
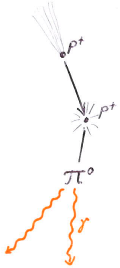
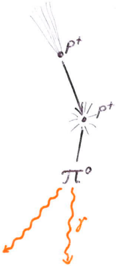


Image credits: 1) D. Darling; 2) N.J. Hammer/MPA; 3) C. Fukushima/TUDelft

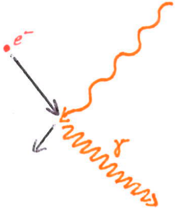
Pion decay



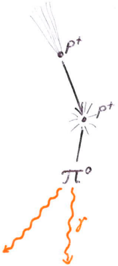
Pion decay



Inverse-Compton



Pion decay



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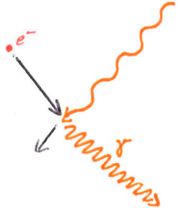
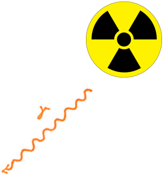
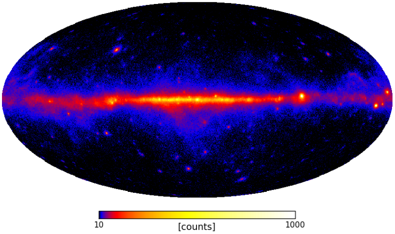


Radioactive decay



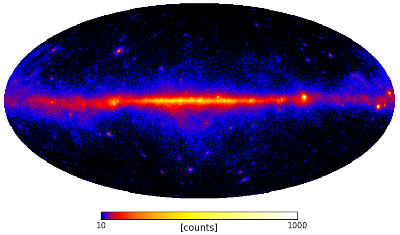
Gamma-ray photon counts:



FERMI data

- non-Gaussian
- always positive
- varying over several orders of magnitude

Gamma-ray photon counts:



FERMI data

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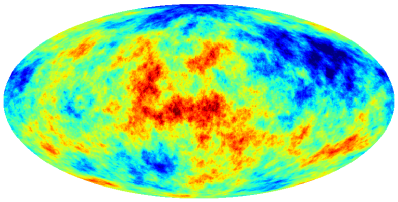
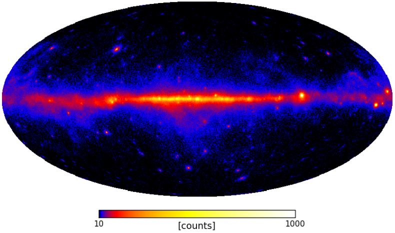
The log-normal model

- use logarithm of photon flux density as signal
- model this as Gaussian random field



s

Gamma-ray photon counts:



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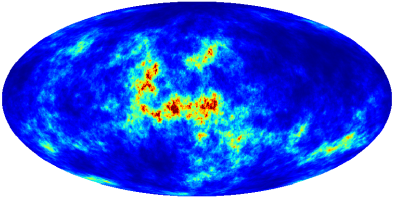
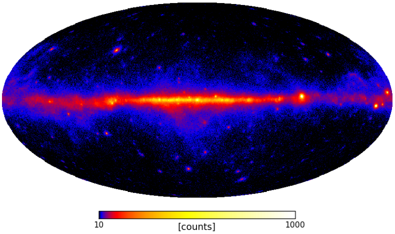


e^s

Gamma-ray photon counts:



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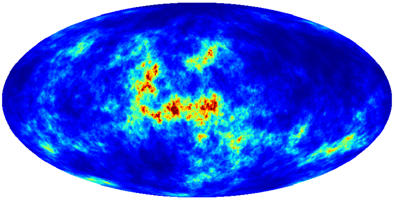
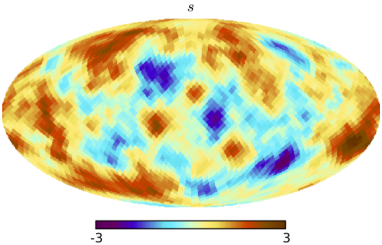


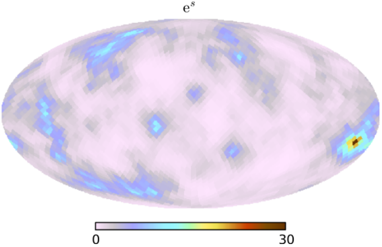
e^{s}

$$d = R\mathrm{e}^{s} + n$$

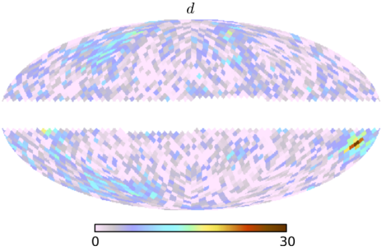
signal



exponentiated signal

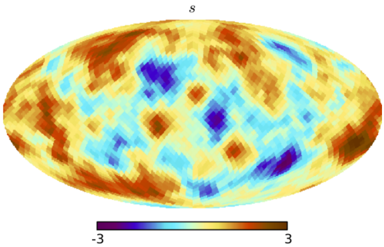


data

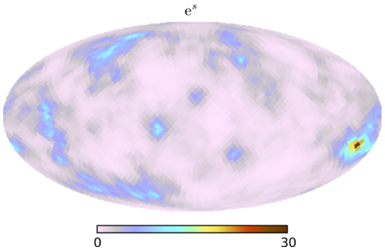


Oppermann et al. (2013)

signal



exponentiated signal



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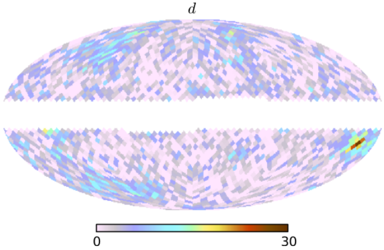
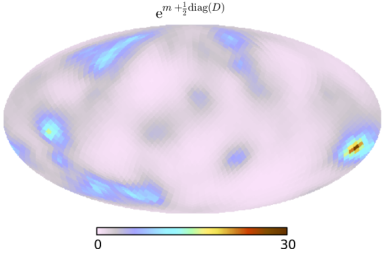
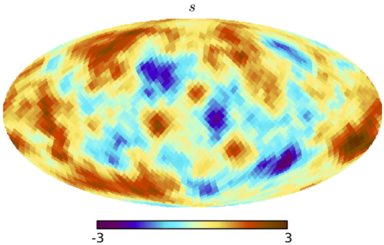


reconstructed exponentiated signal

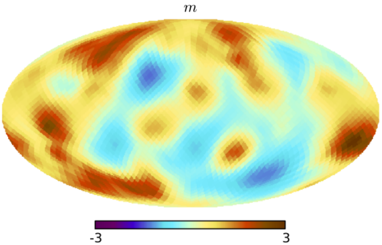


Oppermann et al. (2013)

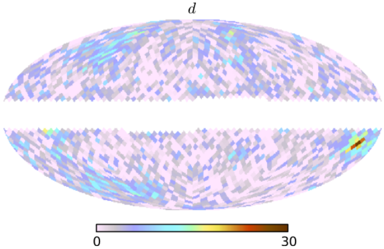
signal



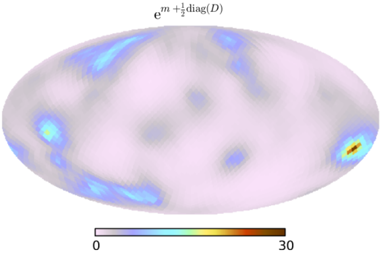
reconstructed signal



data

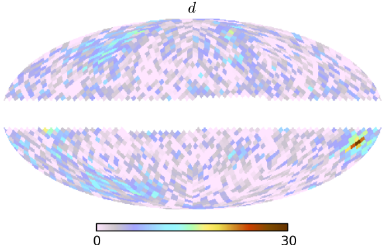


reconstructed exponentiated signal

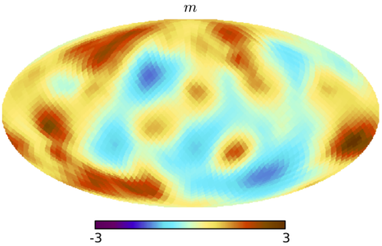


Oppermann et al. (2013)

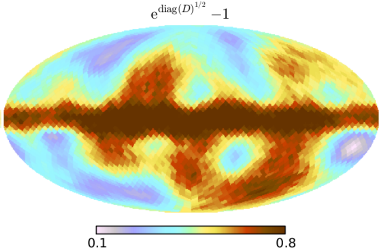
uncertainty



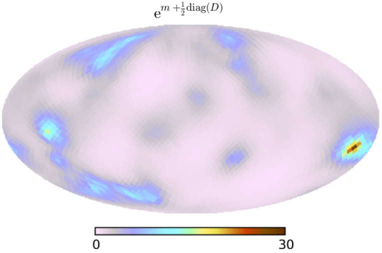
reconstructed signal



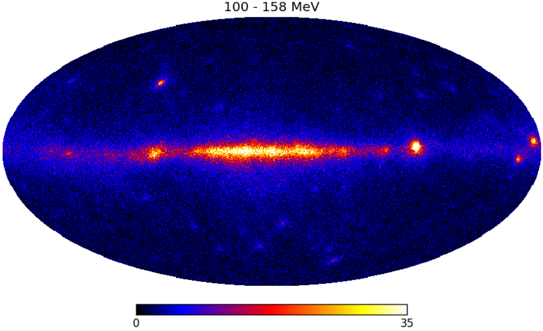
fractional uncertainty

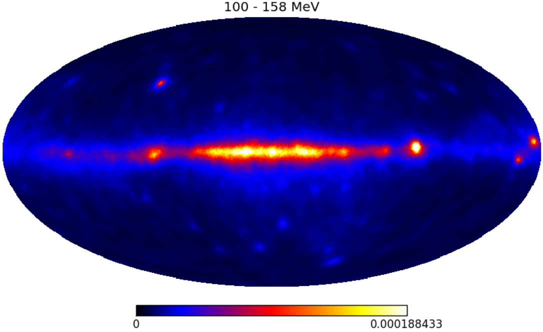


reconstructed exponentiated signal



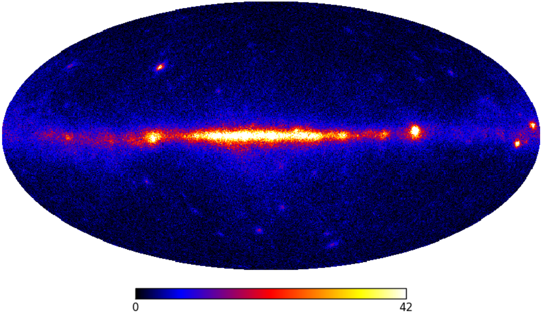
Oppermann et al. (2013)





Selig et al. (in prep.)

158 - 251 MeV



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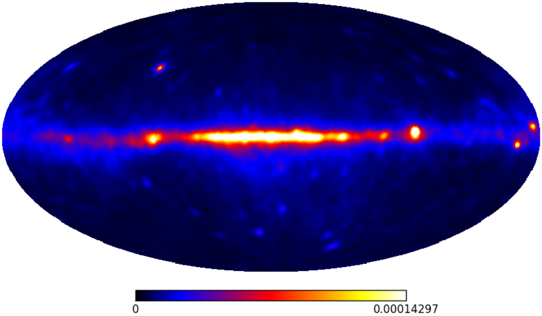

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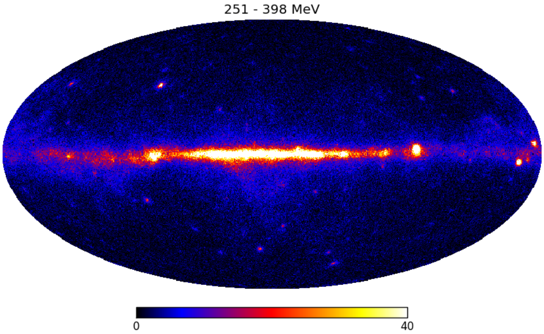

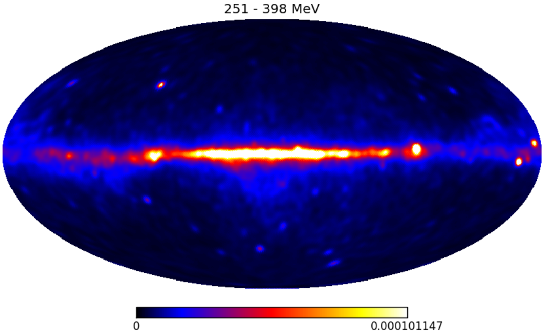

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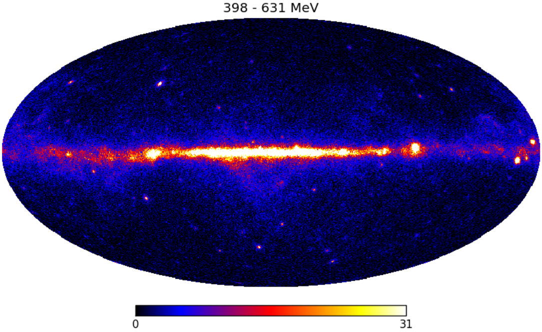

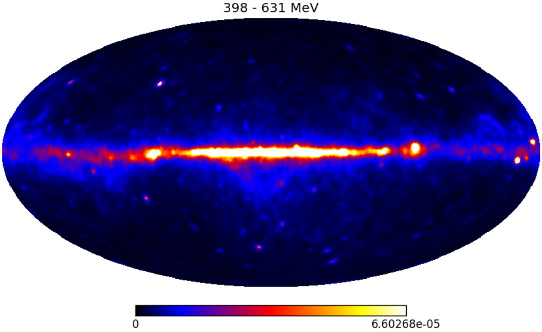

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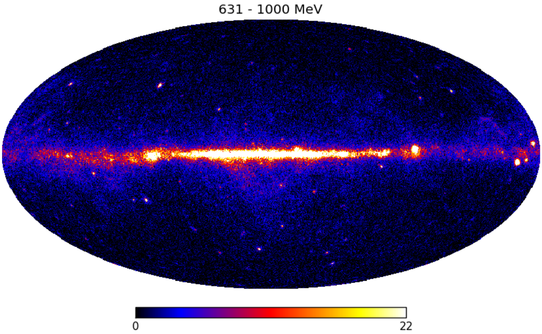

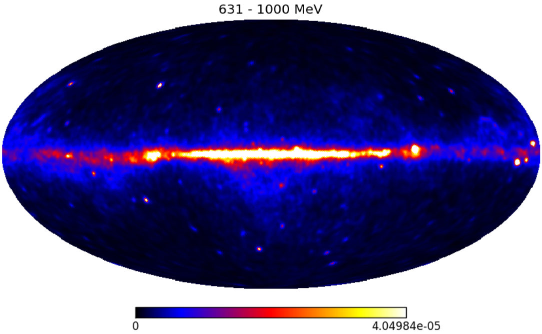

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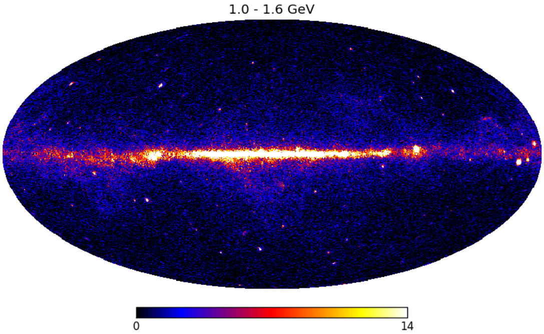

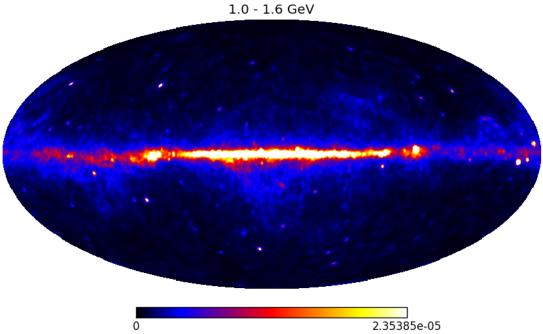

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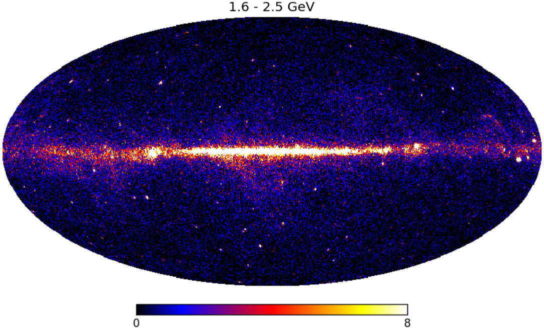

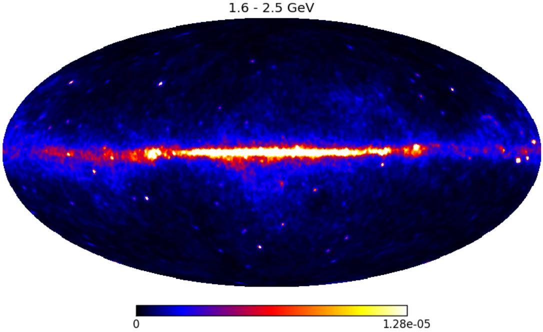

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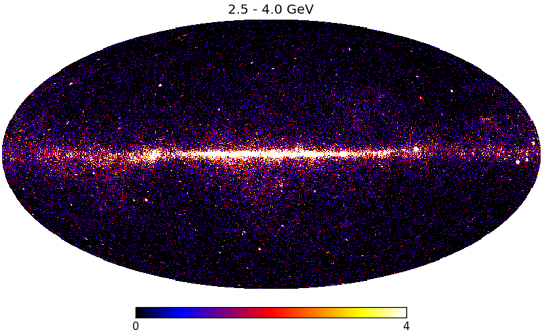

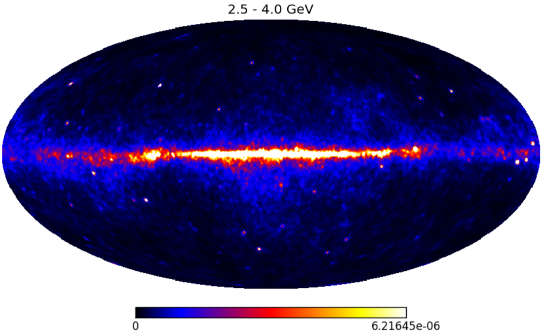

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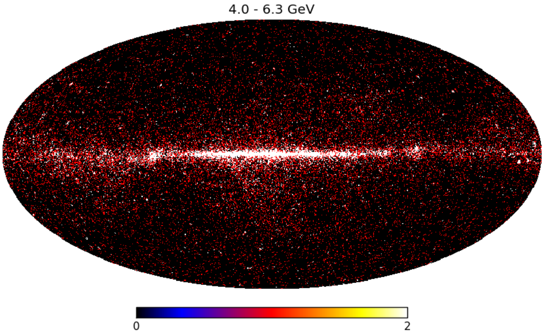

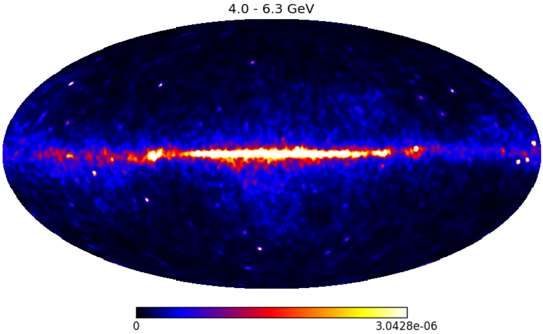

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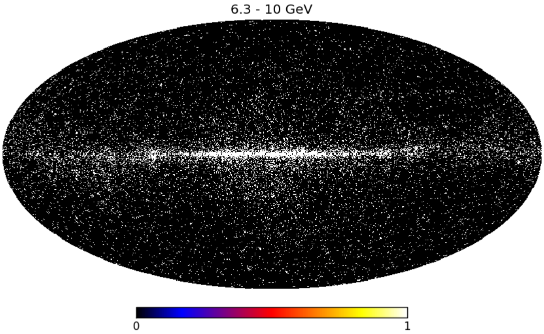

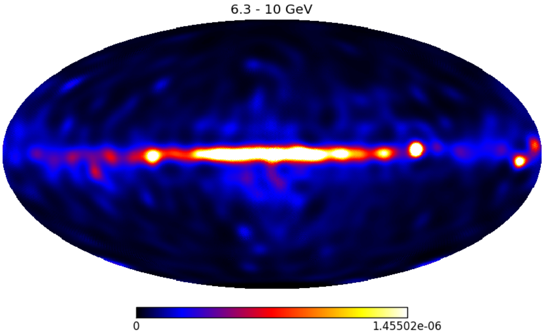

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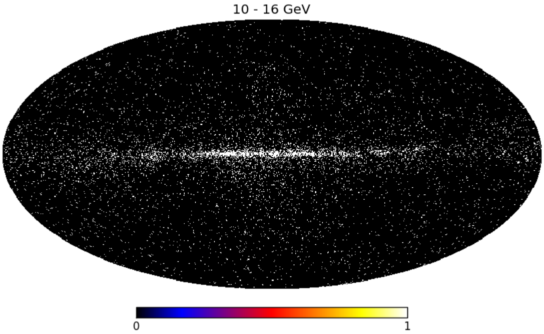

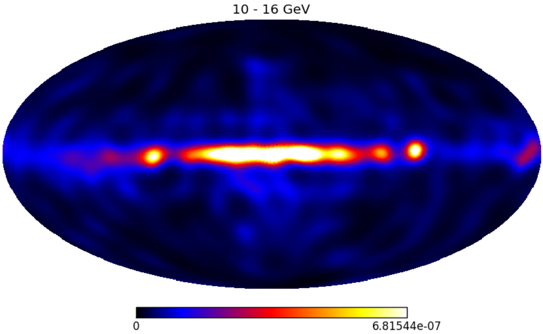

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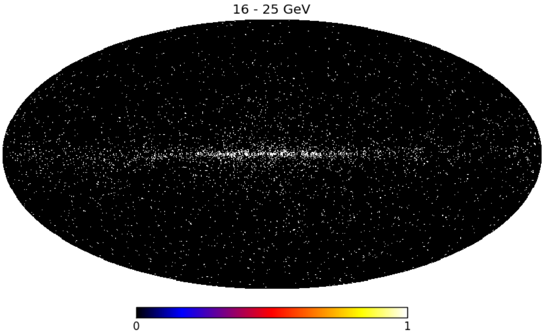

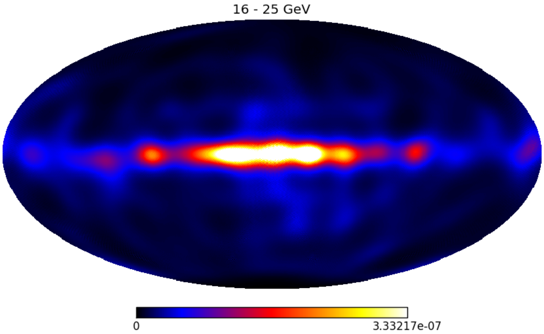

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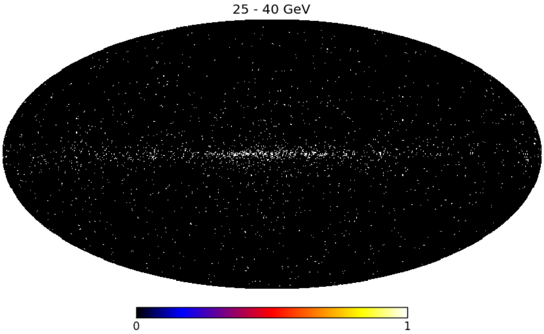

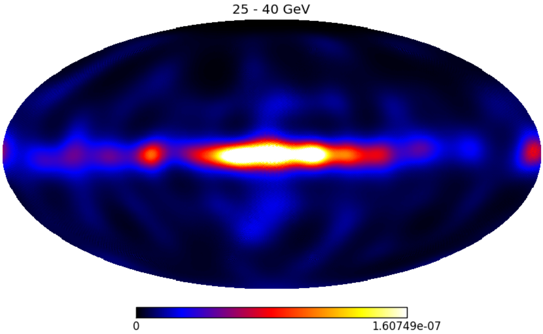

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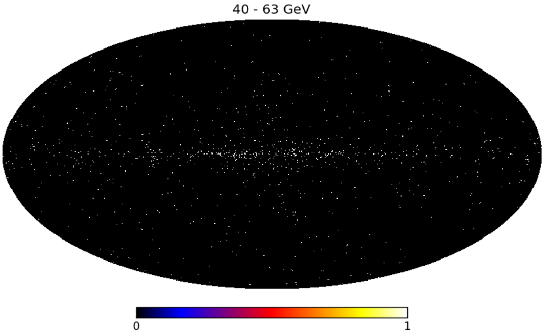

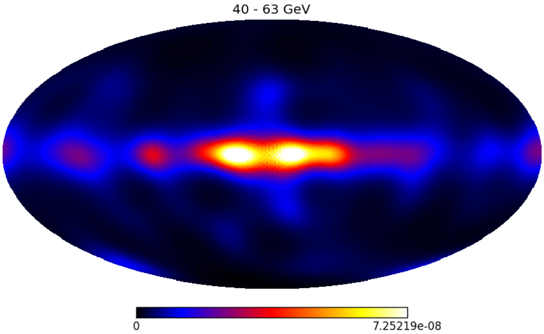

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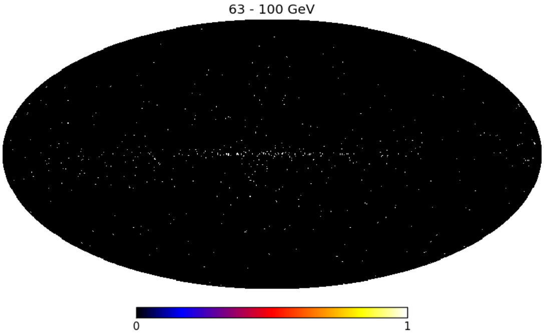

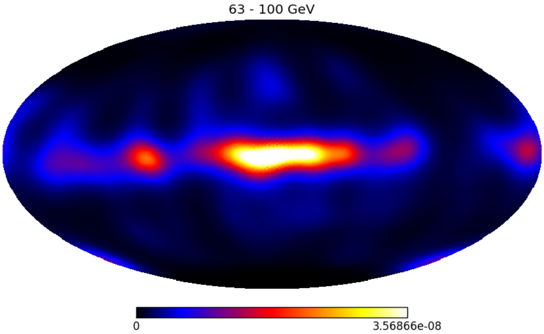

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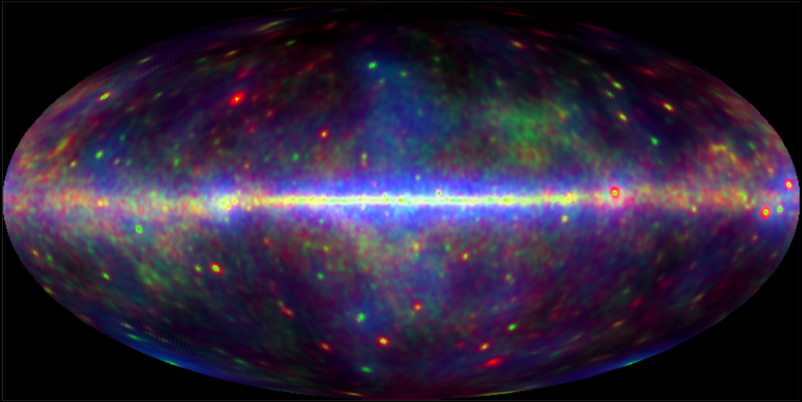


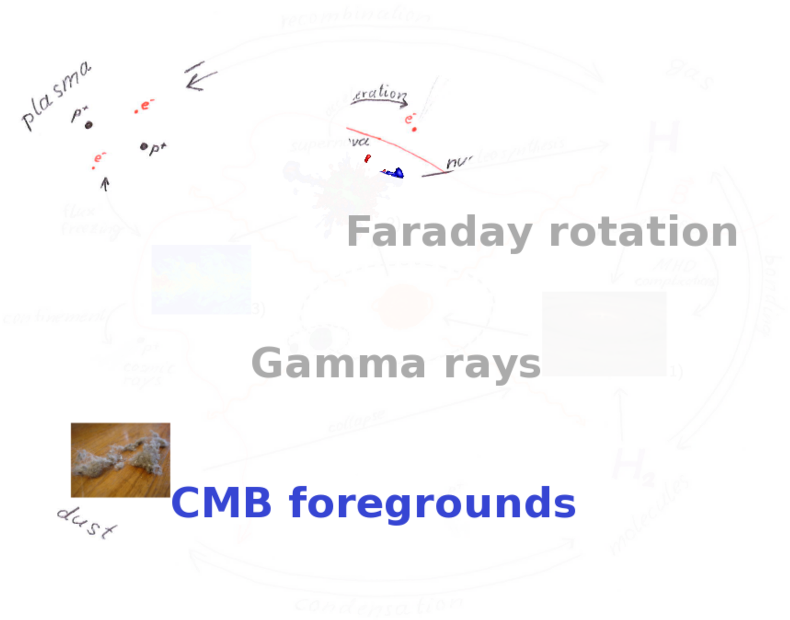
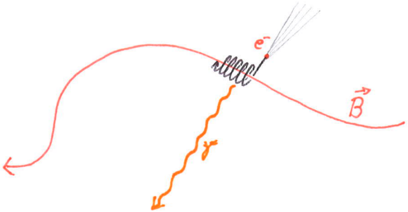


Image credits: 1) D. Darling; 2) N.J. Hammer/MPA; 3) C. Fukushima/TUDelft

Synchrotron radiation



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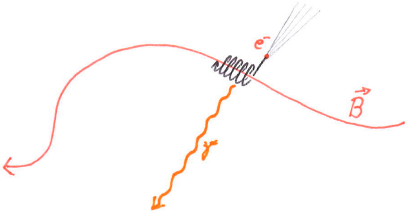
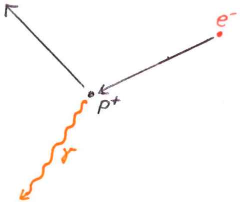


Bremsstrahlung (free-free)



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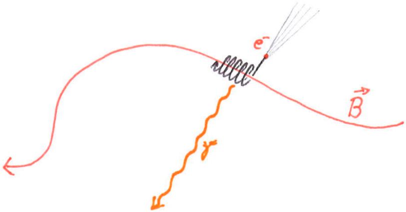
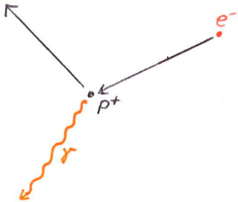
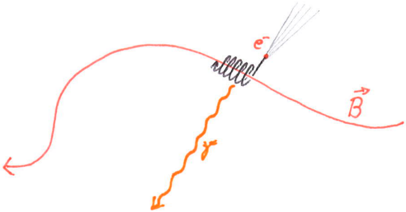


Bremsstrahlung (free-free)

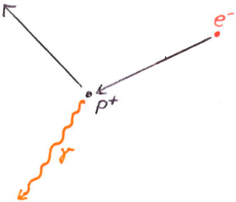


Thermal dust radiation

Synchrotron radiation

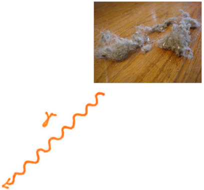


Bremsstrahlung (free-free)



Thermal dust radiation

Radiation from rotating dust grains



data:

- measurements at different frequencies
- inhomogeneous noise

response:

- mixing matrix according to frequency spectra of components

signal:

- different emission mechanisms
- Gaussian (CMB) and log-normal (foregrounds)
- cross-correlated

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix} = \begin{pmatrix} f^{\nu_1}_{\mathrm{CMB}} & f^{\nu_1}_{\mathrm{synch}} & f^{\nu_1}_{\mathrm{ff}} & f^{\nu_1}_{\mathrm{dust}} \\ f^{\nu_2}_{\mathrm{CMB}} & f^{\nu_2}_{\mathrm{synch}} & f^{\nu_2}_{\mathrm{ff}} & f^{\nu_2}_{\mathrm{dust}} \\ f^{\nu_3}_{\mathrm{CMB}} & f^{\nu_3}_{\mathrm{synch}} & f^{\nu_3}_{\mathrm{ff}} & f^{\nu_3}_{\mathrm{dust}} \\ f^{\nu_4}_{\mathrm{CMB}} & f^{\nu_4}_{\mathrm{synch}} & f^{\nu_4}_{\mathrm{ff}} & f^{\nu_4}_{\mathrm{dust}} \\ f^{\nu_5}_{\mathrm{CMB}} & f^{\nu_5}_{\mathrm{synch}} & f^{\nu_5}_{\mathrm{ff}} & f^{\nu_5}_{\mathrm{dust}} \end{pmatrix} \begin{pmatrix} s_{\mathrm{CMB}} \\ s_{\mathrm{synch}} \\ s_{\mathrm{ff}} \\ s_{\mathrm{dust}} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{pmatrix}$$

data:

- measurements at different frequencies
- inhomogeneous noise

response:

- mixing matrix according to frequency spectra of components

signal:

- different emission mechanisms
- Gaussian (CMB) and log-normal (foregrounds)
- cross-correlated

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix} = \begin{pmatrix} f^{\nu_1}_{\mathrm{CMB}} & f^{\nu_1}_{\mathrm{synch}} & f^{\nu_1}_{\mathrm{ff}} & f^{\nu_1}_{\mathrm{dust}} \\ f^{\nu_2}_{\mathrm{CMB}} & f^{\nu_2}_{\mathrm{synch}} & f^{\nu_2}_{\mathrm{ff}} & f^{\nu_2}_{\mathrm{dust}} \\ f^{\nu_3}_{\mathrm{CMB}} & f^{\nu_3}_{\mathrm{synch}} & f^{\nu_3}_{\mathrm{ff}} & f^{\nu_3}_{\mathrm{dust}} \\ f^{\nu_4}_{\mathrm{CMB}} & f^{\nu_4}_{\mathrm{synch}} & f^{\nu_4}_{\mathrm{ff}} & f^{\nu_4}_{\mathrm{dust}} \\ f^{\nu_5}_{\mathrm{CMB}} & f^{\nu_5}_{\mathrm{synch}} & f^{\nu_5}_{\mathrm{ff}} & f^{\nu_5}_{\mathrm{dust}} \end{pmatrix} \begin{pmatrix} s_{\mathrm{CMB}} \\ s_{\mathrm{synch}} \\ s_{\mathrm{ff}} \\ s_{\mathrm{dust}} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{pmatrix}$$

1. Determine mixing matrix.
2. “Invert” equation.

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- measurements at different frequencies
- inhomogeneous noise

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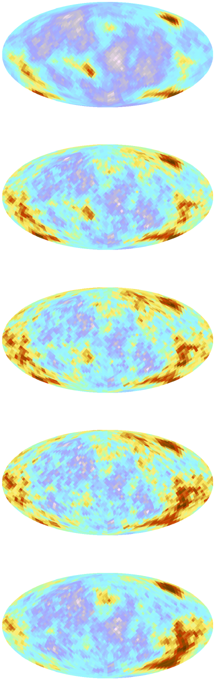
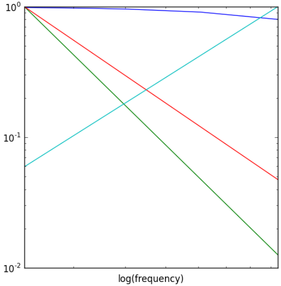
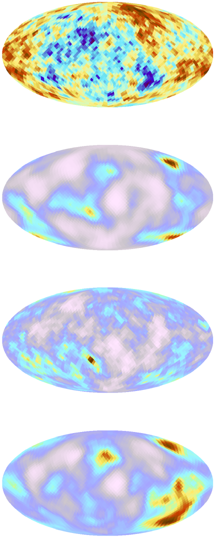
data:



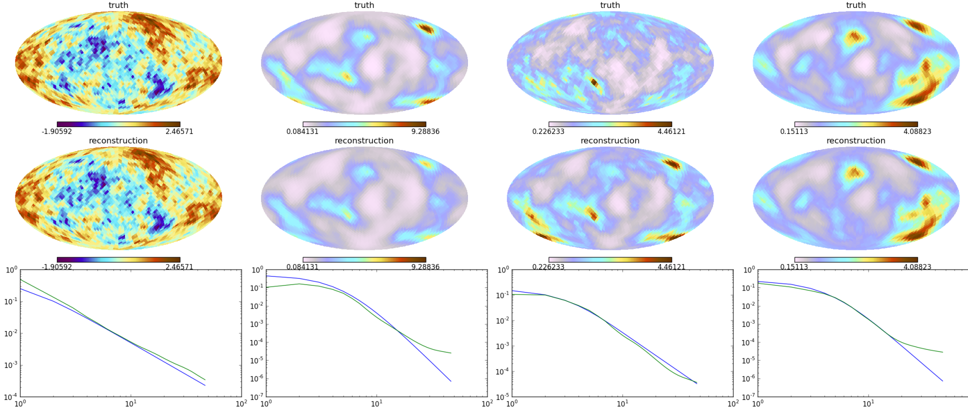
response:



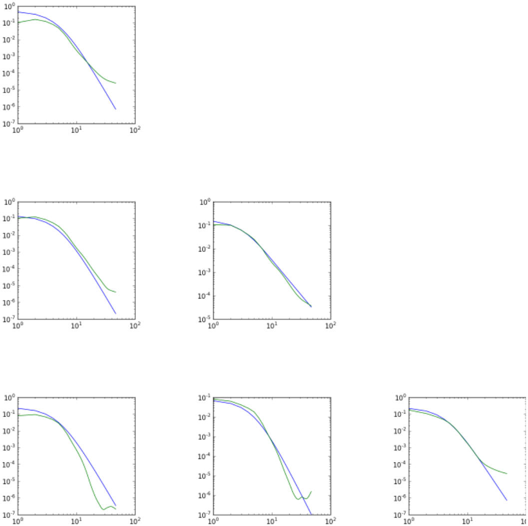
signal:



WORK IN PROGRESS

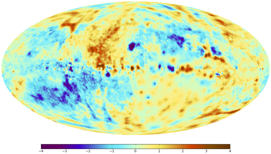


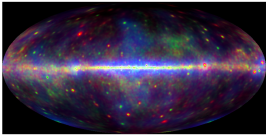
WORK IN PROGRESS

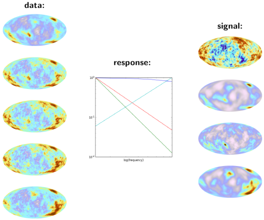


Summary

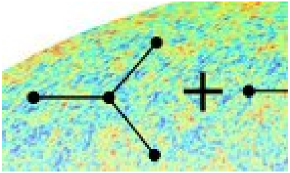
- Probabilistic inference problems
- Use correlation structure to interpolate
- Probabilistic method for dealing with outliers
- Non-linear response / Non-Gaussian signals can be dealt with







Information Field Theory



Numerical IFT for python

NIFTY

http://www.mpa-garching.mpg.de/ift/

http://www.mpa-garching.mpg.de/ift/nifty/

- Lecture on IFT next Wednesday (July 10th)
- NIFTy tutorial next Thursday (July 11th)