Reconstructing CMB Foregrounds and the Faraday Sky

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with T. Enßlin, G. Robbers, H. Junklewitz

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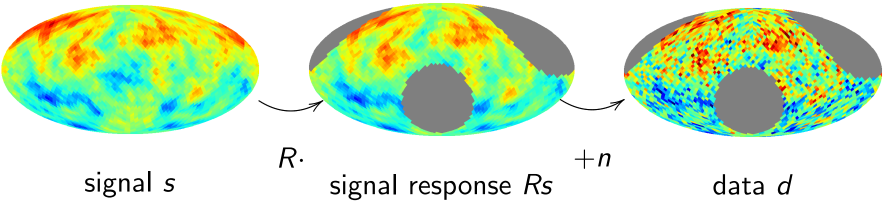
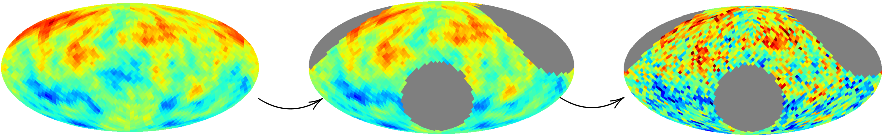


Signal model

$$d = Rs + n$$

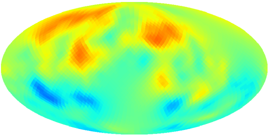
Gaussian signal (s) and noise (n)
Statistically homogeneous and isotropic signal

$$\Rightarrow S_{(\ell,m)(\ell',m')} = \left\langle s_{\ell,m} s_{\ell',m'} \right\rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$



signal s → $R\cdot$ → signal response Rs → $+n$ → data d

↓ $DR^{\dagger}N^{-1}\cdot$



Wiener Filter

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$d = Rs + n$$

$$m = Dj, \text{ where } \quad \begin{array}{l} j = R^{\dagger}N^{-1}d \\ D = \left(S^{-1} + R^{\dagger}N^{-1}R\right)^{-1} \end{array}$$

Wiener Filter

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$d = Rs + n$$

$$m = Dj, \;\; \text{where} \;\; \begin{cases} j = R^\dagger N^{-1} d \\ D = \left(S^{-1} + R^\dagger N^{-1} R\right)^{-1} \\ S_{(\ell,m)(\ell',m')} = \delta_{\ell\ell'}\delta_{mm'} C_\ell \end{cases}$$

Critical Filter

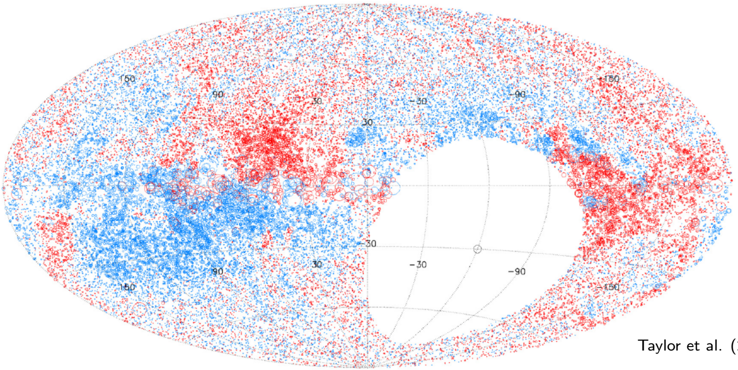
$$m = Dj$$

$$C_\ell = \frac{1}{2\ell+1} \mathrm{tr}\left(\left(mm^\dagger + D\right) P_\ell\right)$$

Enßlin & Frommert (2010), arXiv:1002.2928

Enßlin & Weig (2010), arXiv:1004.2868

The Faraday Sky



Taylor et al. (2009)

Figure 3. Plot of 37,543 RM values over the sky north of $\delta = -40°$. Red circles are positive rotation measure and blue circles are negative. The size of the circle scales linearly with magnitude of rotation measure.

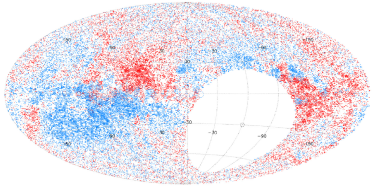
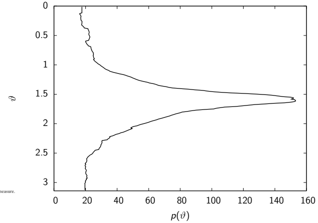
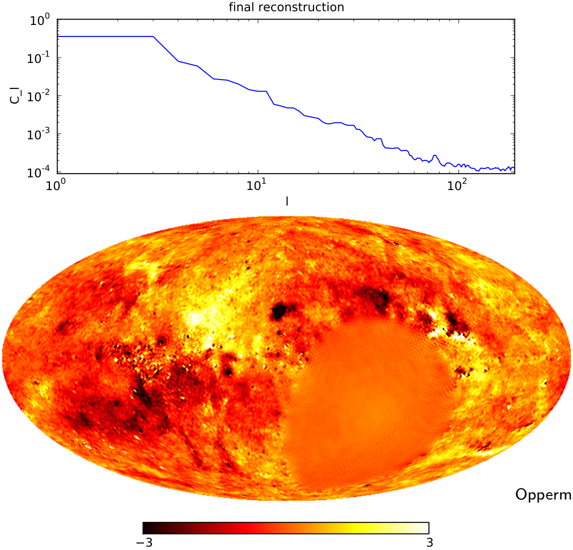


Figure 3. Plot of 37,543 RM values over the sky north of $\delta = -40°$. Red circles are positive rotation measure and blue circles are negative. The size of the circle scales linearly with magnitude of rotation measure.

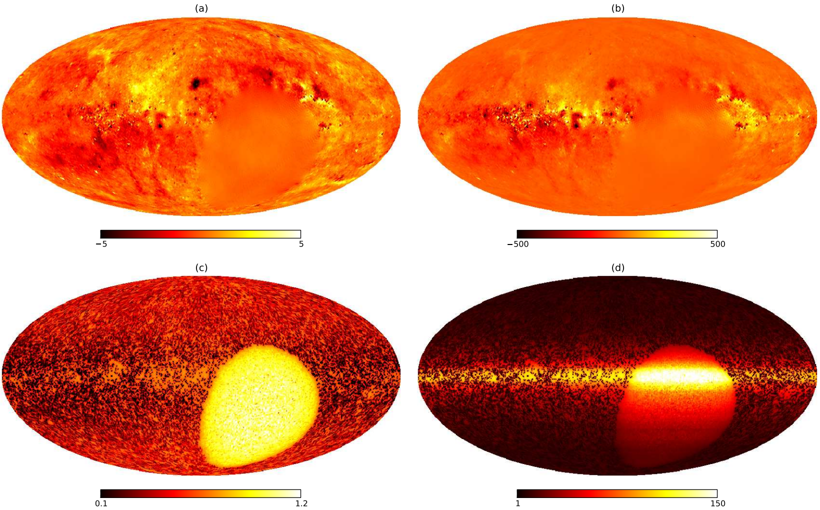


- Approximate $s := \frac{\phi}{p(\vartheta)}$ as a homogeneous, isotropic, Gaussian field
- R: multiplication with $p(\vartheta)$ and projection on directions of clusters
- $N_{ij} = \delta_{ij} \left(\alpha \sigma_i^2 + \sigma_i^{(\mathrm{extr})2} \right)$

$\Rightarrow$ Critical filter formalism can be used directly.



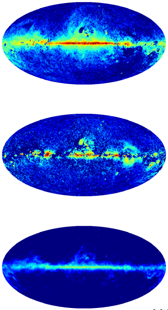
Oppermann et al. (2011)



Oppermann et al. (2011)

CMB Foregrounds

signal



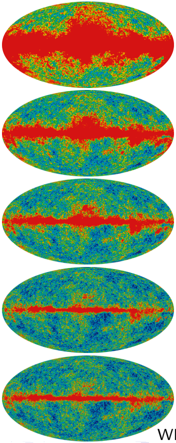
WMAP

⇒

Response
(frequency spectra)
+
Noise
(determined from number of observations per pixel)

⇒

data



WMAP

$$d = Rs + n$$

Data

- frequency maps

Signal

- component maps
- power spectra unknown
- not Gaussian

Response

- mixing matrix
- determined by frequency spectra
- known?

Noise

- Gaussian noise
- spatially inhomogeneous
- correlations?

$$d = Rs + n$$

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- frequency maps

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- component maps
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- additional component for CMB (with known power spectrum)

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- Gaussian noise
- spatially inhomogeneous
- assume uncorrelated noise

$$d = Rs + n$$

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- frequency maps

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- determined by frequency spectra
- assume frequency spectra to be known

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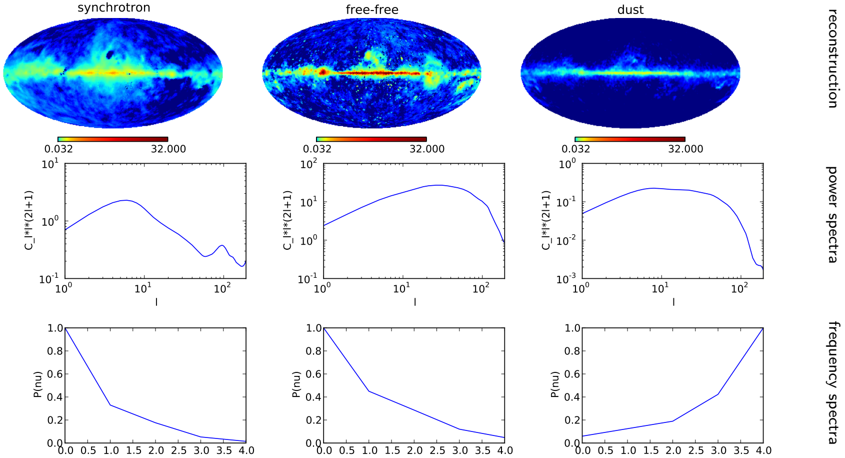
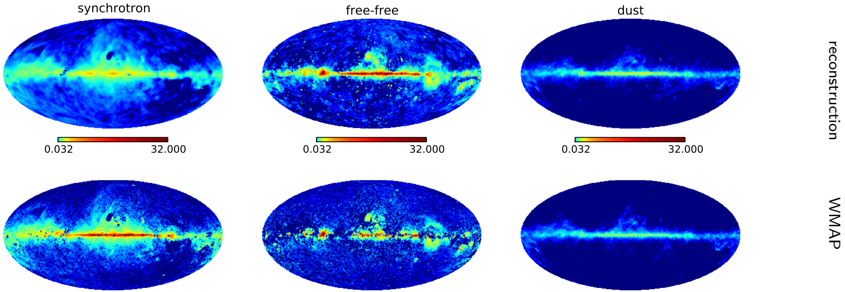

synchrotron
free-free
dust
reconstruction
0.032
32.000
power spectra
C_l*(2l+1)
l
frequency spectra
P(nu)


synchrotron
free-free
dust
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0.032
32.000
0.032
32.000
0.032
32.000
WMAP

Outlook

CMB foregrounds

- Use log-normal data model: $d = R\mathrm{e}^{s} + n$, with Gaussian s
  $\Rightarrow$ positivity of emission processes assured
  $\Rightarrow$ large range of values easily accomodated
  $\Rightarrow$ statistical isotropy better approximation

The Faraday sky

- Include several datasets
- Allow for uncertainty in the uncertainties