The Faraday Sky

Map Making and Helicity Inference

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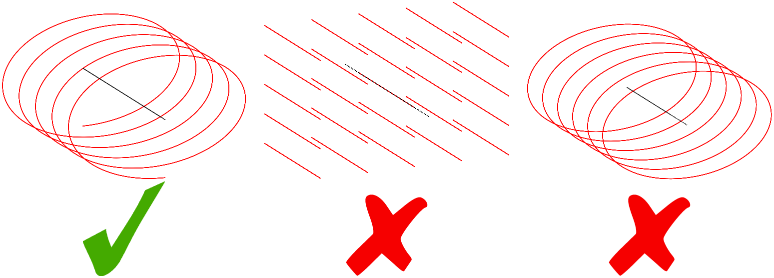
arXiv:1008.1246

Pushchino, May 18, 2011

Local
Inference
Test for
Magnetic fields,
which **U**ncovers
helice**S**

Junklewitz & Enßlin (2010), arXiv:1008.1243

$$H = \int \vec{j} \cdot \vec{B} \, \mathrm{d}V$$



Synchrotron Emission

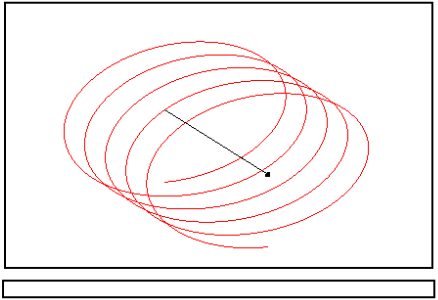
- magnetic field + charged particles
- $\vec{B}$-component $\perp$ LoS
- polarized $\perp$ $\vec{B}_\perp$

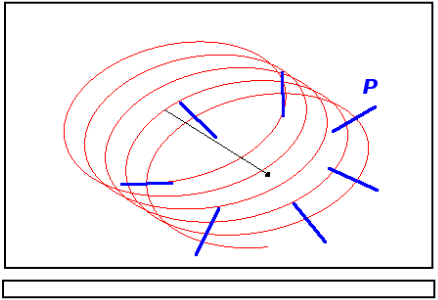
Faraday Rotation

- magnetic field + polarized background source
- $\vec{B}$-component $\parallel$ LoS
- rotation of polarization plane $\propto \lambda^2$
- $\rightarrow$ Faraday depth $\phi = \int n_\mathrm{e} \vec{B} \cdot \mathrm{d}\vec{l}$

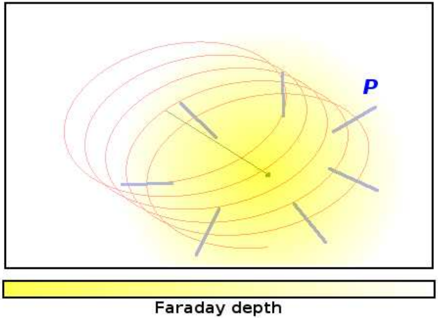
According to Henrik...

$$\langle P\phi\phi\rangle \propto \epsilon_H^2 \;\Rightarrow\; \langle GP^*\rangle \propto \left(\int_0^\infty \mathrm{d}k\, \frac{\epsilon_H(k)}{k}\right)^2$$

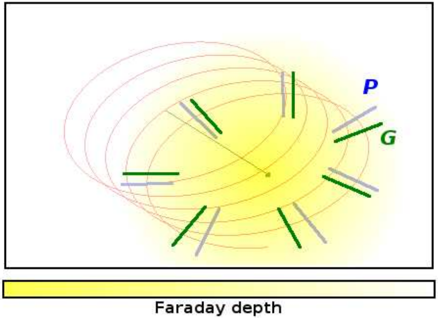




$$P = |P|\, e^{2i\alpha}$$



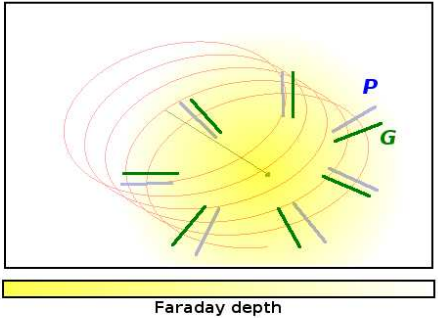
$$P = |P| \, e^{2i\alpha}$$



$$P = |P|\, e^{2i\alpha}$$

$$G = T_2(\nabla\phi) = (\partial_x\phi + i\partial_y\phi)^2 = |G|\, e^{2i\gamma}$$

$$T_2 : \mathbb{R}^2 \to \mathbb{C}$$



$$P = |P|\, e^{2i\alpha}$$

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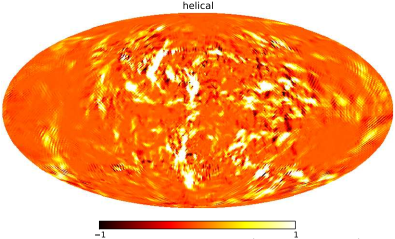
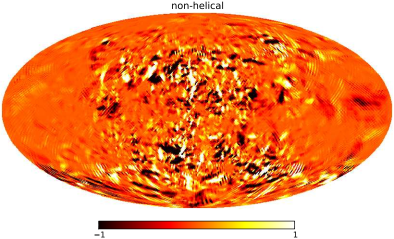
$$T_2 : \ \mathbb{R}^2 \rightarrow \mathbb{C}$$

Helicity

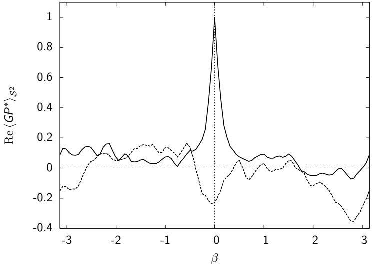
$$\mathrm{Re}\left(GP^*\right) > 0$$

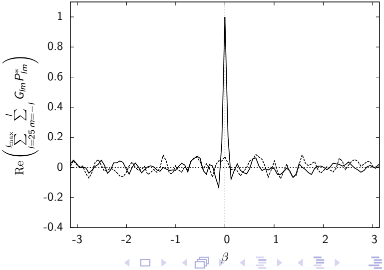




with helicity: $\left\langle \mathrm{Re} \left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = 1.0, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_{\mathcal{S}^2}} = 0.25$

without helicity: $\left\langle \mathrm{Re} \left\langle GP^* \right\rangle_{\mathcal{S}^2} \right\rangle_{\mathrm{samples}} = -0.27, \quad \sigma_{\mathrm{Re}\langle GP^* \rangle_{\mathcal{S}^2}} = 0.23$





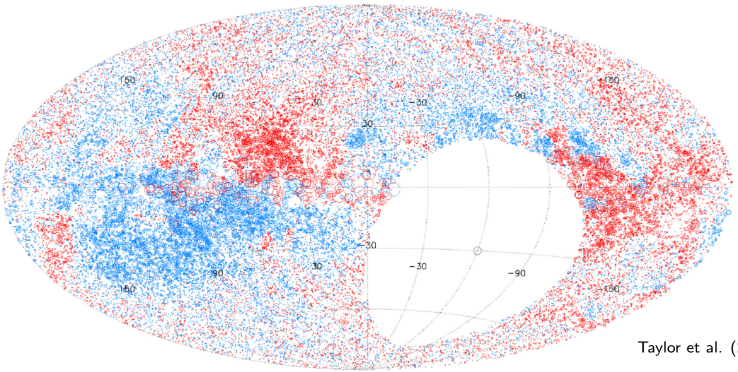


Figure 3. Plot of 37,543 RM values over the sky north of $\delta = -40°$. Red circles are positive rotation measure and blue circles are negative. The size of the circle scales linearly with magnitude of rotation measure.

Wiener Filter

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$d = Rs + n$$

$$m = Dj, \text{ where } \quad \begin{array}{l} j = R^\dagger N^{-1} d \\ D = \left(S^{-1} + R^\dagger N^{-1} R\right)^{-1} \end{array}$$

Assumptions

- signal field $s := \frac{\phi}{p(\vartheta)}$
  - s statistically homogeneous
    $\Rightarrow S(x,y) = \langle s(x)s(y) \rangle = S(x-y)$
  - s statistically isotropic
    $\Rightarrow S(x,y) = \langle s(x)s(y) \rangle = S(|x-y|)$
- $\Rightarrow S_{(\ell,m)(\ell',m')} = \delta_{\ell\ell'}\delta_{mm'}C_\ell$
- s Gaussian field

Wiener Filter

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$d = Rs + n$$

$$m = Dj, \;\; \text{where} \quad \begin{matrix} j = R^{\dagger} N^{-1} d \\ D = \left(S^{-1} + R^{\dagger} N^{-1} R\right)^{-1} \end{matrix}$$

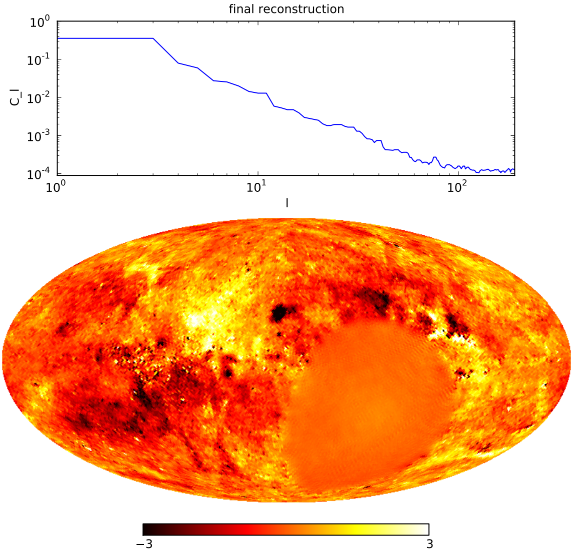
Critical Filter

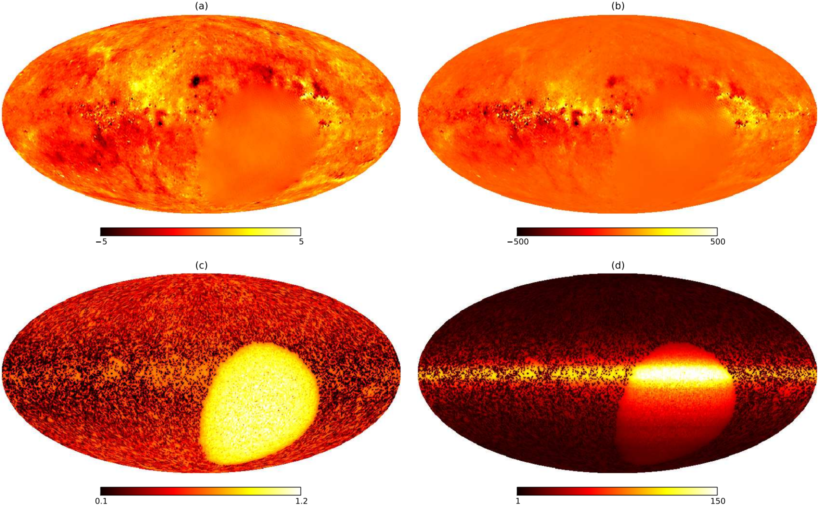
$$m = Dj$$

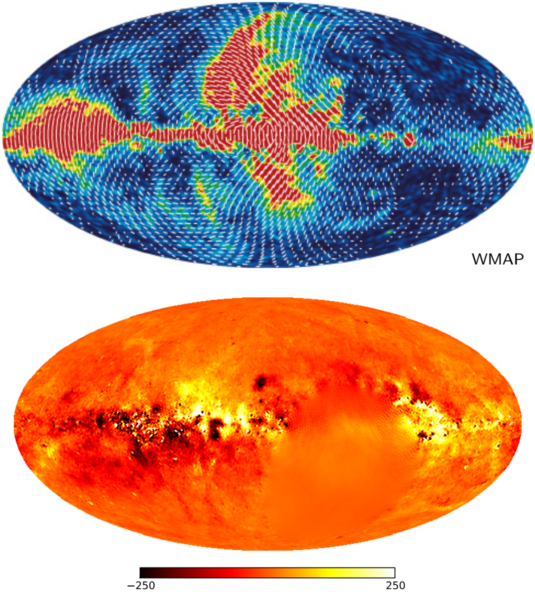
$$C_{\ell} = \frac{1}{2\ell + 1} \mathrm{tr}\left(\left(mm^{\dagger} + D\right) P_{\ell}\right)$$

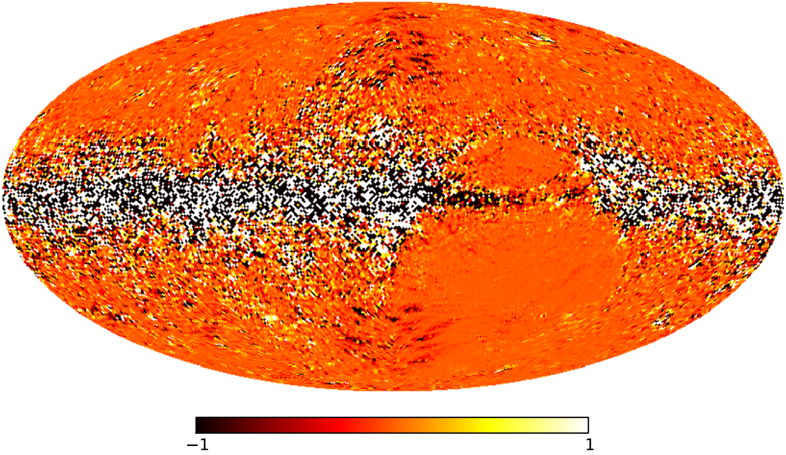
Enßlin & Frommert (2010), arXiv:1002.2928

Enßlin & Weig (2010), arXiv:1004.2868

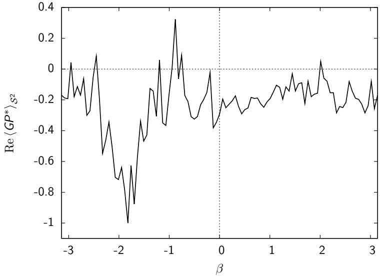

final reconstruction
C_l
l
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^0
10^1
10^2
−3
3


(a)
−5
5
(b)
−500
500
(c)
0.1
1.2
(d)
1
150

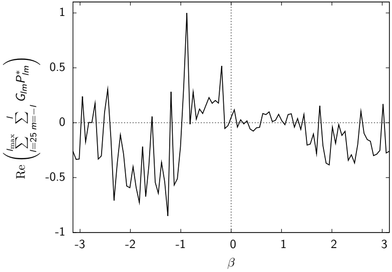

WMAP
−250
250


−1
1

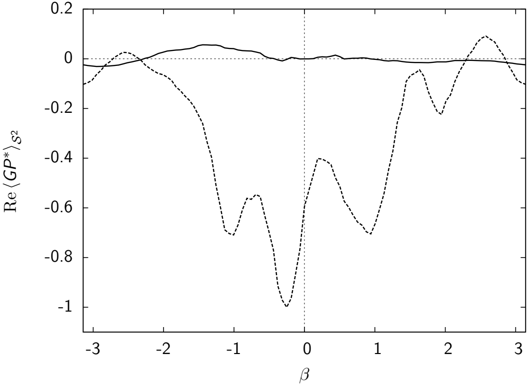
contributions of all scales



small-scale contributions



Test case with non-trivial electron densities:



Conclusions

- *Critical Filter* works.
- *LITMUS* test works, provided the electron densities don't vary too much.
- $\Rightarrow$ *LITMUS* test does **not** work on galactic scales.

Outlook

- Incorporate several datasets.
- Allow for uncertainty in the measurement errors.
- Increase resolution in order to detect helicity on small scales.