Vortices in planetary migration

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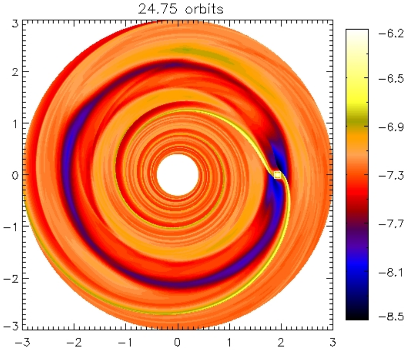
Outline

- Introduction: planet migration types
- Numerical methods, first results and motivation
- Type III migration in an inviscid disc:
  - Formation of vortensity rings
  - Linear stability
  - Non-linear outcome and role in type III
- Conclusions
- Future work

Introduction

- 374 exo-planets discovered (2 October 2009).
- First 'hot Jupiter' around 51 Pegasi, orbital period 4 days (Mayor & Queloz 1995). Fomalhaut b with semi-major axis 115AU.
- Formation difficult in situ, so invoke *migration*: interaction of planet with gaseous disc (Goldreich & Tremaine 1979; Lin & Papaloizou 1986).



Type I and type II

- Type I: linear theory for small planet masses (Earths). Waves from Lindblad resonances ($\Omega(r_L) = \Omega_p \pm \kappa/m$) imply a torque on the disc

$$\Gamma_{\mathrm{LR,m}} = \frac{\mathrm{sgn}(\Omega_p - \Omega)\pi^2\Sigma}{3\Omega\Omega_p} \times \left[r\frac{d\psi_m}{dr} + \frac{2m^2(\Omega - \Omega_p)}{\Omega}\psi_m \right]^2 .$$

  The linear co-rotation torque due to co-rotation resonance ($\Omega(r_C) = \Omega_p$)

$$\Gamma_{\mathrm{CR,m}} = \frac{\pi^2 m\psi_m^2}{2}\left(\frac{d\Omega}{dr}\right)^{-1}\frac{d}{dr}\left(\frac{\Sigma}{B}\right),$$

  where $B = \omega/2$. No Γ_{CR} in Keplerian disc with $\Sigma \propto r^{-3/2}$.
- Type II: gap-opening for massive planets (Jovian). Migration locked with disc viscous evolution. Criteria: $r_p(M_p/3M_*)^{1/3} > H$ or $M_p/M_* > 40\nu/a^2\Omega$.

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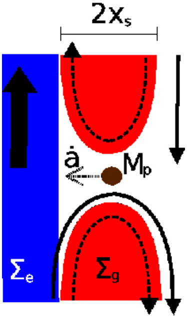
- What about intermediate, Saturn-mass planets with partial gaps? There is another source of torque that depends on migration rate.

Physics of type III migration



- Fluid element orbital radius changes from $a - x_s \rightarrow a + x_s \Rightarrow$ torque on planet:

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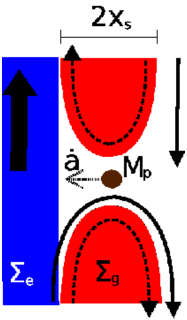

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- Migration rate for $M_p + M_r + M_h$:

$$\dot{a} = \frac{2\Gamma_{\mathrm{L}}}{\Omega a(\underbrace{M_p + M_{\mathrm{r}}}_{M_p'} - \delta m)} \qquad (1)$$

where Γ_L is the total Lindblad torque and

$$\delta m = 4\pi\Sigma_e a x_s - M_h = 4\pi a x_s(\Sigma_e - \Sigma_g)$$

is the density-defined co-orbital mass deficit.

Key ideas in type III

- Co-orbital mass deficit:

  $\boxed{\text{larger } \delta m \Rightarrow \text{faster migration.}}$

- Horse-shoe width: $\boxed{x_s}$, separating co-orbital and circulating region. Take $x_s = 2.5r_h$ for result analysis ($r_h \equiv (M_p/3M_*)^{1/3}a$). Can show $x_s \lesssim 2.3r_h$ in particle dynamics limit.

- Vortensity: $\boxed{\eta \equiv \omega/\Sigma,}$ important for stability properties and η^{-1} also used to define δm (Masset & Papaloizou 2003).

- Modelling assumptions: steady, slow migration ($\boxed{\tau_{\mathrm{lib}}/\tau_{\mathrm{mig}} \ll 1}$), horse-shoe material moves with planet.

Numerics

Standard numerical setup for disc-planet interaction. 2D disc in polar co-ordinates centered on primary but non-rotating. Units $G = M_* = 1$.

- Hydrodynamic equations with local isothermal equation of state:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{v}) = 0,$$
$$\frac{\partial v_r}{\partial t} + \mathbf{v} \cdot \nabla v_r - \frac{v_\phi^2}{r} = -\frac{1}{\Sigma}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r} + \frac{f_r}{\Sigma},$$
$$\frac{\partial v_\phi}{\partial t} + \mathbf{v} \cdot \nabla v_\phi + \frac{v_\phi v_r}{r} = -\frac{1}{\Sigma r}\frac{\partial P}{\partial \phi} - \frac{1}{r}\frac{\partial \Phi}{\partial \phi} + \frac{f_\phi}{\Sigma},$$
$$P = c_s^2(r)\Sigma.$$

  Viscous forces $f \propto \nu = \nu_0 \times 10^{-5}$, temperature $c_s^2 = h^2/r$, $h = H/r$. Φ is total potential including primary, planet (softening $\epsilon = 0.6H$), indirect terms but no self-gravity.

- Method: FARGO code (Masset 2000), finite difference for hydrodynamics, RK5 for planet motion.

Type III in action

Discs: uniform density $\Sigma = 7 \times 10^{-4}$, aspect ratio $h = 0.05$ and different uniform kinematic viscosities.

Planet: Saturn mass $M_p = 2.8 \times 10^{-4}$ initially at $r = 2$.

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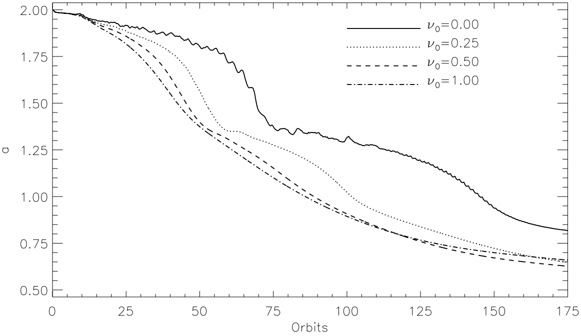
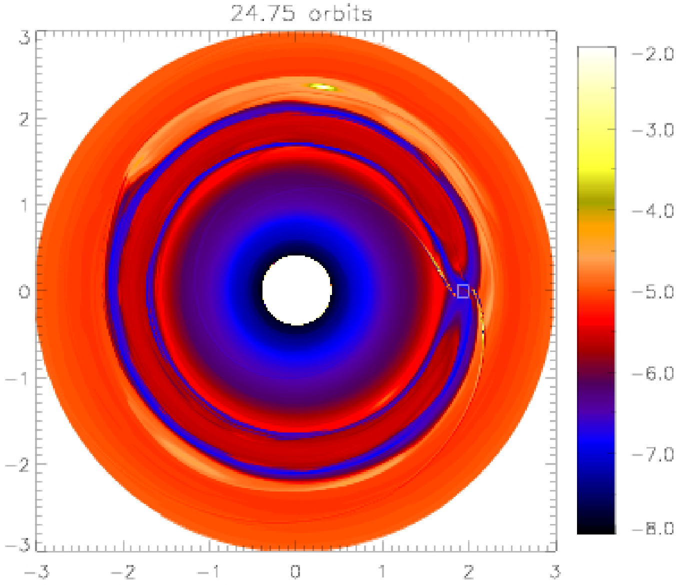


What's going on at low viscosities?

Inviscid case: evolution of Σ/ω:



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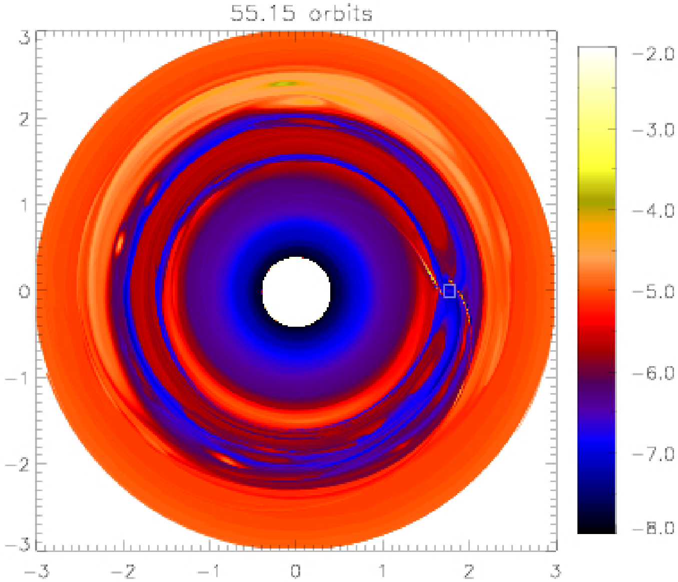

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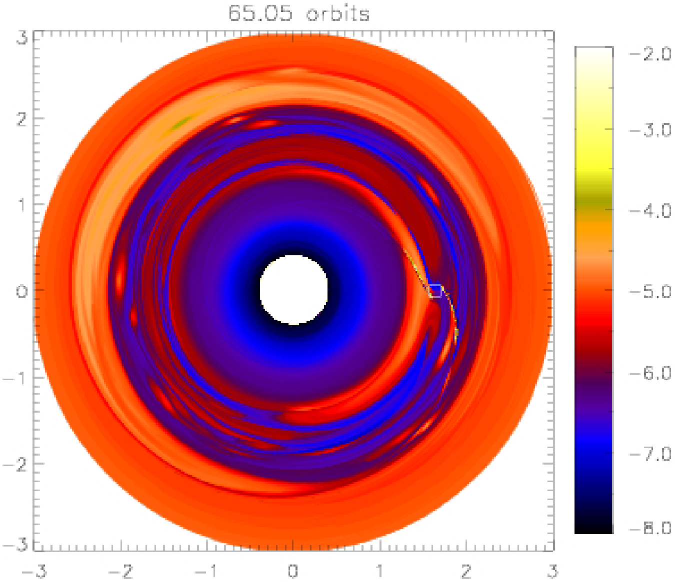

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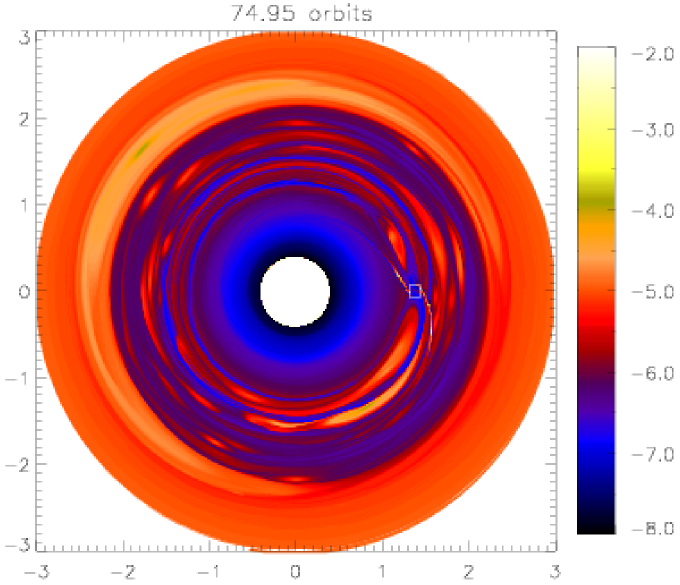

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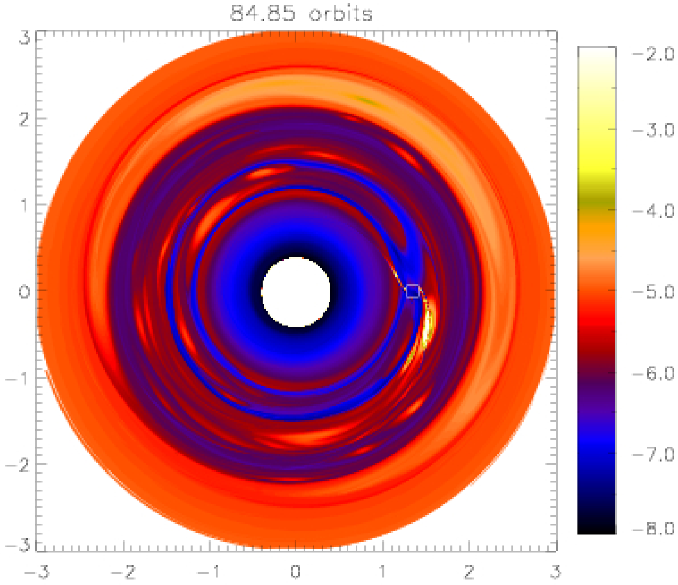

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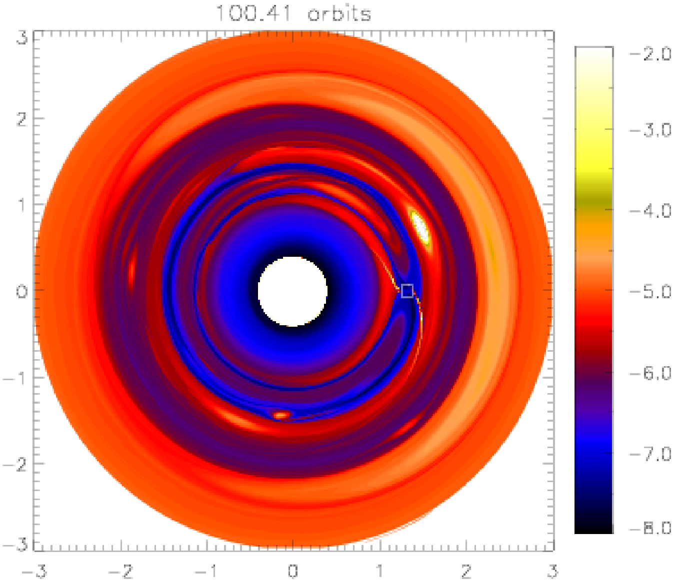
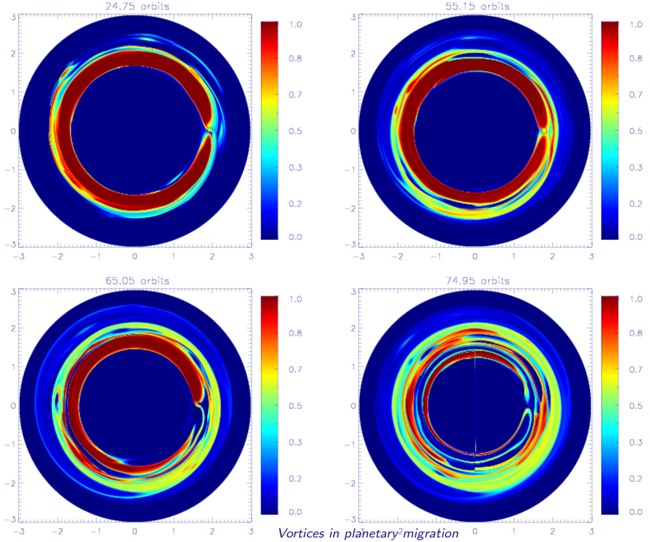


Loss of horse-shoe material

Advection of passive scalar initially in $r = r_p \pm 2r_h$. $t_{\rm lib}/t_{\rm mig} \simeq 0.6$.



Vortensity rings: formation via shocks

- Vortensity equation:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\left(\frac{\omega}{\Sigma}\right) = \frac{dc_s^2}{dr}\frac{\partial}{\partial \phi}\left(\frac{1}{\Sigma}\right).$$

  Axisymmetry and/or barotropic $\Rightarrow$ vortensity conserved *except at shocks*.

- Confirmed by fixed-orbit, high resolution simulation ($r = [1, 3]$, $N_\phi \times N_r = 3072 \times 1024$) $\Delta r \simeq 0.02 r_h$, $r\Delta\phi \simeq 0.05 r_h$.

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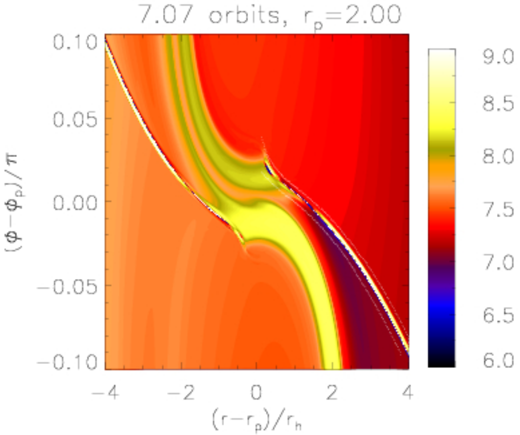

Vortensity generated as fluid elements U-turn during its horse-shoe orbit.

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$$\left[\frac{\omega}{\Sigma}\right] = -\frac{(M^2-1)^2}{\Sigma M^4}\frac{\partial v_\perp}{\partial S} - \left(\frac{M^2-1}{\Sigma M^2 v_\perp}\right)\frac{\partial c_s^2}{\partial S}.$$

RHS is pre-shock. $M = v_\perp/c_s$, S is distance along shock (increasing radius). Additional baroclinic term compared to Li et al. (2005) but has negligible effect ($c_s^2 \propto 1/r$).

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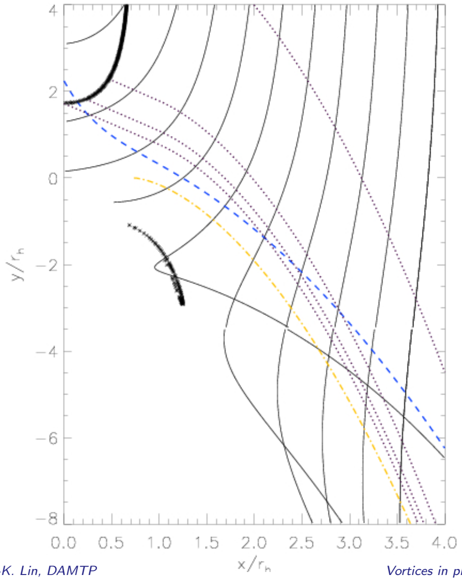
- Shock location : generalised Papaloizou et al. (2004)

$$\frac{dy_s}{dx} = \frac{\hat{v}_y^2 - 1}{\hat{v}_x\hat{v}_y - \sqrt{\hat{v}_x^2 + \hat{v}_y^2 - 1}}.$$

$\hat{v} \equiv v/c_s$.

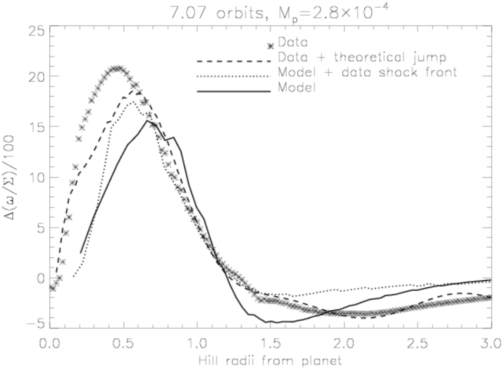
Shock location



- Solid lines: particle paths from the zero-pressure momentum equations,
- Thick lines: sonic points $|\mathbf{v}| = c_s$,
- Dotted lines: theoretical shock fronts;
- Dash-dot: solution for Keplerian flow;
- Dashed : polynomial fit to simulation shock front.

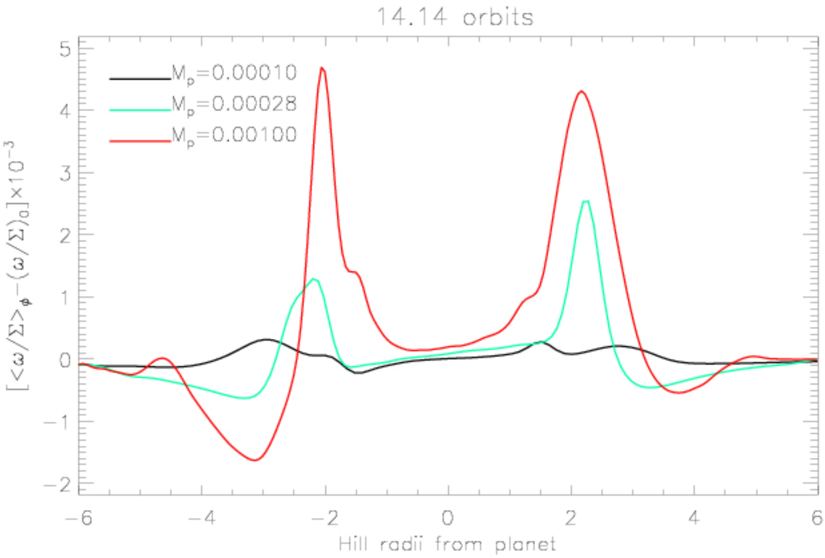
The actual shock front begins around $x = 0.2r_h$, where it crosses the sonic point.

Theoretical jumps



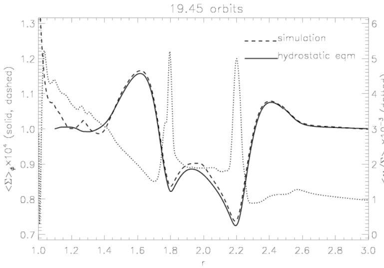
- Vortensity generation near shock tip (horse-shoe orbits) , vortensity destruction further away (circulating region). Variation in flow properties on scales of $r_h \simeq H$.
- Variation in disc profiles on scale-heights enables shear instability $\Rightarrow$ vortices in non-linear stage (Lovelace et al. 1999, Li et al. 2001).

Vortensity generation v.s. M_p

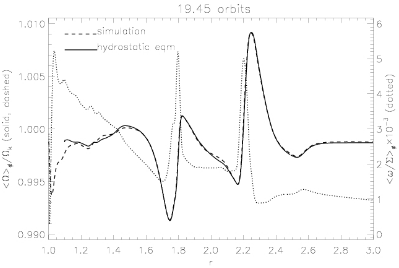


Ring stability

Idea: linear stability analysis of inviscid disc but use simulation vortensity profile as basic state: axisymmetric, $v_r = 0$.



Σ



Ω/Ω_k

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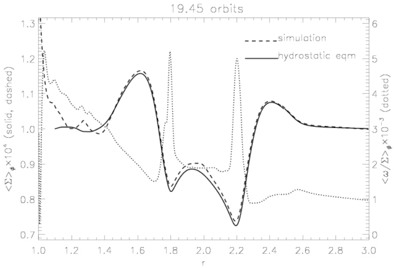

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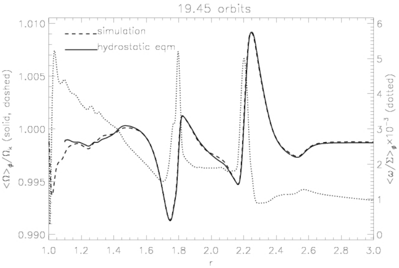

Ω/Ω_k

- In principle can predict gap structure via shock modelling / vortensity generation. Important to check axisymmetric hydrostatic basic state, otherwise linear analysis becomes very difficult.

Linear theory

- Governing equation for isothermal perturbations $\propto \exp i(\sigma t + m\phi)$:

$$\frac{d}{dr}\left(\frac{\Sigma}{\kappa^2 - \bar{\sigma}^2}\frac{dW}{dr}\right) + \left\{\frac{m}{\bar{\sigma}}\frac{d}{dr}\left[\frac{\kappa^2}{r\eta(\kappa^2 - \bar{\sigma}^2)}\right] - \frac{r\Sigma}{h^2} - \frac{m^2\Sigma}{r^2(\kappa^2 - \bar{\sigma}^2)}\right\} W = 0$$

$W = \delta\Sigma/\Sigma$; $\kappa^2 = 2\Sigma\eta\Omega$; $\bar{\sigma} = \sigma + m\Omega(r)$.

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- Simplified equation for "co-rotational modes" ($\kappa^2 \gg |\bar{\sigma}^2|$, $m=O(1)$):

$$\frac{d}{dr}\left(\frac{rc^2\Sigma}{\kappa^2}\frac{dW}{dr}\right)+\left\{\frac{m}{\bar{\sigma}}\frac{d}{dr}\left[\frac{c^2}{\eta}\right]-r\Sigma\right\}W=0.$$

Should have $(c^2/\eta)' \to 0$ as $\bar{\sigma}\to 0$ to stay regular.

Properties of co-rotational modes

- Multiply simplified equation by W^*, integrate then take imaginary part:

$$-i\gamma \int_{r_1}^{r_2} \frac{m}{(\sigma_R + m\Omega)^2 + \gamma^2} \left(\frac{c^2}{\eta}\right)' |W|^2 dr = 0$$

$\sigma = \sigma_R + i\gamma$. Must have $(c^2/\eta)' = 0$ at co-rotation point r_0 for non-neutral modes to exist ($\gamma \neq 0$). Shock-modified protoplanetary disc satisfies neccessary condition.

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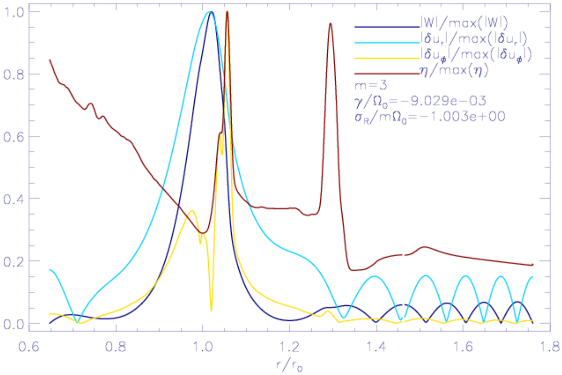
- Semi-circle theorem. Define $W = g\bar{\sigma}$ then multiply by g^* and integrate. Can show (approximately):

$$\gamma^2 + \left[\sigma_R + \frac{1}{2} m(\Omega_+ + \Omega_-)\right]^2 \leqslant m^2 \left(\frac{\Omega_+ - \Omega_-}{2}\right)^2.$$

$\Omega_\pm$ are maximum and minimum angular speed in region of interest. Growth rate limited by local shear.

Example: $m = 3,\ h = 0.05$



- Disturbance focused around vortensity minimum (gap edge), exponential decays either side joined by vortensity term at co-rotation r_0. More extreme minimum $\Rightarrow$ more localised.
- Waves beyond the Lindblad resonances ($\kappa^2 - \bar{\sigma}^2 = 0$) but amplitude not large compared to co-rotation.

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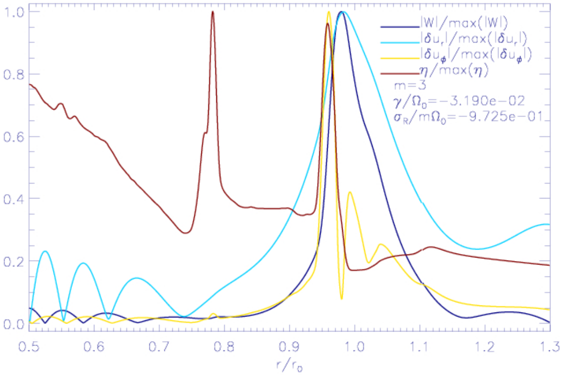

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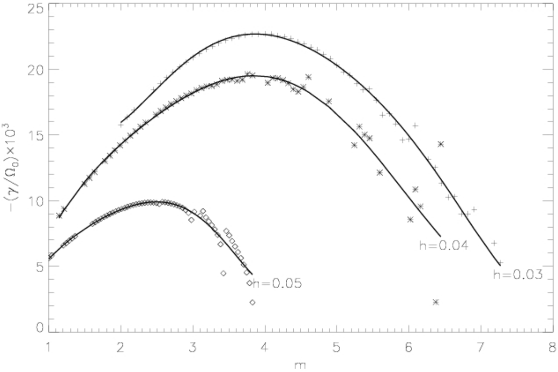
Growth rate v.s. m and h

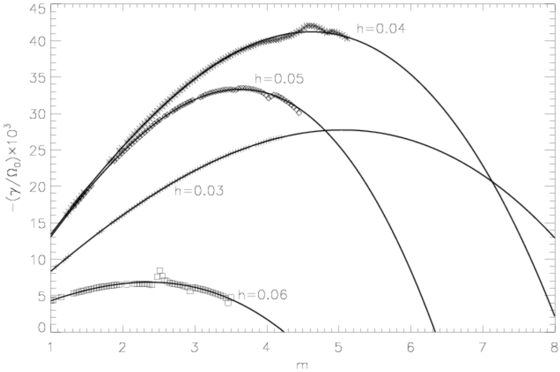
Solve for non-integer *m* to get dependence on γ on azimuthal wave-number. Polynomial fit.



Inner edge

$h = 0.05,\ T_{\rm grow} \sim 10$ orbital periods at $r = 2$.

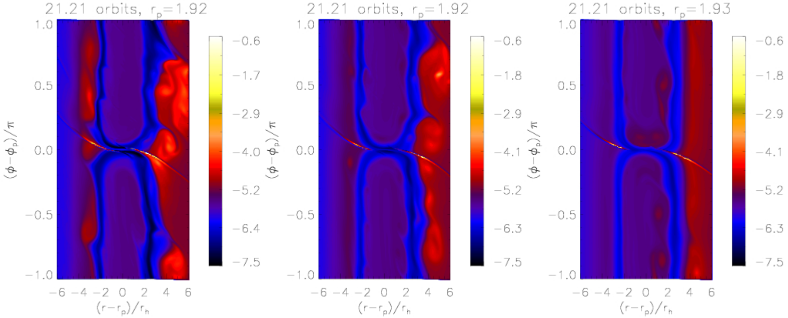
Growth rate v.s. m and h



Outer edge

($h = 0.03$ spurious, $\eta(r)$ not representative of disc.)

Growth rate v.s. m and h



$h = 0.04$ $\quad$ $h = 0.05$ $\quad$ $h = 0.06$

Σ/ω shown here.

Link to type III

Recall co-orbital mass deficit

$$\delta m = 4\pi a x_s (\Sigma_e - \Sigma_g)$$

- Instability can increase δm by increasing Σ_e but not Σ_g (co-rotational modes are localised) $\Rightarrow$ favouring type III. When vortex flows across co-orbital region, Σ_g increases and migration *may* stall.

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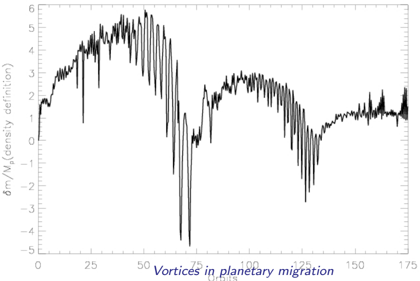
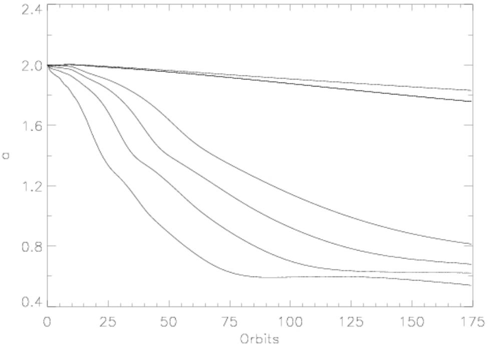
- Can expect interaction when δm of order planet mass.



Implications of linear theory

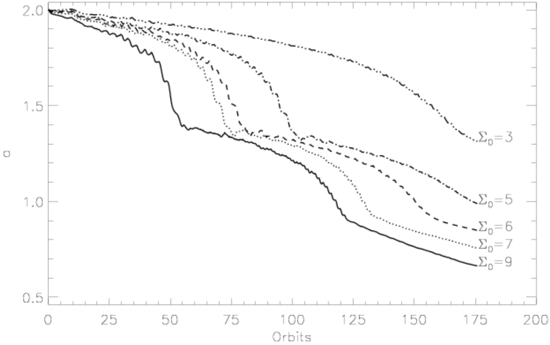
Changing disc masses in standard viscous disc $\nu = 10^{-5}$



Top to bottom: $\Sigma \times 10^4 = 1,\ 2.5,\ 5,\ 7,\ 10,\ 15$

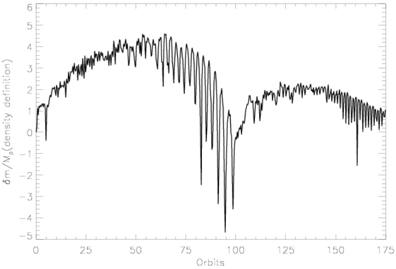
Implications of linear theory

Growth rate indepdent of density scale. Higher density just means less time needed for vortex to grow sufficiently large for interaction.

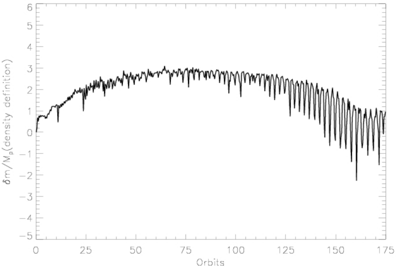


Implications of linear theory

In terms of co-orbital mass deficit...



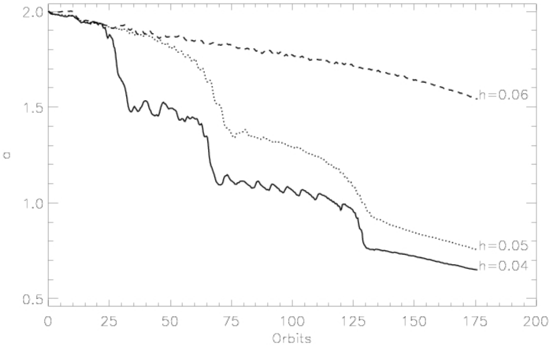
$\Sigma_0 = 5$



$\Sigma_0 = 3$

Implications of linear theory

$c_s^2 = T \propto h^2$. Lower temperature $\Rightarrow$ stronger shocks $\Rightarrow$ profile more unstable $\Rightarrow$ shorter time-scale to vortex-planet interaction.



Require disc profile to be sufficiently extreme and have enough mass to trigger vortex-planet interaction, but the extent of migration during one episode is the same.

Vortex-triggered migration

- Migration may not stall after vortex-planet interaction if Σ_e increases relative to Σ_g. Possible if planet scattered to region of high density.
- Consider discs with $\Sigma \propto r^{-p}$. Note $\delta m(t=0) > 0$ in this case.

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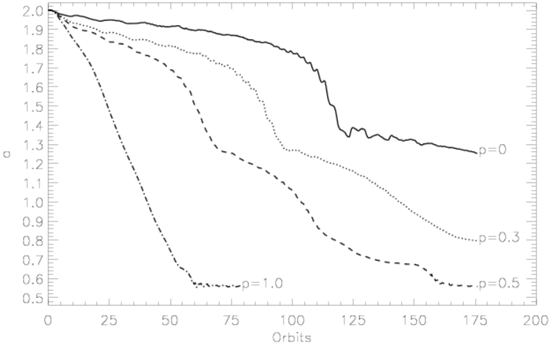
- Migration may not stall after vortex-planet interaction if Σ_e increases relative to Σ_g. Possible if planet scattered to region of high density.
- Consider discs with $\Sigma \propto r^{-p}$. Note $\delta m(t=0) > 0$ in this case.



Vortices can trigger migration, needed for type III.

Summary

- Migration in low viscosity/inviscid discs is non-smooth due to shear instabilities associated with gap edge (vortensity minima).
- Provided an over-all picture of vortex-planet interaction: formation of unstable basic state via shocks, linear stability analysis and hydrodynamic simulations.
- Instability encourages type III by increasing co-orbital mass deficit. Vortex-planet interaction when $\delta m/M_p \sim 4$—5. Associated disruption of co-orbital vortensity structure.
- Vortex-induced migration stalls in uniform density discs but can act as trigger in $\Sigma \propto r^{-p}$ discs.

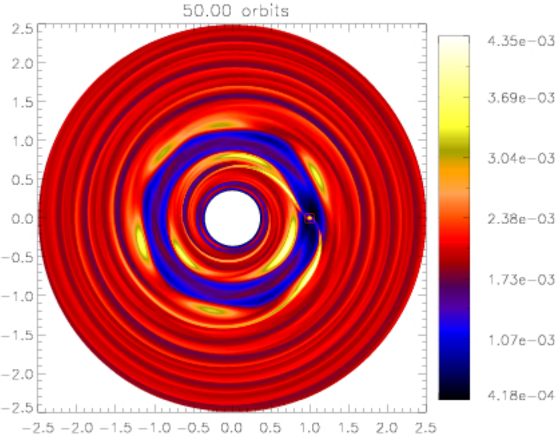
Future work: self-gravity

- Type III, or runaway migration recognised to operate in massive discs (few times MMSN), but conclusion reached using simulations without self-gravity.
- Fiducial case with $\Sigma = 7 \times 10^{-4}$ gives $Q(r_p) \simeq 5.6$. Need to have SG!

- Implement Li et al. (2009) Poisson solver to FARGO for high-resolution studies of co-orbital disc-planet interaction with self-gravity.

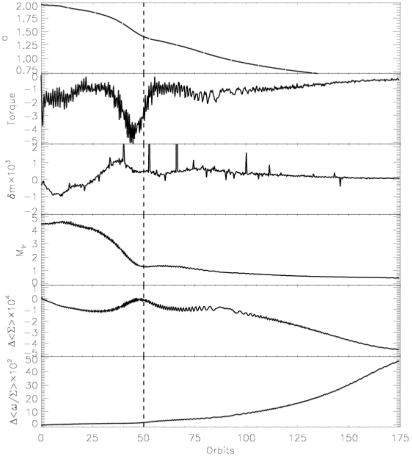
Future work: self-gravity

Thanks

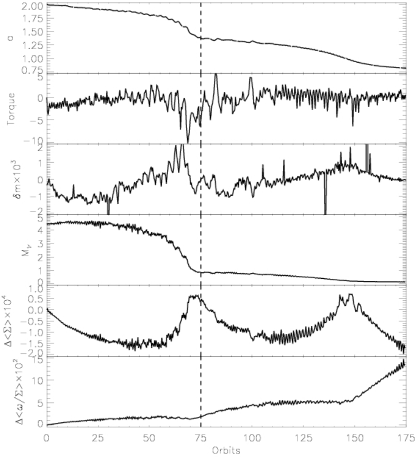


(FARGO with Li et al.'s Poisson solver.)

Bonus slide: evolution of co-orbital region



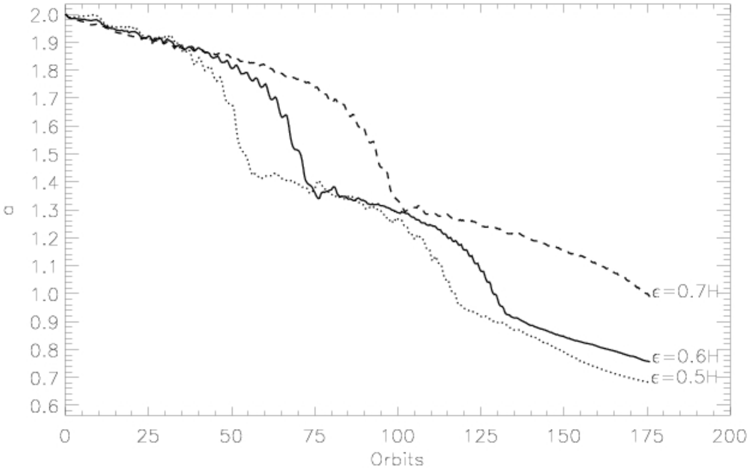
$\nu_0 = 0.5$



$\nu_0 = 0$

Bonus slide: effect of softening

$$\Phi_p = -\frac{M_p}{\sqrt{|\mathbf{x}-\mathbf{x}_p|^2+\epsilon^2}}$$

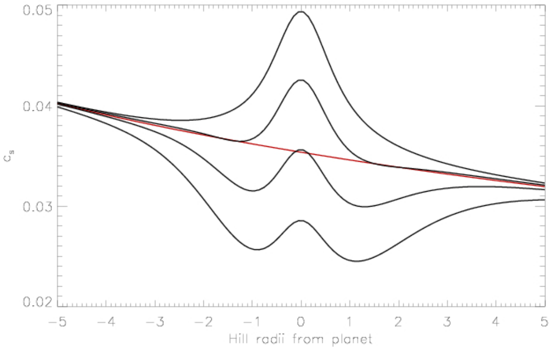


Bonus slide: special equation of state

Peplinski et al. (2008):

$$c_s = \frac{hr_s h_p r_p}{[(hr_s)^n + (h_p r_p)^n]^{1/n}} \sqrt{\Omega_s^2 + \Omega_p^2}$$

$n = 3.5$ and vary h_p to get temperature modifications close to planet.



Top to bottom (black curve) $h_p = 0.7, 0.6, 0.5, 0.4$. Red curve: local isothermal.

Bonus slide: special equation of state