Spin down (?) of protostars through gravitational torques

Min-Kai Lin[1,2], Mark Krumholz[1], Kaitlin Kratter[3]

[1]UC Santa Cruz
[2] University of Cambridge
[3] University of Toronto

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Plan

- Problem
- Numerical modelling
- Simulation 1: hindered spin down
- Simulation 2: spin down
- Discussion and implications
- Problems

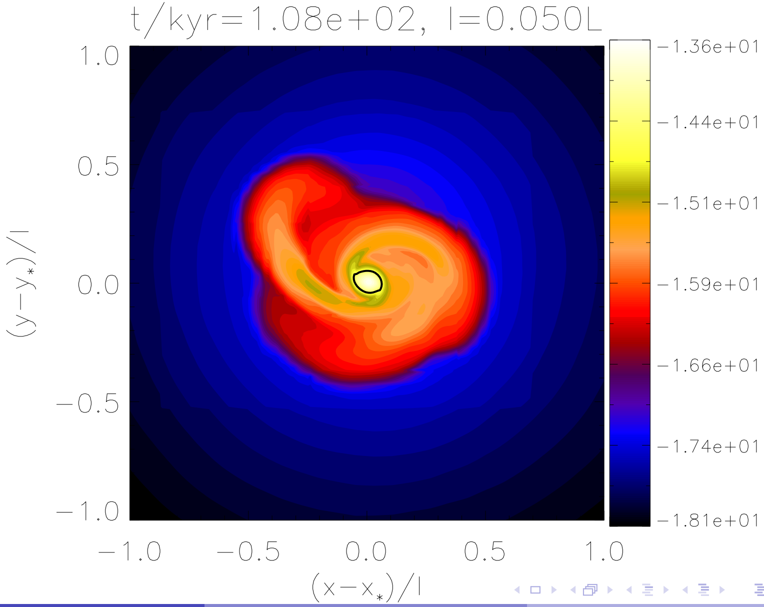
Problem

Angular momentum problem in star formation (Bodenheimer, 1995). Specific angular momenta (cgs) of:

- Molecular cloud core, $j \sim 10^{21}$
- T Tauri, $j \sim 10^{17}$
- Sun, $j \sim 10^{15}$

Need to transport angular momentum out of central region, otherwise star spins too fast. How?

Problem



t/kyr=1.08e+02, l=0.050L
1.0
0.5
0.0
−0.5
−1.0
(y−y*)/l
−1.0
−0.5
0.0
0.5
1.0
(x−x*)/l
−1.36e+01
−1.44e+01
−1.51e+01
−1.59e+01
−1.66e+01
−1.74e+01
−1.81e+01

Governing equations

Inviscid, non-magnetised and self-gravitating fluid with customised equation of state:

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\boldsymbol{v}$$
$$\frac{D\boldsymbol{v}}{Dt} = -\frac{1}{\rho}\nabla P - \nabla\Phi$$
$$\rho\frac{De}{Dt} = -\rho\boldsymbol{v}\cdot\nabla\Phi - \nabla\cdot(P\boldsymbol{v})$$
$$\nabla^2\Phi = 4\pi G\rho,$$
$$P = c^2\rho^{\gamma_1}\left[1 + \left(\frac{\rho}{\rho_*}\right)^{\gamma_2-\gamma_1}\right].$$

$c \simeq 266\text{ms}^{-1}$ is isothermal sound speed of $\mu = 2.33$ gas at 20K, $\gamma_1 = 1$ and $\gamma_2 = 5/3$. Here, e is the sum of internal and kinetic energy densities.

Angular momentum transport

Conservation equation

$$\frac{\partial}{\partial t}(\rho R v_\phi) + \nabla\cdot(\rho R v_\phi \boldsymbol{v}) = -\rho\frac{\partial\Phi}{\partial\phi} - \frac{\partial P}{\partial\phi},$$

integrate over volume V, use $\rho = \nabla^2\Phi/4\pi G$ on RHS to get:

$$\frac{\partial J}{\partial t} + \oint \boldsymbol{F}\cdot\boldsymbol{dS} = 0$$

with

$$\boldsymbol{F} = \boldsymbol{F}_A + \boldsymbol{F}_G$$
$$\boldsymbol{F}_G = \frac{1}{4\pi G}\frac{\partial\Phi}{\partial\phi}\nabla\Phi.$$

Also have the Reynolds stress, $\rho R\delta v_\phi\delta\boldsymbol{v}$.

Collapse into star-disc system

Kratter et al. (2010) description of collapse of spherical, rotating cloud into a disc (mass M_d) with central object (mass M_*). Call $M_* + M_d = M_{\mathrm{sys}}$.

- Infall parameter

$$\xi = \frac{G\dot{M}}{c^3}$$

- Rotation parameter

$$\Gamma = \frac{\dot{M}}{M_{\mathrm{sys}}\Omega_k}$$

Ω_k is Keplerian frequency, due to M_{sys}, of material joining the system from the cloud, assumed to occur at cylindrical radius R_k. Can show disc aspect-ratio

$$h = \left(\frac{\Gamma}{\xi}\right)^{1/3},$$

and $R_k = h^2\xi ct$.

Initial conditions & numerical method

- Start with spherical cloud of radius r_c of density profile

$$\rho(r) = \frac{Ac^2}{4\pi G r^2}$$

  Shu (1977)$\rightarrow$ self-similar collapse.

- Designate a central region $r \leq r_* \equiv qr_c$ to be the 'star'. Set $\rho_* = \rho(r_*)$.
- Set $v_r = v_\theta = 0$ and azimuthal velocity

$$v_\phi = 2Ach \times \begin{cases} R/r_* & R \leq r_* \\ 1 & R > r_*. \end{cases}$$

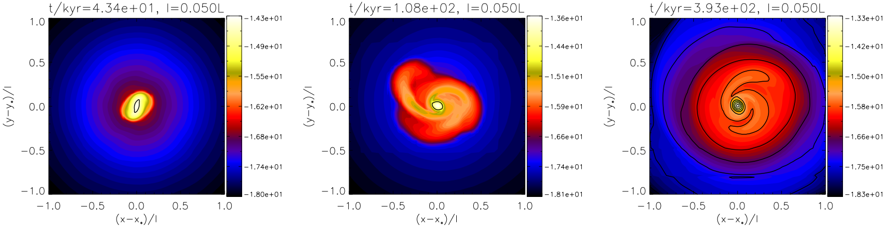
- Need $2h\sqrt{A} < 1$ for below break-up speed.

Initial conditions & numerical method

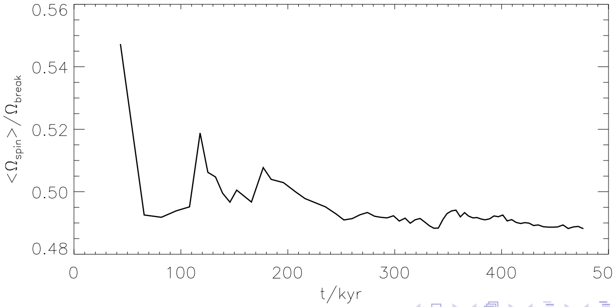
- Solve in Cartesian box of length $L = 4r_c$.
- ORION: Godunov-type code with adaptive mesh refinement.
- Base grid 128^3, 6 refinement levels (effective highest resolution 8192^3).

Case 1: $\xi = 5.58$, $h = 0.1$, $q = 0.005$

Density slices:



Star spin



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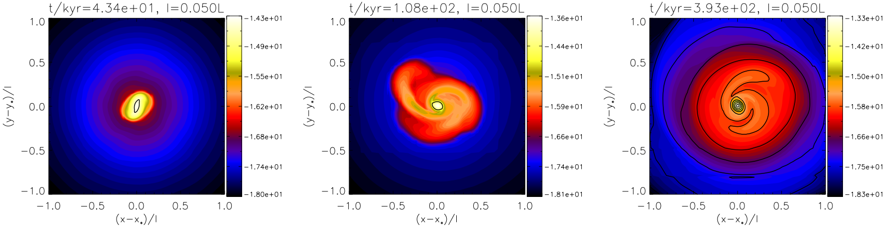
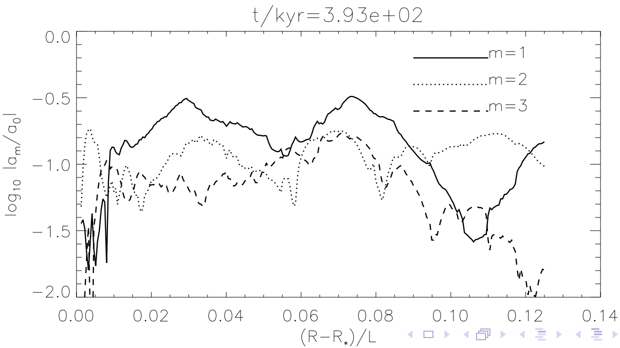


Mode amplitudes



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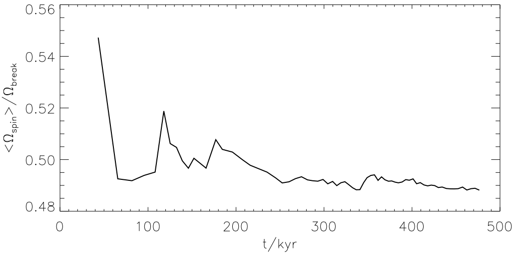
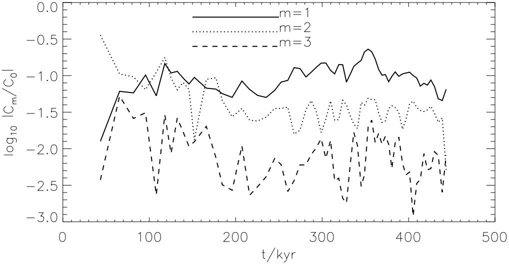
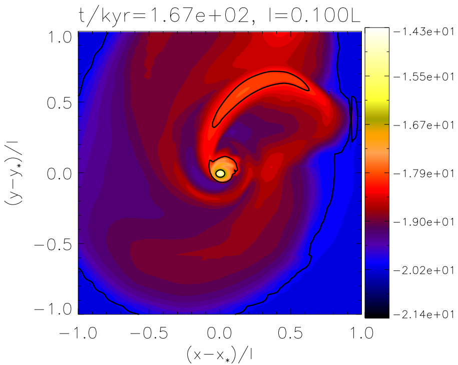
Star spin



Modes evolution

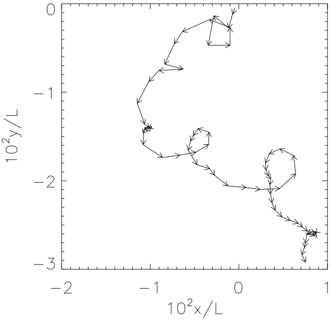


Influence of $m = 1$ modes



- Theoretical studies: Adams et al. (1989); Heemskerk et al. (1992).
- $m = 1$ **displaces star from COM (of box)**
- Require sufficient disc mass.
- Exchange of *orbital* angular momentum.

Case 1: stellar motion & angular momenta



Estimates:

- $\Omega_{\rm spin} \sim 1.5 \times 10^{-10}$
- $\Omega_{\rm orb} \sim 2 \times 10^{-13}$
- $\Omega_{\rm disc}(R_k) \sim 6 \times 10^{-12}$
- $\Omega_{\rm patt} \sim (3\text{—}5) \times 10^{-12}$

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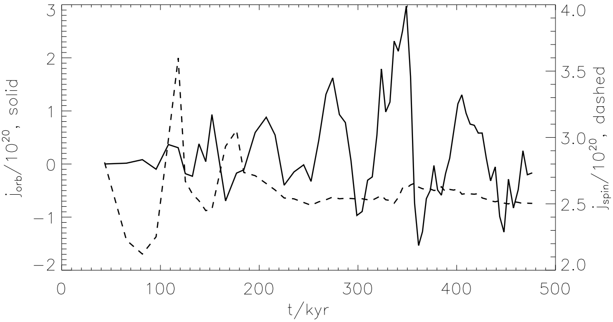

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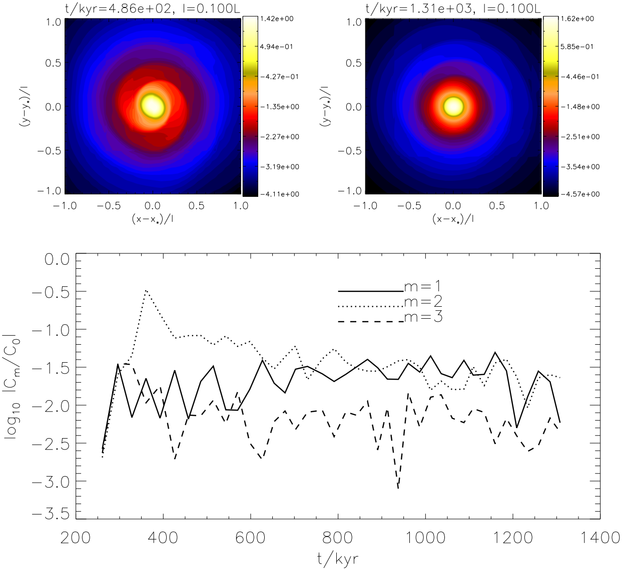

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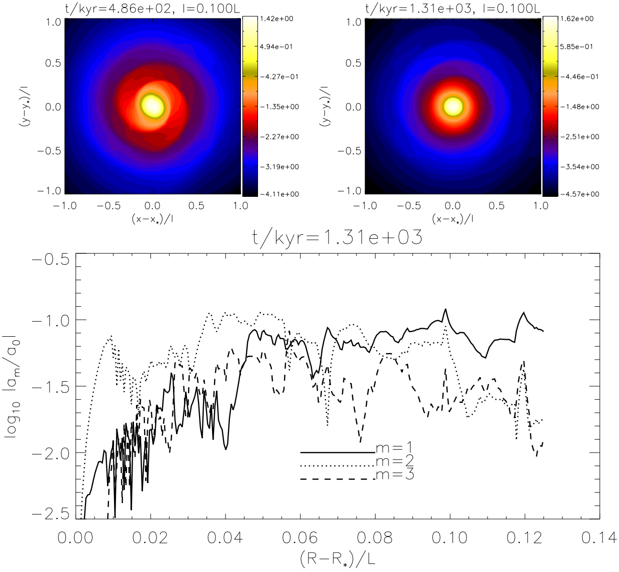

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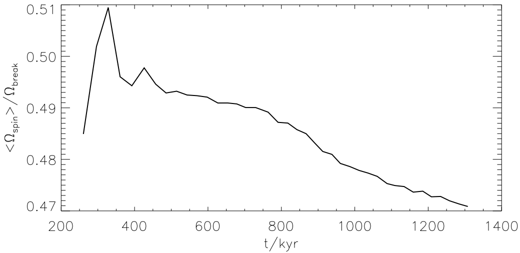

- Spin down around $t = 500$kyr with visible $m = 2$.
- Spin down near the end but no visible $m = 2$, although FT $\rightarrow$ $m = 2$ still the main non-axisymmetry near star.
- FT $\rightarrow$ $m = 1$ becomes important in outer region but limited orbital motion.

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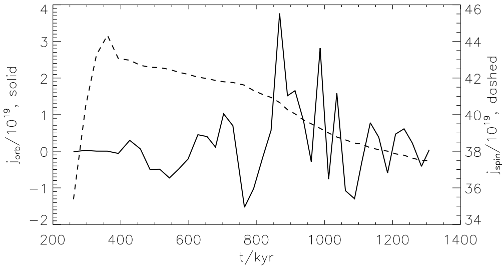

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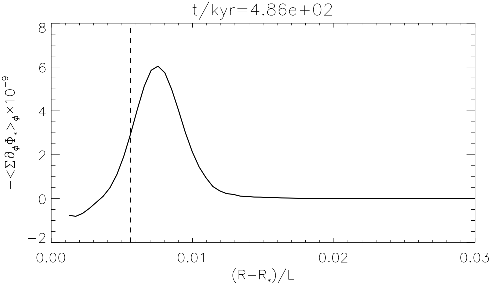
Star torque & gravity flux

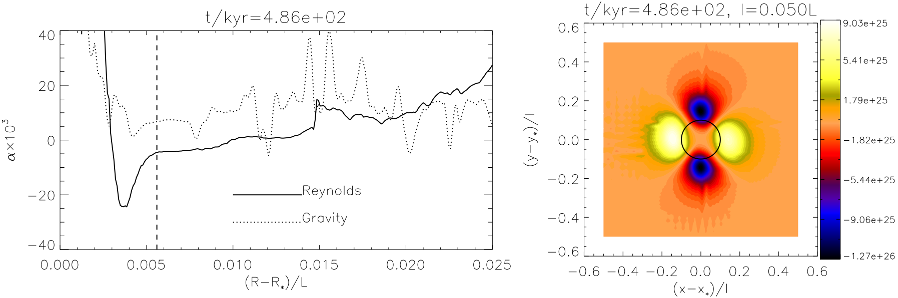
$\nabla^2\Phi_* = 4\pi G\rho_{\rm star} \rightarrow$ get star torque per unit area:



consistent with disc-on-star torques, but...

Star torque & gravity flux

$\partial_t J + \oint \boldsymbol{F} \cdot \boldsymbol{dS} = 0 \rightarrow$ look at radial gravity flux near star

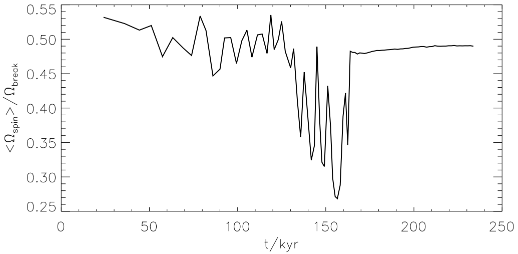


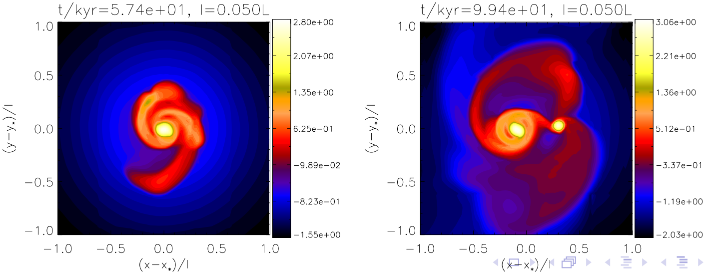
Solid and dashed line: characteristic star size.

- $\alpha < O(10^{-2})$ also reported in Kratter et al. (2010) but is SMALL compared to numerical α!
- Numerical spin down? But why ineffective in Case 1?

Case 3: binary spin down

Theoretical work: Boss (1984).





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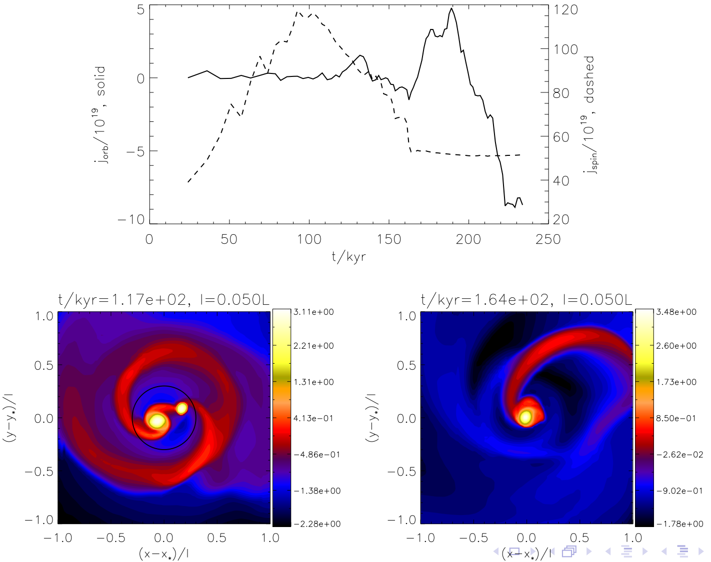
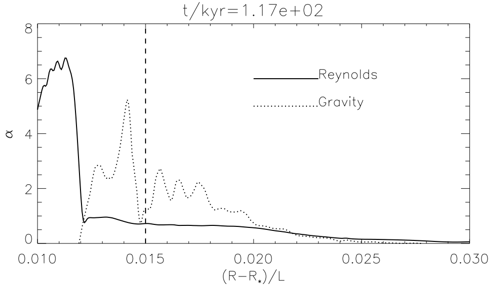
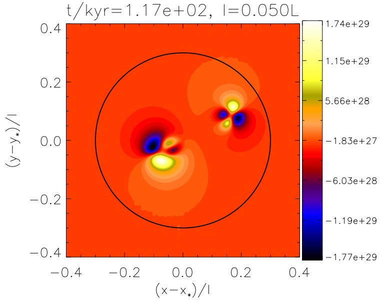


Gravity flux into binary

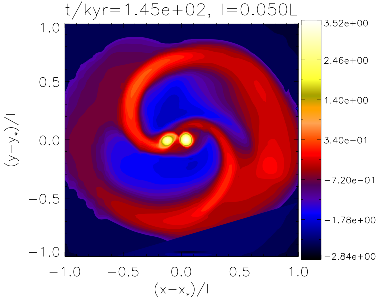




- Gravity $\alpha \sim 2$ from binary $\rightarrow$ torque down
- Numerical $\alpha < 0.5$ in this region
- $\Omega_{\rm spin} > \Omega_p$ on circle

Problems

- Spin down using gravity flux? Maybe, but cannot have influences from $m = 1$.
- Need better experiment designs to overcome numerical spin down.
- Binary results are less unconvincing.



Thank you

mkl23@cam.ac.uk

References

Adams F. C., Ruden S. P., Shu F. H., 1989, ApJ, 347, 959

Bodenheimer P., 1995, ARAA, 33, 199

Boss A. P., 1984, MNRAS, 209, 543

Heemskerk M. H. M., Papaloizou J. C., Savonije G. J., 1992, A&A, 260, 161

Kratter K. M., Matzner C. D., Krumholz M. R., Klein R. I., 2010, ApJ, 708, 1585

Shu F. H., 1977, ApJ, 214, 488