# Edge instabilities in disc-planet interactions

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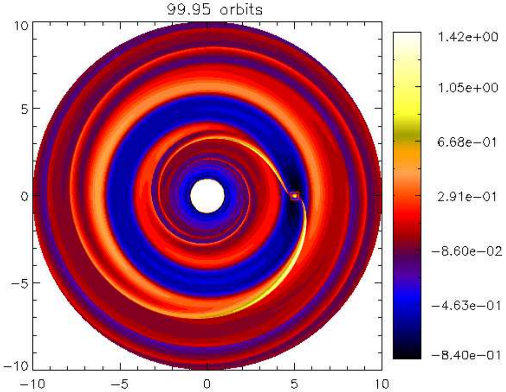
# Introduction

- 455 exo-planets discovered (May 2010).
- First 'hot Jupiter' around 51 Pegasi, orbital period 4 days (Mayor & Queloz 1995). Fomalhaut b with semi-major axis 115AU.
- Formation difficult in situ, so invoke *migration*: interaction of planet with protoplanetary disc (Goldreich & Tremaine 1979; Lin & Papaloizou 1986).



# Model equations

2D disc in polar co-ordinates centered on primary but non-rotating. Units $G = M_* = 1$.

- Hydrodynamic equations with local isothermal equation of state:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\mathbf{u}\Sigma) = 0,$$
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\Sigma}\nabla P - \nabla \Phi + \frac{\mathbf{f}}{\Sigma}.$$

- Viscous forces $f \propto \nu = \nu_0 \times 10^{-5}$, pressure $P = c_s^2 \Sigma$ with $c_s^2 = 0.05^2 GM_*/r$ and total potential $\Phi$ includes disc potential $\Phi_d$:

$$\Phi_d = -\int \frac{G\Sigma(r', \phi')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\phi - \phi') + \epsilon_g^2}} r' dr' d\phi',$$

with $\epsilon_g = 0.015 r'$.

# Model equations

- Self-gravity charaterised by Toomre parameter

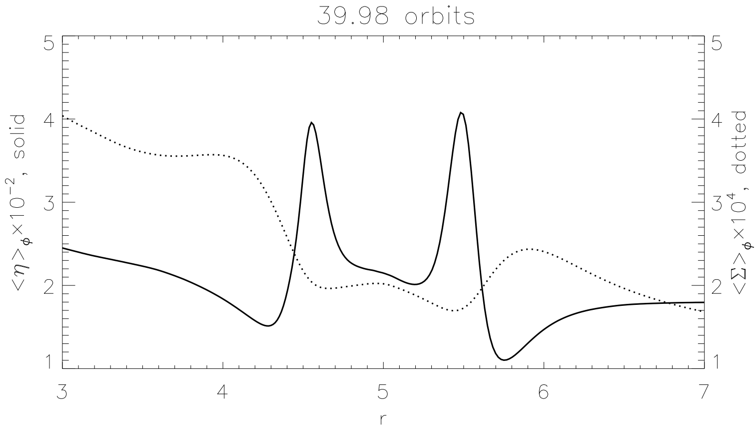
$$Q = \frac{c_s \kappa}{\pi G \Sigma}$$

  $\kappa^2 = 2\Sigma\Omega\eta$ is the epicycle frequency, where $\Omega = u_\phi / r$ and $\eta$ is potential vorticity.

- Smaller $Q$ means stronger self-gravity.

# Unstable disc profiles associated with giant planets

- The planet opens a gap in surface density $\Sigma$ with associated vortensity $\eta$ (potential vorticity) profile that has sharp gradients:




- Local extrema in $\eta$ give rise to various unstable modes.

# Linearised equations

- Perturb the system, e.g. $\Sigma \to \Sigma + \delta\Sigma(r) \exp i(\sigma t + m\phi)$, and linearise to get

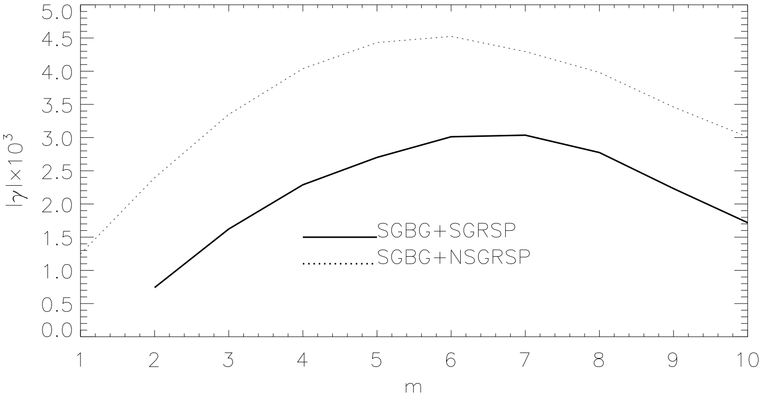
$$\frac{d}{dr}\left[\frac{r\Sigma}{\kappa^2-\bar{\sigma}^2}\left(c_s^2\frac{dW}{dr}+\frac{d\delta\Phi}{dr}\right)\right]+\left[\frac{2m}{\bar{\sigma}}\left(\frac{\Sigma\Omega}{\kappa^2-\bar{\sigma}^2}\right)\frac{dc_s^2}{dr}-r\Sigma\right]W$$
$$+\left[\frac{2m}{\bar{\sigma}}\frac{d}{dr}\left(\frac{\Sigma\Omega}{\kappa^2-\bar{\sigma}^2}\right)-\frac{m^2\Sigma}{r(\kappa^2-\bar{\sigma}^2)}\right]\left(c_s^2W+\delta\Phi\right)=0.$$

- $\bar{\sigma} = \sigma + m\Omega$, $W = \delta\Sigma/\Sigma$ and:

$$\delta\Phi = -G\int K_m(r,\xi)\Sigma(\xi)W(\xi)\xi d\xi.$$

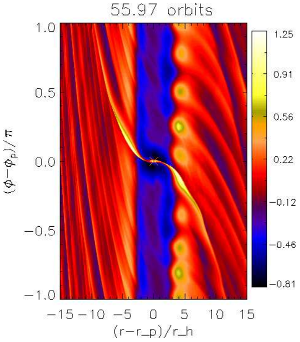
# Numerical solutions

Growth rate $|\gamma|$ as a function of azimuthal wave-number $m$:

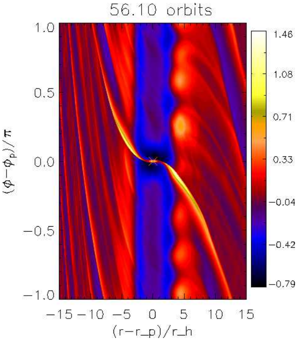



Solid: with self-gravity, dotted: without self-gravity (set $\delta\Phi = 0$ in linearised equations). Modes have co-rotation at vortensity minimum (where $\Re(\bar{\sigma}) = 0$).

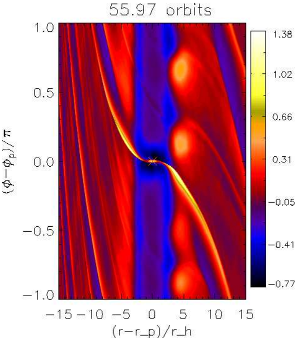
# Vortex formation and self-gravity




(a) $Q_m = 2$



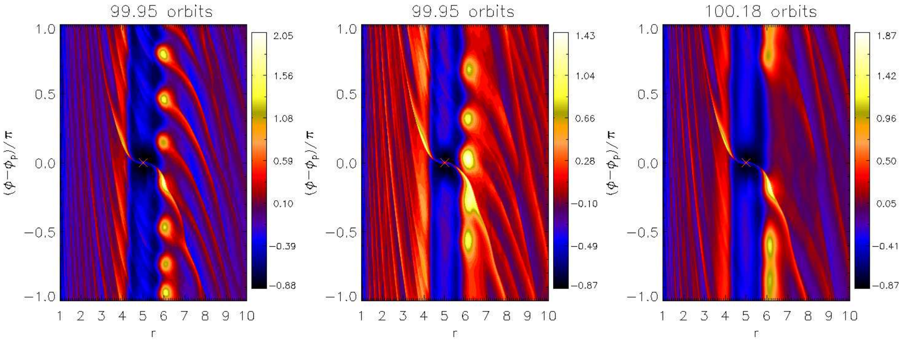

(b) $Q_m = 4$




(c) $Q_m = 8$

- Fewer (and larger) vortices as self-gravity is decreased.

# Vortex evolution and self-gravity

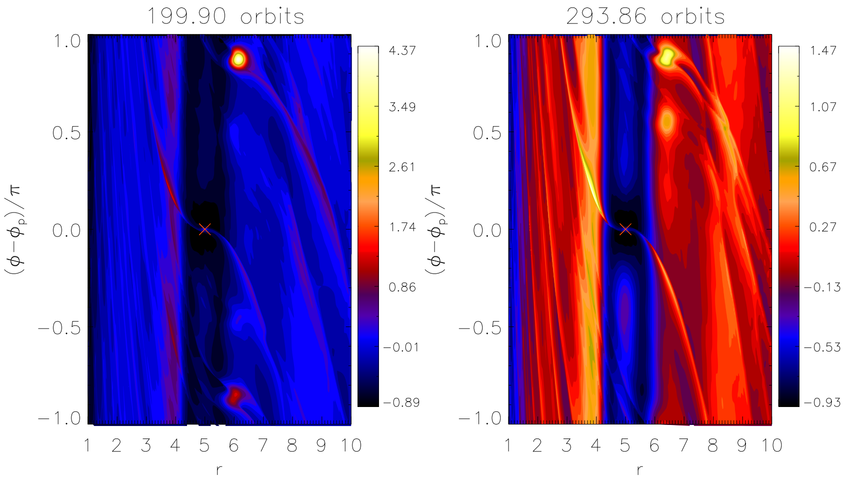



(a) $Q_m = 2.5$ (b) $Q_m = 3.5$ (c) $Q_m = 4.0$

- Vortex merging on dynmical time-scales when self-gravity is weak.

# Vortex evolution and self-gravity

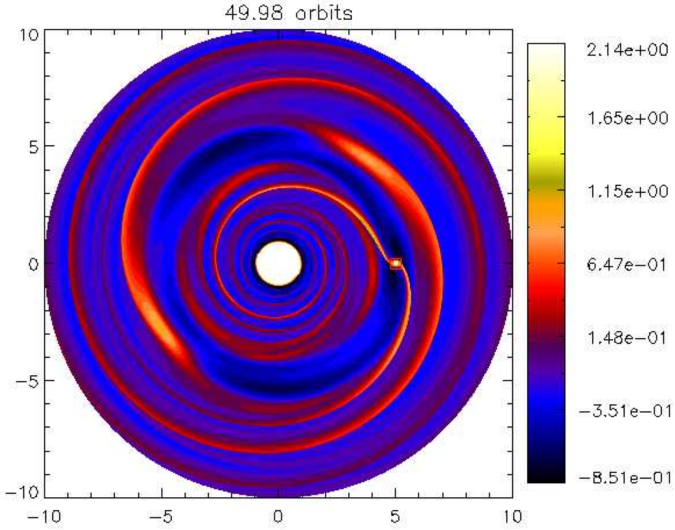
A case with $Q_m = 3$, or $M_d = 0.032M_*$:




- Vortex mass can be $\sim 1/3$ Saturn.

# Strong self-gravity

A case with $Q_m = 1.5$. Co-rotation radius at $r \simeq 5.5$, local vortensity *maximum*.



# Summary

- Self-gravity affects onset of instability at gap edge, higher $m$ modes preferred as SG becomes important.
- Self-gravity delays vortex merging. Final configuration without SG is a single large vortex. With SG, final vortex is compact and has local scale.
- When self-gravity is strong, get different type of mode altogehter: $m = 2$ global spirals. Sometimes have strong affects on planet migration.