Large-scale hydrodynamic instabilities in protoplanetary disks

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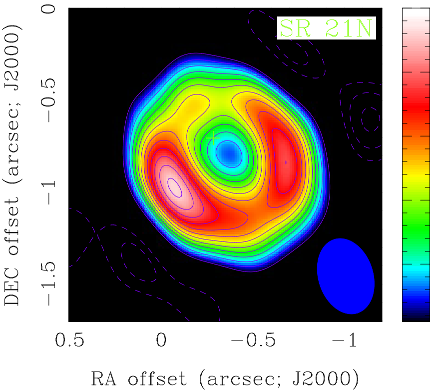
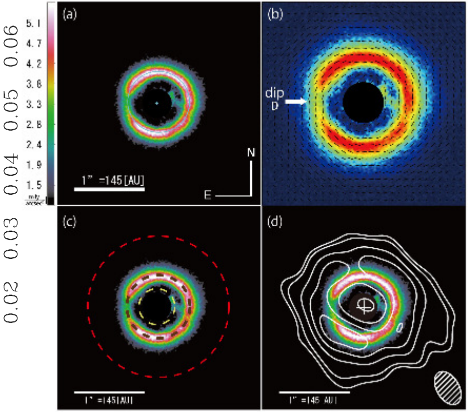
Observational motivation



(Brown et al., 2009)

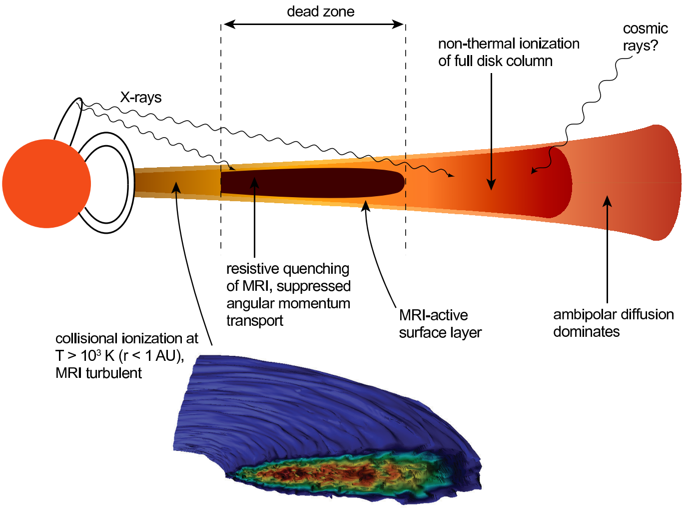


(Mayama et al., 2012)

Theoretical motivations

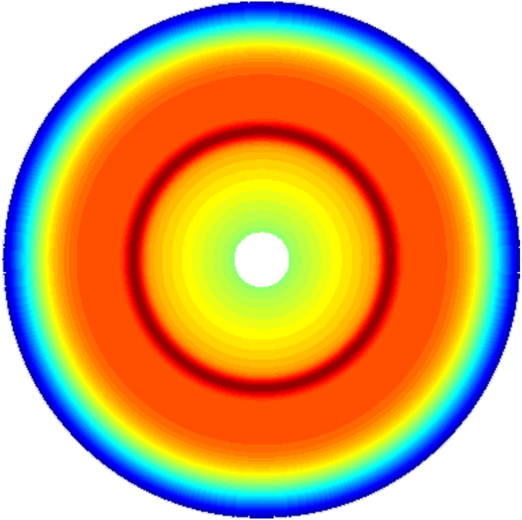
- Angular momentum transport: by vortices and non-local transport by waves
- Dust concentration by vortices $\longrightarrow$ planetesimal formation
- Modifying planet migration
- Instabilities may be naturally associated with disk structure
  e.g. planet gaps and 'dead zones' $\longrightarrow$ localized radial gradients

Theoretical motivations



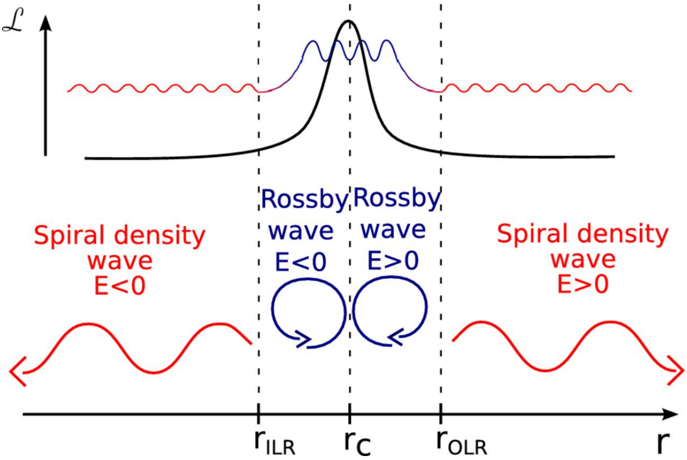
(Armitage, 2011)

Toy model: axisymmetric over-dense ring



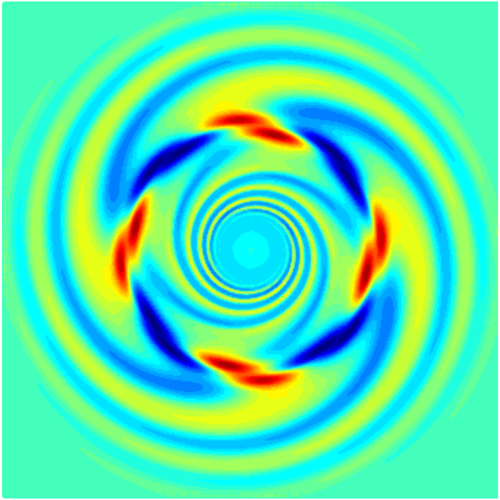
Rossby wave instability

- Kelvin-Helmholtz instability in a rotating disk (Lovelace et al., 1999)
- Thin-disk version of the Papaloizou-Pringle instability (Papaloizou & Pringle, 1985)



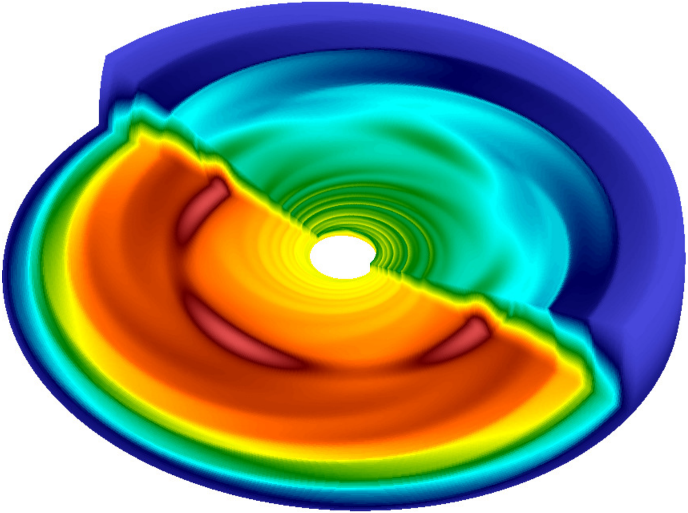
(Meheut et al., 2013)

Non-linear examples



ATHENA code: 3D disk in a Cartesian box

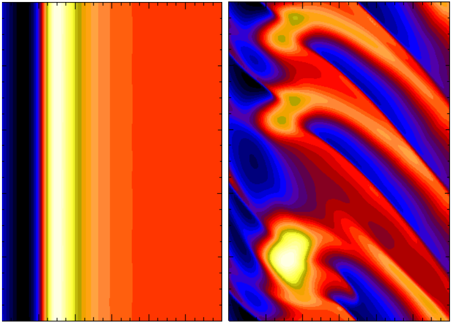
Non-linear examples



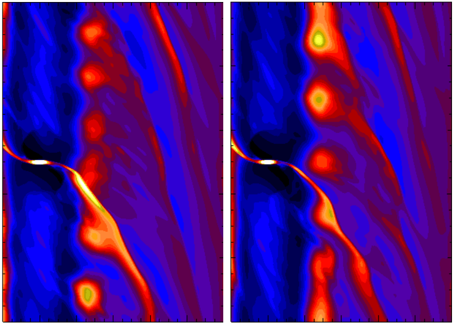
ZEUS code: 3D self-gravitating adiabatic disk

Non-linear examples

PLUTO code



3D disk with viscosity jump in radius

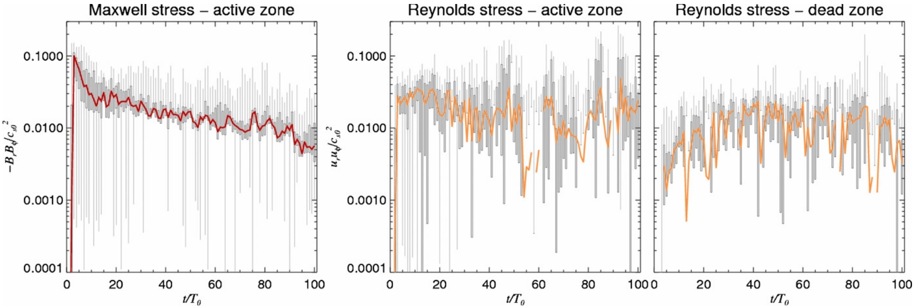


3D self-gravitating disk-planet simulation

[Note: global simulations plotted in a box ($r \to x$, $\phi \to y$)]

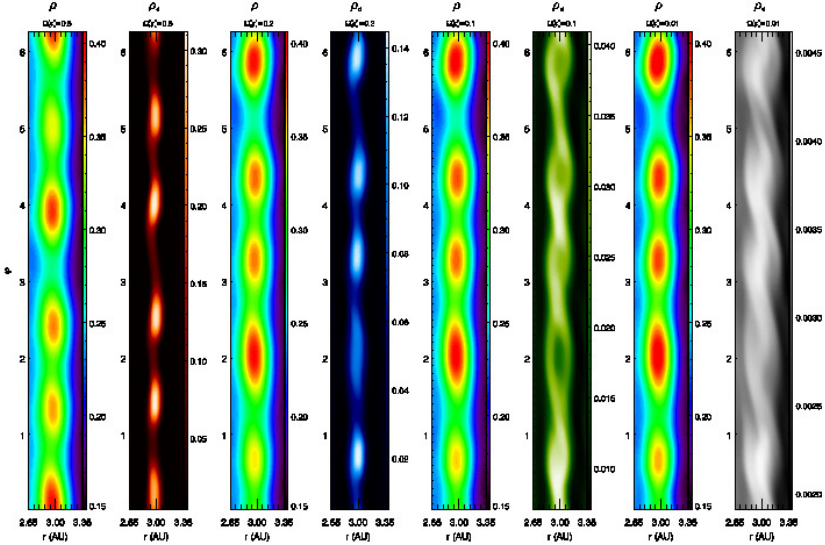
Application I: angular momentum transport

Lyra & Mac Low (2012): non-ideal MHD simulation with jump in resistivity to mimic the dead zone/active zone boundary → vortex formation in DZ



Application II: planetesimal formation

Meheut et al. (2012): add dust to RWI-unstable disk



Starting point: linear stability

Linear problem by Lovelace et al. (1999):

adiabatic non-self-gravitating 2D disk

Recent generalizations:

- Self-gravity 2D (Lin & Papaloizou, 2011a,b; Lovelace & Hohlfeld, 2013)
- Magnetic fields 2D (Yu & Li, 2009; Yu & Lai, 2013)
- Isothermal 3D (Meheut et al., 2012)

This talk:

- Polytropic 3D (Lin, 2012a, 2013a)
- Adiabatic 3D (Lin, 2013b)

Starting point: linear stability

After some manipulation, we have the basic equation for $\chi(=\delta p/\rho)$ as

$$\left[\frac{\partial}{\partial r}\left(a_{rr}\frac{\partial}{\partial r}+a_{rz}\frac{\partial}{\partial z}+b_r\right)+\frac{\partial}{\partial z}\left(a_{zz}\frac{\partial}{\partial z}+a_{rz}\frac{\partial}{\partial r}+b_z\right)+d_r\frac{\partial}{\partial r}+d_z\frac{\partial}{\partial z}+f\right]\chi=0,$$

with

$$a_{rr}=\frac{\rho\sigma r}{D}\left(1+\frac{\mu g_r^2}{DH}\right),\qquad a_{zz}=\frac{\rho r}{\sigma}\left(1+\frac{\mu g_z^2}{\sigma^2 H}\right),\qquad a_{rz}=\frac{\mu\rho g_r g_z r}{DH\sigma},$$

$$b_r=\frac{\mu\rho g_r}{DH}\left(\sigma r-\frac{2m\Omega g_r}{D}\right)-\frac{2m\Omega\rho}{D},\qquad b_z=\frac{\mu\rho g_z r}{\sigma H}\left(1-\frac{2m\Omega g_r}{\sigma D r}\right),$$

$$d_r=\frac{m\kappa^2\rho}{2\Omega D}-\left(\sigma r-\frac{m\kappa^2 g_r}{2\Omega D}\right)\frac{\mu\rho g_r}{DH},\qquad d_z=-\left(\sigma r-\frac{m\kappa^2 g_r}{2\Omega D}\right)\frac{\mu\rho g_z}{\sigma^2 H},$$

$$f=-\frac{m^2\sigma\rho}{Dr}-\left(\sigma r-\frac{m\kappa^2 g_r}{2\Omega D}\right)\left(1-\frac{2m\Omega g_r}{D\sigma r}\right)\frac{\mu\rho}{H}+\frac{(\mu+1)\sigma r\rho}{c^2},$$

(Kojima et al., 1989)

Linear problem for 3D polytropic disks ($p \propto \rho^{1+1/n}$)

1. Steady, axisymmetric, vertically hydrostatic density bump at $r = r_0$
2. Perturb fluid equations, e.g. $\rho \to \rho + \delta\rho(r,z)\exp i(m\phi + \sigma t)$
3. Combine linear equations to get equation for $W \equiv \delta p/\rho$:

$$L(r, z; \sigma)W = 0.$$

- $W \to$ eigenfunction; $\sigma \to$ eigenvalue
- Note: σ appears through $\bar{\sigma} = \sigma + m\Omega(r)$
- RWI: $\mathrm{Re}[\bar{\sigma}(r_0)] \simeq 0$ *and* $\left.\frac{d\eta}{dr}\right|_{r_0} \simeq 0$ ($\eta = \kappa^2/2\Omega\Sigma$ is the vortensity)

Very complicated PDE even for numerical work!

Application of orthogonal polynomials

$L(r, z; \sigma)$ only depends on z through $\rho(r, z)$. For thin polytropic disks:

$$\rho(r, z) = \rho_0(r)\left[1 - \frac{z^2}{H^2(r)}\right]^n .$$

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Ansatz:

$$W(r, z) = \sum_{l=0}^{\infty} W_l(r)\mathcal{C}_l^{\lambda}(z/H),$$

where $\mathcal{C}_l^{\lambda}(x)$ are Gegenbauer polynomials (generalization of Legendre and Chebyshev polynomials). Then

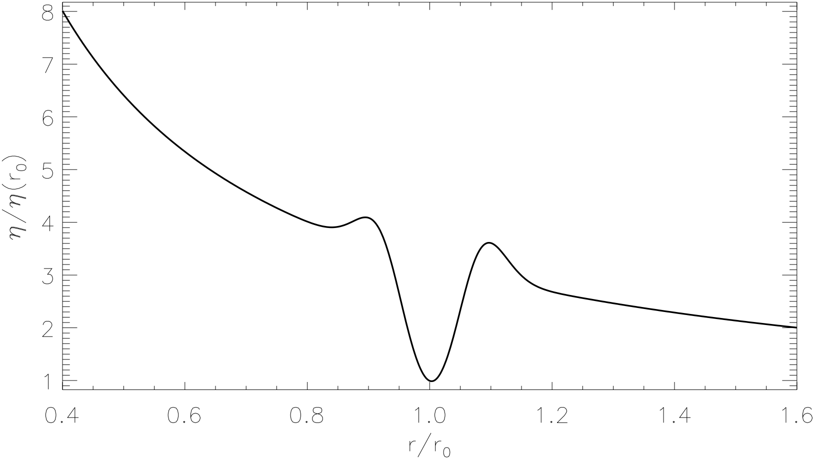
$$L(r, z; \sigma)W = 0 \to A_l(W_l) + B_l(W_{l-2}) + C_l(W_{l+2}) = 0,$$

where differential operators A_l, B_l and C_l only depend on r and σ

$\to$ vertical dependence *exactly* removed, but this is a lot of work!

Example problem

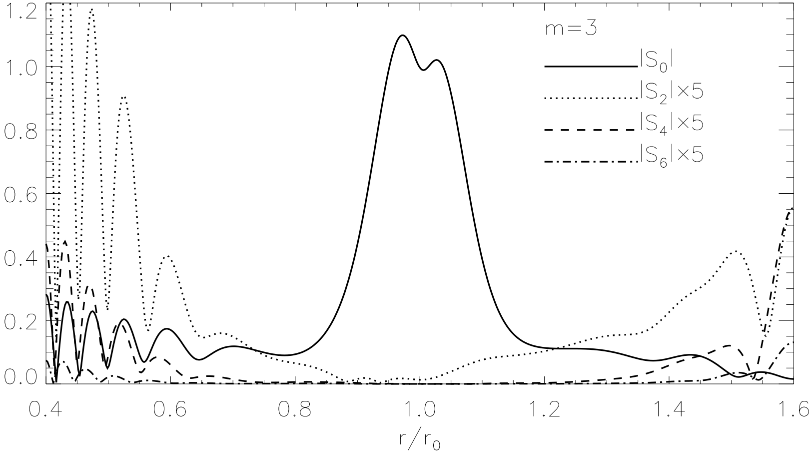
$n = 1.5$ polytrope with a surface density bump



Recall $\eta = \frac{1}{r\Sigma}\frac{d}{dr}\left(r^2\Omega\right)$ is the potential vorticity (note: RWI for PV minima only)

Example solution

$$W(r,z) = W_0(r) + W_2(r)\mathcal{C}_2^{\lambda}(z/H) + \cdots$$

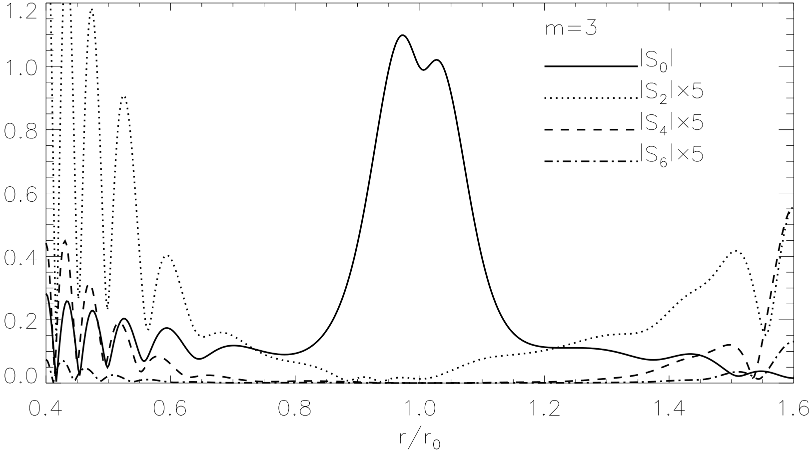


Growth rate $\sim 0.1\Omega$, same as 2D ($l_{\max} \equiv 0$).

Instability is 2D.

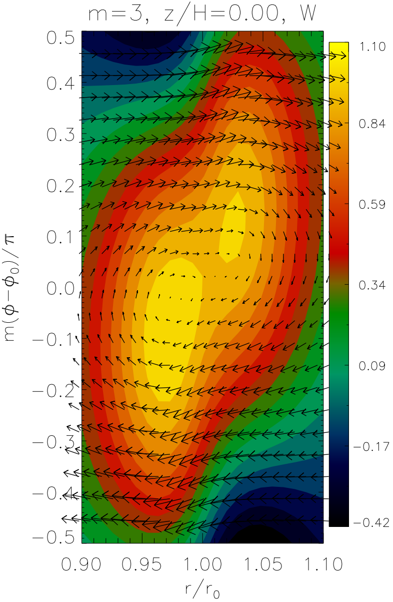
Example solution

$$W(r,z) = W_0(r) + W_2(r)\mathcal{C}_2^{\lambda}(z/H) + \cdots$$



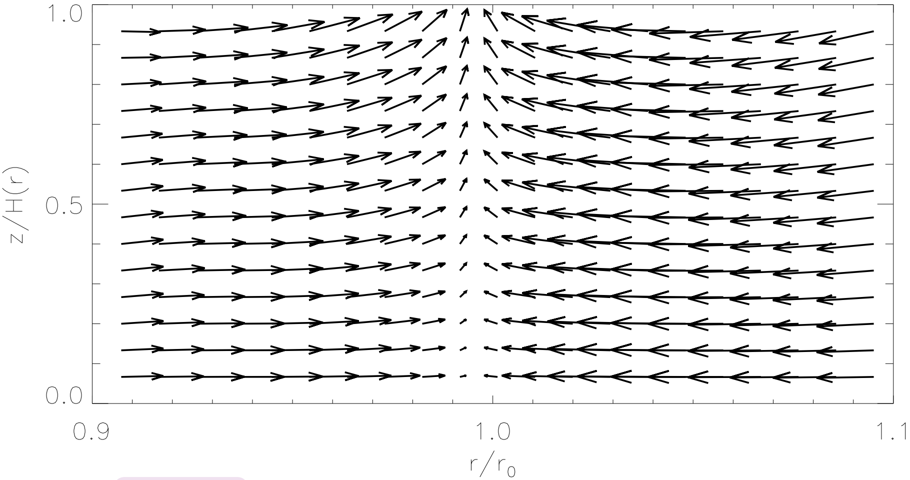
Note $\delta v_z = \mathrm{i}\left(\partial W/\partial z\right)/\bar{\sigma}$ but $|\bar{\sigma}| \sim 0$ at $r \sim r_0$

Horizontal flow



Anti-cyclonic motion associated with over-density

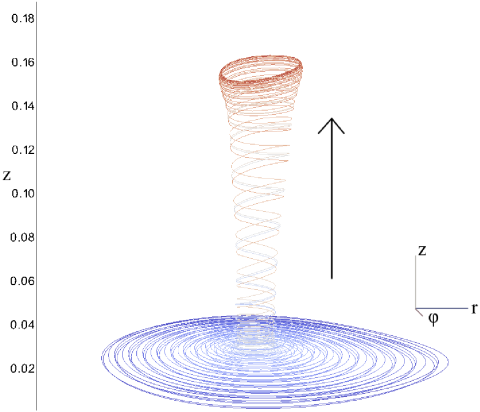
Vertical motion



Motion is upwards at (r_0, ϕ_0, z).

Vertical motion

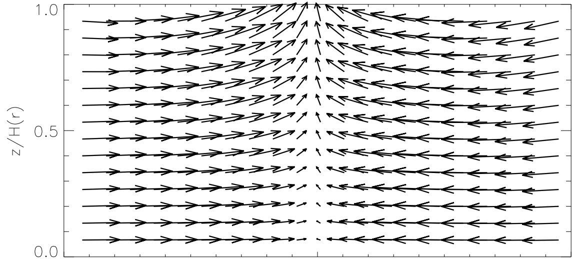
Upward motion seen in non-linear hydrodynamic simulations of Meheut et al. (2012):



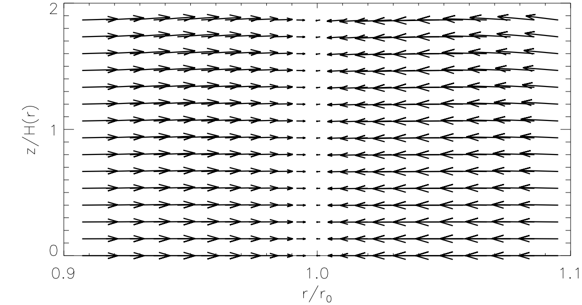
Meheut et al. (2012) → mm dust lifted to disk surface

Back to linear problem: equation of state

Magnitude of vertical motion decreases with increasing n (more compressible)



← $n = 1.0$ polytrope



← vertically isothermal disk ($n = \infty$, special treatment with Hermite polynomials)

Extension to adiabatic 3D disks

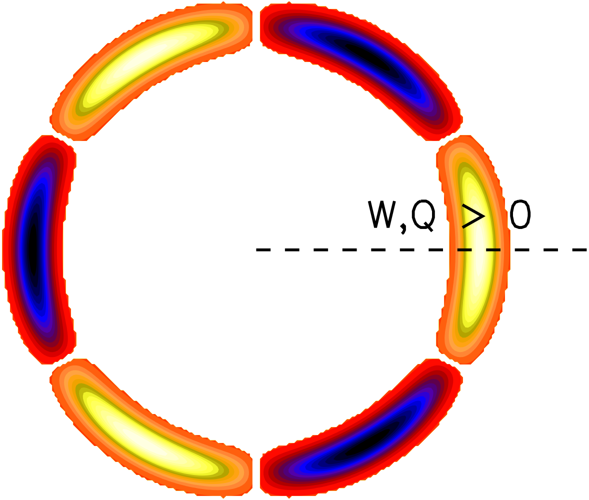
- $p \propto \rho^{\Gamma}$ in basic state only
- Energy equation $Ds/Dt = 0$, $s \equiv p/\rho^{\gamma} \propto \rho^{\Gamma-\gamma}$
- $\gamma \geq \Gamma \geq 1$, density bump $\rightarrow$ entropy dip

$$V_1 W + \bar{V}_1 Q = 0$$
$$V_2 W + \bar{V}_2 Q = 0$$

- $W = \delta p/\rho \rightarrow$ pressure perturbation
- $Q = c_s^2 \delta\rho/\rho \rightarrow$ density perturbation
- $S \equiv W - Q \rightarrow$ entropy perturbation

What should we look for?

$$\bar{S} \equiv Q - \frac{\gamma}{\Gamma} W = \left(1 - \frac{\gamma}{\Gamma}\right) W - S$$



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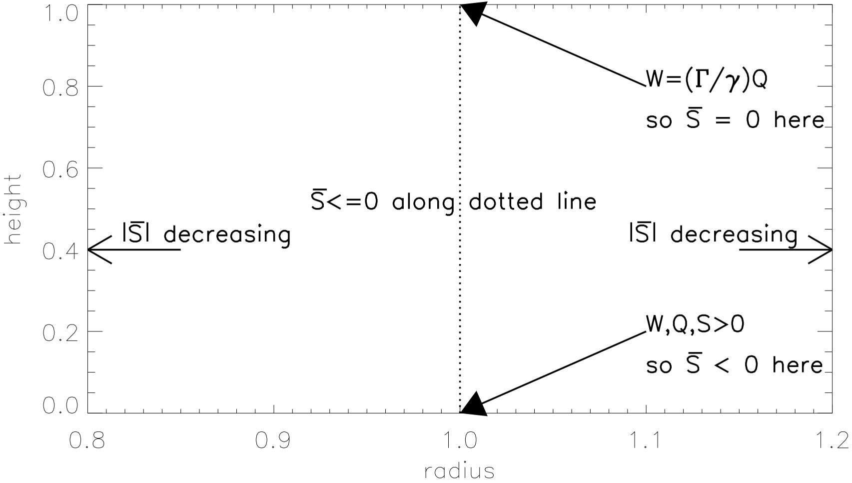
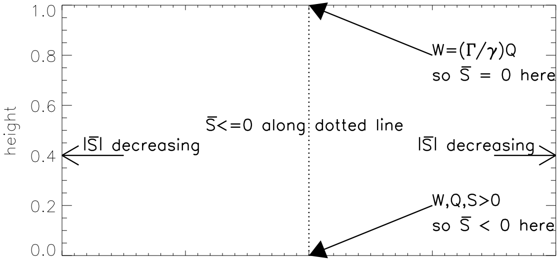
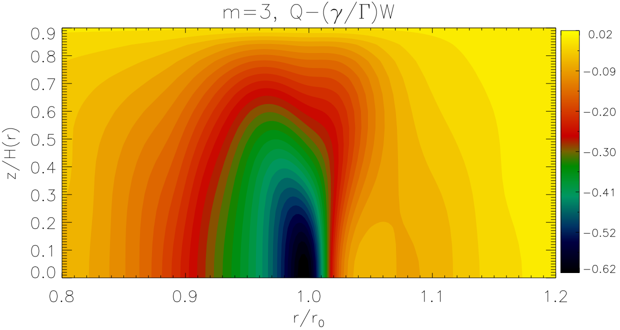


Expectation and reality





- $\bar{S} \Rightarrow \delta v_z$
- $\nabla \bar{S} \Rightarrow (\nabla \times \boldsymbol{\delta v})_\phi$

PDE eigenvalue problem: numerical approach

Finite-difference in r, pseudo-spectral in $Z \equiv z/H$:

$$W(r_i, z) \equiv W_i(Z) = \sum_{k=1}^{N_z} w_{ki} \psi_k(Z/Z_{\max})$$

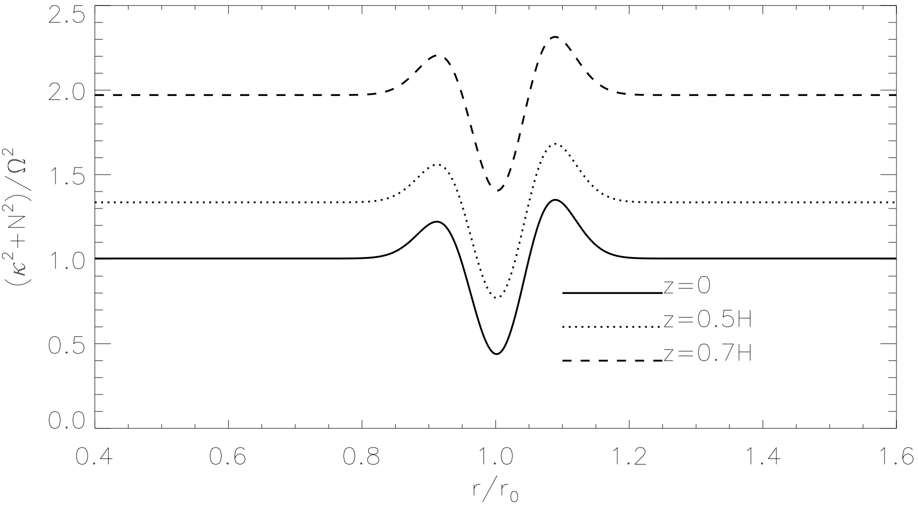
$$[V_1 - \bar{V}_1(\bar{V}_2^{-1} V_2)]W = 0 \rightarrow \mathbf{U}(\sigma)\mathbf{w} = \mathbf{0}$$

- $\mathbf{U} \rightarrow$ matrix representation of PDE operator
- $\mathbf{w} \rightarrow$ vector to store the w_{ki}
- Vertical boundary condition: $\Delta P = 0$, $\delta v_z = 0$ or $\delta v_\perp = 0$ at $Z = Z_{\max}$
- *Much easier* to derive and implement than previous method, and allows for different vertical b.c., but need an accurate initial guess for σ

See Lin (2013a) for method recipe.

Non-homentropic example

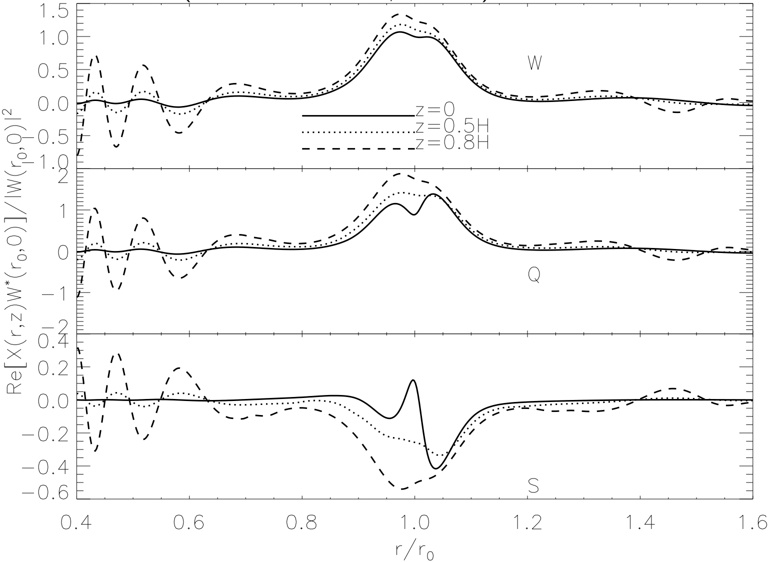
$\Gamma = 1.67$, $\gamma = 2.5$



N is the buoyancy frequency

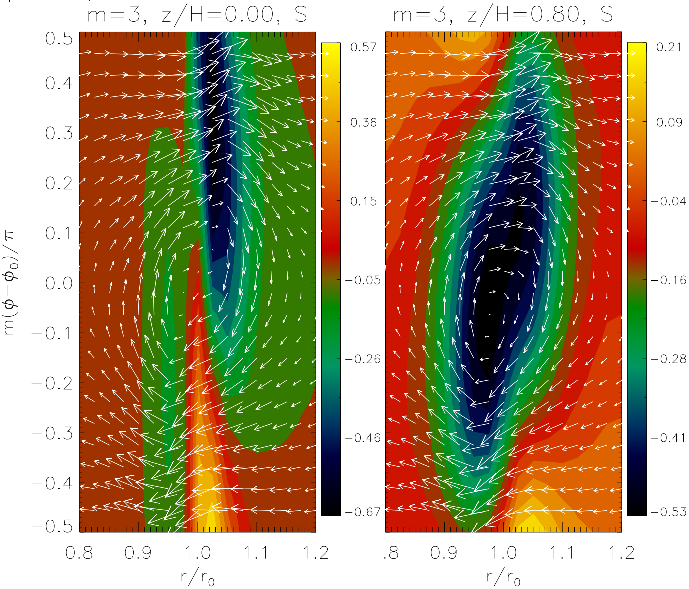
Non-homentropic example

$\Gamma = 1.67$, $\gamma = 2.5$, $m = 3$ along $\phi = \phi_0$.
Growth rate $0.1099\Omega_0$ (cf. $0.1074\Omega_0$ for $\gamma = 1.67$)



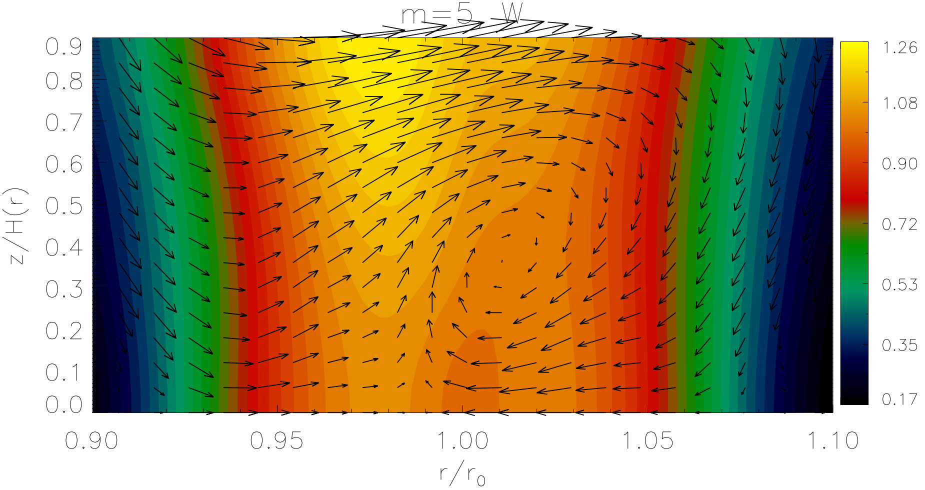
Entropy perturbation

$\Gamma = 1.67$, $\gamma = 2.5$, $m = 3$



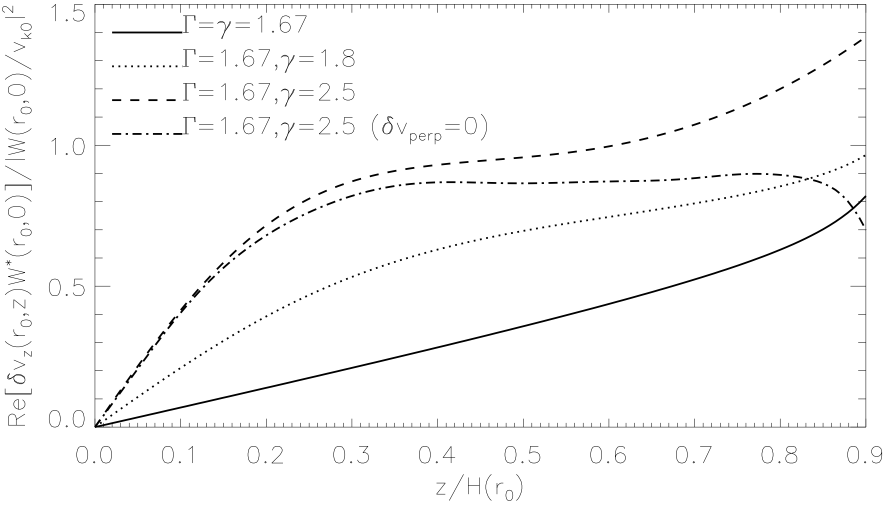
Meridional vortical motion

$\Gamma = 1.67$, $\gamma = 2.5$, $m = 5$ along $\phi = \phi_0$



Vertical motion

Fix $\Gamma = 1.67$, vary γ, plot δv_z along (r_0, ϕ_0, z).



Vertical motion

Kato (2001):

$$\delta v_z \sim -\frac{\nu}{N_z^2}\frac{\partial W}{\partial z} - \nu\rho\left(\frac{\partial p}{\partial z}\right)^{-1} W, \quad N_z^2 \neq 0$$

at co-rotation radius, and ν here is the growth rate. Compared to

$$\delta v_z \sim -\frac{1}{\nu}\frac{\partial W}{\partial z}, \quad N_z^2 \equiv 0.$$

Notice for $N_z^2 \neq 0$

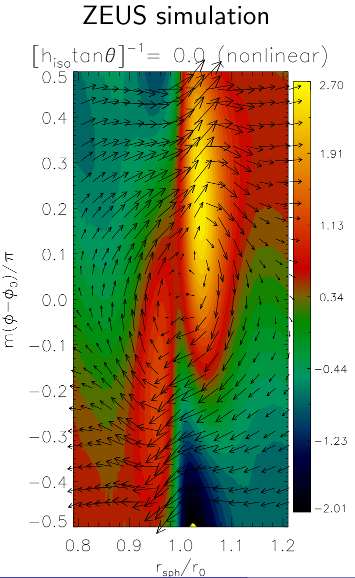
$$\frac{\text{pressure}}{\text{buoyancy}} \sim \frac{\Omega^2}{N_z^2}\frac{\partial \ln W}{\partial \ln z},$$

i.e. buoyancy dominates at large z as N_z^2 increases with height.

Origin of δv_z is different between homentropic and non-homentropic flow

Comparison with hydrodynamic simulations

- Isothermal disk, adiabatic evolution ($\Gamma \equiv 1$, $\gamma = 1.4$)



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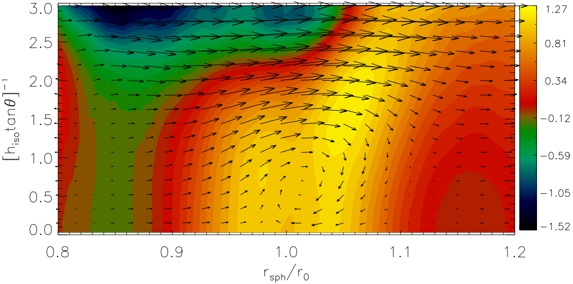
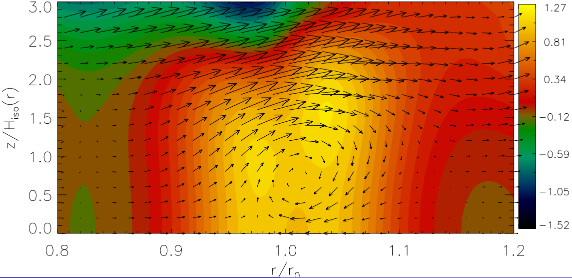


← ZEUS simulation

$$\mathrm{Re}(\sigma) = -0.99 m\Omega_0$$
$$\mathrm{Im}(\sigma) = -0.194\Omega_0$$



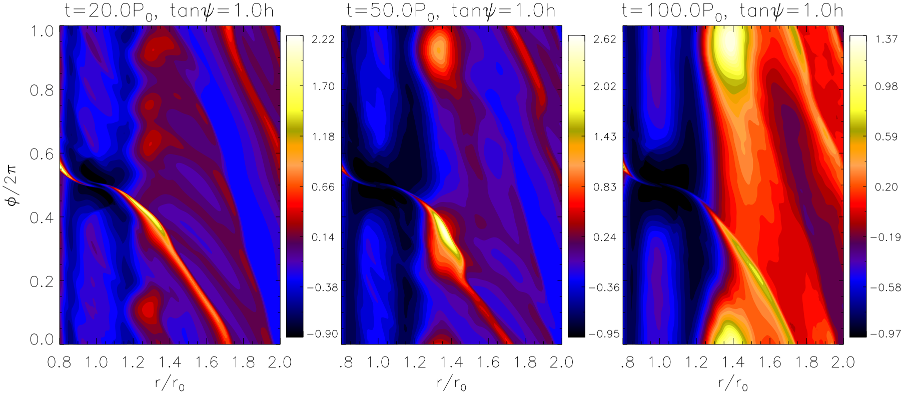
← linear code

$$\mathrm{Re}(\sigma) = -0.9896 m\Omega_0$$
$$\mathrm{Im}(\sigma) = -0.1937\Omega_0$$

Vortex-formation in layered-accretion disks?

PLUTO disk-planet experiments

Imposed viscosity $\alpha \sim 10^{-4}$ everywhere



[Lin and Umurhan (in preparation)]

Vortex-formation in layered-accretion disks?

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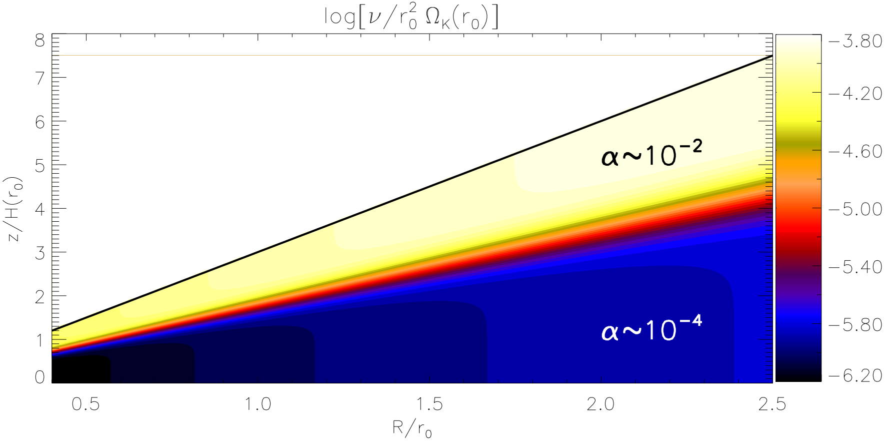
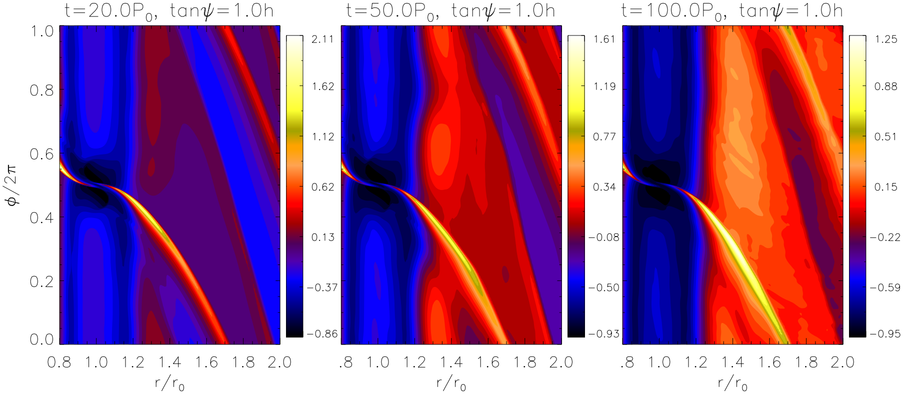


[Lin and Umurhan (in preparation)]

Vortex-formation in layered-accretion disks?

PLUTO disk-planet experiments

$\alpha \sim 10^{-4}$ in bulk of the disk, $\alpha \sim 10^{-2}$ in atmosphere



[Lin and Umurhan (in preparation)]

Self-gravity

- Vortices are over-dense blobs
- Vortensity η and Toomre Q_T are related: $Q_T = (c_s/\pi G)\sqrt{2\Omega\eta/\Sigma}$
- *Stabilization* of low m vortex modes, see Lin & Papaloizou (2011a) for formal proof and linear calculations

The 2D linear problem with self-gravity:

$$L(S) = \delta\Sigma, \quad S = c_s^2\delta\Sigma/\Sigma + \delta\Phi.$$

$$\int rS^* L(S)dr = \int rS^*\delta\Sigma dr = \mathrm{energy}.$$

For modes associated with vortensity extrema:

$$\underbrace{\int \frac{m|S|^2}{\bar{\sigma}}\frac{d}{dr}\left(\frac{1}{\eta}\right)dr}_{>0 \text{ for } \min(\eta) \text{ at } r=r_c \text{ (RWI)}} \sim \underbrace{\int rc_s^2\frac{|\delta\Sigma|^2}{\Sigma}dr}_{\text{thermal energy} > 0} + \underbrace{\int r\delta\Phi^*\delta\Sigma dr}_{\text{gravitational energy} < 0}$$

Balance does not work for strong SG (RHS < 0, gravitational disturbance)

Self-gravity

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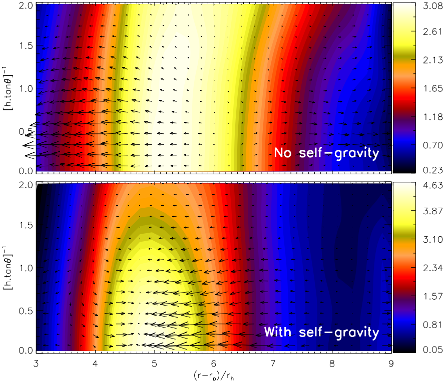
- *Stabilization* of low m vortex modes, see Lin & Papaloizou (2011a) for formal proof and linear calculations

Self-gravity in 3D $[\min(Q_T) = 8]$:

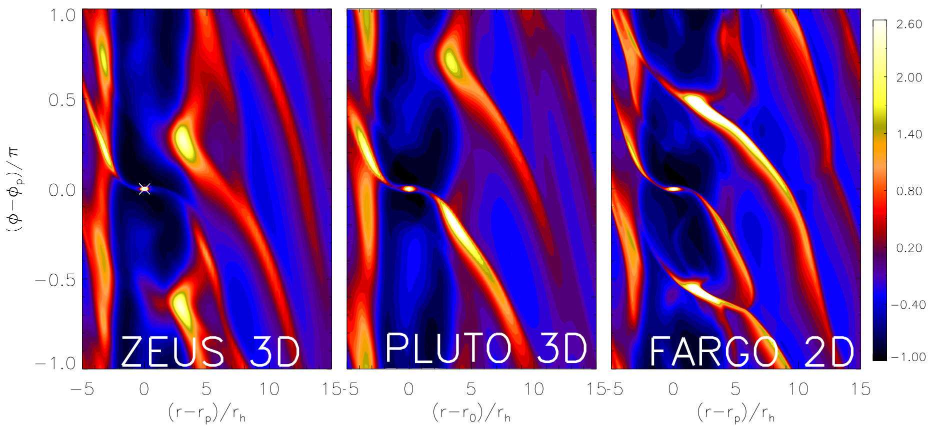


(Global 3D ZEUS simulations, Lin, 2012b). What about massive disks?

Gravitational edge instabilities

GI associated with gaps or edges even when Toomre stability criterion satisfied ($Q_T > 1$ everywhere)

- Lovelace & Hohlfeld (1978); Sellwood & Kahn (1991): galactic/stellar disks
- Meschiari & Laughlin (2008): gaps in gaseous protoplanetary disks
- Lin & Papaloizou (2011b): confirmation of GEI for planet gaps (PV max.)



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A necessary condition is

$$\Lambda = \beta \times \left| \frac{d^2}{dr^2} \underbrace{\left(\frac{\Omega\Sigma}{\kappa^2}\right)}_{\sim Q^{-1}} \right|_{\mathrm{edge}} > 1$$

$\rightarrow$ Don't need small Q_{edge}. See Lin & Papaloizou (2011b) for details.

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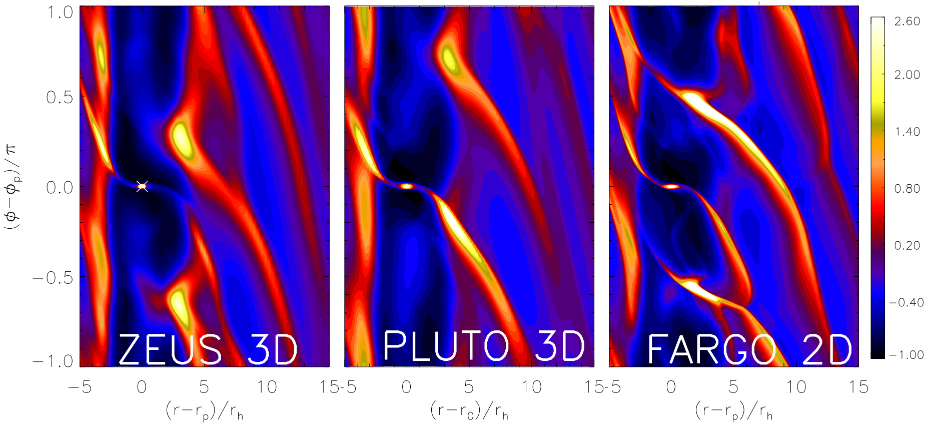
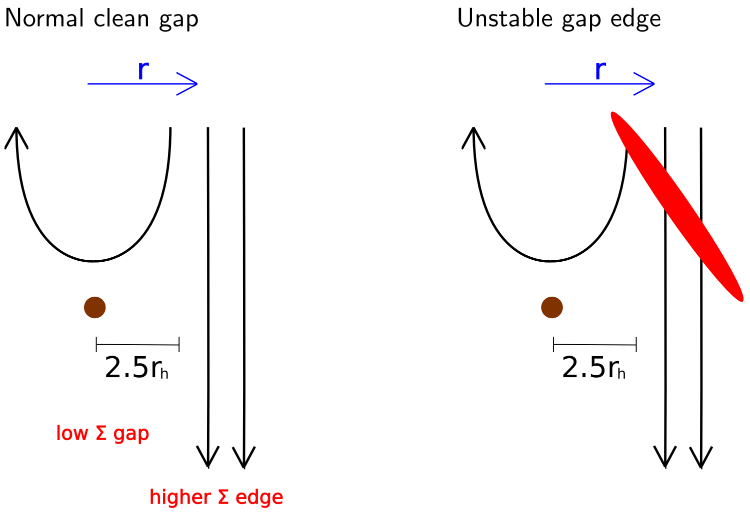


Influence of GEI on disk-planet torques

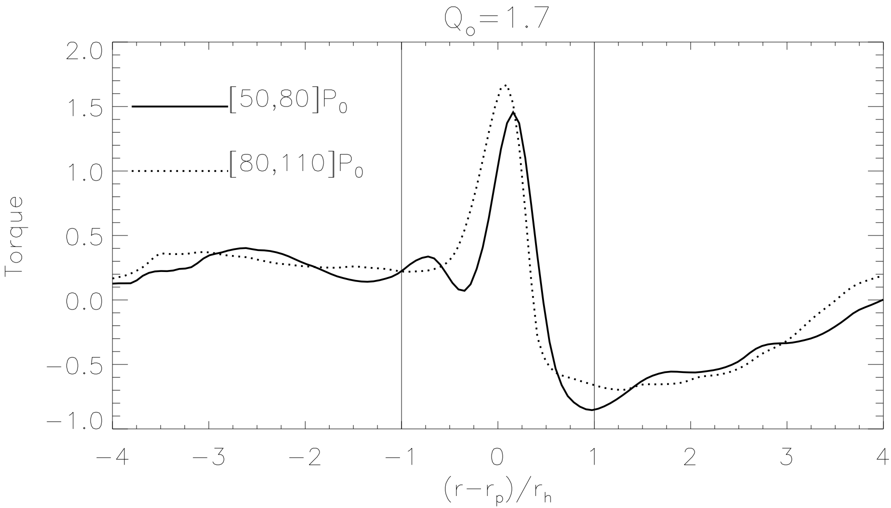
Spirals supply material to execute horseshoe turns ahead of planet



→ positive co-orbital torques

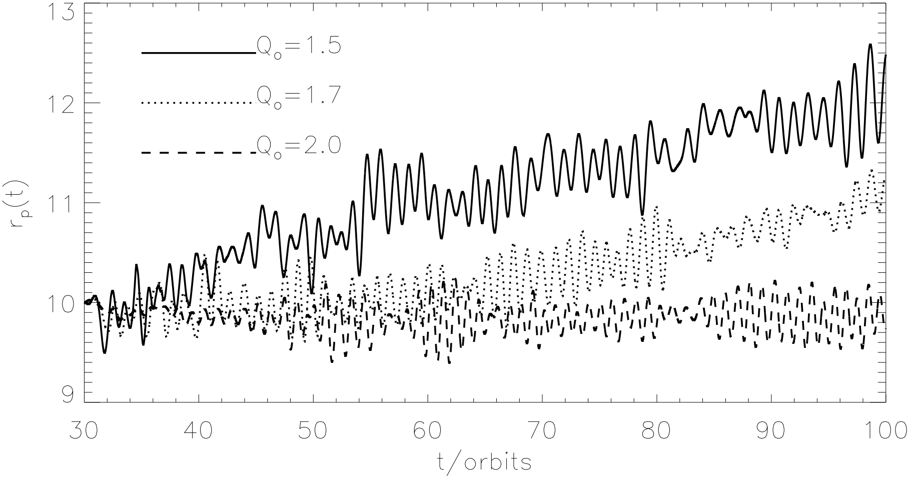
Influence of GEI on disk-planet torques

Spirals supply material to execute horseshoe turns ahead of planet



(Lin & Papaloizou, 2012)

Outward migration induced by an unstable gap

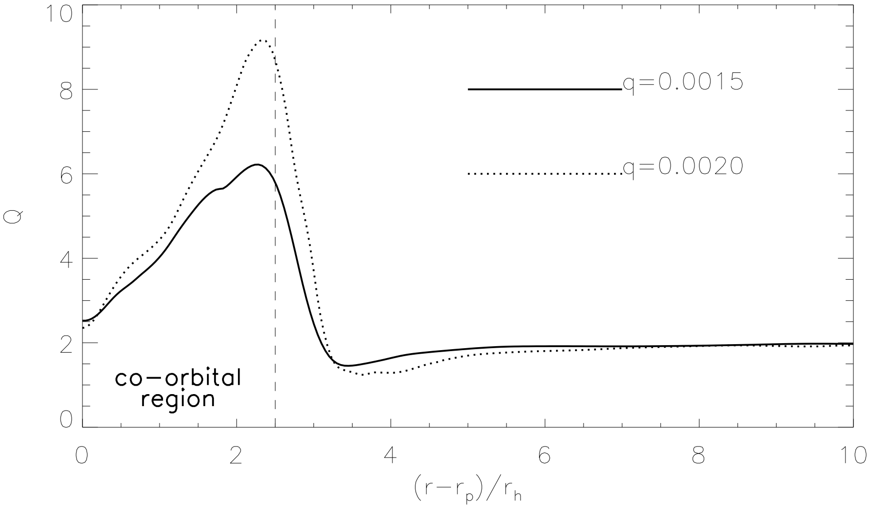


$Q_o = 1.5$ and $Q_o = 1.7$ have GEI, $Q_o = 2.0$ does not (Lin & Papaloizou, 2012)

Dependency on planet mass

Instability ↔ gap structure ↔ planet mass ↔ orbital migration

[2012 CITA summer student project (Cloutier and Lin, 2013, submitted)]



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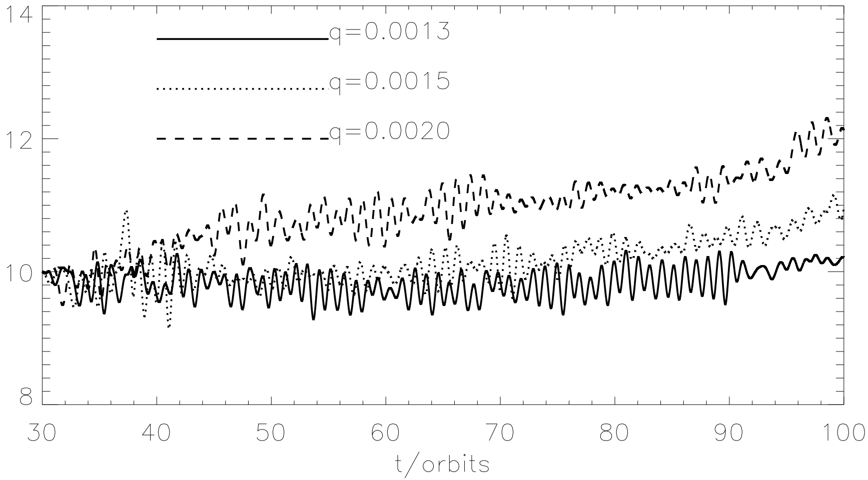
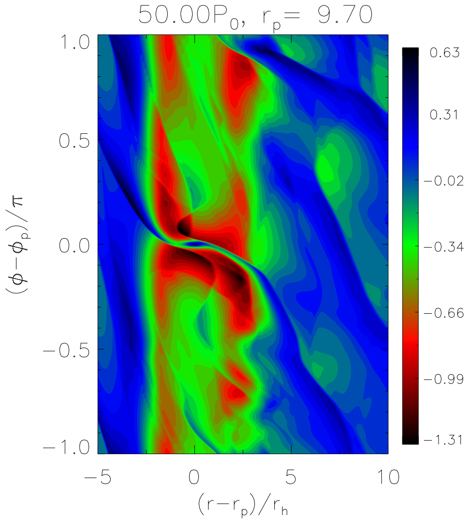


Torque balance?

Can positive torques counter-act inward type II migration → no migration?



Cloutier and Lin (2013, submitted)

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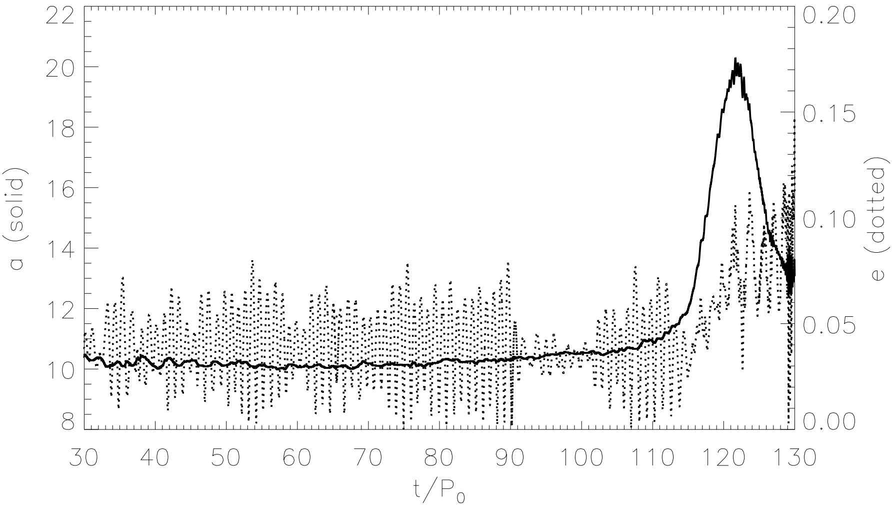
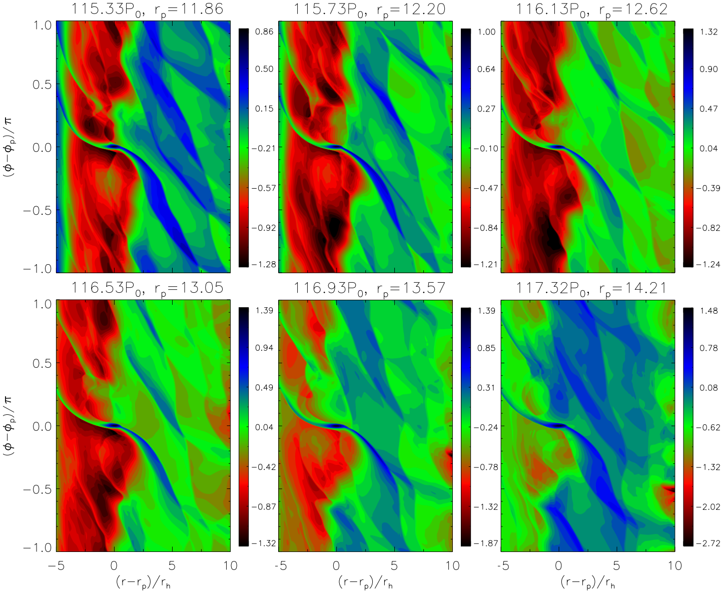


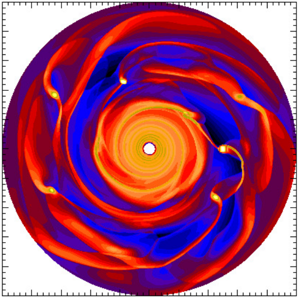
Cloutier and Lin (2013, submitted)

Type III migration triggered by the unstable gap



Cloutier and Lin (2013, submitted)

Wide-orbit giant planet formation by disk fragmentation



E.g. HR 8799bcd, Fomalhault b (?)

- Zhu et al. (2012); Vorobyov (2013): most clumps fall in, but occasionally can survive by opening gaps
- Our simulations → gap stability may be another issue
- Zhu et al.: additional clump formation along edge of a gap opened by a previous clump; Vorobyov: clump migrates outward

On the other hand:

- Move planets to large distances by inducing outward type III migration?

Future

- Gap formation/stability in non-isothermal disks (Lin and Cloutier, in preparation)

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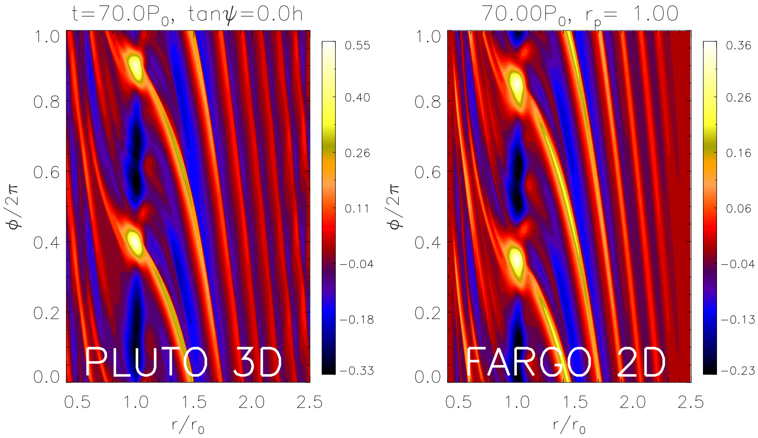
- Dead zone boundary GI (global transport)



Future

- Gap formation/stability in non-isothermal disks (Lin and Cloutier, in preparation)
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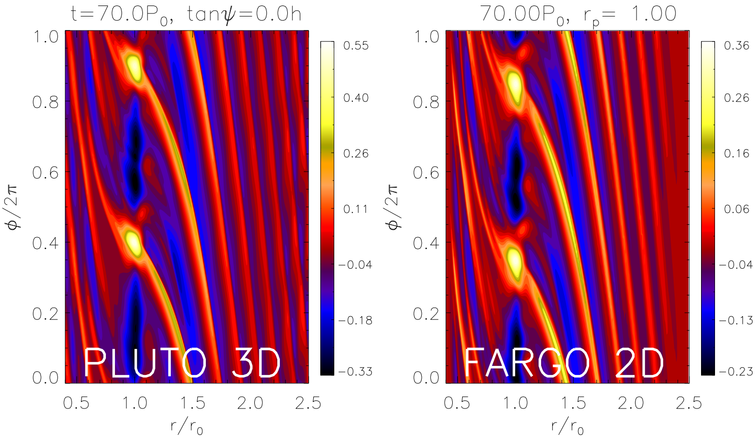

- Magneto-gravitational instabilities

References

Armitage P. J., 2011, ARAA, 49, 195

Brown J. M., Blake G. A., Qi C., Dullemond C. P., Wilner D. J., Williams J. P., 2009, ApJ, 704, 496

Kato S., 2001, PASJ, 53, 1

Kojima Y., Miyama S. M., Kubotani H., 1989, MNRAS, 238, 753

Lin M.-K., 2012a, ApJ, 754, 21

Lin M.-K., 2012b, MNRAS, 426, 3211

Lin M.-K., 2013a, MNRAS, 428, 190

Lin M.-K., 2013b, ApJ, in press

Lin M.-K., Papaloizou J. C. B., 2011a, MNRAS, 415, 1426

Lin M.-K., Papaloizou J. C. B., 2011b, MNRAS, 415, 1445

Lin M.-K., Papaloizou J. C. B., 2012, MNRAS, 421, 780

Lovelace R. V. E., Hohlfeld R. G., 1978, ApJ, 221, 51

Lovelace R. V. E., Hohlfeld R. G., 2013, MNRAS, 429, 529

Lovelace R. V. E., Li H., Colgate S. A., Nelson A. F., 1999, ApJ, 513, 805

Lyra W., Mac Low M.-M., 2012, ApJ, 756, 62

Mayama et al. 2012, ApJL, 760, L26

Meheut H., Keppens R., Casse F., Benz W., 2012, A&A, 542, A9

Meheut H., Lovelace R. V. E., Lai D., 2013, MNRAS, 430, 1988

Meheut H., Meliani Z., Varniere P., Benz W., 2012, A&A, 545, A134

Meheut H., Yu C., Lai D., 2012, MNRAS, 422, 2399

Meschiari S., Laughlin G., 2008, ApJL, 679, L135

Papaloizou J. C. B., Pringle J. E., 1985, MNRAS, 213, 799

Sellwood J. A., Kahn F. D., 1991, MNRAS, 250, 278

Vorobyov E. I., 2013, A&A, 552, A129

Yu C., Lai D., 2013, MNRAS, 429, 2748

Yu C., Li H., 2009, ApJ, 702, 75

Zhu Z., Hartmann L., Nelson R. P., Gammie C. F., 2012, ApJ, 746, 110