Mon. Not. R. Astron. Soc. **000**, 1–7 (1994) Printed 2 March 2002 (MN LATEX style file v1.4)

Dust envelopes around RV Tauri stars

A. V. Raveendran
Indian Institute of Astrophysics, Bangalore 560034, India

Accepted 1988 December 15. Received 1988 December 14; in original form 1988 October 11

ABSTRACT
In the *IRAS* [12]–[25], [25]–[60] colour–colour diagram, RV Tauri stars are found to populate cooler temperature regions ($T < 600\,\mathrm{K}$), distinctly different from those occupied by the oxygen and carbon Miras. The *IRAS* fluxes are consistent with the dust density in the envelope varying as the inverse square of the radial distance, implying that the grain formation processes in these objects are most probably continuous and not sporadic. It is found that the spectroscopic subgroups A and B are well separated in the far-infrared two-colour diagram, with group B objects having systematically cooler dust envelopes. We interpret this as being due to a difference in the nature of grains, including the chemical composition, in the two cases.

Key words: circumstellar matter – infrared: stars.

1 INTRODUCTION

It has been well established that RV Tauri variables possess infrared emission far in excess of their expected blackbody continuum, arising from their extended cool dust envelopes (Gehrz & Woolf 1970; Gehrz 1972; Gehrz & Ney 1972). Recently, Lloyd Evans (1985) and Goldsmith et al. (1987) have given detailed descriptions of the near-infrared properties of RV Tauri stars. In this paper we present an analysis of the *IRAS* data of RV Tauri stars with the help of the far-infrared two-colour diagram and a grid computed using a simple model of the dust envelope. Such two-colour plots have already been employed extensively by several investigators to study the circumstellar envelopes around oxygen-rich and carbon-rich objects which are in the late stages of stellar evolution (Hacking et al. 1985; Zuckerman & Dyck 1986; van der Veen & Habing 1988; Willems & de Jong 1988).

Table 1 summarizes the basic data on the 17 objects detected at 60 μm. Apart from the *IRAS* identification and the flux densities at 12-, 25-, 60- and 100-μm wavebands, it gives the spectroscopic groups of Preston et al. (1963), the light-curve classes of Kukarkin et al. (1969) and the periods of light variation. The list, which contains about 20 per cent of all the known RV Tauri stars, is essentially the same as that given by Jura (1986). The spectroscopic subgroups are from either Preston et al. (1963) or Lloyd Evans (1985).

2 DESCRIPTION OF THE ENVELOPE MODEL

If we assume that the dust grains in the envelope are predominantly of the same kind and are in thermal equilibrium, the luminosity at frequency ν in the infrared is given by

$$L(\nu) = \int_{\text{envelope}} \rho(r) Q_{\text{abs}}(\nu) B[\nu, T_{\text{g}}(r)] \exp[-\tau(\nu, r)] \,\text{dV}, \qquad (1)$$

where $Q_{\text{abs}}(\nu)$ is the absorption efficiency at frequency ν, $\rho(r)$ is the dust grain density, $T_{\text{g}}(\nu)$ is the grain temperature, $B[\nu, T_{\text{g}}(r)]$ is the Planck function, and $\tau(\nu, r)$ is the optical depth at distance r from the centre of the star.

The temperature $T_{\text{g}}(r)$ is determined by the condition of energy balance: amount of energy radiated = amount of energy absorbed. The amount of energy absorbed at any point is proportional to the total available energy at that point, which consists of:

(i) the attenuated and diluted stellar radiation;
(ii) scattered radiation, and
(iii) reradiation from other grains.

Detailed solutions of radiative transfer in circumstellar dust shells by Rowan-Robinson & Harris (1983a,b) indicate that the effect of heating by other grains becomes significant only at large optical depths at the absorbing frequencies [$\tau(\text{UV}) \gg 10$], and at optical depths $\tau(\text{UV}) < 1$ the grains have approximately the same temperature that they would have if they were seeing the starlight unattenuated and no other radiation.

The Planck mean optical depths of circumstellar envelopes around several RV Tauri stars, derived from the ratios of the luminosities of the dust shell (at infrared wavelengths) and the star, range from 0.07 to 0.63 (Goldsmith et al. 1987). There is much uncertainty in the nature of the optical properties of dust grains in the envelope. The carbon-rich RV Tauri stars are also reported to show the 10-μm silicate emission feature typical of oxygen-rich objects (Gehrz & Ney 1972; Olnon & Raimond 1986). The pure terrestrial

Table 1. Data on the RV Tauri stars detected by *IRAS*.

Name Variable	*IRAS*	Flux density (Jy)[a] 12 μm	25 μm	60 μm	100 μm	Sp. group	Period (d)	Light-curve type	T_0 (K)
TW Cam	04166+5719	8.27	5.62	1.82	<1.73	A	85.6	a	555
RV Tau	04440+2605	22.53	18.08	6.40	2.52	A	78.9	b	460
DY Ori	06034+1354	12.44	14.93	4.12	<11.22	B	60.3		295
CT Ori	06072+0953	6.16	5.57	1.22	<1.54	B	135.6		330
SU Gem	06108+2734	7.90	5.69	2.16	<11.66	A	50.1	b	575
UY CMa	06160−1701	3.51	2.48	0.57	<1.00	B	113.9	a	420
U Mon	07284−0940	124.30	88.43	26.28	9.24	A	92.3	b	480
AR Pup	08011−3627	131.33	94.32	25.81	11.65	B	75.0	b	450
IW Car	09256−6324	101/06	96.24	34.19	13.07	B	67.5	b	395
GK Car	11118−5726	2.87	2.48	0.78	<12.13	B	55.6		405
RU Cen	12067−4508	5.36	11.02	5.57	2.01	B	64.7		255
SX Cen	12185−4856	5.95	3.62	1.09	<1.50	B	32.9	b	590
AI Sco	17530−3348	17.68	11.46	2.88	<45.62	A	71.0	b	480
AC Her	18281+2149	41.47	65.33	21.12	7.79	B	75.5	a	260
R Sct	18448−0545	20.88	9.30	8.10	<138.78	A	140.2	a	
R Sge	20117+1634	10.63	7.57	2.10	<1.66	A	70.6	b	455
V Vul	20343+2625	12.39	5.72	1.29	<6.96	A	75.7	a	690

[a] Observed by *IRAS*.

silicates or lunar silicates are found to be completely unsuitable to account for the infrared emission from circumstellar dust shells around M-type stars (Rowan-Robinson & Harris 1983a). We assume that the absorption efficiency $Q_{\rm abs}(\nu)$ in the infrared varies as ν^{γ}. $\gamma = 1$ appears to provide a reasonable fit in a variety of sources (Harvey, Thronson & Gatley 1979; Jura 1986). Under these circumstances the condition of energy balance implies that the dust temperature $T_{\rm g}$ will vary as r^{β}.

In view of the low value of the observed Planck mean optical depth for the stellar radiation and the nature of the assumed frequency dependence of the absorption efficiency, the extinction of the infrared radiation by the dust envelope can be neglected. If we consider the envelope to be spherically symmetric, equation (1) reduces to

$$L(\nu) = \int_{r_1}^{r_2} 4\pi r^2 \rho(r)\, Q_{\rm abs}(\nu) B[\nu, T_{\rm g}(r)]\, {\rm d}r, \qquad (2)$$

where r_1 and r_2 are the inner and outer radii of the shell. For a dusty density distribution $\rho(r) \propto r^{\alpha}$ and $r_2 \gg r_1$, equation (2) reduces to

$$L(\nu) \propto \nu^{2+\gamma-Q} \int_{X_0}^{\infty} \frac{x^Q}{{\rm e}^x - 1} {\rm d}x, \qquad (3)$$

where $Q = -(\alpha+\beta+3)/\beta$ and $X_0 = (h\nu/kT_0)$. T_0 represents the temperature at the inner boundary of the dust shell where grains start condensing. In a steady radiation pressure driven mass outflow in the optically thin case, values of α lie near -2 (Gilman 1972). γ and β are related by $\beta = -2/(\gamma+4)$.

In the *IRAS* Point Source Catalog (PSC, Beichman et al. 1985a), the flux densities have been quoted at the effective wavelengths 12, 25, 60 and 100 μm, assuming a flat energy spectrum $[\nu F(\nu) = 1]$ for the observed sources. For each model given by equation (3), using the relative system response, the colour-correction factors (Beichman et al. 1985b) in each of the *IRAS* passbands were calculated and the fluxes were converted into flux densities expected for a flat energy distribution, as assumed in the *IRAS* PSC, so that the computed colours can be directly compared with the colours determined from the catalogue quantities. Such a procedure is more appropriate than correcting the *IRAS* colours for the energy distribution given by a particular model and then comparing them with those computed by the model.

2.1 Colour–colour diagram

The IR colour is defined as

$$[\nu_1] - [\nu_2] = -2.5 \log[f(\nu_1)/f(\nu_2)],$$

where ν_1 and ν_2 are any two wavebands and $f(\nu_1)$ and $f(\nu_2)$ are the corresponding flux densities assuming a flat energy spectrum for the source. In Fig. 1, we have plotted the [25]–[60] colours of RV Tauri stars against their corresponding [12]–[25] colours derived from the *IRAS* data. Filled circles represent stars of group A and open circles stars of group B. The two sets of near-parallel lines represent the loci of constant inner shell temperature T_0 and the quantity Q defined above. The models correspond to the case of absorption efficiency $Q_{\rm abs}(\nu)$ varying as ν (with $\gamma = 1$ and hence $\beta = -0.4$). We have omitted R Sct in Fig. 1 because it shows a large deviation from the average relation shown by all the other objects. R Sct has a comparatively large excess at 60 μm, but the extent of a possible contamination by the infrared cirrus (Low et al. 1984) is unknown. Goldsmith et al. (1987) found no evidence of the presence of a dust envelope at near-IR wavelengths and the spectrum was consistent with a stellar continuum. This explains why R Sct lies well below the mean relation shown by stars of groups A and C between the [3.6]–[11.3] colour excess and the photometrically determined (Fe/H) (Dawson 1979). R Sct has the

Figure 1. Plot of [25]–[60] colours of RV Tauri stars against their [12]–[25] colours after normalizing as indicated in Beichman et al. (1985b). Some of the objects are identified by their variable-star names. Typical error bars are shown in the bottom right-hand corner. The lines represent the loci for constant inner shell temperature and the quantity Q. Note the separation of group A and B stars at $T_0 \sim 460$ K. Positions occupied by a sample of carbon and oxygen Miras are also shown. The $Q = 1.0$ line differs from the blackbody line by a maximum of ~ 0.05.

longest period of 140 d among the RV Tauri stars detected at far-infrared wavelengths and does not have the 10-μm emission feature seen in other objects (Gehrz 1972; Olnon & Raimond 1986). R Sct is probably the most irregular RV Tauri star known (McLaughlin 1932).

The inner shell temperatures (T_0) derived for the various objects are also given in Table 1 and we find the majority of them to have temperatures in the narrow range 400–600 K. If the dependences of $Q_{\rm abs}(\nu)$ on ν and $\rho(r)$ on r are similar in all the objects considered, then in the colour–colour diagram they all should lie along a line corresponding to different values of T_0 and in Fig. 1 we find that this is essentially the case. In view of the quoted uncertainties in the flux measurements, we cannot attach much significance to the scatter in Fig. 1.

At 100 μm the infrared sky is characterized by emission, called infrared cirrus, from interstellar dust on all spatial scales (Low et al. 1984), thereby impairing the measurements at far-infrared wavelengths. In Fig. 2, we have plotted the [60]–[100] colours of the six RV Tauri stars detected at 100 μm against their [25]–[60] colours, along with the grid showing the regions of different values for inner shell temperature T_0 and the quantity Q, as in Fig. 1. The results indicated by Fig. 2 are consistent with those derived from Fig. 1. AR Pup shows a large excess at 100 μm but, in view of the large values for the cirrus flags given in the catalogue, the intrinsic flux at 100 μm is uncertain.

2.2 Radial distribution of dust

From Fig. 1, it is evident that all RV Tauri stars lie between the lines corresponding to $Q = 1.5$ and 0.5. With

$$\alpha = -(1+Q)\beta - 3,$$

these values suggest limits of $r^{-2.0}$ and $r^{-2.4}$ for the dust density variation, indicating a near-constant mass-loss rate. Jura (1986) has suggested that the density in the circumstellar envelope around RV Tauri stars varies as r^{-1}, implying a mass-loss rate that was greater in the past than it is currently. By fitting a power law to the observed fluxes, such that $f(\nu)$ varies as ν^q, values of q determined by him for the various objects given in Table 1 lie in the range 0.6–1.2, with a mean $\bar{q} = 0.98$. The assumption of a power law corresponds to the case of $X_0 = 0$ in equation (3) and hence we get

$$q = 2 + \gamma - Q.$$

Since we assume that $Q_{\rm abs}(\nu)$ varies as ν, the resulting value for Q=2.0. None of the objects is found to lie in the corresponding region in the colour–colour diagram. Even this extreme value for Q implies a density which varies as $r^{-1.8}$.

Goldsmith et al. (1987) have reported that the simultaneous optical and near-IR data of AC Her can be fitted by a combination of two blackbodies at 5680 and 1800 K, representing, respectively, the stellar and dust shell temperatures, and suggested that in RV Tauri stars the grain formation is a sporadic phenomenon and not a continuous process. Apparently, they have been influenced by the remark by Gehrz & Woolf (1970) that their data in the 3.5–11 μm region of AC Her indicated a dust temperature of $\sim$300 K. We find that the K–L colours given by Gehrz (1972), Lloyd Evans (1985) and Goldsmith et al. (1987) are all consistent with each other. Surely, hot dust ($\sim$ 1800 K), if present at the time of observations by Goldsmith et al. (1987), would have affected the K–L colour significantly. AC Her, like other members of its class, is found to execute elongated loops in the (U–B), (B–V) plane (Preston et al. 1963), indicating that significant departure of the stellar continuum from the blackbody is to be expected. Further, their data show only a marginal excess at the near-IR wavelengths. We feel that the case for the existence of hot dust around AC Her and hence for the sporadic grain formation around RV Tauri stars is not strong. In Fig. 3 we find that AC Her and RU Cen lie very close to R Sct which, according to Goldsmith et al. (1987), shows no evidence for the presence of a hot dust envelope.

2.2.1 Comparison with oxygen and carbon Miras

In Fig. 1 we have also shown the positions of a sample of oxygen-rich and carbon-rich Miras. At the low temperatures characteristic of the Miras, a part of the emission at 12 μm comes from the photosphere. For a blackbody at 2000 K, the ratio of fluxes at wavelengths of 12 and 2 μm (f_{12}/f_2) $\sim$ 0.18. The Miras shown in Fig. 1 have (f_{12}/f_2) ratios larger than

Figure 2. Plot of the [60]–[100] colours of RV Tauri stars against their [25]–[60] colours after normalizing as indicated in Beichman et al. (1985b). The solid lines represent the loci for constant inner shell temperature and the quantity Q. The dashed line shows the locus for a blackbody distribution.

twice the above value. It is clear that the three groups of objects populate three different regions of the diagram. Hacking et al. (1985) have already noticed that there are distinct differences between the *IRAS* colours of oxygen-rich and carbon-rich objects. On the basis of an analysis, using a bigger sample of bright giant stars in the *IRAS* catalogue, this has been interpreted by Zuckerman & Dyck (1986) as being due to a systematic difference in the dust grain emissivity index. U Mon shows the 10-μm silicate emission convincingly and, in most of the other objects for which low-resolution spectra in the near-infrared have been reported (Gehrz 1972; Olnon & Raimond 1986), the 10-μm emission may be partly attributed to silicates. Hence it is reasonable to expect that, in the envelopes around at least some of the RV Tauri stars, the dust grains are predominantly of silicates, as in the case of oxygen Miras (Rowan-Robinson & Harris 1983a). The fact that none of the RV Tauri stars is found in the region of the two-colour diagram occupied by the oxygen Miras indicates that the emissivity indices of the silicate grains in the two cases are different. Because of the higher temperatures and luminosities, the environment of grain formation will be different in RV Tauri stars.

2.2.2 Correlation with subgroups

Preston et al. (1963) have identified three spectroscopic subgroups, which are designated as groups A, B and C. Objects of group A are metal-rich; group C are metal-poor; group B objects are also metal-poor, but show carbon enhancements (Preston et al. 1963; Lloyd Evans 1974; Dawson 1979; Baird 1981). It is interesting to see that Table 1 contains no group C objects and that in Fig. 1 there is a clear separation of the two spectroscopic subgroups A and B, with the demarcation occurring at an inner shell temperature of about 450 K, group B stars having lower temperatures than group A. SX Cen is the only exception. Lloyd Evans (1974) has reported that metal lines are stronger in SX Cen than in other group B objects. It may be worth noting that SX Cen has the shortest period among the 100 or so objects with the RV Tauri classification. RU Cen has the coolest inner

Figure 3. Plot of (K–L) colours of RV Tauri stars detected by *IRAS* against their corresponding (J–K) colours. The position of AR Pup is indicated. The three objects lying close to the blackbody line are AC Her, RU Cen and R Sct.

shell temperature, as already suggested by the near-infrared spectrum (Gehrz & Ney 1972).

Group B objects follow a different mean relationship from those of group A, having systematically larger 11-μm excess for a given excess at 3 μm (Lloyd Evans 1985). For a general sample of RV Tauri stars, the distinction between the oxygen-rich and carbon-rich objects is not that apparent in the *JHKL* bands. In Fig. 3 we have plotted the near-IR magnitudes of the objects given in Table 1 (except V Vul which has no available measurements) in the J–K, K–L plane. The colours, taken from Lloyd Evans (1985) and Goldsmith et al. (1987), are averaged if more than one observation exists, because the internal agreements are found to be often of the order of observational uncertainties, in accordance with the earlier finding by Gehrz (1972) that variability has relatively little effect on colours. Barring RU Cen and AC Her, it is evident that stars belonging to group B show systematically larger excesses at L band for a given excess at K. The low excesses at near-IR wavelengths for AC Her and RU Cen are

Landscape figure to go here. This figure was not part of the original paper and is inserted here for illustrative purposes.
See the author guide for details on how to handle landscape figures or tables, and `mnland.tex`.

Figure 4.

consistent with the very low dust temperatures indicated by the far-infrared colours.

It is already well established that from *UBV* photometry one can distinguish between groups A and B, members of group A being significantly redder than those of group B (Preston et al. 1963). Similarly, Dawson (1979) has found that the two spectroscopic groups are well separated in the DDO colour–colour diagrams when mean colours are used for the individual objects.

The clear separation of the spectroscopic subgroups A and B in the IR two-colour diagram suggests that the natures of dust grains in the envelopes in the two cases are not identical. This is to be expected because of the differences in the physical properties of the stars themselves. The average colours of group B stars are bluer than group A, but the envelope dust temperatures of B are cooler than those of A. The near-IR spectra of AC Her and RU Cen are extremely similar (Gehrz & Ney 1972). The striking similarities in the optical spectra of AC Her and RU Cen have been pointed out by Bidelman (O'Connell 1961). We feel that the physical properties, including the chemical composition, of the grains formed in the circumstellar envelope strongly depend on those of the embedded star. This, probably, explains the diversity of the energy distributions of RV Tauri stars in the near-infrared found by Gehrz & Ney (1972). On the basis of the observed differences in chemical abundances and space distribution of RV Tauri stars, Lloyd Evans (1985) has already pointed out that there is no direct evolutionary connection between group A and group B objects, thus ruling out the possibility that group B objects are the evolutionary successors of group A, in which grain formation has stopped and the cooler temperatures for the former are caused by an envelope expansion.

Kukarkin et al. (1969) have subdivided RV Tauri stars into two classes, RVa and RVb, on the basis of their light curves; the former shows a constant mean brightness, whereas the latter shows a cyclically varying mean brightness. Extensive observations in the near-infrared show that, on average, RVb stars are redder than RVa stars, and Lloyd Evans (1985) has suggested that in RVb stars dust shells are denser in the inner regions and hence radiate strongly in the 1–3 μm region. Fig. 3 confirms this; RVb objects show systematically larger (J–K) and (K–L) colours than RVa objects. Apparently, there is no distinction between objects of the two light-curve types at far-infrared wavelengths (Fig. 1).

3 CONCLUSIONS

In the [12]–[25], [25]–[60] colour diagram, RV Tauri stars populate cooler temperature regions ($T < 600$ K), distinctly different from those occupied by the oxygen and carbon Miras. Using a simple model in which

(i) the envelope is spherically symmetric,

(ii) the IR-emitting grains are predominantly of the same kind, and

(iii) in the infrared the absorption efficiency $Q_{\rm abs}(\nu) \propto \nu$,

we find that the *IRAS* fluxes are consistent with the density in the envelope $\rho(r) \propto r^{-2}$, where r is the radial distance. Such a dependence for the dust density implies that the mass-loss rates in RV Tauri stars have not reduced considerably during the recent past, contrary to the suggestion by Jura (1986). In the two-colour diagram, the blackbody line and the line corresponding to $\rho(r) \propto r^{-2.2}$ nearly overlap and the present data are insufficient to resolve between the two cases. The latter case is more physically reasonable, however.

The spectroscopic subgroups A and B are well separated in the *IRAS* two-colour diagram, with group B objects having systematically cooler dust envelopes. If we consider only the objects detected by *IRAS*, we find that stars belonging to group B show systematically larger excess at L band for a given excess at K. Apparently, there is no correlation between the light-curve types (RVa and RVb) and the far-infrared behaviour of these objects. It is fairly certain that the physical properties, including the chemical composition, of the embedded stars are directly reflected by those of the dust grains. Most probably, the grain formation process in RV Tauri stars is continuous and not sporadic as suggested by Goldsmith et al. (1987).

ACKNOWLEDGMENTS

I thank Professor N. Kameswara Rao for some helpful suggestions, Dr H. C. Bhatt for a critical reading of the original version of the paper and an anonymous referee for very useful comments that improved the presentation of the paper.

REFERENCES

Baird S.R., 1981, ApJ, 245, 208
Beichman C.A., Neugebauer G., Habing H.J., Clegg P.E., Chester T.J., 1985a, *IRAS* Point Source Catalog. Jet Propulsion Laboratory, Pasadena
Beichman C.A., Neugebauer G., Habing H.J., Clegg P.E., Chester T.J., 1985b, *IRAS* Explanatory Supplement. Jet Propulsion Laboratory, Pasadena
Dawson D.W., 1979, ApJS, 41, 97
Gerhz R.D., 1972, ApJ, 178, 715
Gerhz R.D., Ney E.P., 1972, PASP, 84, 768
Gerhz R.D., Woolf N.J., 1970, ApJ, 161, L213
Gilman R.C., 1972, ApJ, 178, 423
Goldsmith M.J., Evans A., Albinson J.S., Bode M.F., 1987, MNRAS, 227, 143
Hacking P. et al., 1985, PASP, 97, 616
Harvey P.M., Thronson H.A., Gatley I., 1979, ApJ, 231, 115
Jura M., 1986, ApJ, 309, 732
Kukarkin B.V. et al., 1969, General Catalogue of Variable Stars. Moscow
Lloyd Evans T., 1974, MNRAS, 167, 17P
Lloyd Evans T., 1985, MNRAS, 217, 493
Low F.J. et al., 1984, ApJ, 278, L19
McLaughlin D.B., 1932, Publ. Univ. Obs. Mich., 4, 135
O'Connell J.K., 1961, Specola Vaticana Ric. Astron., 6, 341
Olnon F.M., Raimond E., 1986, A&AS, 65, 607
Preston G.W., Krzeminski W., Smak J., Williams J.A., 1963, ApJ, 137, 401
Rowan-Robinson M., Harris S., 1983a, MNRAS, 202, 767
Rowan-Robinson M., Harris S., 1983b, MNRAS, 202, 797
van der Veen W.E.C.J., Habing H.J., 1988, A&A, 194, 125
Willems F.J., de Jong T., 1988, A&A, 196, 173
Zuckerman B., Dyck H.M., 1986, ApJ, 311, 345

APPENDIX A: LARGE GAPS IN Lyα FORESTS DUE TO FLUCTUATIONS IN LINE DISTRIBUTION

(This appendix was not part of the original paper by A.V. Raveendran and is included here just for illustrative purposes.)

Spectroscopic observations of bright quasars show that the mean number density of Lyα forest lines, which satisfy certain criteria, evolves like $\mathrm{dN/dz} = \mathrm{A}(1+\mathrm{z})^{\gamma}$, where A and γ are two constants. Given the above intrinsic line distribution we examine the probability of finding large gaps in the Lyα forests. We concentrate here only on the statistics and neglect all observational complications such as the line blending effect (see Ostriker, Bajtlik & Duncan 1988).

Suppose we have observed a Lyα forest between redshifts z_1 and z_2 and found $N-1$ lines. For high-redshift quasars z_2 is usually the emission redshift z_{em} and z_1 is set to $(\lambda_{\mathrm{Ly}\beta}/\lambda_{\mathrm{Ly}\alpha})(1+z_{\mathrm{em}}) = 0.844(1+z_{\mathrm{em}})$ to avoid contamination by Lyβ lines. We want to know whether the largest gaps observed in the forest are significantly inconsistent with the above line distribution. To do this we introduce a new variable x:

$$x = \frac{(1+z)^{\gamma+1} - (1+z_1)^{\gamma+1}}{(1+z_2)^{\gamma+1} - (1+z_1)^{\gamma+1}}. \qquad \text{(A1)}$$

x varies from 0 to 1. We then have $\mathrm{dN/dx} = \lambda$, where λ is the mean number of lines between z_1 and z_2 and is given by

$$\lambda \equiv \frac{A[(1+z_2)^{\gamma+1} - (1+z_1)^{\gamma+1}]}{\gamma+1}. \qquad \text{(A2)}$$

This means that the Lyα forest lines are uniformly distributed in x. The probability of finding $N-1$ lines between z_1 and z_2, P_{N-1}, is assumed to be the Poisson distribution.

Figure A1. $P(>x_{\mathrm{gap}})$ as a function of x_{gap} for, from left to right, $N = 160$, 150, 140, 110, 100, 90, 50, 45 and 40.

A1 Subsection title

We plot in Fig. A1 $P(>x_{\mathrm{gap}})$ for several N values. We see that, for $N = 100$ and $x_{\mathrm{gap}} = 0.06$, $P(>0.06) \approx 20$ per cent. This means that the probability of finding a gap with a size larger than six times the mean separation is not significantly small. When the mean number of lines is large, $\lambda \sim N >> 1$, our $P(>x_{\mathrm{gap}})$ approaches the result obtained by Ostriker et al. (1988) for small (but still very large if measured in units of the mean separation) x_{gap}, i.e., $P(>x_{\mathrm{gap}}) \sim N(1-x_{\mathrm{gap}})^{N-1} \sim N\exp(-\lambda x_{\mathrm{gap}})$.

This paper has been produced using the Royal Astronomical Society/Blackwell Science LaTeX style file.