Constraining the Ultra-Large Scale Structure of the Universe

Jonathan Braden

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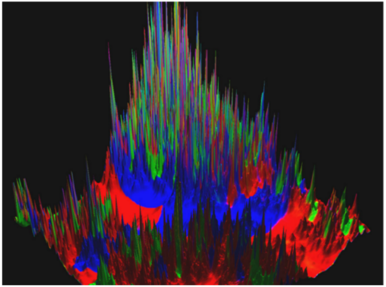
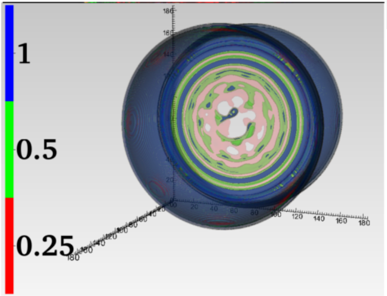
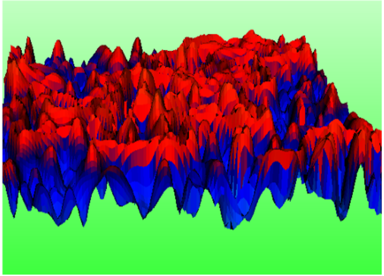
ICG Portsmouth, May 25, 2016

w/ Hiranya Peiris, Matthew Johnson and Anthony Aguirre
based on arXiv:1604.04001 and *in progress*

Inhomogeneous Nonlinear Ultra-Large Scale Cosmology







… A Mosaic of Interesting Dynamics

- **Strongly inhomogeneous and nonlinear cosmological initial conditions** (this talk) [JB, Peiris, Johnson, Aguirre]
- Isocurvature mode conversion into intermittent density perturbations [JB, Bond, Frolov, Huang]
  - Caustic formation in chaotic long wavelength dynamics
  - Generalised form of local nonGaussianity
- First order phase transitions [JB, Bond, Mersini-Houghton]
- Entropy production in highly inhomogeneous nonlinear field theories [JB, Bond]

Spatially intermittent dynamics, fundamental issues in QFT, novel and poorly constrained observables

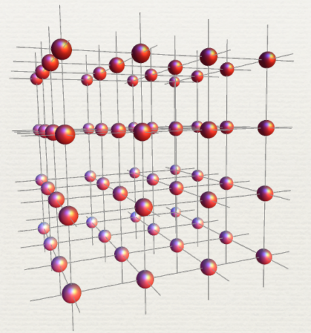
Numerical Approach is Essential [JB, in preparation]

Hybrid MPI/OpenMP Lattice Code

- Solve field equation (e.g.)

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + V'(\vec{\phi}) = 0$$

- 10th order Gauss-Legendre integration (general) or 8th order Yoshida (nonlinear sigma models)
- Finite-difference (fully parallel) or Pseudospectral (OpenMP)
- Optional absorbing boundaries
- Quantum fluctuations $\rightarrow$ realization of random field



- Energy conservation $\mathcal{O}(10^{-9} - 10^{-14})$

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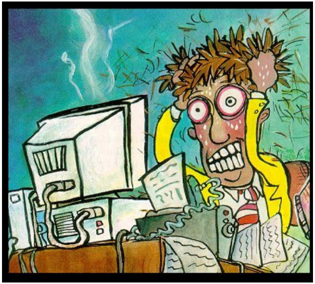

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Dawn of Numerical Relativity in Cosmology

- Bentivegna, **Bruni**
- Mertens, Giblin, Starkman
- Kleban, Linde, Senatore, West
- Peiris, Johnson, Feeney, Aguirre, Wainwright (symmetry reduced bubbles)
- Adamek, Daverio, Durrer, Kunz (weak field subhorizon)

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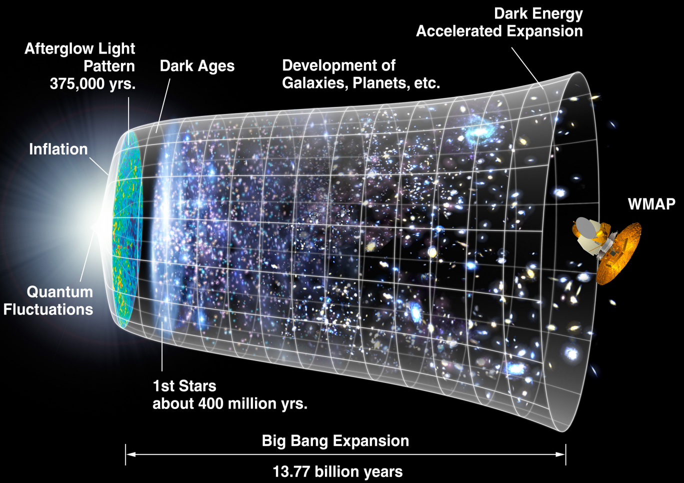
Connection of Initial Conditions with Observables

- Requires Monte Carlo sampling of Initial Conditions
- Highly accurate integrators to achieve machine precision

Outline

- Brief Review of Cosmology and Role of Inflation
- Inflationary Models and Initial Conditions
- Choice of Initial Conditions
- Planar Symmetric Dynamics
- Projection of Dynamics into Observations (CMB quadrupole)
- Semi-analytic Approximation (work in progress)

History of the Universe



NASA/WMAP Science Team

Model Choices

$$V(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_P}}\right)^2$$

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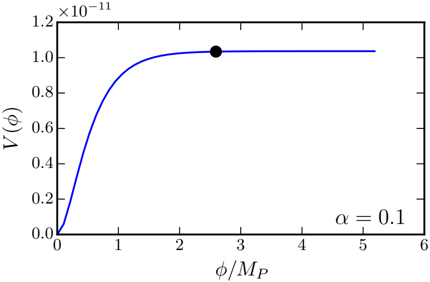

- $\alpha \ll 1 \rightarrow$ small-field model

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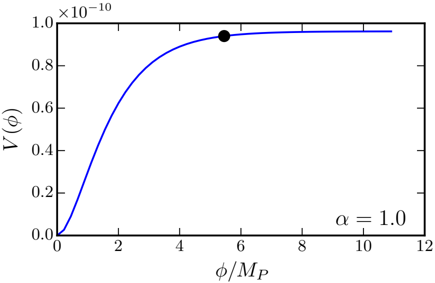

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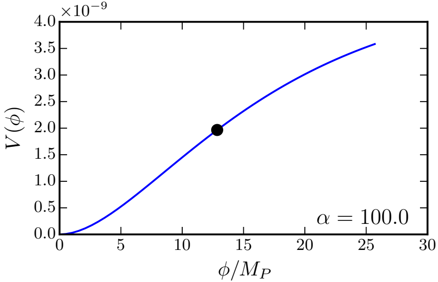

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- $\alpha \gg 1 \rightarrow m^2\phi^2/2$

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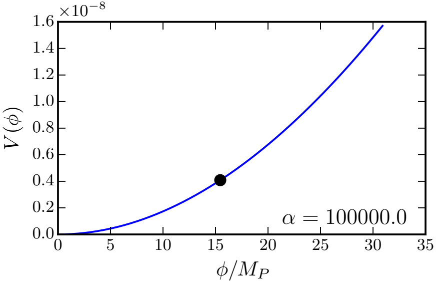

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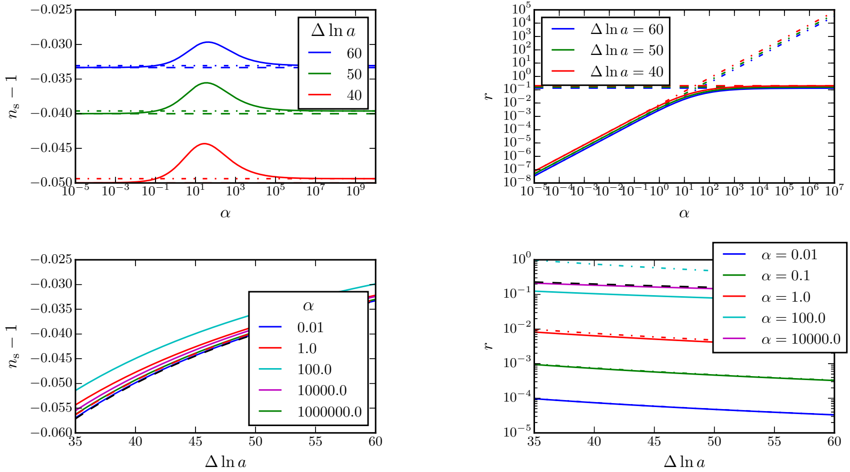
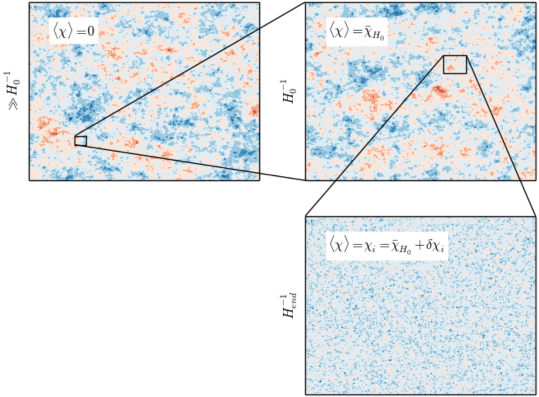
CMB Parameter Predictions



Ultra-Large Scale Structure



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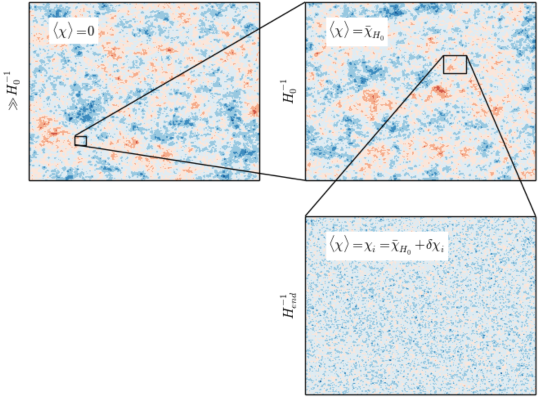

Local Remnants of Ultra-Large Scale Structure?

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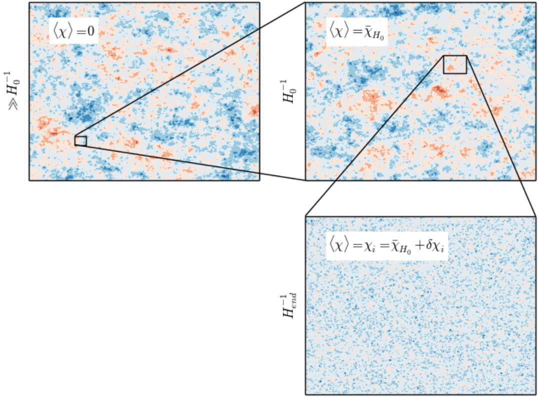

Local Remnants of Ultra-Large Scale Structure?

- Structure present at start of inflation
- Conversion of structure during or after inflation

Dimensional Reduction: Planar Symmetry

Synchronous Gauge

$$ds^2 = -d\tau^2 + a_{\parallel}^2(x,\tau)dx^2 + a_{\perp}^2(x,\tau)(dy^2 + dz^2)$$

Residual gauge freedom : $a_{\parallel}(x,\tau=0)$, $a_{\perp}(x,\tau=0)$

Isotropised Expansion

$$a \equiv \det(\gamma_{ij})^{1/6} = \left(a_{\parallel} a_{\perp}^2\right)^{1/3}$$

$$H \equiv -\frac{1}{3}\gamma^{ij}K_{ij} = -\frac{1}{3}\left(K^x{}_x + 2K^y{}_y\right)$$

Initial Conditions: Field

Work in Spatially Flat Gauge $a_{\|}(\tau = 0) = 1 = a_{\perp}(\tau = 0)$

$$\phi(x) = \bar{\phi} + \delta\hat{\phi}$$

$\bar{\phi}$ gives desired $\mathcal{N}$ e-folds of inflation in homogeneous limit
$$3H_{\mathrm{I}}^2 \equiv V(\bar{\phi})$$

Field Fluctuations

$$\delta\hat{\phi}(x_i) = A_\phi \sum_{n=1} \hat{G} e^{ik_n x_i} \sqrt{P(k_n)} \qquad \hat{G} = \sqrt{-2\ln\hat{\beta}}\, e^{2\pi i \hat{\alpha}}$$

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$$P(k) = \Theta(k_{\mathrm{max}} - k) \qquad H_{\mathrm{I}}^{-1} k_{\mathrm{max}} = 2\pi\sqrt{3}$$

Initial Conditions: Metric

Work in Spatially Flat Gauge $a_{\|}(\tau = 0) = 1 = a_{\perp}(\tau = 0)$

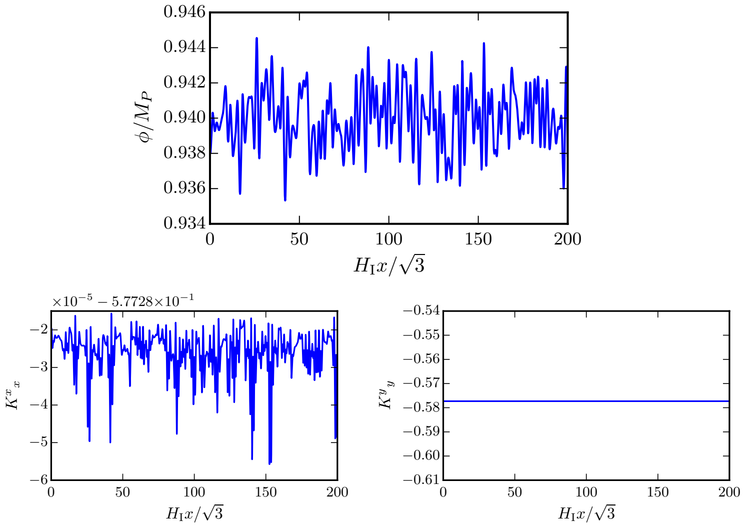
Momentum Constraint Sews Neighbouring Grid Sites Together

$0 = \mathcal{P} = {K^y}_y{}' - \frac{a'_\perp}{a_\perp}\left({K^x}_x - {K^y}_y\right) - \frac{\phi' \Pi_\phi}{2 a_\| M_P^2}$

Hamiltonian Constraint Enforces Energy Conservation

$$0 = \mathcal{H} = \frac{2 a_\perp a'_\| a'_\perp - a_\| a'^2_\perp - 2 a_\| a_\perp a''_\perp}{a_\|^3 a_\perp^2} + 2 {K^x}_x {K^y}_y + {K^y}_y{}^2 - M_P^{-2}\left(\frac{\phi'^2 + \Pi_\phi^2}{2 a_\|^2} + V\right) = 0$$

Distribution of Initial Conditions



What's the response in the curvature perturbation?

Time Evolution

$$\dot{a_{\parallel}} = -a_{\parallel} K^{x}{}_{x}, \qquad \dot{a_{\perp}} = -a_{\perp} K^{y}{}_{y},$$

$$\dot{K^{x}{}_{x}} = \frac{a_{\perp}'^{2}}{a_{\perp}^{2} a_{\parallel}^{2}} + {K^{x}{}_{x}}^{2} - {K^{y}{}_{y}}^{2} + \frac{\left(\Pi_{\phi}^{2} - \phi'^{2}\right)}{2 a_{\parallel}^{2} M_{P}^{2}},$$

$$\dot{K^{y}{}_{y}} = -\frac{a_{\perp}'^{2}}{2 a_{\perp}^{2} a_{\parallel}^{2}} + \frac{3}{2} {K^{y}{}_{y}}^{2} - \frac{V(\phi)}{2 M_{P}^{2}} + \frac{\left(\Pi_{\phi}^{2} + \phi'^{2}\right)}{4 a_{\parallel}^{2} M_{P}^{2}},$$

$$\dot{\Pi}_{\phi} = 2 K^{y}{}_{y} \Pi_{\phi} + \frac{1}{a_{\parallel}} \phi'' + \left(\frac{2 a_{\perp}'}{a_{\parallel} a_{\perp}} - \frac{a_{\parallel}'}{a_{\perp}^{2}} \right) \phi' - a_{\perp} \partial_{\phi} V(\phi),$$

$$\dot{\phi} = \frac{\Pi_{\phi}}{a_{\parallel}}$$

Numerical Approach

Machine precision accuracy

- Gauss-Legendre time-integrator ($\mathcal{O}(dt^{10})$, symplectic)
- Fourier pseudospectral discretisation (exponential convergence)

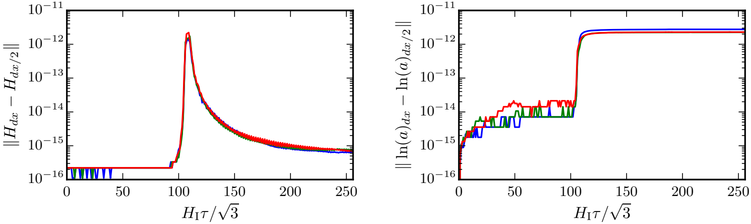
Fast to allow sampling

- Adaptive time-stepping
- Adaptive grid spacing

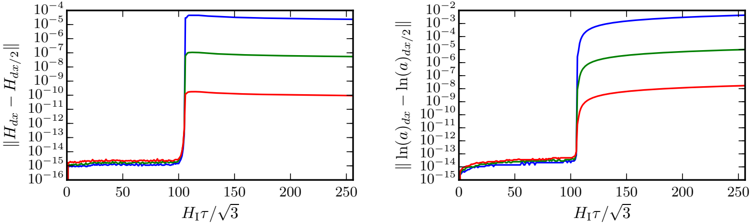
$\mathcal{O}(1-10)$**s to evolve through 60 e-folds of inflation**

Convergence Testing : Dynamical Variables

Vary grid spacing dx at fixed dx/dt

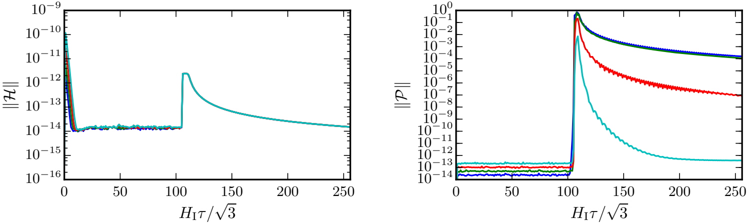


Vary time-step dt at fixed dx

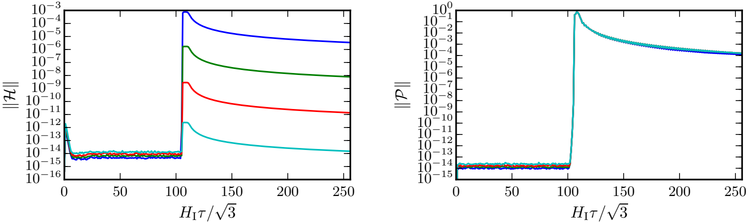


Convergence Testing : Constraint Preservation

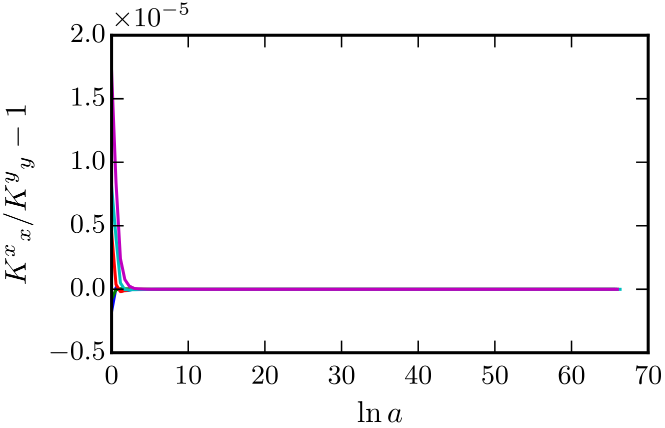
Vary grid spacing *dx* at fixed *dx*/*dt*



Vary time-step *dt* at fixed *dx*

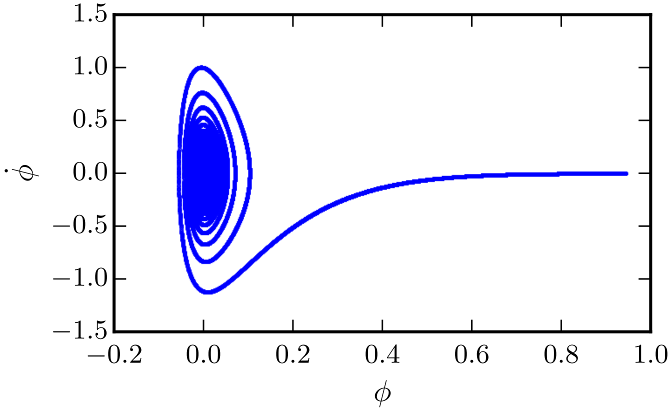


Self-Gravitating Dynamics: Isotropisation of Expansion

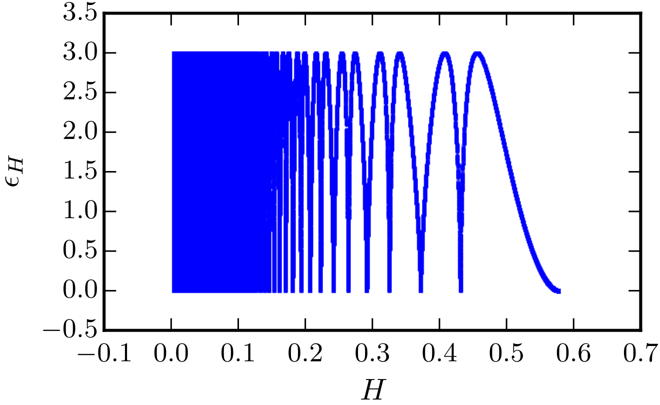


Expansion quickly isotropises itself

Self-Gravitating Dynamics: Attractor Behaviour



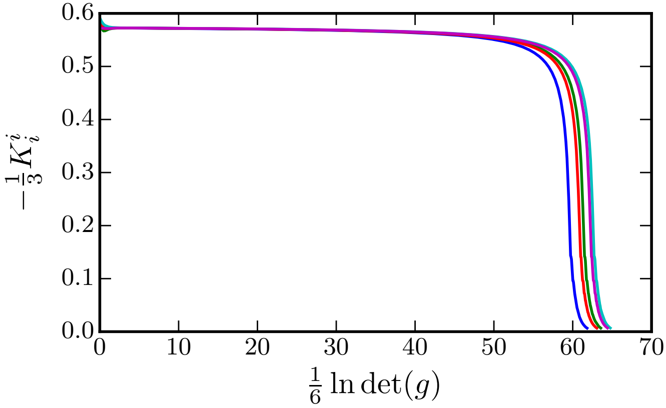
Self-Gravitating Dynamics: Geometry Attractor



Individual Lattice Sites Evolve Along Attractor

Fixed H, ϵ_H are equivalent

Self-Gravitating Dynamics: Spatially Dependent Expansion History



Observations: Relation to Local Multipoles

$$\zeta(H) \equiv \delta \ln a|^{H}_{a_{\|}=1=a_{\perp}}$$

Large-scale perturbations $\rightarrow$
evaluate ζ on last-scattering surface around a point x_0

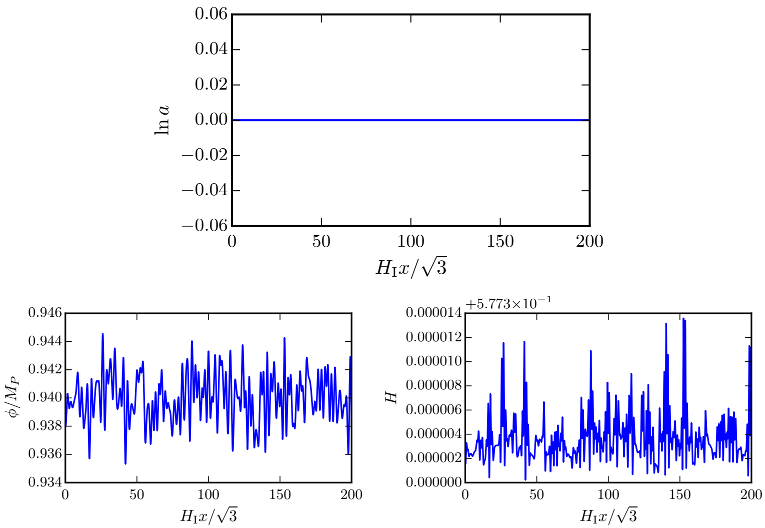
Expand for Large-Scale Fluctuations

$$\zeta(x_0+r_{\rm ls}) \approx \zeta(x_0)+(H_I r_{ls})\frac{\partial\zeta}{\partial(H_I x_p)}(x_0)+\frac{(H_I r_{ls})^2}{2}\frac{\partial^2\zeta}{\partial(H_I x_p)^2}(x_0)+\ldots$$

Matches onto Multipoles

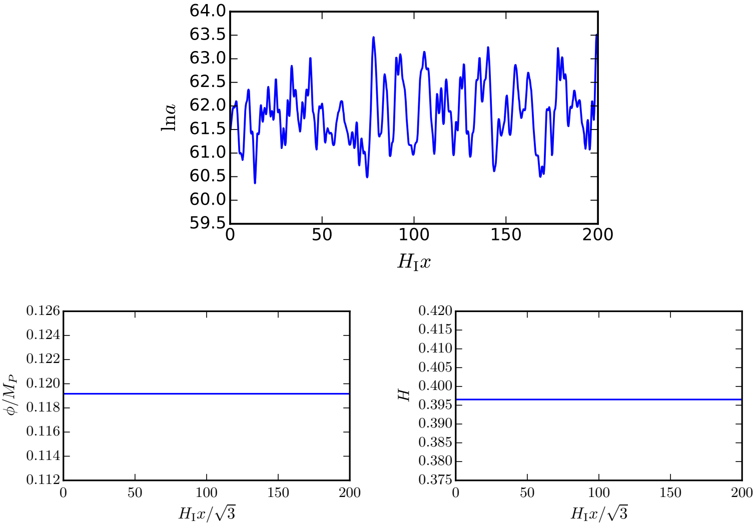
$$a_{20}^{(UL)} \approx F(L_{\rm obs}H_{\rm I})^2\partial^2_{x_p}\zeta \simeq F(L_{\rm obs}H_{\rm I})^2\frac{1}{a_{\|}}\partial_x\left[\frac{\partial_x\zeta}{a_{\|}}\right]$$

Required Evolution



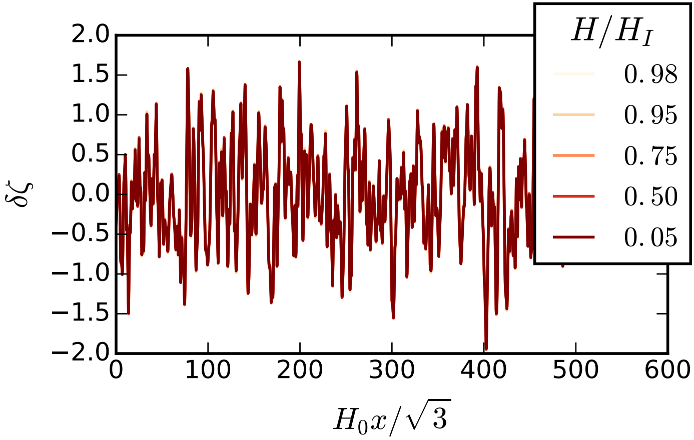
Initial Conditions ($\tau = 0$)

Required Evolution



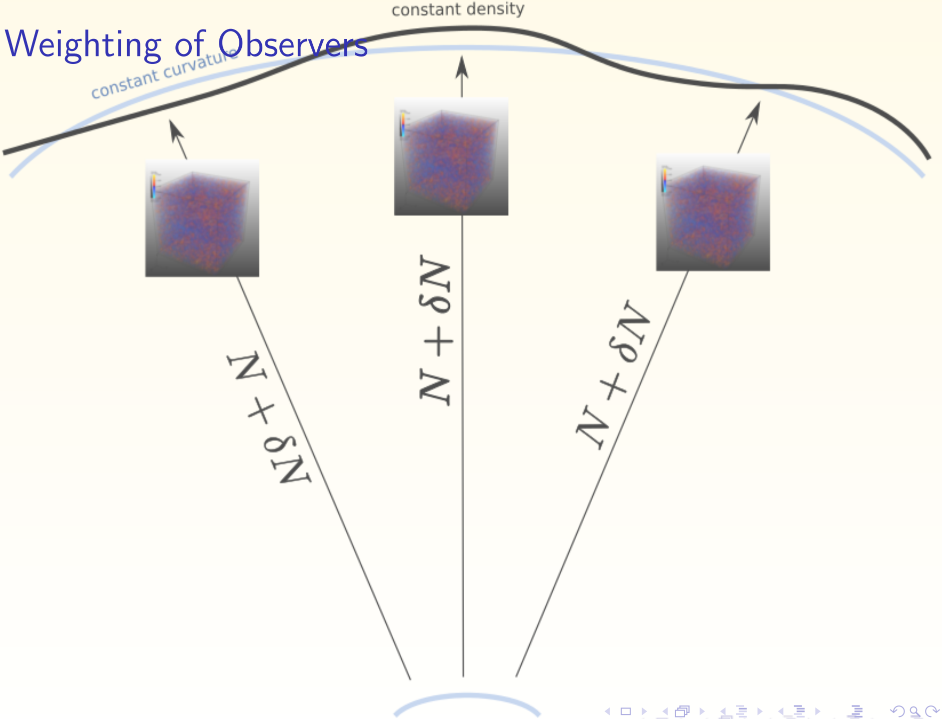
End of Inflation ($\epsilon_H = -d \ln H / d \ln a = 1$)

Freeze-in of Superhorizon Perturbations

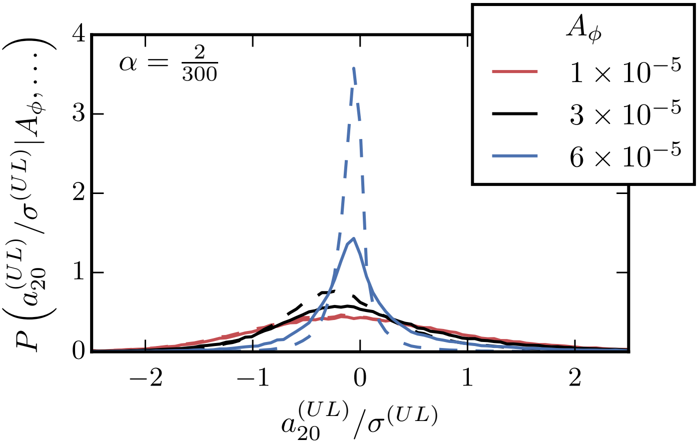


Initial transient modifies result from separate universe approximation

Weighting of Observers



Distribution of Superhorizon a_{20}



Statistical Framework

$$\mathrm{Pr}(A_\phi, L_{\rm obs}|C_2^{\rm obs}) \propto \mathcal{L}(A_\phi, L_{\rm obs}, \ldots)\mathrm{Pr}(A_\phi, L_{\rm obs}, \ldots)$$

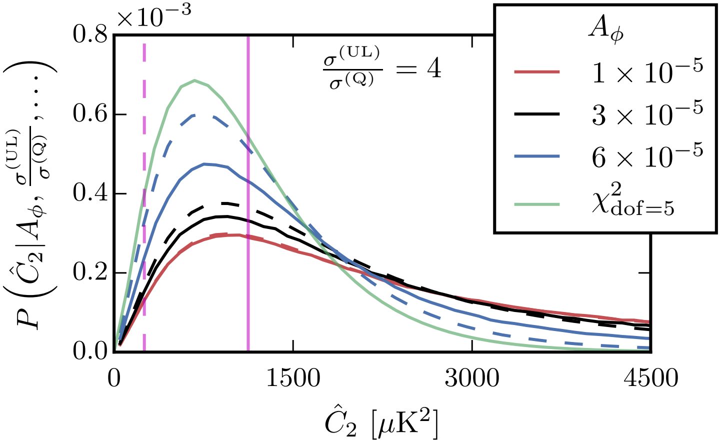
$$\mathcal{L}(A_\phi, L_{\rm obs}) = \mathrm{Pr}(C_2^{\rm obs}|A_\phi, L_{\rm obs}, \ldots)$$

Planck high-ℓ "theory" and low-ℓ "observed"

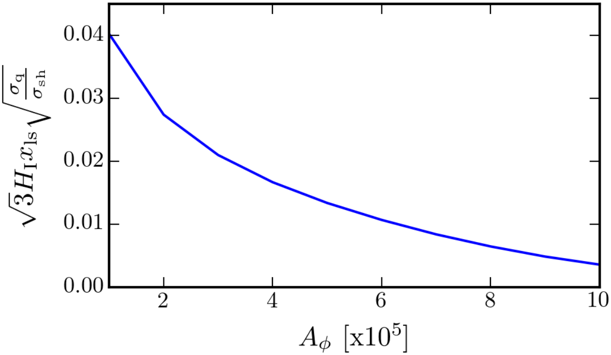
$C_2^{\rm obs} = 253.6\mu\mathrm{K}^2$ $\quad$ $C_2^{\rm theory} = 1124.1\mu\mathrm{K}^2$

Distribution of CMB quadrupole

$$\hat{C}_2 = \frac{1}{5}\left[\left(a_{20}^{(\mathrm{UL})} + a_{20}^{(\mathrm{Q})}\right)^2 + \sum_{m=-2,\ m\neq 0}^{2} \left(a_{2m}^{(\mathrm{Q})}\right)^2\right]$$



Relative RMS of superhorizon contribution depends of distance to last-scattering surface



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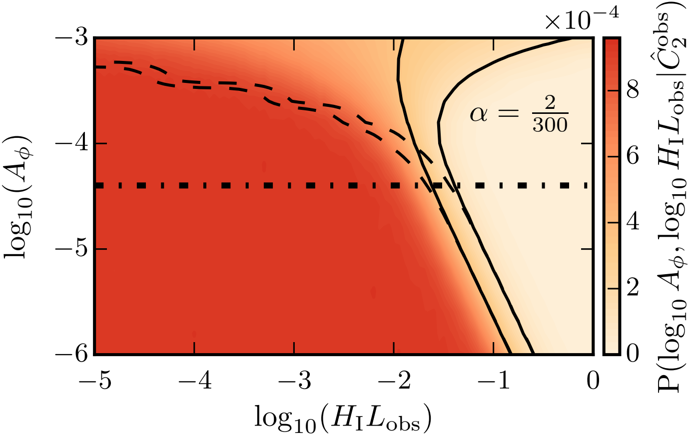
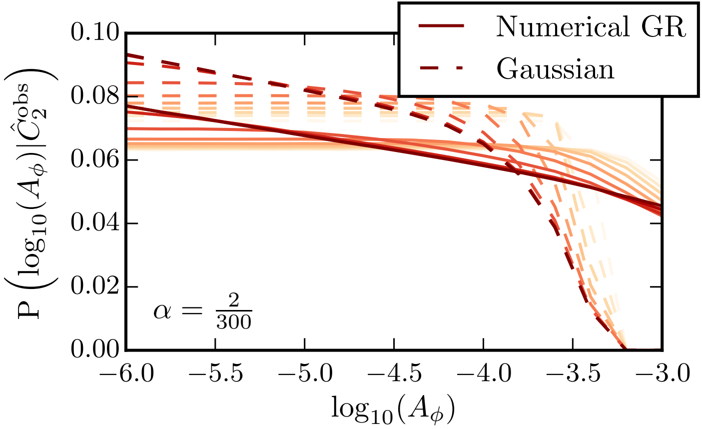
Constraints from the CMB Quadrupole



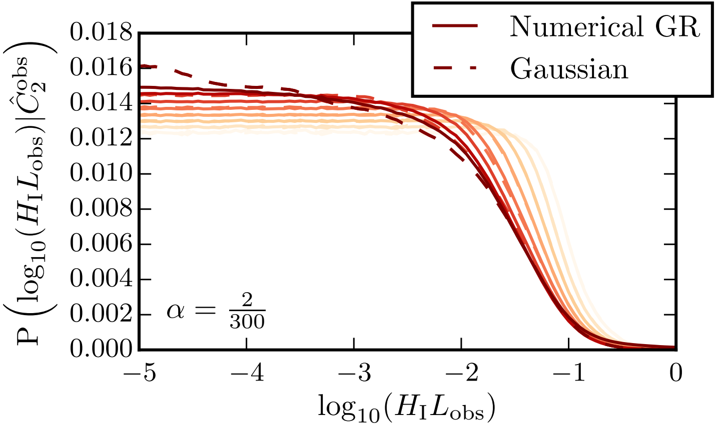
Strong Modification from Gaussian Ansatz at Large A_ϕ

Marginalised Constraints



Constraint of A_ϕ as we marginalise over L_{obs}

Marginalised Constraints



Constraint of L_obs as we marginalise over A_ϕ

Heuristic Picture of Large Amplitude Effects

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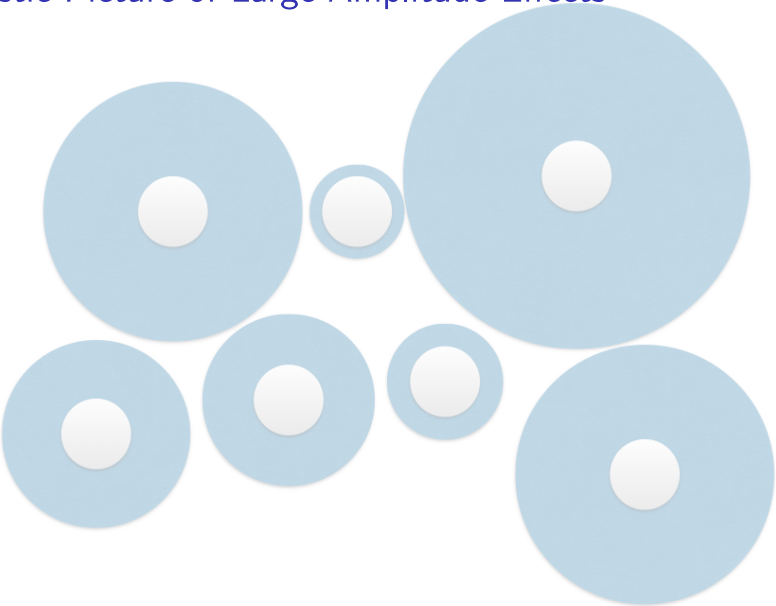
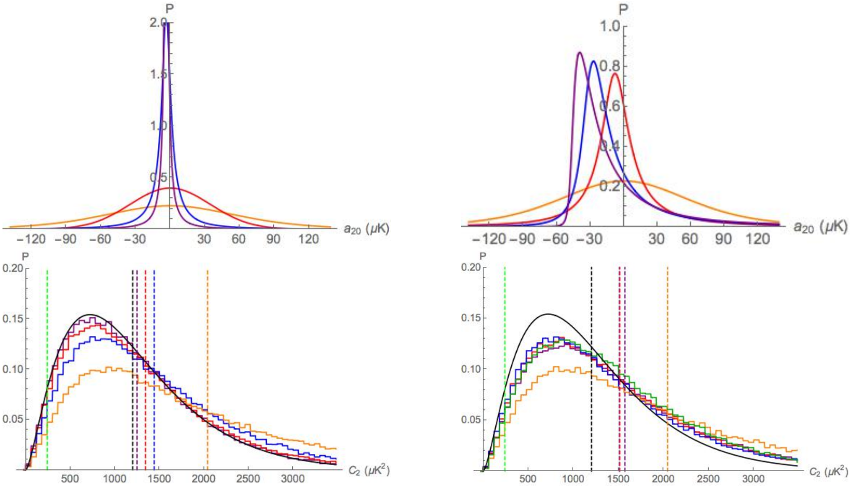
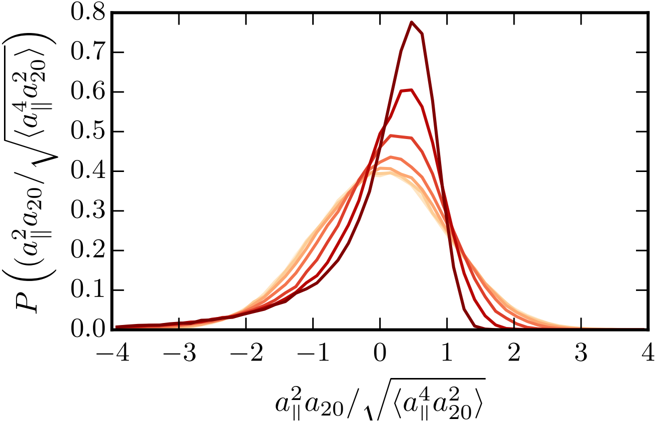


Modelling With Toy Distributions



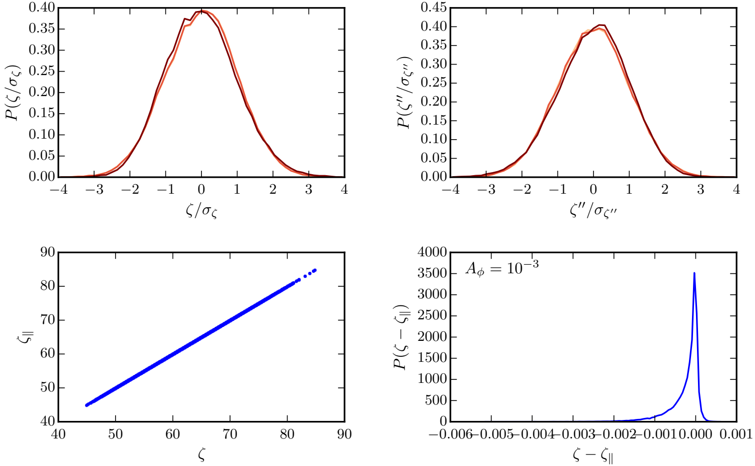
Model behaviour with Johnson distribution

Removing Inhomogeneous Expansion



Statistics of ζ

ζ and comoving derivatives nearly Gaussian



Analytical Modelling

$$\zeta(H, x_{\rm com}) \sim \rm GRF$$

Large-Scale Approximation for a_{20}

$$a_{20}(x_0) \approx \mathcal{A} e^{-2\zeta_{||}(x_0)} \left(\zeta''(x_0) - \zeta'_{||}(x_0)\zeta'(x_0) \right)$$

ζ, ζ', ζ'' are correlated Gaussian random deviates, with covariance

$$C_\zeta = \begin{bmatrix} \sigma_0^2 & 0 & -\sigma_1^2 \\ 0 & \sigma_1^2 & 0 \\ -\sigma_1^2 & 0 & \sigma_2^2 \end{bmatrix}$$

$$\sigma_i = \int dk k^{2i} \left\langle \left| \tilde{\zeta}_k \right|^2 \right\rangle$$

Analytical Modelling

$$\zeta(H, x_{\mathrm{com}}) \sim \mathrm{GRF}$$

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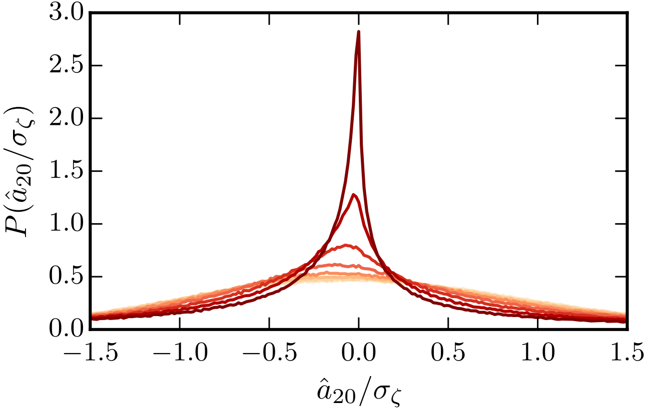
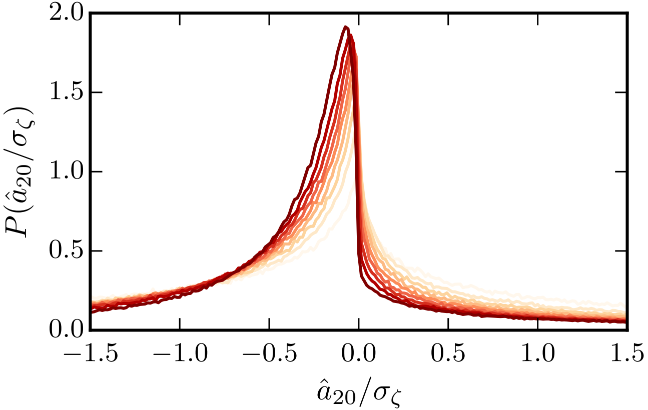
Anaytic a_{20} Distributions



Vary σ_ζ at fixed $\sigma_{\zeta(p)}/\sigma_\zeta$

Anaytic a_{20} Distributions



Vary $\sigma_{\zeta'}/\sigma_\zeta$ at fixed σ_ζ and $\sigma_{\zeta''}/\sigma_\zeta$

Future Steps

- Three-dimensional simulations (in progress)
- Multi-field models (additional isocurvature modes, non-attractor, etc.)
- Inclusion of stochastic effects from subhorizon fluctuations (technical challenge)
- Correlated anomalies

Conclusions

- Fully relativistic treatment of highly inhomogeneous initial conditions for inflation
- Constraints obtained from CMB quadrupole based on full Bayesian analysis
- Gravitational nonlinearities extremely important in determining final distributions of observables
- Very large amplitude initial inhomogeneities more poorly constrained than intermediate amplitudes
- Comoving curvature perturbation ζ approximately GRF in appropriate coordinate system