The Shock-in-Time : Generating the Entropy of the Early Universe

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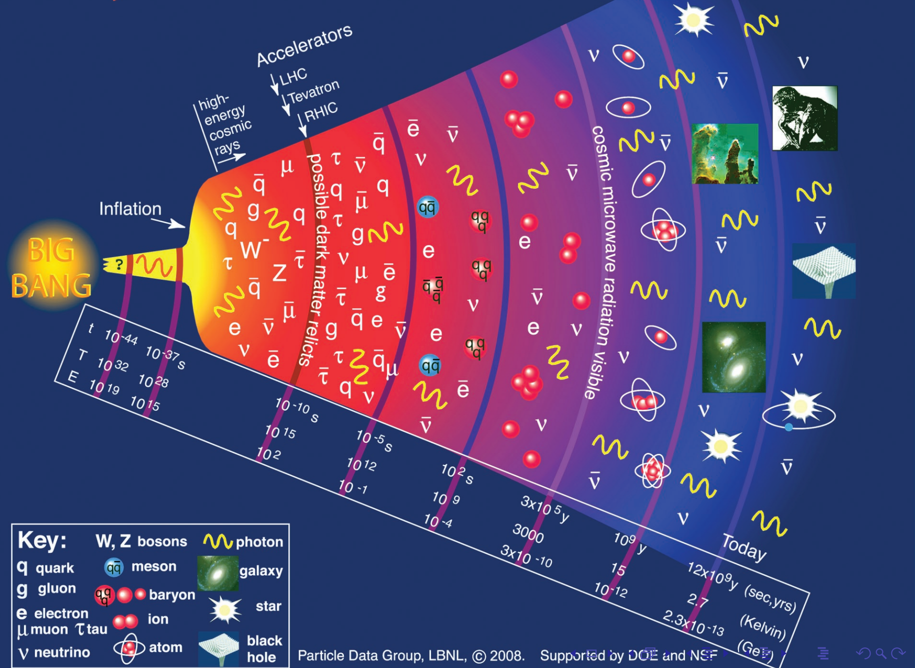
University College London

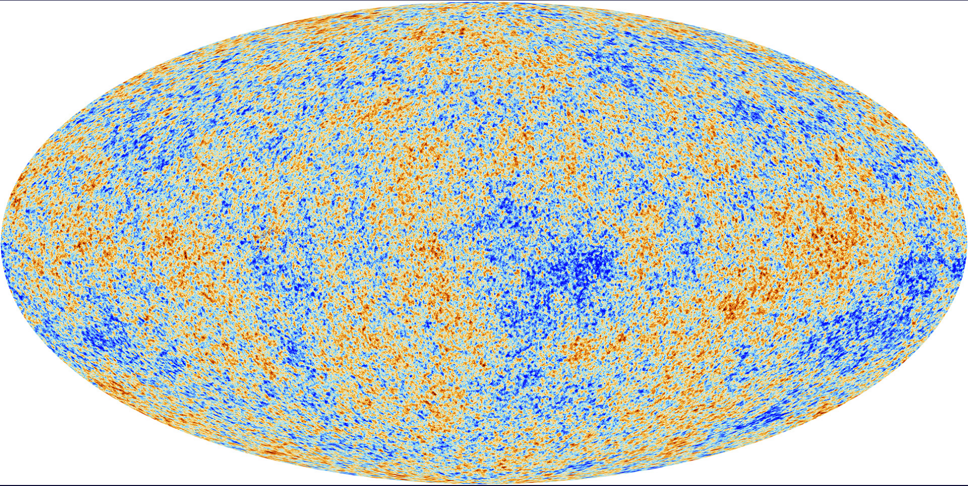
UK QFT VI, Kings College London, March 14, 2017

in collaboration with Dick Bond, Andrei Frolov and Zhiqi Huang (in preparation)

History of the Universe



Observational Evidence for Inflation



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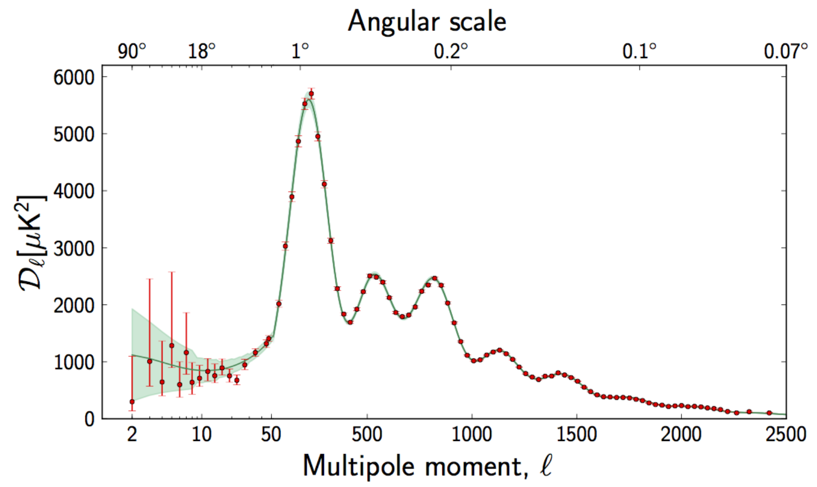
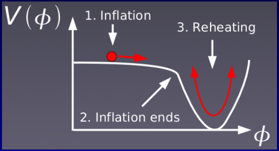


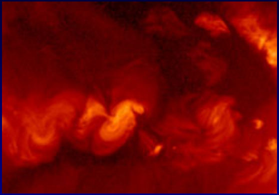
[Planck Collaboration]

Starting the Big Bang

Inflation



Hot Big Bang



- Cold ($T \sim 0$), $\frac{S}{V} \approx 0$
- Few active d.o.f.

- Hot ($T > MeV$), $\frac{S}{V} \propto g_{eff}(T)T^3$
- Many active d.o.f.

Huge entropy production (information processing)

But how does it happen?

The Cosmic Recipe?

Why Is This Regime Interesting

Theoretical Consistency

- Inflationary cosmology is incomplete without this transition
- Understand nonequilibrium quantum field theory

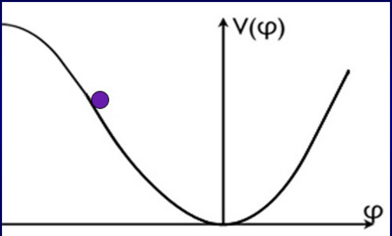
More practical concerns

- $N \equiv \ln(a_0/a_{end})$ needed to match observations to inflationary models
- Production of
  - nonGaussian density perturbations [Bond,Frolov,Huang,Kofman],[**Rajantie**,Chambers]
  - tensors [Easter,Giblin,Lim],[Figueroa,Garcia-Bellido],[Dufaux,Felder,Kofman,Huang]
  - defects, Higgs, EOS, etc. [**Rajantie, Markkanen**, ...], [Amin,Lozanov], [**Hardwick**,Vennin,...]
- Linear structure growth depends on background expansion
- Nonequilibrium - baryogenesis?, nonthermal DM production?

A Theorist's View of the Universe

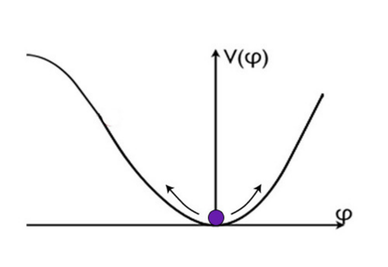
$$\mathcal{L} = -\frac{G_{IJ}(\vec{\phi})}{2}\partial_\mu \phi^I \partial^\mu \phi^J - V(\vec{\phi})$$

During Inflation



- Subhorizon Homogeneity
- (Small) Superhorizon Inhomogeneity

End of Inflation



$$\left[\delta\phi, \delta\dot{\phi}\right] \neq 0$$

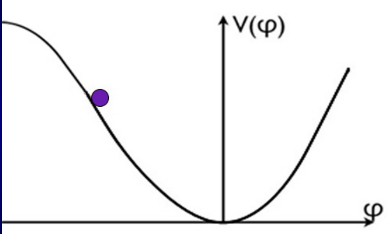
$$\implies \langle|\delta\tilde{\phi}_k|^2\rangle, \langle|\delta\tilde{\dot{\phi}}_k|^2\rangle > 0$$

- Variety of instabilities

A Theorist's View of the Universe

$$\phi(x,t) = \bar{\phi}(t) + \delta\hat{\phi}(x,t) \qquad \bar{\phi} : \mathrm{Long-Wavelength}$$

During Inflation



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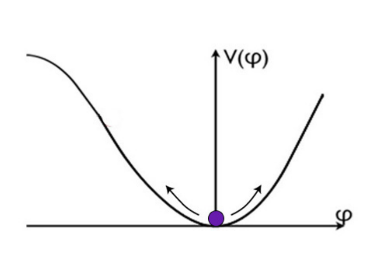

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$$\Longrightarrow \quad \langle|\delta\tilde{\phi}_k|^2\rangle, \langle|\delta\tilde{\dot{\phi}}_k|^2\rangle > 0$$

- Variety of instabilities

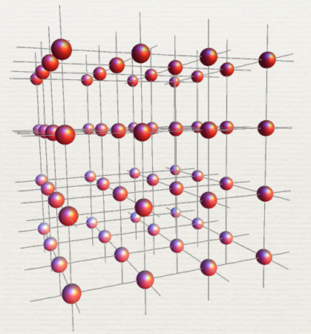
Numerical Approach is Essential [JB, in preparation]

Hybrid MPI/OpenMP Lattice Code

- Solve field equation (e.g.)

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + V'(\vec{\phi}) = 0$$

- 10th order Gauss-Legendre integration (general) or 8th order Yoshida (nonlinear sigma models)
- Finite-difference (fully parallel) or Pseudospectral (OpenMP)
- Optional absorbing boundaries
- Quantum fluctuations $\rightarrow$ realization of random field



- Energy conservation $\mathcal{O}(10^{-9} - 10^{-14})$

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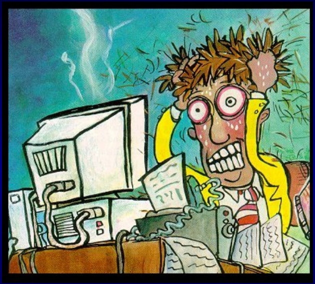

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Developing Complexity of $\ln(\rho/\bar{\rho})$

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Entropy and Information

Shannon Entropy

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Entropy and Information

Shannon (or von Neumann) Entropy

$$S_{shannon} \equiv -\int \mathcal{D}\varphi f[\varphi] \ln f[\varphi] \qquad S_{vN} = -\mathrm{Tr}\hat{\rho}(\hat{\varphi}) \ln \hat{\rho}(\hat{\varphi})$$

Entropy and Information

Entropy : Expectation Value of Information

$$S = -\langle \ln f \rangle_f = -\langle \ln \hat{\rho} \rangle$$

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Relative Entropy (KL-Divergence) - Continuum Variables

$$S_{KL} \equiv \int \mathcal{D}\varphi f[\varphi] \ln\left(\frac{f[\varphi]}{Q[\varphi]}\right) = \left\langle \ln\left(\frac{f}{Q}\right)\right\rangle_f$$

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Approximating $f[\varphi]$: Maximum Entropy Coarse Graining

Maximise S Subject to Measured $\mathcal{C}_{\varphi\vartheta}(x,y) = \langle \varphi(x)\vartheta(y) \rangle$

$$S_{ME} = \frac{1}{2}\ln\det(\mathcal{C}) + \frac{N_{\rm dof}}{2} + \frac{N_{\rm dof}}{2}\ln 2\pi$$

Entropy of set of Gaussian Random Fields with given covariance

$$\det\mathcal{C} \sim V_{\rm fluc}^2$$

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Homogeneous Field Statistics

$$S_{ME} = \frac{1}{2}\sum_i \ln\Delta^2(k_i) + \frac{N_{\rm dof}}{2} + \frac{N_{\rm dof}}{2}\ln 2\pi$$

Entropy of set of Gaussian Random Fields with given covariance

$$\Delta^2(k) = \det(\tilde{\mathcal{C}}_{ij}(k)) \qquad \tilde{\mathcal{C}}_{ij}(k) = \frac{1}{2}\left\langle \varphi_i(k)\varphi_j^*(k) + {\rm c.c.}\right\rangle$$

Entropy Generation

Higher Order Correlators $\rightarrow$ Entropy Generation

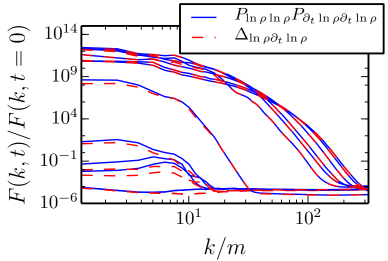
$$\frac{dS}{dt} \sim a_{23}\langle 3 - pt\rangle + a_{24}\langle 4 - pt\rangle + \ldots$$

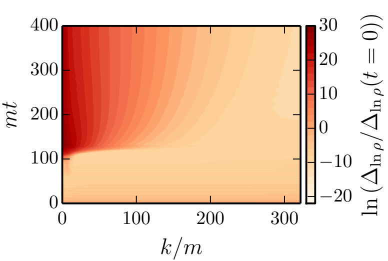
Information Flow from System (2-pt) to Environment (higher points)

$\frac{dS}{dt} = 0$ for linear fluctuation evolution of canonical fields

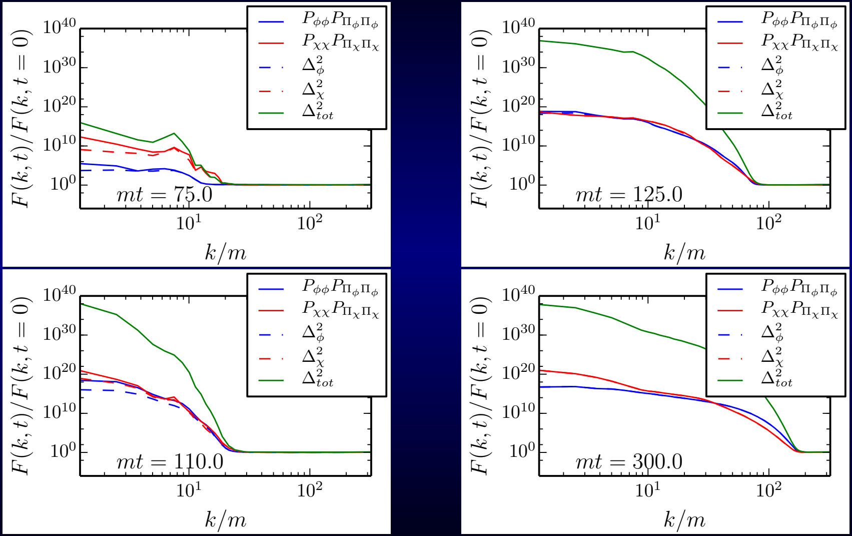
Evolution of Power Spectra of Fluctuations

Evolution of Determinants: Phonons





Evolution of Determinants: Fundamental Fields



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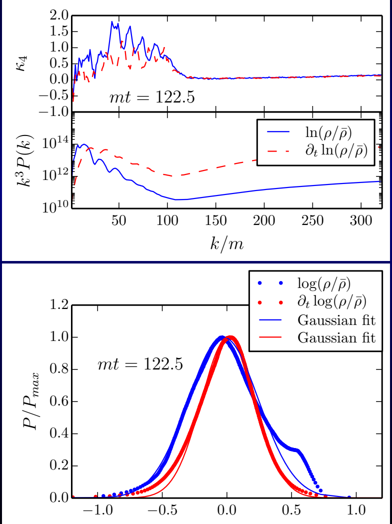
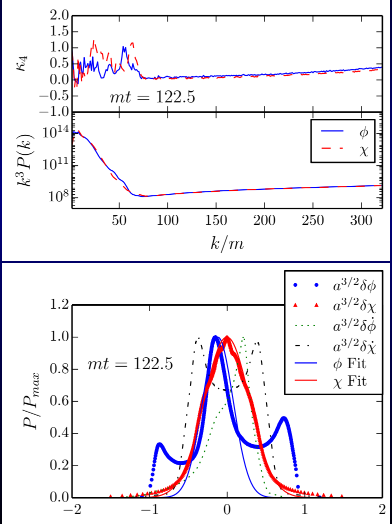
What is φ - Phonons as Collective Variables : In Shock

$\ln \rho$ Phonons



Fundamental Fields



What is φ - Phonons as Collective Variables : Post Shock

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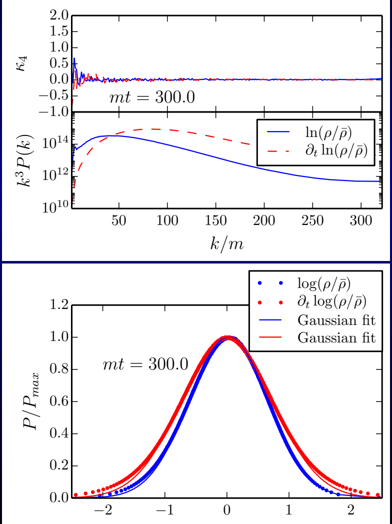

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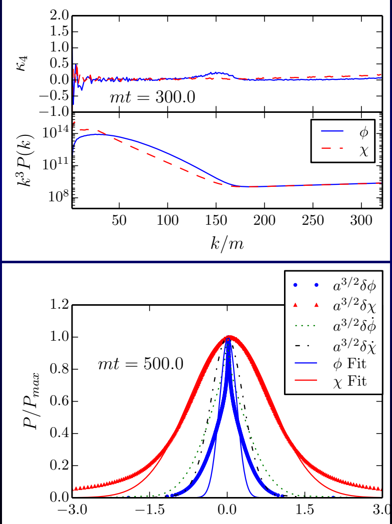

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NonCanonical Variables $(\mathcal{Q} \to \mathcal{J})$

$$S_{ME}^{\rm nc} = \frac{1}{2}\ln\left(\frac{\det\mathcal{C}}{\mathcal{J}^2}\right) + \dots$$

Same as entropy of a Gaussian Random Field with same covariance

$$\det\mathcal{C} \sim V_{\rm fluc}^2 \qquad \mathcal{J}^2 = \left|\frac{\partial\varphi}{\partial\varphi_{\rm can}}\right|^2 \sim V_{\rm quantum}^2$$

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NonCanonical Variables and Phase Space Discretisation

Q represents partitioning of phase space

Choice of Phase Space Discretisation and Quantum Theory

$$C_{\vartheta,\varphi}^{\text{quantum}}(x,y) = \left\langle \hat{\vartheta}(x)\hat{\varphi}(y) \right\rangle = \frac{1}{2}\left\langle \left\{ \hat{\vartheta}, \hat{\varphi} \right\} \right\rangle + \frac{1}{2}\left\langle \left[\hat{\vartheta}, \hat{\varphi} \right] \right\rangle = C^{\mathrm{S}} + C^{\mathrm{A}}$$

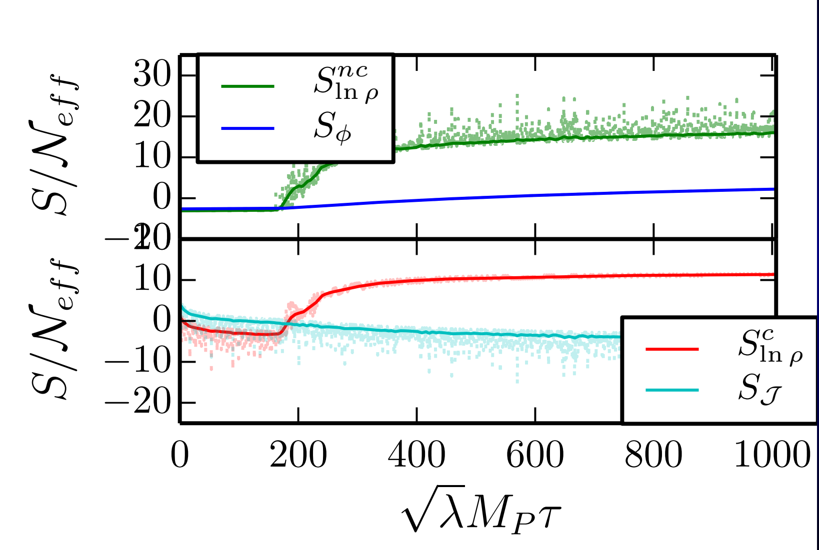
$$[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A} \text{ and } \{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$$

Semi-Classical Limit $\hbar \to 0$

$$C^{S} \to C_{\vartheta,\varphi}^{\text{classical}}$$

$$C^{A} \to \left\langle \left\{ \hat{\vartheta}, \hat{\varphi} \right\}_{PB} \right\rangle = \left\langle \left| \frac{\partial(\varphi)}{\partial(\varphi_{\text{can}})} \right|^{2} \right\rangle$$

Accounting for NonCanonical Nature



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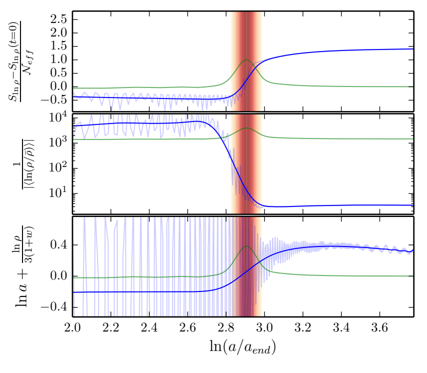
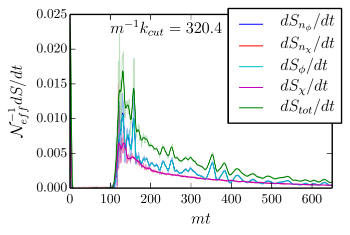
The Shock-in-Time

$\ln\rho$ Phonon DOF





Field DOF

The Shock-in-Time

The Analogy

Spatial Shock

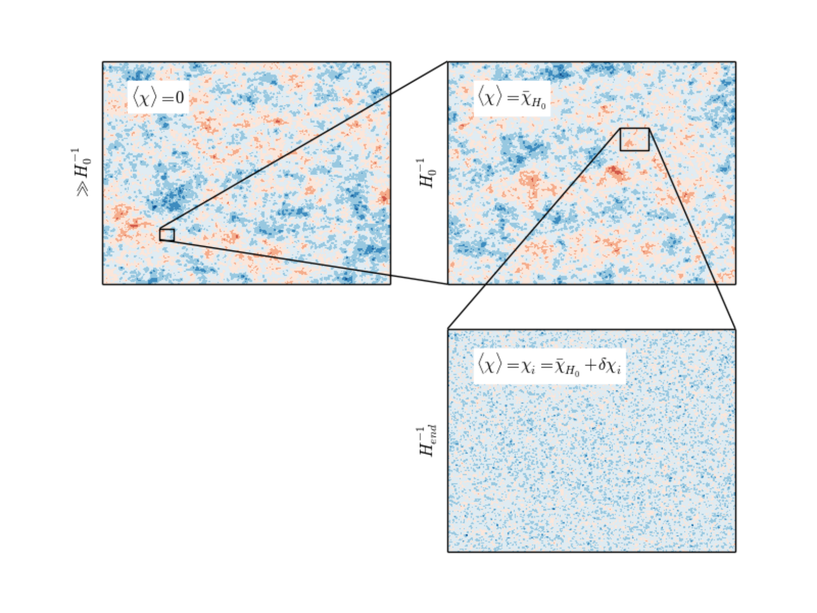
- $v_{bulk}^2 > c_s^2 \rightarrow v_{bulk}^2 < c_s^2$
- Characteristic spatial scale
- Mediated by viscosity or collisionless dynamics
- Randomizing : shock front ΔS
- Post-shock evolution towards thermalization
- Jump in conserved quantities
- Timelike surface

Shock-in-Time

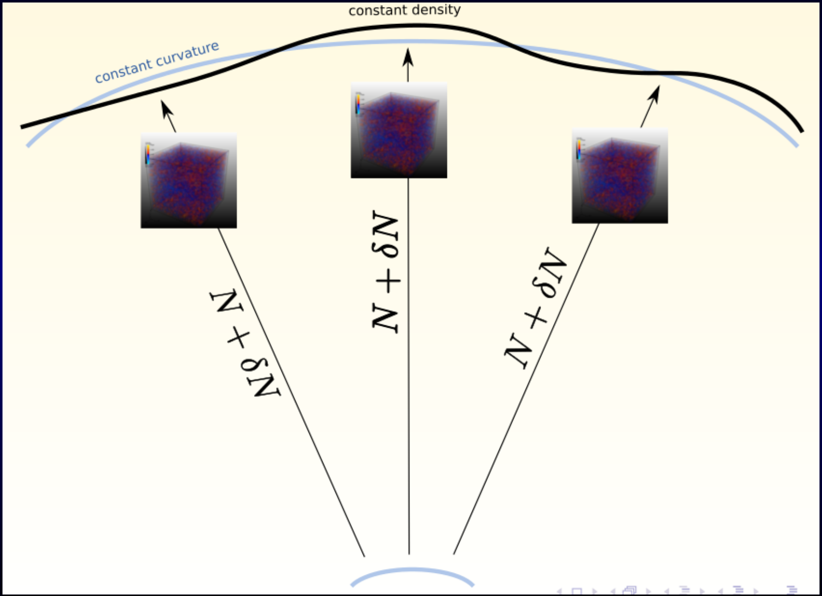
- $ln(\frac{\rho}{\bar{\rho}})^{-1} \gg 1 \rightarrow \ln(\frac{\rho}{\bar{\rho}})^{-1} \sim 1$
- Characteristic time scale
- Mediated by gradients and nonlinearities
- Randomizing : cascade/part. production ΔS
- Slow post-shock evolution
- Jump in $a^{3(1+w)}\rho$
- Can be spacelike surface

Density Perturbations from the Shock-in-Time

Ultra Large Scale Modulating Isocurvature Field



$$\zeta = \zeta_{\text{inf}} + F_{NL}(\chi) \qquad \zeta = (\delta \ln a)\,|_{\rho}$$



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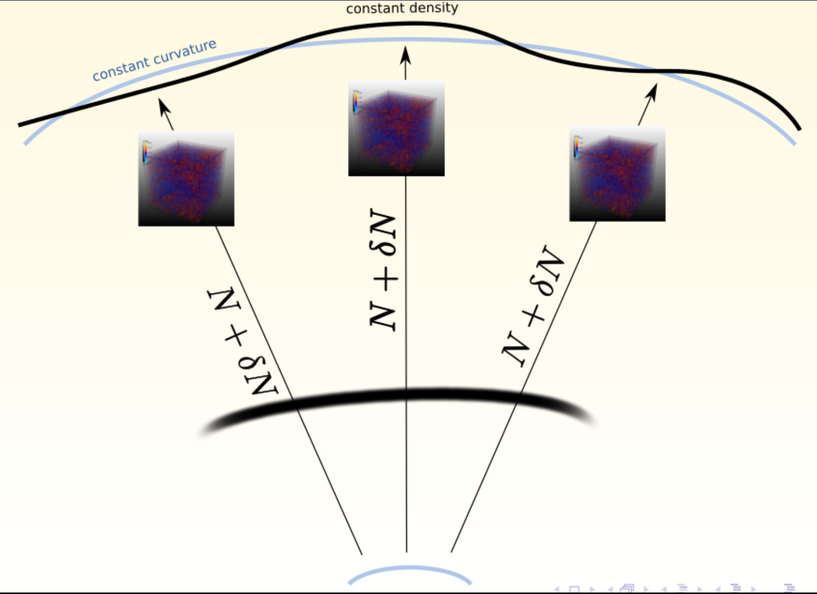
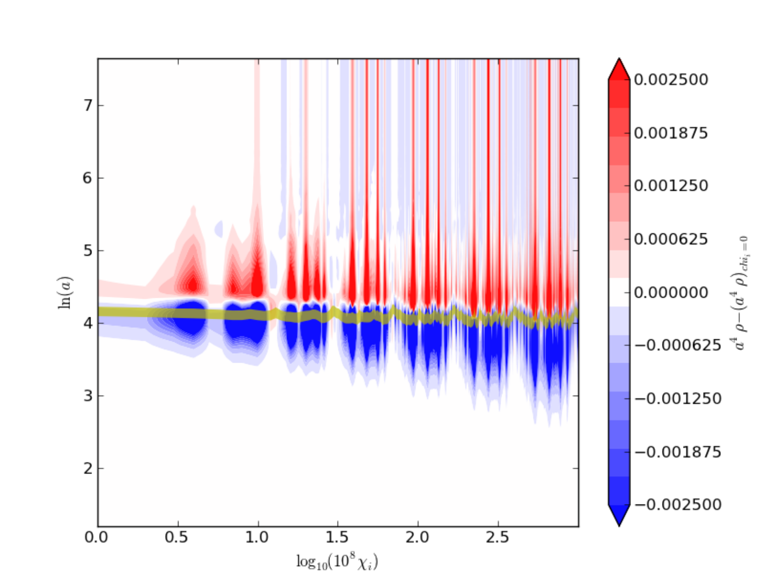
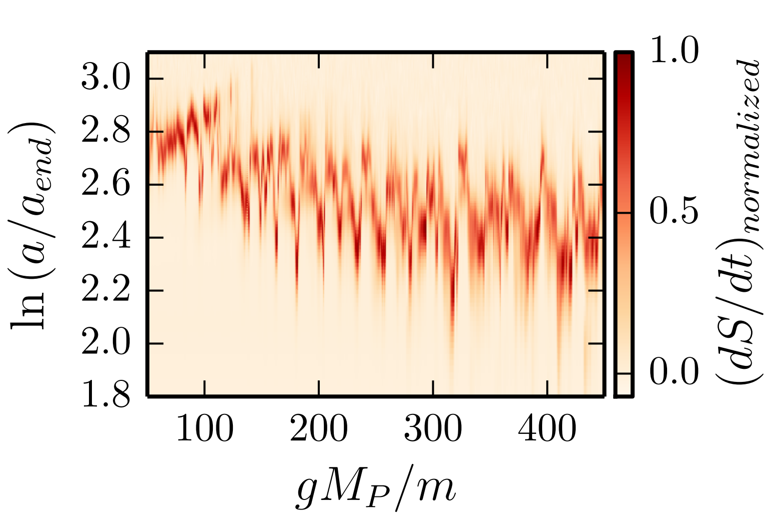


In-Shock Modulation [Bond, **JB**, Frolov, Huang]

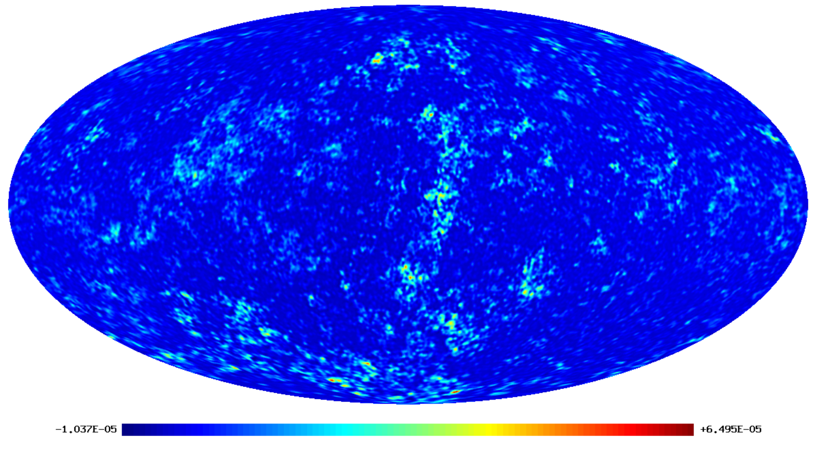


Shock Surface Modulation [Bond, **JB**]



There are dynamical ways to generate $F_{NL}(\chi)$

$F_{NL}(\chi)$ on the CMB Sky



[Couresty of Andrei Frolov]

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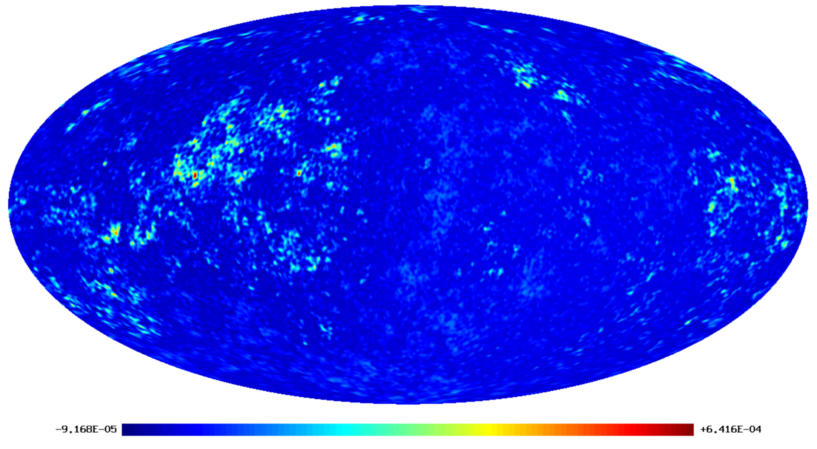

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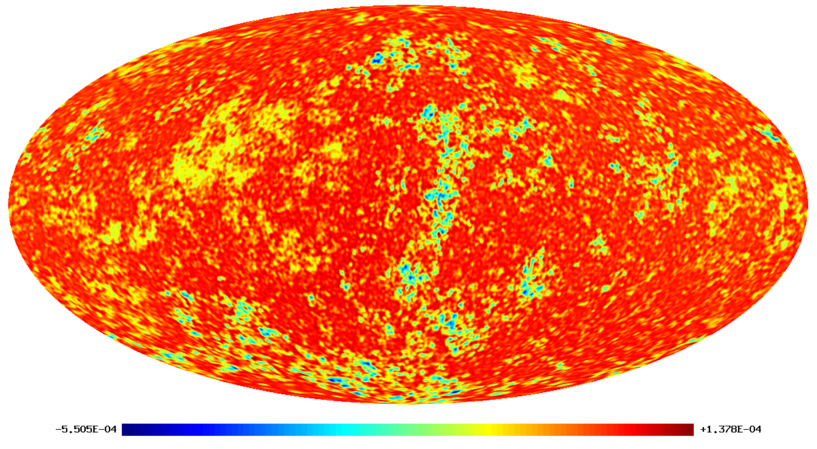

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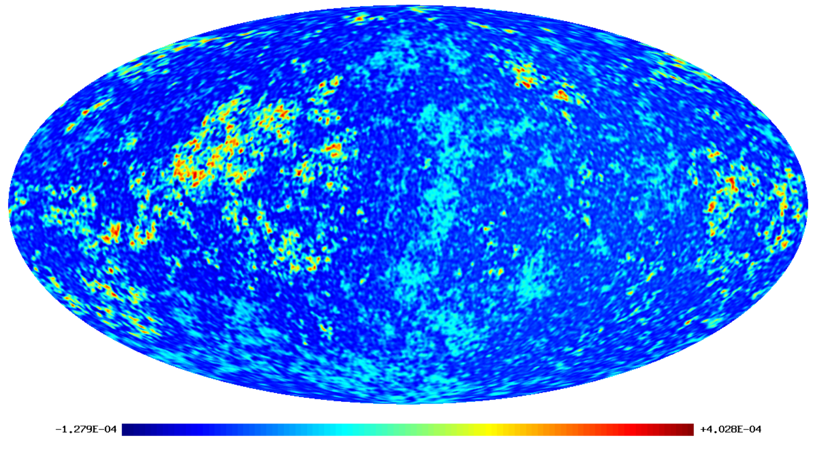

[Couresty of Andrei Frolov]

Conclusions

- Cosmology is incomplete without end-of-inflation dynamics
- Highly nonequilibrium and strongly mode-mode coupled dynamics of a QFT
- Entropy is a useful Information Theoretic measure of this period
- $\ln \rho$ phonons more Gaussian than fundamental fields
- Entropy is primarily produced in a short burst during onset of mode-mode coupling
- This shock-in-time can imprint itself on the CMB
  - Spatial intermittency, requires new pattern matching approaches

Thanks