Intermittency from Nonlinear Scalar Field Dynamics in the Early Universe

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Completed Work

Nonequilibrium Physics on the Lattice [Braden]

BLattice : Noncanonical Scalar Field Lattice Code

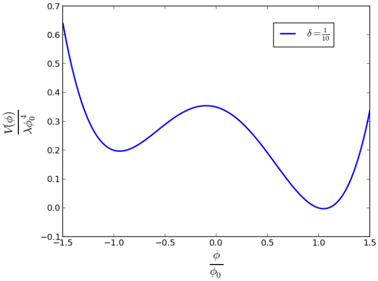
- Solve field equation self-consistently coupled to gravity (e.g.)

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + V'(\vec{\phi}) = 0 \qquad ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

- Fully parallel (MPI & OpenMP hybrid) (tested up to $N_{lat} = 2048^3$)
- Fourier pseudospectral or finite-difference derivatives
- Symplectic evolution ($\mathcal{O}(dt^{10})$) for **non-canonical** scalar fields
  - Gauss-Legendre stepping w/ coupled lattice sites (general)
  - Yoshida splitting w/ GL on decoupled lattice sites

    ($\mathcal{L} \sim -G_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi)$)
- Optional absorbing boundary conditions
- Quantum fluctuations $\rightarrow$ realization of random field
- Hamiltonian constraint $\mathcal{O}(10^{-14})$ achievable

Scalar Field Bubble Collisions [Bond, Braden, Mersini]



What are the dynamics of individual collisions

Standard Lore

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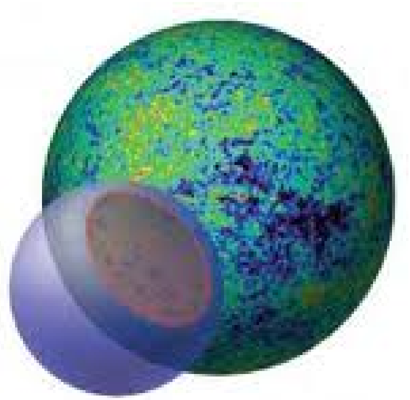

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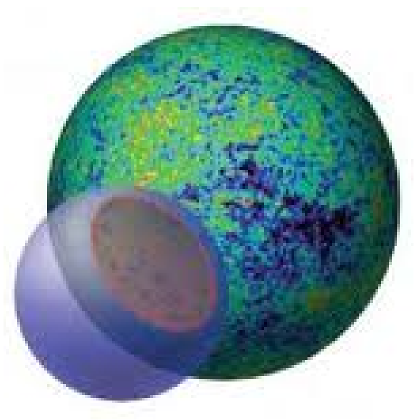

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- Two bubbles : 2 boost + 1 rotational symmetry

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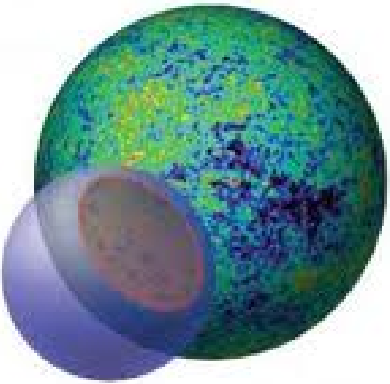

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$\Longrightarrow$ 1+1-d field evolution

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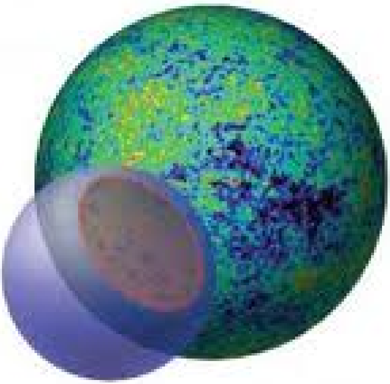

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$\Longrightarrow$ 1+1-d field evolution

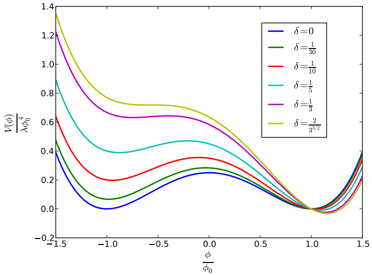
What about fluctuations (the bubbles nucleated)

Initial Conditions (Instantons)

Instanton Equation $(\phi(|\mathbf{x}-\mathbf{x_0}|, t=0) = \phi_{inst}(r_E))$

$$\frac{\partial^2 \phi_{inst}}{\partial r_E^2} + \frac{3}{r_E}\frac{\partial \phi_{inst}}{\partial r_E} - V'(\phi_{inst}) = 0$$

$$\phi_{inst}(r_E = \infty) = \phi_{false} \qquad \frac{\partial \phi_{inst}(r_E = 0)}{\partial r_E} = 0$$

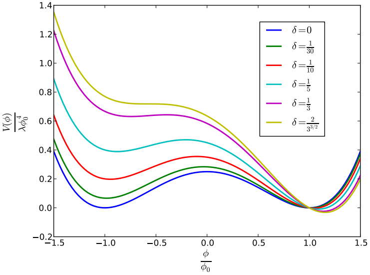


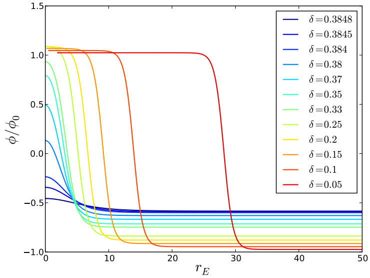
Initial Conditions (Instantons)

Pseudospectral Approach

$$\phi(r_E) = \sum_i c_i B_{2i}\left(h\left(\frac{r_E}{\sqrt{r_E^2 + L^2}}\right)\right)$$

$$h(x) \equiv \frac{1}{\pi}\tan^{-1}\left(d^{-1}\tan\left(\pi\left[x - \frac{1}{2}\right]\right)\right) + \frac{1}{2}$$





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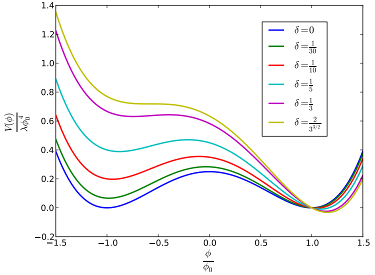

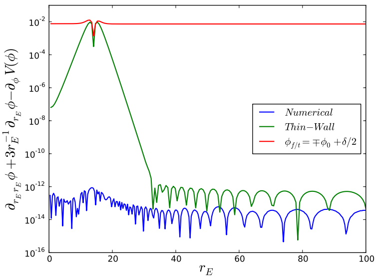

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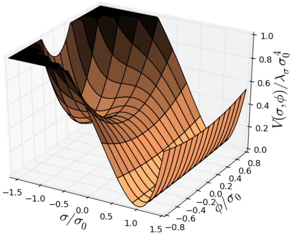
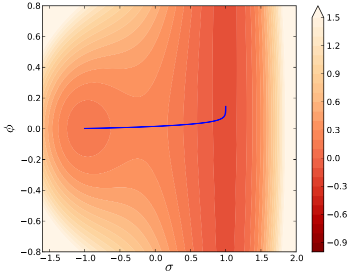
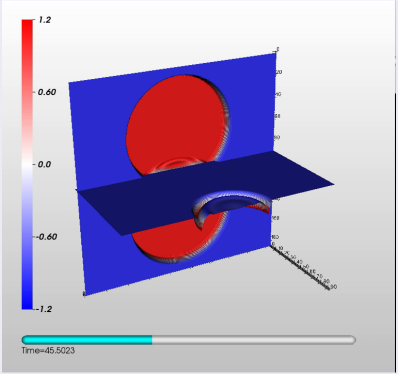




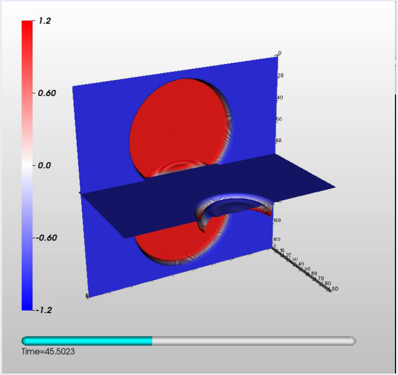
Bubble Collisions Without SO(2,1) Assumed

$$\phi_{init} = \sum_i \phi_{CdL}(|\mathbf{x} - \mathbf{x_i}|) - (N_{bub} - 1)\phi_{false} + \delta\phi(\mathbf{x}, t)$$

SO(2,1) Initial Conditions



SO(2,1) + Small Fluctuations



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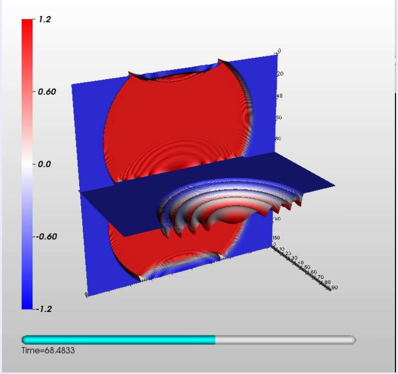

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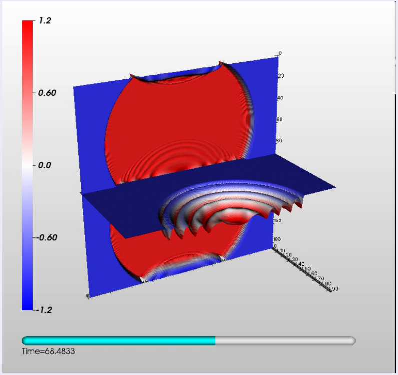

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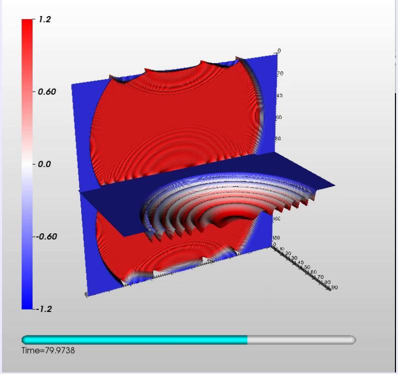

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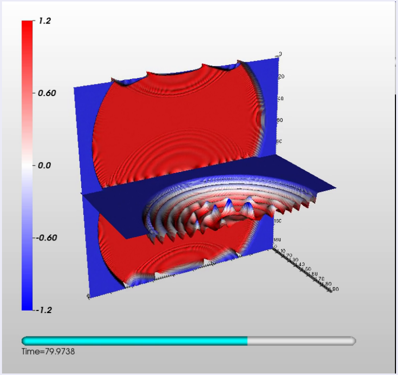

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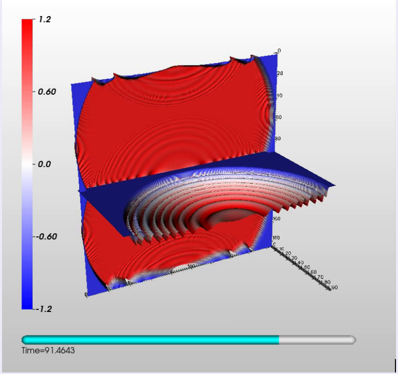

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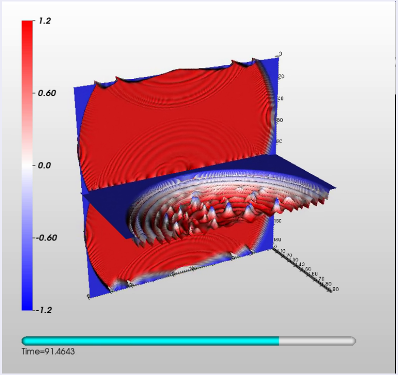
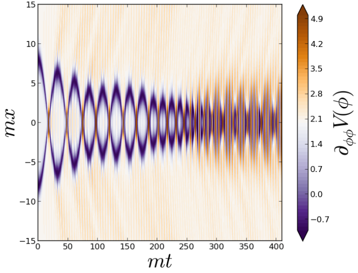


Parametric Resonance Around Domain Walls

Linear Perturbations to SO(2,1) Collision

$$\frac{\partial^2\delta\phi}{\partial s^2} + \frac{2}{s}\frac{\partial\delta\phi}{\partial s} - \frac{\partial^2\delta\phi}{\partial x^2} + \left(\frac{\kappa^2}{s^2} + V''(\phi_{bg}(s,x)\right)\delta\phi = 0$$

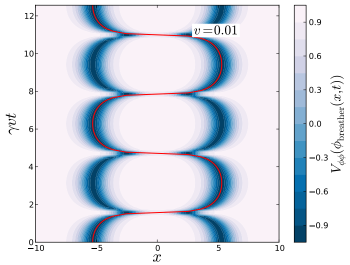


Planar Walls

Parametric Resonance Around Domain Walls

Planar Limit $\to V'' \sim$ periodic $\to$ Floquet Theory

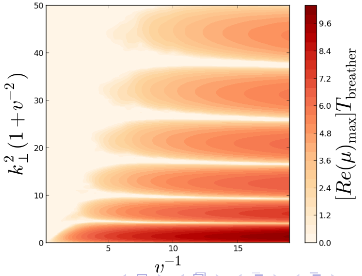
$$\frac{\partial^2\delta\phi}{\partial t^2} - \frac{\partial^2\delta\phi}{\partial x^2} + (k_\perp^2 + V''(\phi_{bg}(x,t)))\delta\phi = 0$$



$v = 0.01$

$V(\phi) = 1 - \cos(\phi)$

$$\phi_{\rm breather} = 4\tan^{-1}\left(\frac{\cos(\gamma vt)}{v\cosh(\gamma x)}\right)$$



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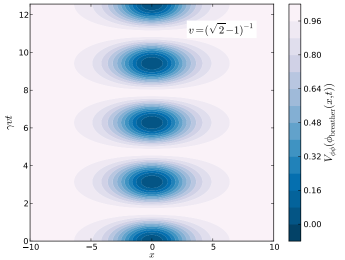

$$v = (\sqrt{2}-1)^{-1}$$

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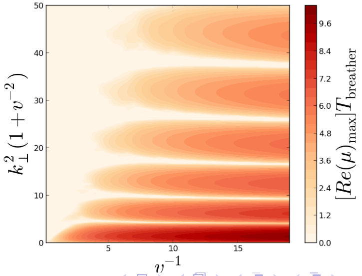
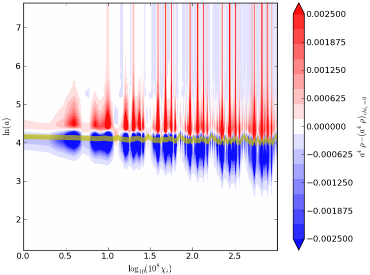
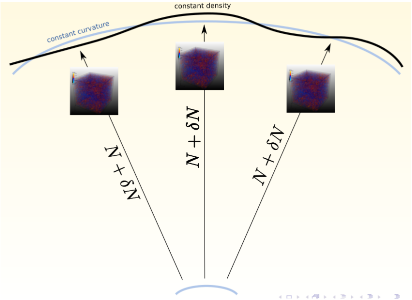


Density Perturbations from Preheating Caustics [Bond, Braden, Frolov, Huang]

$$V(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \xi\phi^2 R + \frac{g^2}{2}\phi^2\chi^2$$



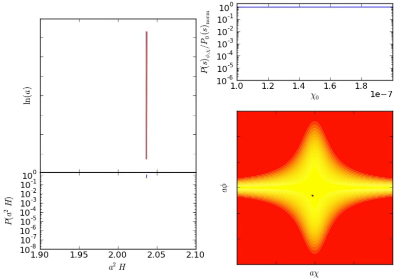


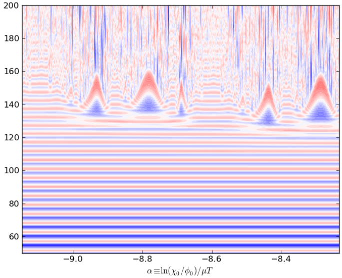
Full Lattice Sim

$$\zeta = \zeta_{inf} + F_{NL}(\chi)$$

Density Perturbations from Preheating Caustics [Bond, Braden, Frolov, Huang]

$$V_{Einstein}(\phi,\chi) = \left(1+\xi\phi^2\right)^{-2}\left(\frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\chi^2\right)$$





Decoupled Subhorizon Ballistic Trajectories

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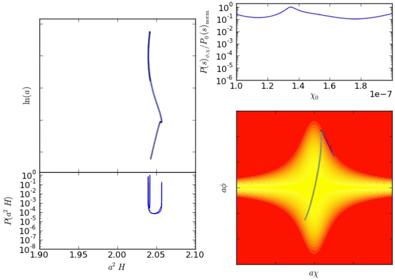

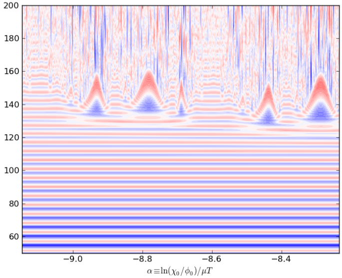

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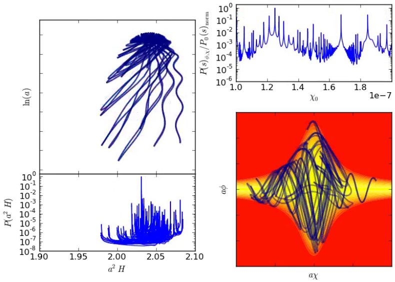

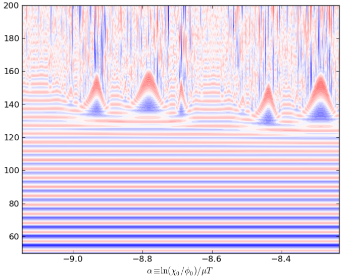

Decoupled Subhorizon Ballistic Trajectories

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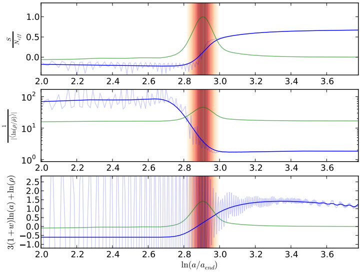
Entropy Production : The Shock-in-Time [Bond, Braden]

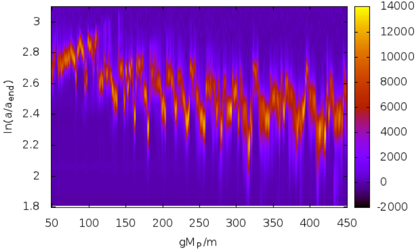
Shannon Entropy (Classical Version of Von Neumann Entropy)

$S_{shannon} = -\int \mathcal{D}\mathbf{f}(Q[\mathbf{f}(x)] \log Q[\mathbf{f}(x)])$ $\quad Q[\mathbf{f}(x)]$:PDF of fields $\mathbf{f}$

$\log(\rho/3H^2)$ as f



Shock Modulation →
Density Perturbations



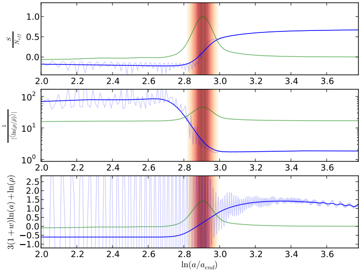
$$V(\phi,\chi) = \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$

Entropy Production : The Shock-in-Time [Bond, Braden]

MaxEnt Subject to Measured 2-Point Correlator

$$S = \sum_{\mathbf{k}} \log(\det(\Delta_{ij}(\mathbf{k}))) + \mathrm{const} \qquad \Delta_{ij}(\mathbf{k}) = \langle \tilde{f}_i(\mathbf{k}) \tilde{f}_j^*(\mathbf{k}) \rangle$$

$\log(\rho/3H^2)$ as f



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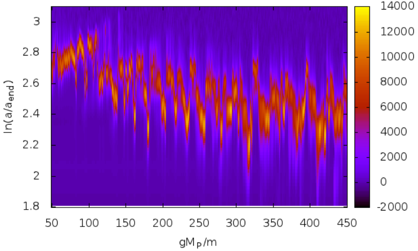

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Future Work

Signatures of SO(2,1) Breaking Collisions

Subset of Recent Related Work (SO(2,1) assumed)

- Wainwright, Johnson, **Peiris**, Aguirre, Lehner, Leibling (1112.4487, 1312.1357)
- **Feeney**, Johnson, **McEwan**, Mortlock, **Peiris** (1210.2725, 1206.5035, 1203.1928, 1202.2861)
- Osborne, Senatore, Smith

New Possibilities

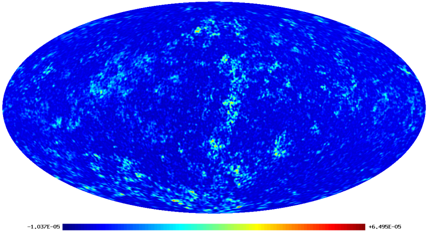
- Production of Tensor Modes - Signature in Polarization
- Direct Imprint of Oscillons? (Hard to avoid beam smoothing)
- Inhomogeneous start to inflation in collision region

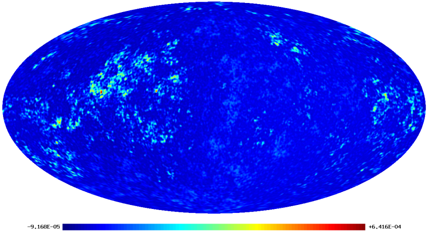
Extend this to other scenarios involving bubbles

Spatially Modulated Signals and Preheating

Preheating is a Local (Subhorizon) Process

$$\zeta = \zeta_\phi + F_{NL}(\chi) \qquad F_{NL} \text{ independent of } \zeta_\phi$$





More General Problem

How to characterize and constrain a random field of the form $F_{NL}(\chi)$

- Peak statistics
- Local measures of nongaussianity
- Statistics of other critical points

Self-Gravitating Scalar Fields

Extend lattice calculations to inhomogeneous metrics

Possible Applications

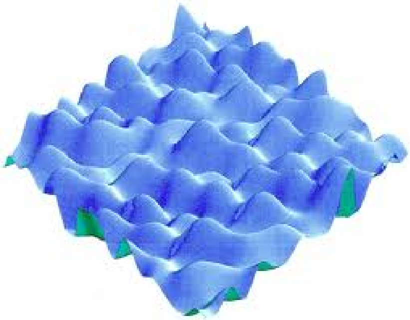
- Full three-dimensional bubble collisions with interior cosmology
- Cosmological phase transitions
- Preheating in conformal inflation models
- "Turbulence" in AdS/CFT
- Implications of B-mode detection for preheating?
- Coupling to unresolved degrees of freedom (model as a fluid)

First Step

$$ds^2 = -e^{2\nu(x,t)}dt^2 + e^{2\alpha(x,t)}d\mathbf{x}^2$$

Linear response for tensor modes

False Vacuum Decay



- Time-dependent description of the tunnelling process
- Properties of instantons in multifield potentials
- How does the system evolve, including effects of CdL instantons, Hawking-Moss instantons, stochastic inflation, etc.
- Are there potentials where a lattice simulation is feasible?