Starting the Hot Big Bang Entropy Generation and Simplicity from Complexity in Nonequilibrium Field Theory

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University of St. Andrews, June 20, 2016

with Dick Bond, Andrei Frolov and Zhiqi Huang (in preparation)





Observational Evidence for Inflation



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Observational Evidence for Inflation



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A Theorist's Description of the Universe Perturbed FRW Metric + Scalar Fields

$$ds^{2} = -e^{2\nu(x,t)}dt^{2} + a^{2}(t)e^{2\zeta(x,t)}\left(\delta_{ij} + h_{ij}\right)dx^{i}dx^{j}$$
$$\mathcal{L} = \sqrt{|g|}\left(-\frac{G_{IJ}(\phi)}{2}\partial_{\mu}\phi^{I}\partial^{\mu}\phi^{J} - V(\phi) + \frac{M_{P}^{2}}{2}R\right)$$



Leading Order: Homogeneous Evolution

$$ds^2 = -dt^2 + a^2(t)dx^2$$
$$\ddot{\phi}_i + 3H\dot{\phi} + \frac{\partial V}{\partial \phi_i} = 0 \qquad H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2} \left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$



 $\ddot{a} > 0 \implies \text{inflation}$

What About Inhomogeneity? Long-Short Split

$$\phi_i = \phi_i^{\text{long}} + \delta\phi_i$$



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$$\phi_i = \phi_i^{\text{long}} + \delta\phi_i$$

$$\phi_i^{\text{long}} = \int d^d x' W(x - x') \phi_i(x')$$

$$\phi_i^{\text{long}}: k \lesssim H \text{ modes}$$

 $\phi_i^{\rm long}$ coherent "classical" condensate



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 $\phi_i^{ ext{long}}: k \lesssim H ext{ modes} \ \delta \phi: k \gtrsim H ext{ modes}$

 ϕ_i^{long} coherent "classical" condensate $\delta\phi$ incoherent "quantum" noise



Evolution of Length Scales



Modes exiting horizon act as a noise term on the long-wavelength condensate

Multiresolution View of the Universe



The Takeaway Message From Inflation

Post-Inflation Universe is Nearly Homogeneous



- Subhorizon Homogeneity
- (Small) Superhorizon Inhomogeneity

End of Inflation



$$\begin{split} & \left[\delta\phi,\delta\dot{\phi}\right]\neq 0\\ \Longrightarrow \ \langle |\delta\tilde{\phi}_k|^2\rangle, \langle |\delta\tilde{\phi}_k|^2\rangle > 0 \end{split}$$

► Variety of instabilities

Starting the Hot Big Bang

Hot Big Bang

Inflation







- Cold $(T \sim 0)$, $\frac{S}{V} \approx 0$
- Few active d.o.f.

- Hot (T > MeV), $\frac{S}{V} \propto g_{eff}(T)T^3$
- Many active d.o.f.

Huge entropy production (information processing)

But how does it happen?

The Cosmic Recipe?



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Why Is This Regime Interesting

Theoretical Consistency

Inflationary cosmology is incomplete without this transition

Understand nonequilibrium quantum field theory

More practical concerns

- ▶ $N \equiv \ln(a_0/a_{end})$ needed to match observations to inflationary models
- Production of
 - nonGaussian density perturbations
 - $[{\sf Bond}, {\sf Frolov}, {\sf Huang}, {\sf Kofman}], [{\sf Rajantie}, {\sf Chambers}]$
 - tensors [Easter,Giblin,Lim],[Figueroa,Garcia-Bellido],[Dufaux,Felder,Kofman,Huang]
- Linear structure growth depends on background expansion
- Nonequilibrium baryogenesis?, nonthermal DM production?

Linear Instability Analysis: Preheating

 $\phi(x,t) = \bar{\phi}(t) + \delta\hat{\phi}(x,t)$ $\delta\ddot{\phi}_k + 3H(t)\delta\dot{\phi}_k + m_{\text{eff}}^2(\bar{\phi}(t))_{ij}\delta\phi_j = 0$





 $m_{\rm eff}^2(t)$ oscillatory



Linear Instability Analysis: Preheating

Floquet Theory for $m_{\rm eff}^2$ approximately periodic

 $\dot{\vec{y}} = \mathbb{M}(t)\vec{y}$ $\mathbb{M}(t+T) = \mathbb{M}(t)$



 $\vec{y}(t) = e^{\mu t} \mathbb{P}(t) \vec{y}_0$

The Many Realms of Nonequilibrium Field Theory







Preheating: A Zoo of Interesting Phenomena





Numerical Approach is Essential [JB, in preparation]

Hybrid MPI/OpenMP Lattice Code

► Solve field equation (e.g.)

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + V'(\vec{\phi}) = 0$$

- 10th order Gauss-Legendre integration (general) or 8th order Yoshida (nonlinear sigma models)
- Finite-difference (fully parallel) or Pseudospectral (OpenMP)
- Optional absorbing boundaries
- ► Quantum fluctuations → realization of random field



• Energy conservation $\mathcal{O}(10^{-9} - 10^{-14})$

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Developing Complexity of $\ln(\rho/\bar{\rho})$

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Evolution of Power Spectra of Fluctuations

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Evolution of One-Point PDFs



How Do We Characterize This Transition?

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Shannon Entropy $S_{shannon} \equiv -\int \mathcal{D} arphi f[arphi] \ln f[arphi]$

Shannon (or von Neumann) Entropy

$$S_{shannon} \equiv -\int \mathcal{D}\varphi f[\varphi] \ln f[\varphi] \qquad S_{vN} = -\text{Tr}\hat{\rho}(\hat{\varphi}) \ln \hat{\rho}(\hat{\varphi})$$

Entropy : Expectation Value of Information

 $S=-\langle \ln f\rangle_f=-\langle \ln \hat{\rho}\rangle$

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Relative Entropy (KL-Divergence) - Continuum Variables

$$S_{KL} \equiv \int \mathcal{D}\varphi f[\varphi] \ln\left(rac{f[\varphi]}{Q[\varphi]}
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Approximating $f[\varphi]$: Maximum Entropy Coarse Graining Maximise S Subject to Measured $C_{\varphi\vartheta}(x,y) = \langle \varphi(x)\vartheta(y) \rangle$

$$S_{ME} = \frac{1}{2}\ln\det(\mathcal{C}) + \frac{N_{\text{dof}}}{2} + \frac{N_{\text{dof}}}{2}\ln 2\pi$$

Same as entropy of a Gaussian Random Field with same covariance

$$\det \mathcal{C} \sim V_{\rm fluc}^2 \qquad \mathcal{J}^2 = \left| \frac{\partial \varphi}{\partial \varphi_{\rm can}} \right|^2 \sim V_{\rm quantum}^2$$

 $\frac{dS}{dt} = 0 \text{ for linear fluctuation evolution of canonical fields}$
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Homogeneous Field

$$S_{ME} = \frac{1}{2} \sum_{k} \ln \det \tilde{\mathcal{C}}_{k} + \frac{N_{\text{dof}}}{2} + \frac{N_{\text{dof}}}{2} \ln 2\pi$$

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Evolution of Determinants: Fundamental Fields





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Evolution of Determinants: Fundamental Fields



Information Stored In Cross-Correlations





Information Content of Cross-Correlations



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Evolution of Determinants: Phonons





The Shock-in-Time

$\ln \rho$ Phonon DOF





Field DOF

The Shock-in-Time



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The Analogy

Spatial Shock

$$\blacktriangleright v_{bulk}^2 > c_s^2 \to v_{bulk}^2 < c_s^2$$

- Characteristic spatial scale
- Mediated by viscosity or collisionless dynamics
- Randomizing : shock front ΔS
- Post-shock evolution towards thermalization
- Jump in conserved quantities
- Timelike surface

Shock-in-Time

- $\ \, \blacktriangleright \ \, ln(\frac{\rho}{\bar{\rho}})^{-1} \gg 1 \rightarrow \\ ln(\frac{\rho}{\bar{\rho}})^{-1} \sim 1$
- Characteristic time scale
- Mediated by gradients and nonlinearities
- ► Randomizing : cascade/part. production ΔS
- Slow post-shock evolution
- ▶ Jump in $a^{3(1+w)}\rho$
- Can be spacelike surface

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- 1. What is $f[\varphi]$? (MaxEnt Coarse Graining)
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What is φ - Phonons as Collective Variables : In Shock

$\ln \rho$ Phonons



Fundamental Fields



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What is φ - Phonons as Collective Variables : Post Shock

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Fundamental Fields



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Fluid-Like Description





Fourier Mode Distributions: Pre-Shock





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Fourier Mode Distributions: In-Shock





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Fourier Mode Distributions: In-Shock





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Fourier Mode Distributions: Post-Shock





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- 1. What is $f[\varphi]$? (MaxEnt Coarse Graining)
- 2. What fields φ should we use? (ln ρ Phonons)
- 3. What is Q? (phase space partitioning)

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NonCanonical Variables $(\mathcal{Q} \rightarrow \mathcal{J})$

$$S_{ME}^{\rm nc} = \frac{1}{2} \ln \left(\frac{\det \mathcal{C}}{\mathcal{J}^2} \right) + \dots$$

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NonCanonical Variables and Phase Space Discretisation *Q* represents partitioning of phase space

Choice of Phase Space Discretisation and Quantum Theory

$$\begin{split} C^{\text{quantum}}_{\vartheta,\varphi}(x,y) &= \left\langle \hat{\vartheta}(x)\hat{\varphi}(y) \right\rangle = \frac{1}{2} \left\langle \left\{ \hat{\vartheta}, \hat{\varphi} \right\} \right\rangle + \frac{1}{2} \left\langle \left[\hat{\vartheta}, \hat{\varphi} \right] \right\rangle = C^{\text{S}} + C^{\text{A}} \\ & [\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A} \text{ and } \{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A} \end{split}$$

Semi-Classical Limit $\hbar \to 0$

$$\begin{split} C^S &\to C^{\text{classical}}_{\vartheta,\varphi} \\ C^A &\to \left\langle \left\{ \hat{\vartheta}, \hat{\varphi} \right\}_{PB} \right\rangle = \left\langle \left| \frac{\partial(\varphi)}{\partial(\varphi_{\text{can}})} \right|^2 \right\rangle \end{split}$$

Accounting for NonCanonical Nature





Accounting for NonCanonical Nature



Density Perturbations from the Shock-in-Time

DAG

Ultra Large Scale Modulating Isocurvature Field



 $\zeta = \zeta_{\text{inf}} + F_{NL}(\chi) \qquad \zeta = (\delta \ln a) |_{\rho}$



Dependence of Shock on Model Parameters: Coupling Constants



Dependence of Shock on Model Parameters: Initial Fields



Density Perturbations



Dependence of Density Perturbations on Parameters



Dependence of Density Perturbations on Parameters



Dependence of Density Perturbations on Parameters




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Conclusions

- Inflation is a crucial ingredient in modern cosmology, confirmed by CMB observations
- ► The inflationary phase *must* end, the stage of (p)reheating
- Leads to highly nonthermal slowly evolving state
- Transition characterised by short burst of entropy production
- Many interesting dynamical phenomena