Three Dimensional Quantum Bubble Collisions

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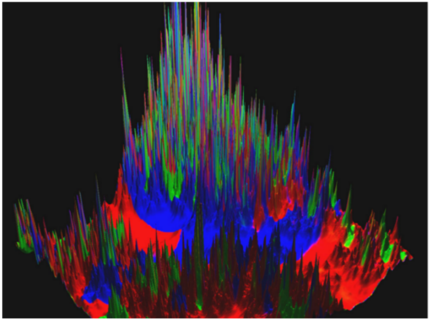
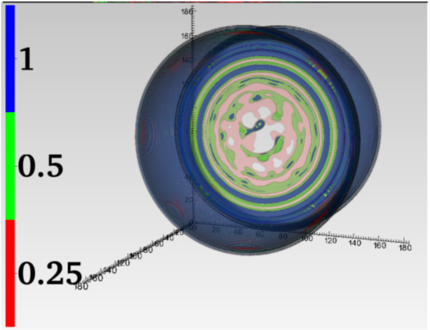
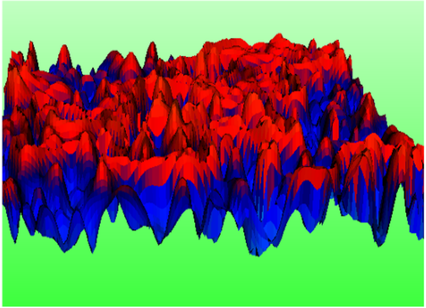
1412.5591 : Cosmic Bubbles and Domain Walls I : Parametric Amplification of Linear Fluctuations
15XX.XXXXX : Cosmic Bubbles and Domain Walls II : Nonlinear Fracturing of Colliding Walls
15XX.XXXXX : Cosmic Bubbles and Domain Walls III : The Role of Oscillons in Three-Dimensional Bubble Collisions

Videos at www.cita.utoronto.ca/~jbraden/

Inhomogeneous Nonlinear Cosmology







Inhomogeneous Nonlinear Cosmology

- **First order phase transitions** (this talk)
- Conversion of isocurvature modes into intermittent density perturbations in preheating [JB, Bond, Frolov, Huang]
  - Caustic formation in chaotic long wavelength dynamics
  - Generalized form of local nonGaussianity with localized spatial properties
- Entropy production in highly inhomogeneous nonlinear field theories (such as the end of inflation) [JB, Bond]
- Strongly inhomogeneous and nonlinear initial conditions for cosmology [JB, Peiris, Johnson, Aguirre (in progress)]

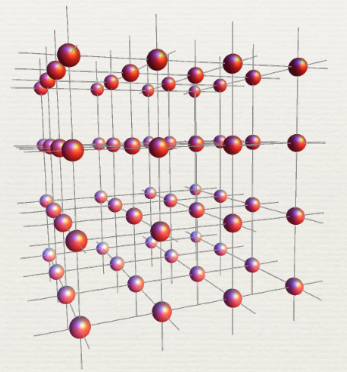
Numerical Approach is Essential

Massively Parallel Lattice Simulation

- Solve field equation (e.g.)

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + V'(\vec{\phi}) = 0$$

- 10th order Gauss-Legendre integration (general) or 8th order Yoshida (nonlinear sigma models)
- Finite-difference (fully parallel) or Pseudospectral (OpenMP)
- Optional absorbing boundaries
- Quantum fluctuations → realization of random field

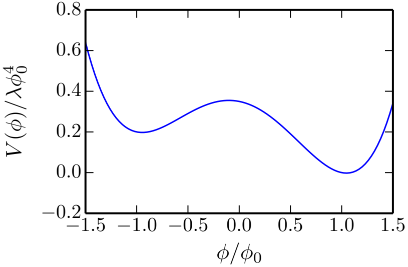


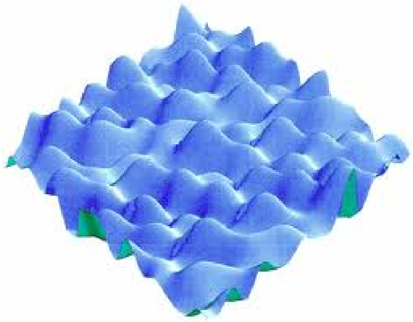
- Energy conservation $\mathcal{O}(10^{-9} - 10^{-14})$

Outline

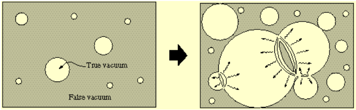
- Bubbly Overview and Review of SO(2,1) Framework
- Setting Initial Conditions (solution of bounce equation)
- Full Nonlinear 3D Dynamics
    1. double-well with slightly broken Z_2 (symmetry breaks)
    2. double-well with strongly broken Z_2 (symmetry remains)
    3. single-well with plateau (symmetry remains)
    4. two-field potential supporting inflation (symmetry breaks)
- Linear Fluctuation Analysis
- Application to Planar Domain Walls
- Implications for Cosmology/Observations

The Bubbly Universe





$$N_{col} \sim \sqrt{\Omega_k}\left(\frac{H_{false}}{H_{inflation}}\right)^2 \frac{\Gamma}{\mathcal{V}} H_{false}^{-4}$$



[Aguirre,Johnson],[Freivogel,Kleban,Nicolis,Sigurdson]

$$\frac{\Gamma}{\mathcal{V}} \sim B^2 |det(\delta^2 S)|^{-1/2} e^{-B}$$

What are the dynamics of individual collisions?

Large Body of Past Work

Single Instantons

- Coleman, deLuccia
- Hawking, Moss
- Turok
- Sasaki, Linde, Tanaka, Yamamoto
- Garriga, Vilenkin, Montes, Garcia-Bellido
- Guth, Guven
- Freese, Adams
- Susskind et al
- ...

Vacuum Bubble Collisions

- Hawking, Moss, Stewart
- Kosowski, Turner, Watkins, Kamionkowski
- **Johnson**, Aguirre, Tysanner, Larfors
- Chang, Kleban, Levy, Sigurdson, **Gobbetti**
- Easther, Giblin, Lim, Lau
- **Johnson**, Lehner, **Peiris**,. . . (GR)
- ...

Observations

- **Johnson**, **Peiris**, Mortlock, McEwan, Feeney,...
- Smith, Senatore, Osborne

Assume (Spacetime) Symmetries

Standard Framework SO(2,1) Symmetry [Hawking,Moss,Stewart],many others

- *Most likely* bubble has SO(3,1) symmetry
- Second bubble breaks
  - Boosts along axis connecting centers
  - Rotations about any axis in plane orthogonal to axis connecting centers
- Preserve SO(2,1)

Standard Framework SO(2,1) Symmetry [Hawking,Moss,Stewart],many others

$$t = s\cosh(\psi)$$
$$x = x$$
$$y = s\sinh(\psi)\cos(\theta)$$
$$z = s\sinh(\psi)\sin(\theta)$$

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1+1-Dimensional Dynamics (e.g. in Minkowski)

$$\frac{\partial^2\phi}{\partial s^2} + \frac{2}{s}\frac{\partial\phi}{\partial s} - \frac{\partial^2\phi}{\partial x^2} - V'(\phi) = 0$$

Standard Framework SO(2,1) Symmetry [Hawking,Moss,Stewart],many others

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Should We Trust This When Quantum Fluctuations are Included?

Effect of Fluctuations on the Collision

Dilute Gas Initial Conditions

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{bounce}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false} + \delta\phi(x, y, z)$$

$\delta\phi$ is not SO(2,1) symmetric and **must** be included in quantum theory

- The bubbles nucleate
- Inflation amplifies subhorizon fluctuations

Is $\delta\phi$ dynamically important?

Need simulations with more than one spatial dimension

Initial Conditions -
Improved Calculation of the Bounce

Setting Initial Conditions : Instantons [Coleman],[Coleman,DeLuccia]

SO(4) Bounce Equation

$$\frac{\partial^2\phi}{\partial r_E^2} + \frac{3}{r_E}\frac{\partial\phi}{\partial r_E} - V'(\phi) = 0$$

$$\phi(r_E = \infty) = \phi_{false} \qquad \frac{\partial\phi(r_E = 0)}{\partial r_E} = 0$$

Pseudospectral Solution

$$\phi(r_E) = \sum_i c_i B_{2i}\left(h\left(\frac{r_E}{\sqrt{r_E^2 + L^2}}\right)\right)$$

$$h(x) \equiv \frac{1}{\pi}\tan^{-1}\left(d^{-1}\tan\left(\pi\left[x - \frac{1}{2}\right]\right)\right) + \frac{1}{2}$$

Global Expansion → Extremely Accurate

Mapping Parameters

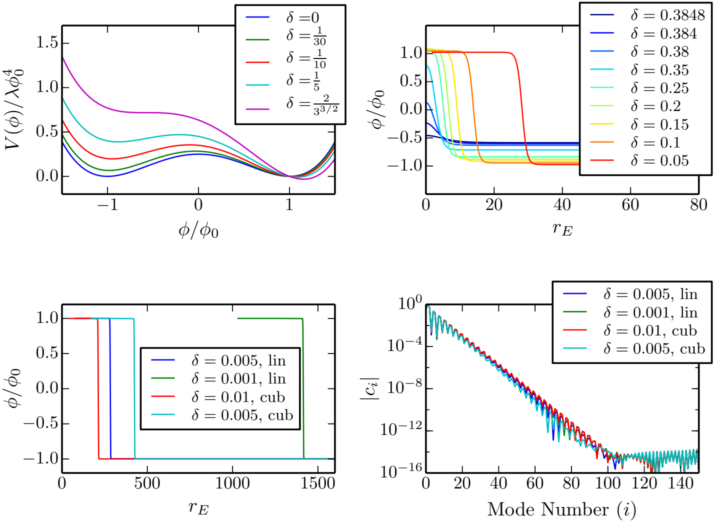
L : $\sim$ radius of bubble

d : $\sim$ width / radius

Extendable to ...

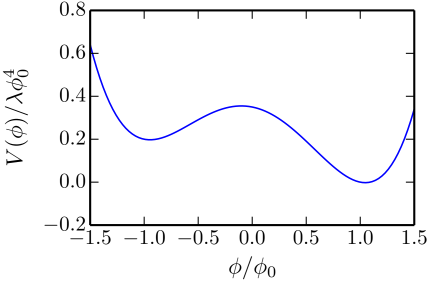
- dynamical metric
- multiple fields (shooting hard)
- PDEs (shooting breaks)

Versatile and Accurate (Even for Very Thin Walls)

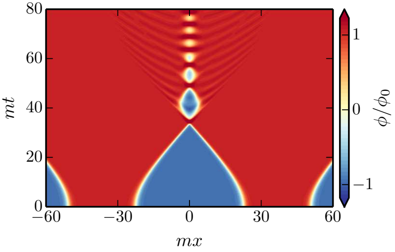


Collision Dynamics

Collisions in a Double Well Potential



Exactly SO(2,1) Invariant Collision



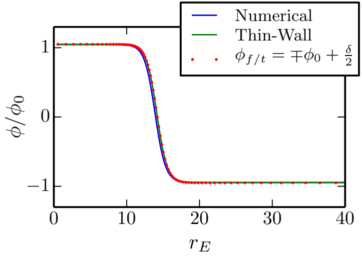
$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \phi_0^2\right)^2 - \delta\lambda\phi_0^3(\phi - \phi_0)$$

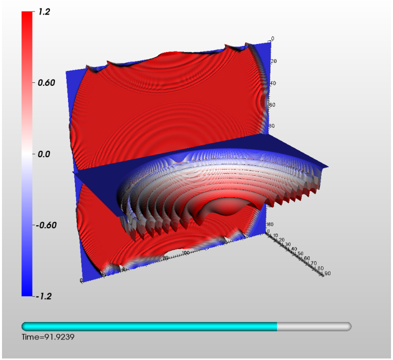
$$s^2 = t^2 - y^2 - z^2$$

Numerical Test : Exact Instanton Initial Conditions

SO(2,1) Invariant Initial Condition

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{bounce}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false}$$

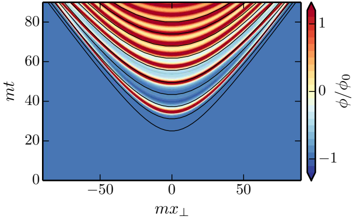


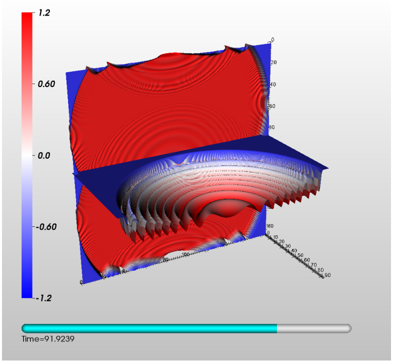


Numerical Test : Exact Instanton Initial Conditions

SO(2,1) Invariant Initial Condition

$$\delta\phi = 0$$

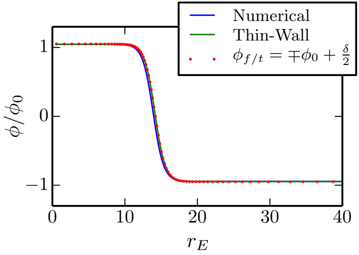




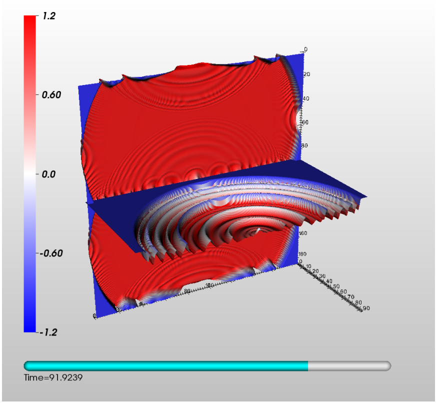
Thin-Wall Approximation

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{tw}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false}$$

Break boost invariance of time-evolved solution



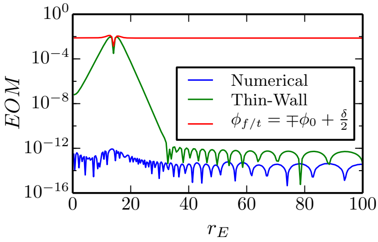
$$\phi_{tw} = \phi_0 \tanh\left(\frac{m(r - R_0)}{\sqrt{2}}\right) + \frac{\delta}{2}$$



Thin-Wall Approximation

$$\delta\phi = \sum_{\mathbf{r_i}} \left(\phi_{bounce}(|\mathbf{x}-\mathbf{r_i}|) - \phi_{tw}(|\mathbf{x}-\mathbf{r_i}|)\right)$$

Break boost invariance of time-evolved solution



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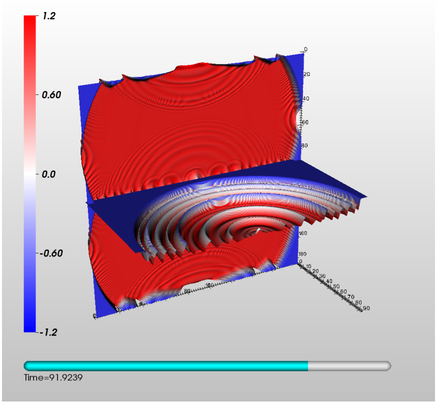

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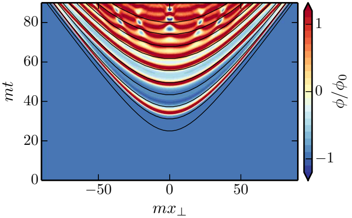

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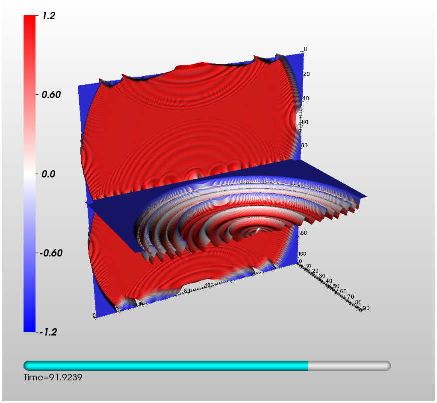
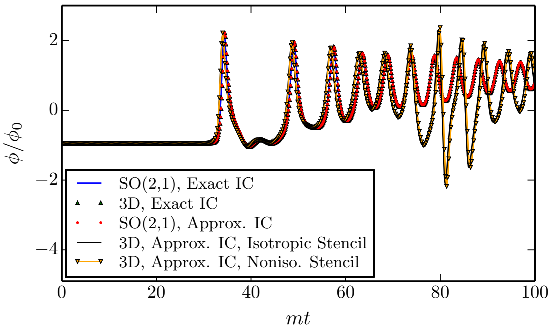


Comparison with One-Dimensional Simulation

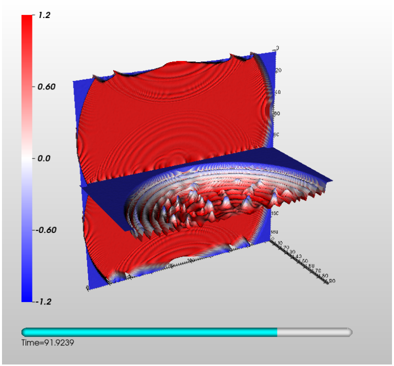
This Effect is Missed by Assuming SO(2,1)



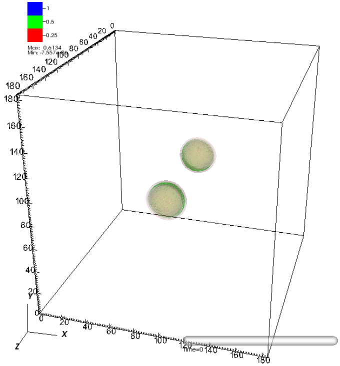
Field Evolution with Fluctuations ...

Bulk Vacuum Fluctuations

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{bounce}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false} + \delta\phi(x, y, z)$$



without Hubble

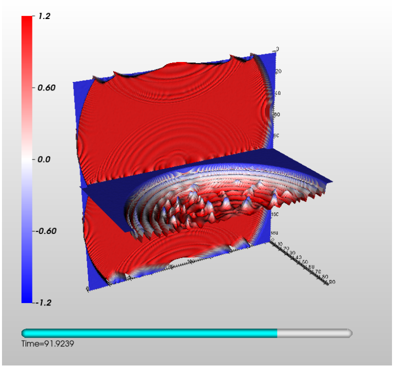


with Hubble

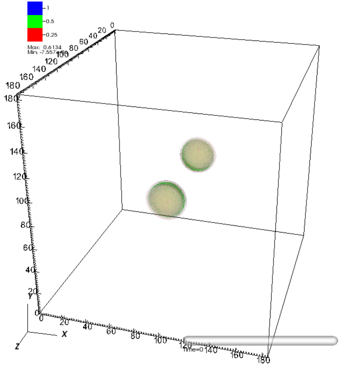
Field Evolution with Fluctuations ...

Bulk Vacuum Fluctuations

$$\delta\tilde{\phi}_{\mathbf{k}} \sim \frac{a_k}{\sqrt{k^2 + V''(\phi_{false})}} \qquad \delta\dot{\tilde{\phi}}_{\mathbf{k}} \sim b_k\sqrt{k^2 + V''(\phi_{false})}$$



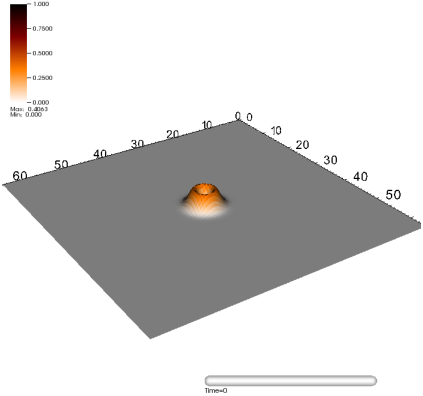
without Hubble

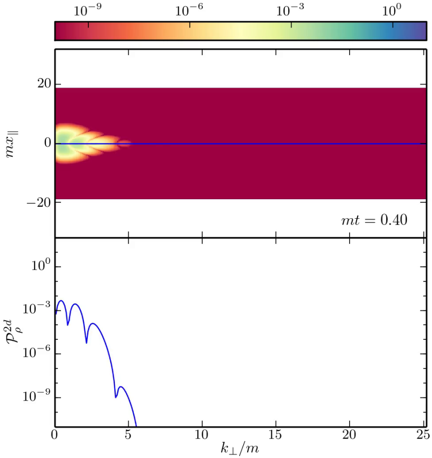


with Hubble

... Produces Oscillons

[Bogolubsky and Makhanov],[Gleiser,Copeland,et al][Guth,Farhi,et al][Amin,Easther,Finkel,Shirokoff][Hertzberg],...

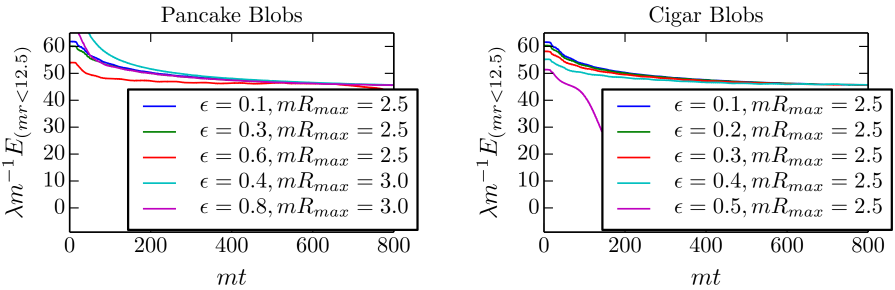




Oscillons Form from Asymmetric Blobs

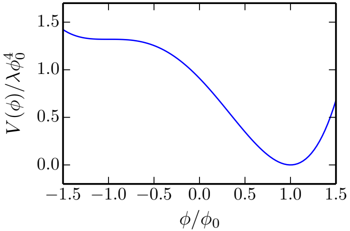
Initial Blob of Field

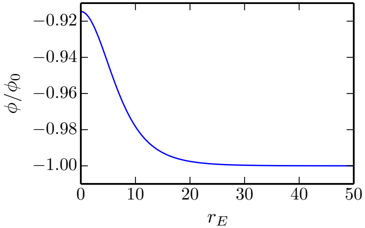
$$\phi_{init} = \phi_{true} + (\phi_{false} - \phi_{true}) exp\left(-\frac{x^2}{a^2} - \frac{y^2+z^2}{b^2}\right)$$

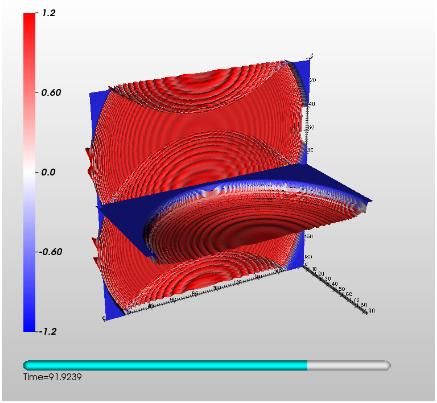


$a^2 < b^2$ $\qquad$ $a^2 > b^2$

A Model without Amplification







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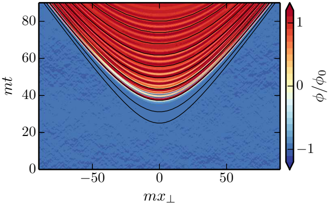
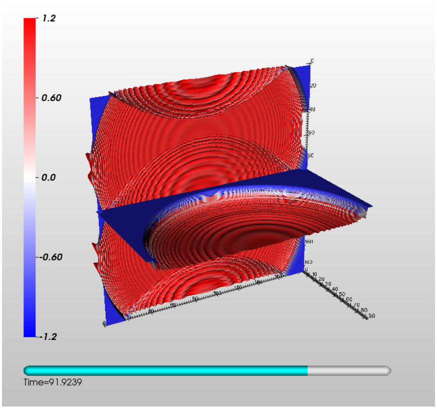
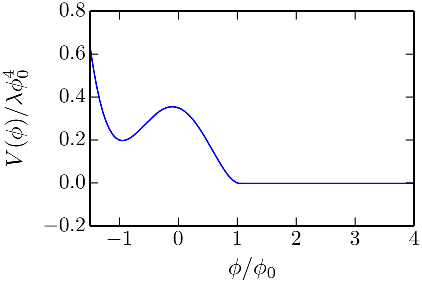
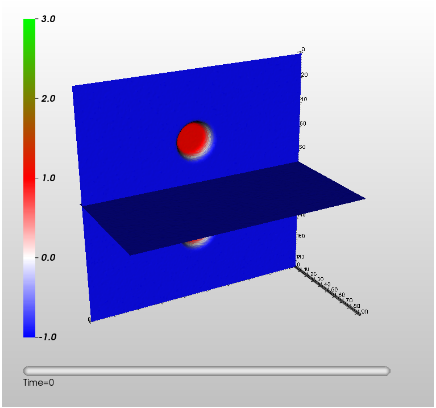


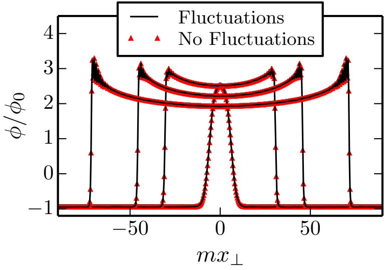


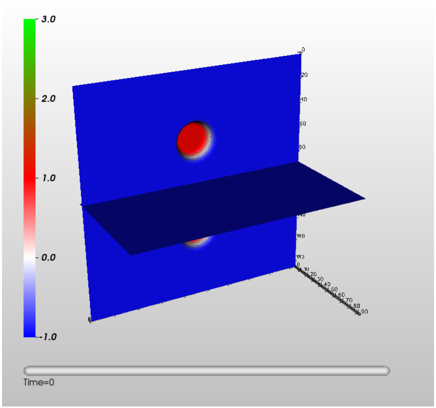
Oscillons with Inflation?



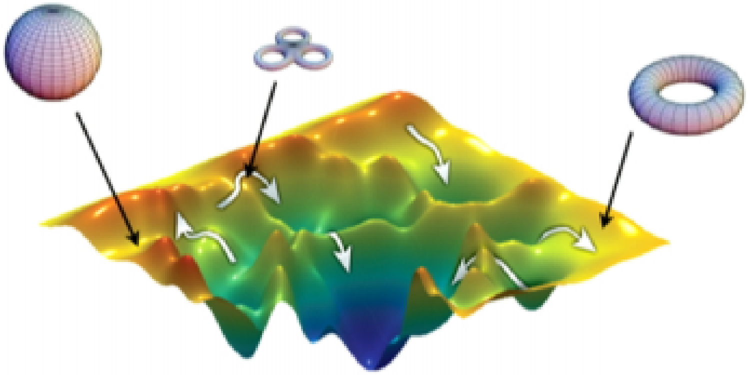


Oscillons with Inflation?





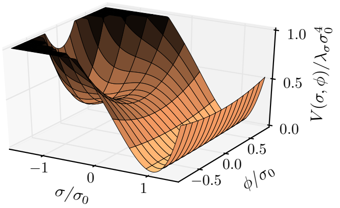
The landscape is not one-dimensional



An Inflationary Model with Oscillons

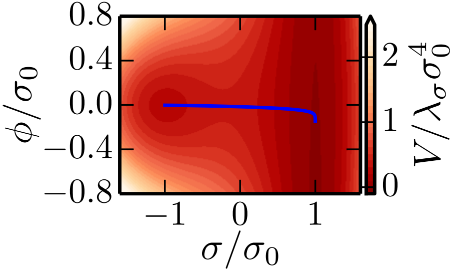
Two Field Model

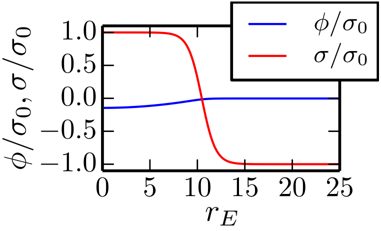
$$V(\sigma,\phi) = \lambda_\sigma \sigma_0^4 \left[\frac{1}{4}\left(\frac{\sigma^2}{\sigma_0^2} - 1\right)^2 + \delta\left(\frac{\sigma^3}{3\sigma_0^3} - \frac{\sigma}{\sigma_0} + \frac{2}{3}\right)\right] + \frac{g^2\lambda_\sigma\sigma_0^2}{2}(\sigma-\sigma_0)^2\phi^2 + \lambda\sigma_0^3\epsilon\phi + V_0$$

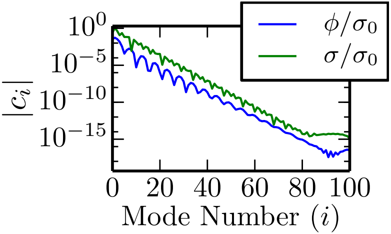


$$V(\sigma,\phi) = V_{tunnel}(\sigma) + V_{coupling}(\sigma,\phi) + V_{inflation}(\phi)$$

Instanton in Two-Field Model



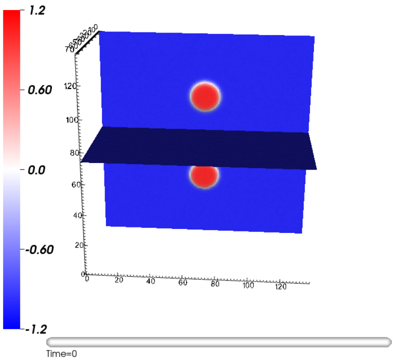




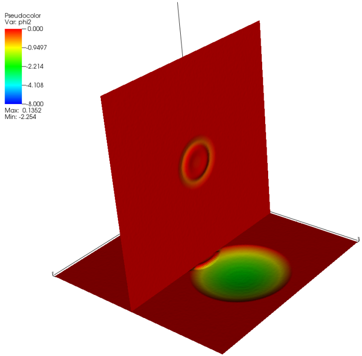
This solution has only a single negative eigenmode in the O(4) symmetric sector

Field Evolution in Two-Field Model

σ Evolution



ϕ Evolution



Theory of Linear Fluctuations -

Parametric Resonance in Inhomogeneous Backgrounds

Parametric Resonance for Linear Fluctuations

Linear Fluctuations Around SO(2,1) Solution

$$\phi(s,x,\psi,\theta) = \phi_{bg}(s,x) + \delta\phi(s,x,\psi,\theta)$$

$$\frac{\partial^2\phi_{bg}}{\partial s^2} + \frac{2}{s}\frac{\partial\phi_{bg}}{\partial s} - \frac{\partial^2\phi_{bg}}{\partial x^2} + V'(\phi_{bg}) = 0$$

$$\frac{\partial^2}{\partial s^2}(s\delta\phi_\kappa) - \frac{\partial^2}{\partial x^2}(s\delta\phi_\kappa) + \left(\frac{\kappa^2}{s^2} + V''(\phi_{bg})\right)(s\delta\phi_\kappa) = 0$$

Parametric Resonance for Linear Fluctuations

Linear Fluctuations Around Planar Solution

$$\phi(t,x,y,z) = \phi_{bg}(t,x) + \delta\phi(t,x,y,z)$$

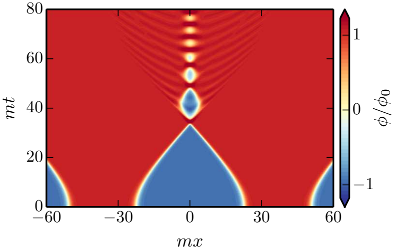
$$\frac{\partial^2 \phi_{bg}}{\partial t^2} - \frac{\partial^2 \phi_{bg}}{\partial x^2} + V'(\phi_{bg}) = 0$$

$$\frac{\partial^2}{\partial t^2}\left(\delta\phi_{k_\perp}\right) - \frac{\partial^2}{\partial x^2}\left(\delta\phi_{k_\perp}\right) + \left(k_\perp^2 + V''(\phi_{bg})\right)\left(\delta\phi_{k_\perp}\right) = 0$$

Planar Limit

- $s \gg 1$
- Time scales much shorter than s
- $\kappa^2 \gg s^2$

Form of V''



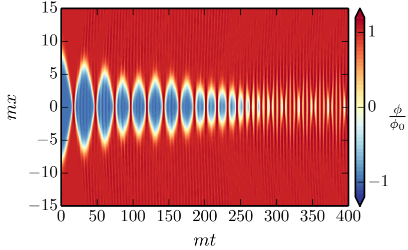
Oscillating Background → Floquet Theory

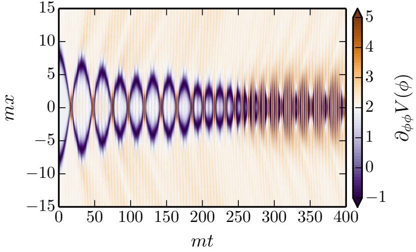
Exactly Periodic Effective Mass

$$\delta\phi_{Floquet} = P(x,t)e^{\mu t} \qquad P(x,t+2T) = P(x,t) \qquad P \in \mathbb{R}$$

Details in Braden et al arXiv:1412.5591

Form of V''





Two wells that repeatedly annihilate

Oscillating Background → Floquet Theory

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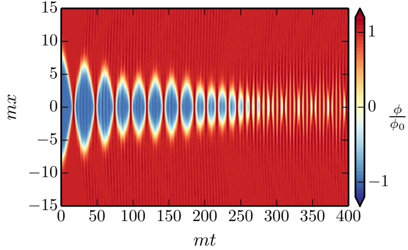
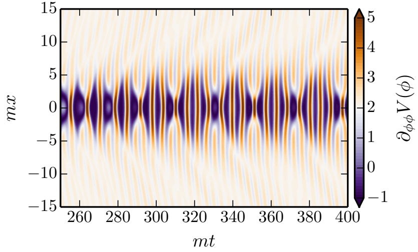
Form of V''





One well oscillating up and down

Oscillating Background $\rightarrow$ Floquet Theory

Exactly Periodic Effective Mass

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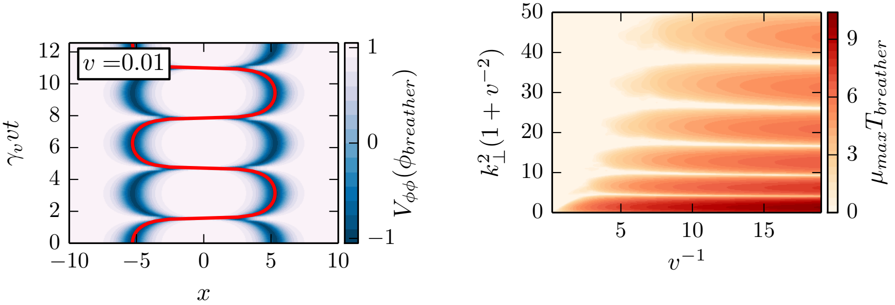
Instability in Sine-Gordon Model

Exactly Periodic Backgrounds

$$\phi_{breather} = 4\tan^{-1}\left(\frac{\cos(\gamma_v vt)}{v\cosh(\gamma_v x)}\right) \qquad \gamma_v \equiv (1+v^2)^{-1/2}$$

$v \ll 1$

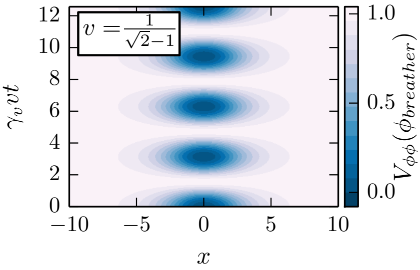


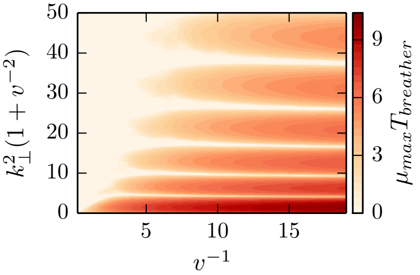
Instability in Sine-Gordon Model

Exactly Periodic Backgrounds

$$\phi_{breather} = 4\tan^{-1}\left(\frac{\cos(\gamma_v vt)}{v\cosh(\gamma_v x)}\right) \qquad \gamma_v \equiv (1+v^2)^{-1/2}$$

$v \gtrsim 1$





Broad Resonance : Well-Defined Wall Collisions c.f.

[Kofman,Linde,Starobinski] for homogeneous bg

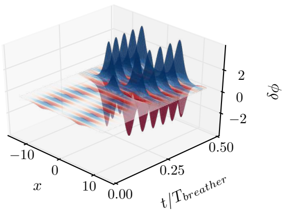
Single Wall, $\phi_{kink,SG} = 4tan^{-1}(e^x)$, $\phi_{kink,DW} = \tanh(x/\sqrt{2})$

In 1d theory $\delta\phi = \partial_x \phi_{kink}(x)$ is a zero mode

In 3d theory $\rightarrow$ bound fluctuations with $\omega = k_\perp$

Very General : Goldstone for spontaneously broken translation invariance

During the collision, there is a short interval when these are not eigenmodes



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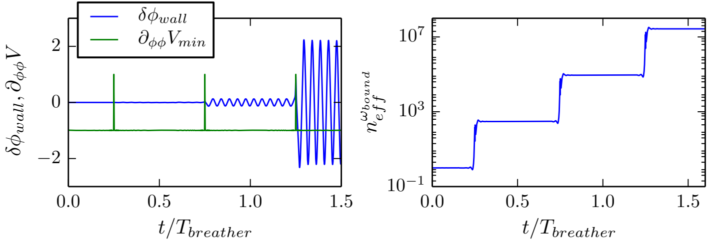
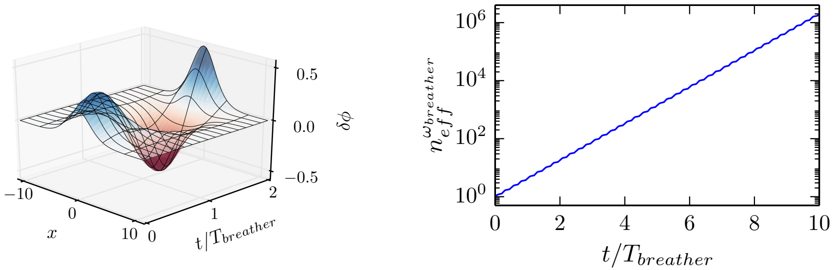
Very General : Goldstone for spontaneously broken translation invariance

$$n_{eff}^{\omega_{bound}} + \frac{1}{2} = \int dx \frac{1}{2k_\perp}\left(k_\perp^2 \delta\phi_{k_\perp}^2 + \delta\dot{\phi}_{k_\perp}^2\right)$$

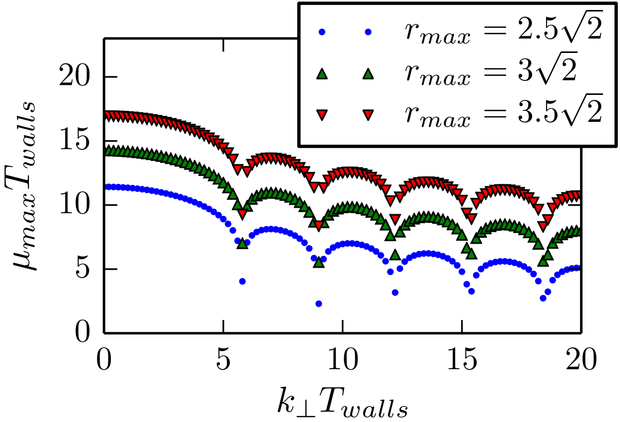


Weak Resonance : Oscillating Blob



$$n_{eff}^{(\omega_{breather})} + \frac{1}{2} \equiv \frac{1}{2\omega_{breather}} \int dx \left(\delta\dot{\phi}^2 + \omega_{breather}^2 \delta\phi^2\right)$$

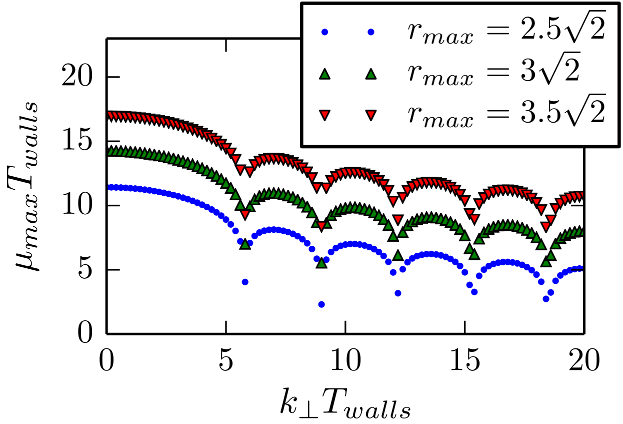
Resonance in Double Well (with approximate backgrounds)



Collisions of Walls

$$\frac{\phi_{bg}}{\phi_0} = -\tanh\left(\frac{\gamma}{\sqrt{2}}(x - r(t))\right) + \tanh\left(\frac{\gamma}{\sqrt{2}}(x + r(t))\right) - 1$$

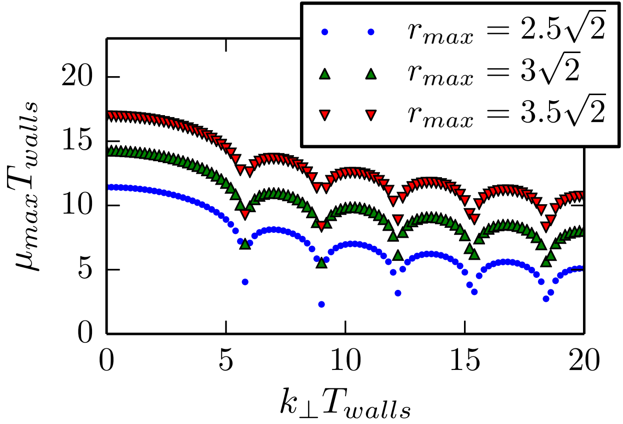
Resonance in Double Well (with approximate backgrounds)



Collisions of Walls

$$r(t) = r_{max} + \frac{1}{2\sqrt{2}} \log\left(\cos^2\left(\frac{\pi t}{T_{walls}}\right) + e^{-2\sqrt{2}(r_{max} - r_{min})}\right)$$

Resonance in Double Well (with approximate backgrounds)



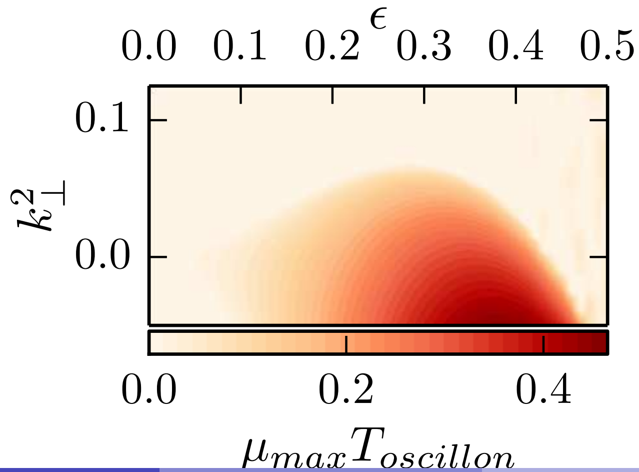
Collisions of Walls

$$T_{walls} = \frac{\pi}{2\sqrt{6}} e^{\sqrt{2} r_{max}}$$

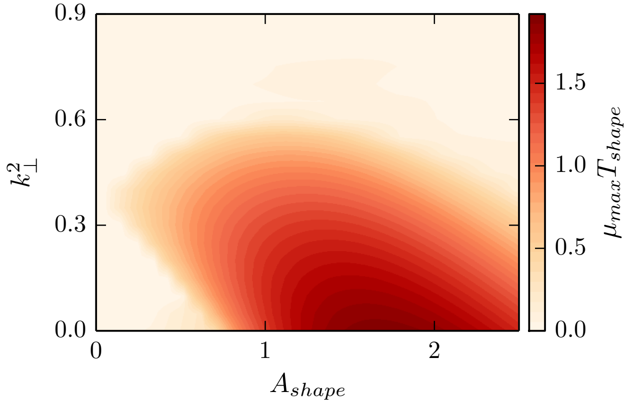
Resonance in Double Well (with approximate backgrounds)

Oscillating Blob

$$\frac{\phi_{bg}}{\phi_0} = 1 + \frac{4\epsilon}{\sqrt{\alpha}}\mathrm{sech}(\sqrt{2}\epsilon m x)\cos(\sqrt{2}\sqrt{1-\epsilon^2}mt)$$



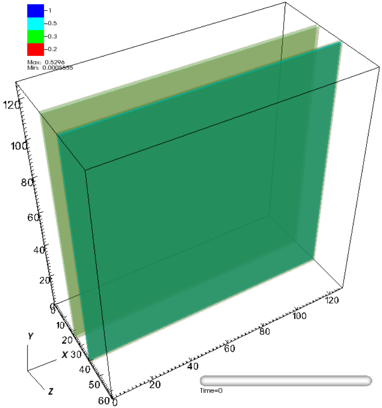
Resonance in Double Well (with approximate backgrounds)



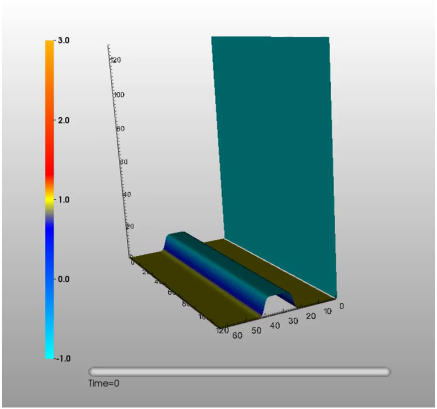
Oscillating Wall Width

$$\frac{\phi_{bg}}{\phi_0} = \tanh\left(\frac{mx}{\sqrt{2}}\right)\left(1 + A_{shape}\mathrm{sech}\left(\frac{mx}{\sqrt{2}}\right)\sin\left(\sqrt{\frac{3}{2}}mt\right)\right)$$

Demonstration in Full Lattice Simulation

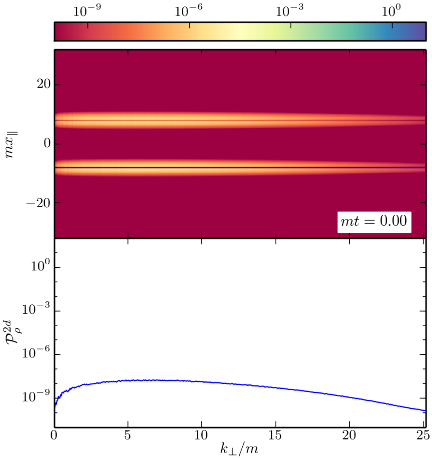


Contours of $\rho/\lambda\phi_0^4$

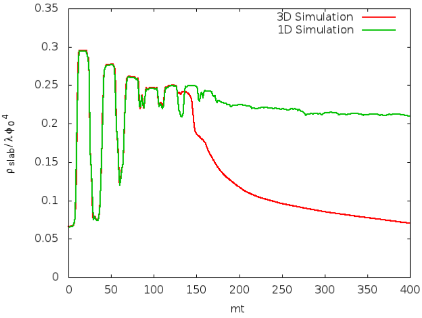


Evolution of ϕ

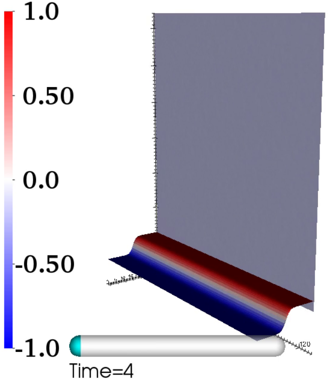
Spectrum of Growing Instabilities

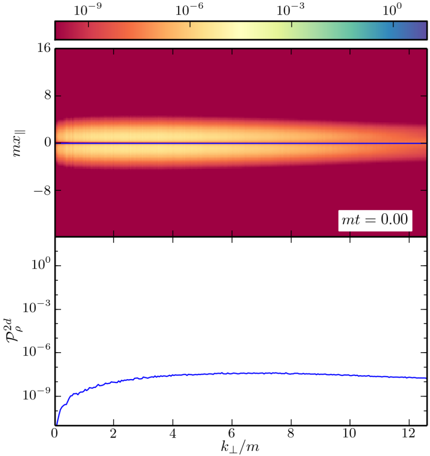


Distribution of Energy



Resonance on a Single Wall





Review of Mechanism

SO(2,1) symmetry can be badly broken by amplified quantum fluctuations

1. Initial state evolves as a piece that preserves SO(2,1) plus a small perturbation that doesn't
2. Perturbations are unstable in the evolving symmetric background
3. Grow ripples and bumps on the bubble walls
4. Large ripples and bumps lead to a random field with blobs whose characteristic size is determined by linear instability
5. Nonlinearities condense these blobs into oscillons

Can have oscillons and inflation in multifield models

Implications

SO(2,1) symmetry can be badly broken

Observables don't necessarily have azimuthal symmetry

- Beam smoothing versus inhomogeneity scale
- Tensor modes are produced by fracturing of walls
- Sign of $\zeta = \delta \ln(a)$ in one field versus two field model

Qualitative conclusions don't depend on inflationary scenario

- Oscillons as nonequilibrium environment for baryogenesis?
- Oscillons dilute as $a^{-3} \rightarrow$ perturbed EOS during phase transition?
- Application to braneworlds with colliding walls
- Preheating in unwinding inflation?
- Bubble baryogenesis

These signals are spatially **intermittent**...