# Collisions of Vacuum Bubbles with Quantum Fluctuations

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Perimeter Institute, Waterloo, Ontario

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in collaboration with Dick Bond, Laura Mersini-Houghton

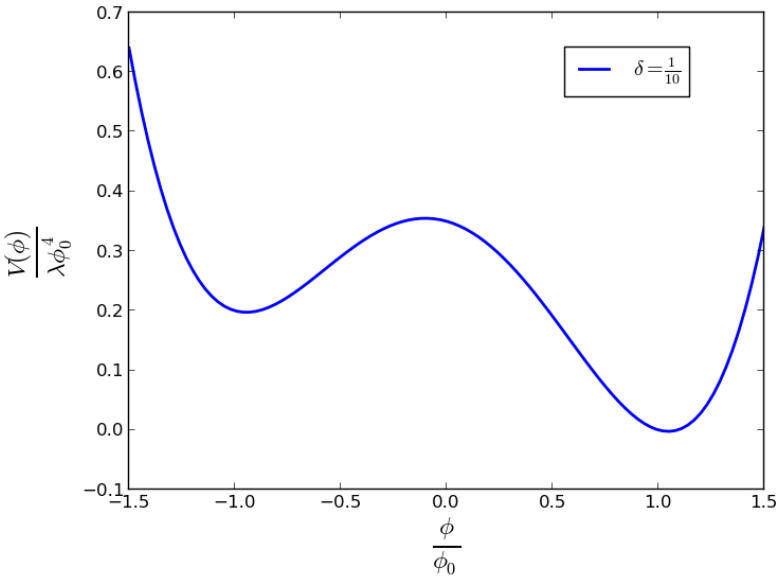
1312.XXXX : Cosmic Bubbles and Domain Walls I : Parametric Amplification of Linear Fluctuations
1312.XXXX : Cosmic Bubbles and Domain Walls II : Nonlinear Fracturing of Colliding Walls
1312.XXXX : Cosmic Bubbles and Domain Walls III : The Role of Oscillons in Three-Dimensional Bubble Collisions

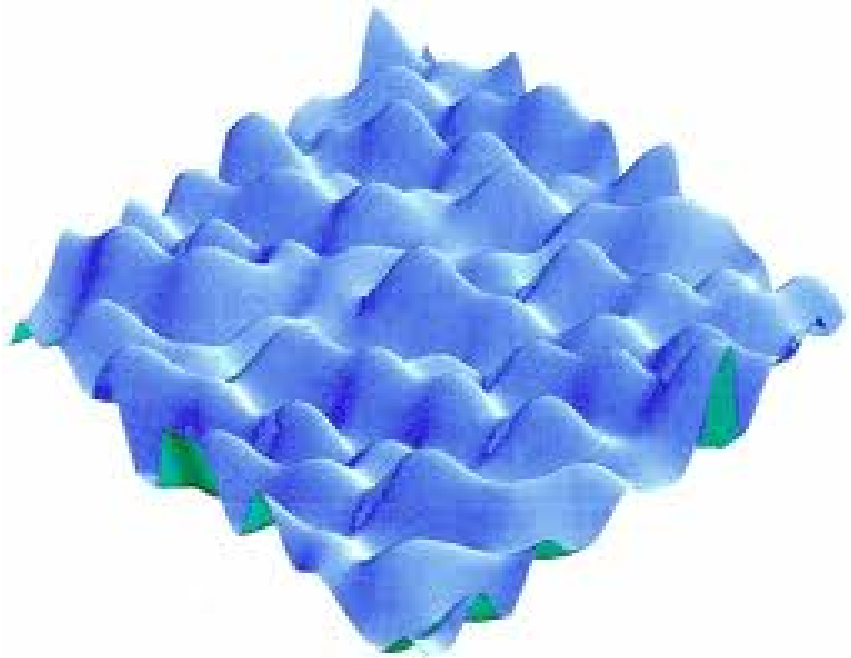
Videos at `www.cita.utoronto.ca/~jbraden/`

# Outline

- Bubbly Overview and Review of SO(2,1) Framework
- Setting Initial Conditions (solution of bounce equation)
- Full Nonlinear 3D Dynamics
  1. double-well with slightly broken $Z_2$ (symmetry breaks)
  2. double-well with strongly broken $Z_2$ (symmetry remains)
  3. single-well with plateau (symmetry remains)
  4. two-field potential supporting inflation (symmetry breaks)
- Linear Fluctuation Analysis
- Implications for Observations

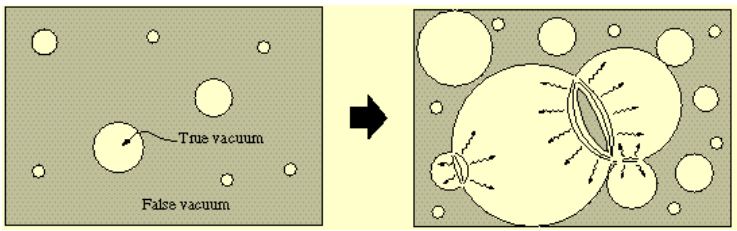
# The Bubbly Universe



$$N_{col} \sim \sqrt{\Omega_k} \left( \frac{H_{false}}{H_{inflation}} \right)^2 \frac{\Gamma}{\mathcal{V}} H_{false}^{-4}$$

[Aguirre,Johnson],[Freivogel,Kleban,Nicolis,Sigurdson]




$$\frac{\Gamma}{\mathcal{V}} \sim B^2 |det(\delta^2 S)|^{-1/2} e^{-B}$$

What are the dynamics of individual collisions?

# Large Body of Past Work

Single Instantons

- Coleman, deLuccia
- Hawking, Moss
- **Turok**
- Sasaki, Linde, Tanaka, Yamamoto
- Garriga, Vilenkin, Montes, Garcia-Bellido
- Guth, Guven
- Freese, Adams
- Susskind et al
- ...

Vacuum Bubble Collisions

- Hawking, Moss, Stewart
- Kosowski, Turner, Watkins, Kamionkowski
- **Johnson**, Aguirre, Tysanner, Larfors
- Chang, Kleban, Levy, Sigurdson
- Easther, Giblin, Lim, Lau (3D, but symmetric IC's)
- **Johnson**, **Lehner**, Peiris,. . . (GR)
- ...

Observations

- **Johnson**, Peiris, Mortlock, McEwan, Feeney,...
- **Smith**, Senatore, Osborne

**Assume (Spacetime) Symmetries**

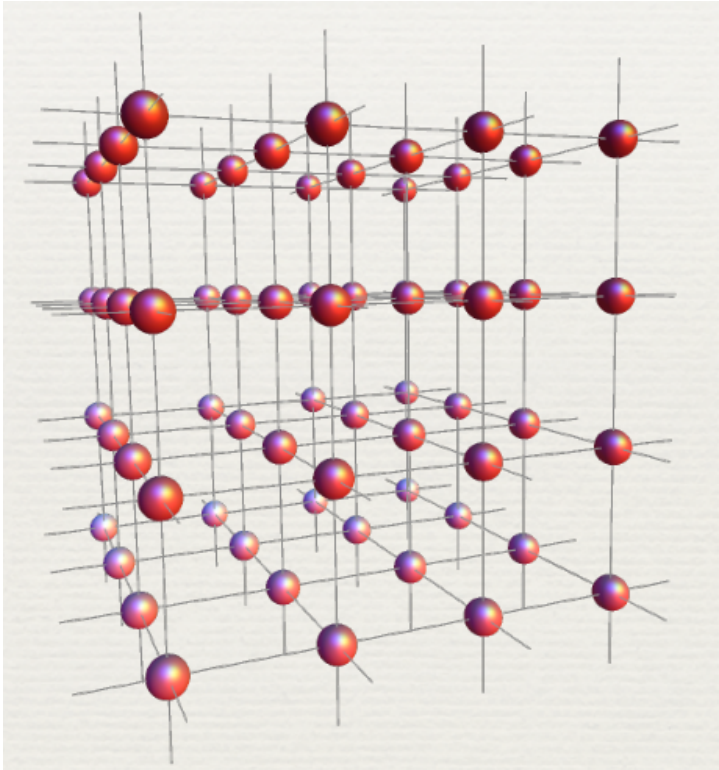
# Numerical Approach

## Massively Parallel Lattice Simulation

- Solve field equation (e.g.)

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + V'(\vec{\phi}) = 0$$

- 10th order Gauss-Legendre integration (general) or 8th order Yoshida (nonlinear sigma models)
- Finite-difference (fully parallel) or Spectral (OpenMP, but MPI version in the works)
- Optional absorbing boundaries
- Quantum fluctuations $\rightarrow$ realization of random field



- Energy conservation $\mathcal{O}(10^{-9} - 10^{-14})$

# Standard Framework SO(2,1) Symmetry [Hawking,Moss,Stewart],many others

- *Most likely* bubble has SO(3,1) symmetry
- Second bubble breaks
  - Boosts along axis connecting centers
  - Rotations about any axis in plane orthogonal to axis connecting centers
- Preserve SO(2,1)

# Standard Framework SO(2,1) Symmetry [Hawking,Moss,Stewart],many others

$$t = s\cosh(\psi)$$
$$x = x$$
$$y = s\sinh(\psi)\cos(\theta)$$
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## 1+1-Dimensional Dynamics (e.g. in Minkowski)

$$\frac{\partial^2\phi}{\partial s^2} + \frac{2}{s}\frac{\partial\phi}{\partial s} - \frac{\partial^2\phi}{\partial x^2} - V'(\phi) = 0$$

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**Should We Trust This When Quantum Fluctuations are Included?**

# Effect of Fluctuations on the Collision

## Dilute Gas Initial Conditions

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{bounce}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false} + \delta\phi(x, y, z)$$

$\delta\phi$ is not SO(2,1) symmetric and **must** be included in quantum theory

- The bubbles nucleate
- Inflation amplifies subhorizon fluctuations

**Is $\delta\phi$ dynamically important?**

Need simulations with more than one spatial dimension

# Setting Initial Conditions : Instantons [Coleman],[Coleman,DeLuccia]

## SO(4) Bounce Equation

$$\frac{\partial^2\phi}{\partial r_E^2} + \frac{3}{r_E}\frac{\partial\phi}{\partial r_E} - V'(\phi) = 0$$

$$\phi(r_E = \infty) = \phi_{false} \qquad \frac{\partial\phi(r_E = 0)}{\partial r_E} = 0$$

Pseudospectral Solution

$$\phi(r_E) = \sum_i c_i B_{2i}\left(h\left(\frac{r_E}{\sqrt{r_E^2 + L^2}}\right)\right)$$

$$h(x) \equiv \frac{1}{\pi}\tan^{-1}\left(d^{-1}\tan\left(\pi\left[x - \frac{1}{2}\right]\right)\right) + \frac{1}{2}$$

Global Expansion $\rightarrow$ Extremely Accurate

## Mapping Parameters

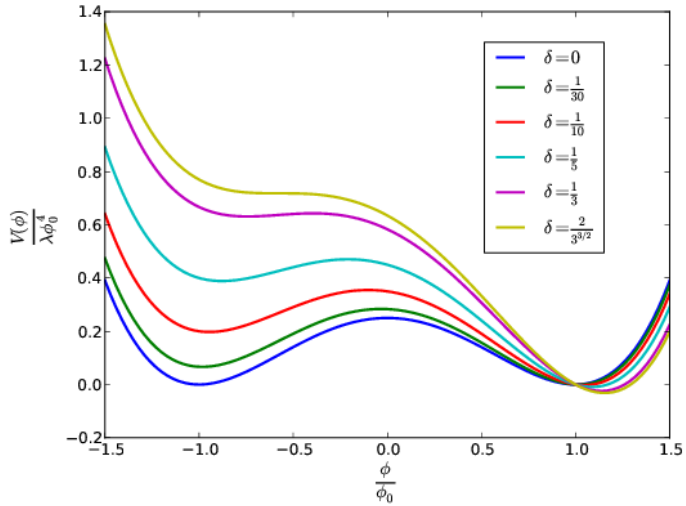
$L$ : $\sim$ radius of bubble

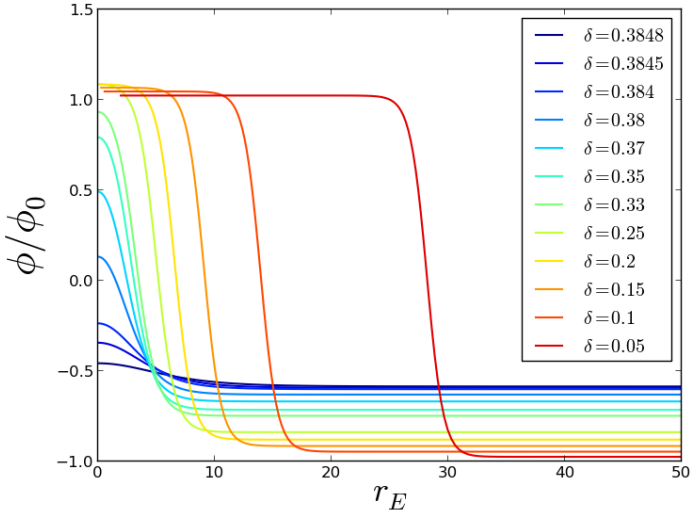
$d$ : $\sim$ width / radius

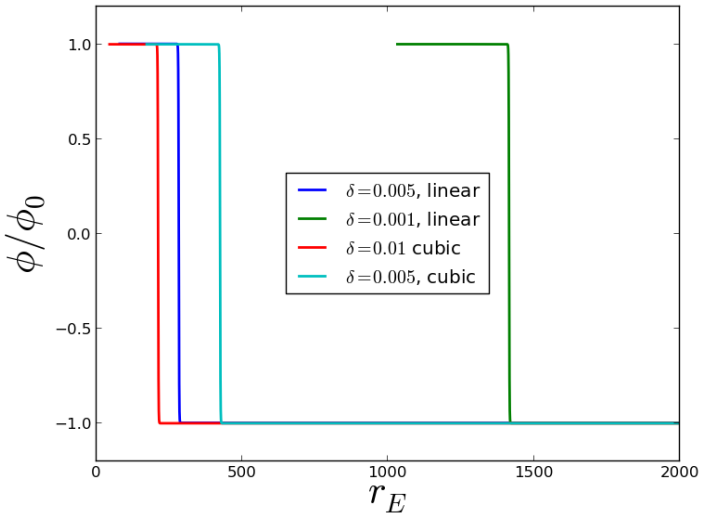
Extendable to . . .

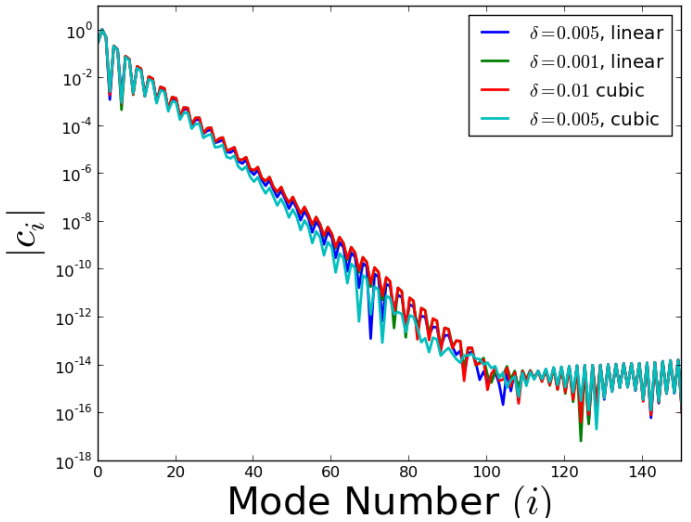
- dynamical metric
- multiple fields (shooting hard)
- PDEs (shooting breaks)

# Versatile and Accurate (Even for Very Thin Walls)

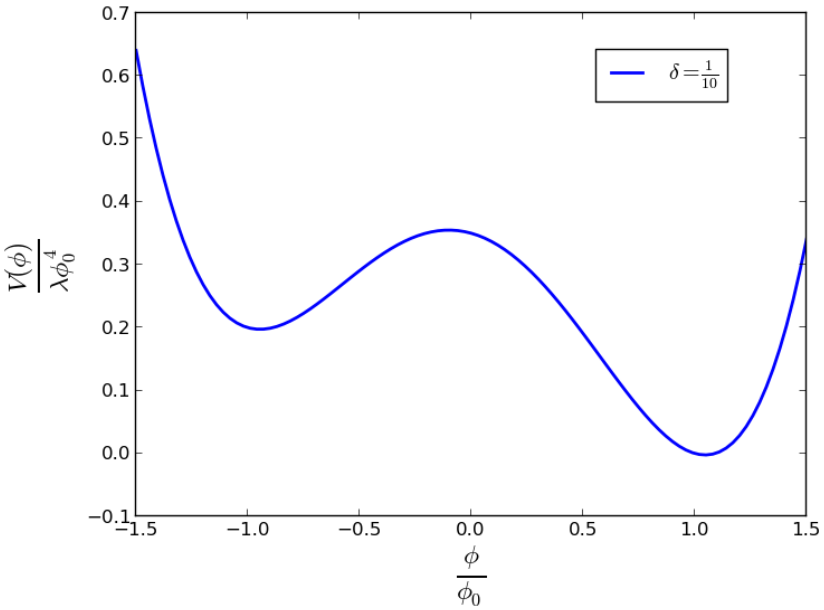


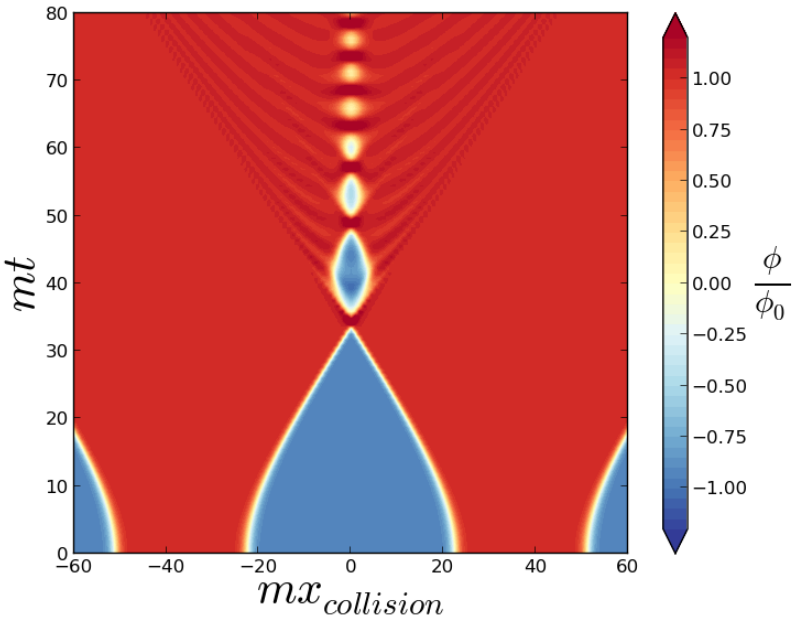



# Collisions in a Double Well Potential




$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \phi_0^2\right)^2 - \delta\lambda\phi_0^3(\phi - \phi_0)$$

Exactly SO(2,1) Invariant Collision

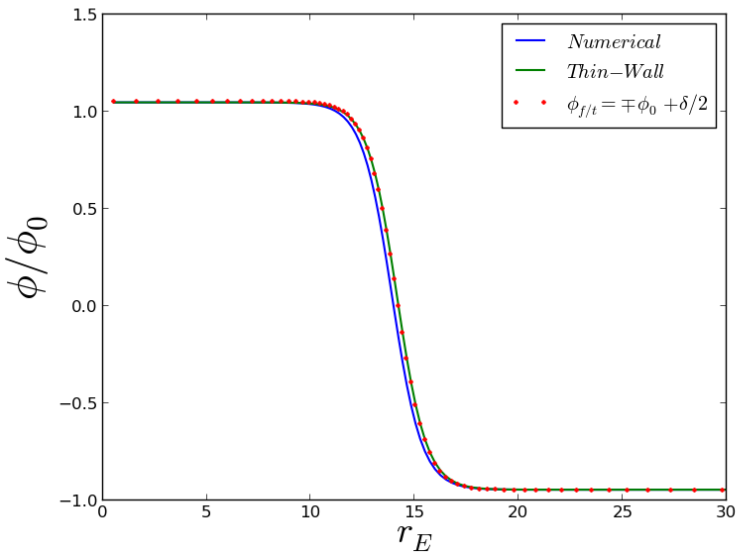


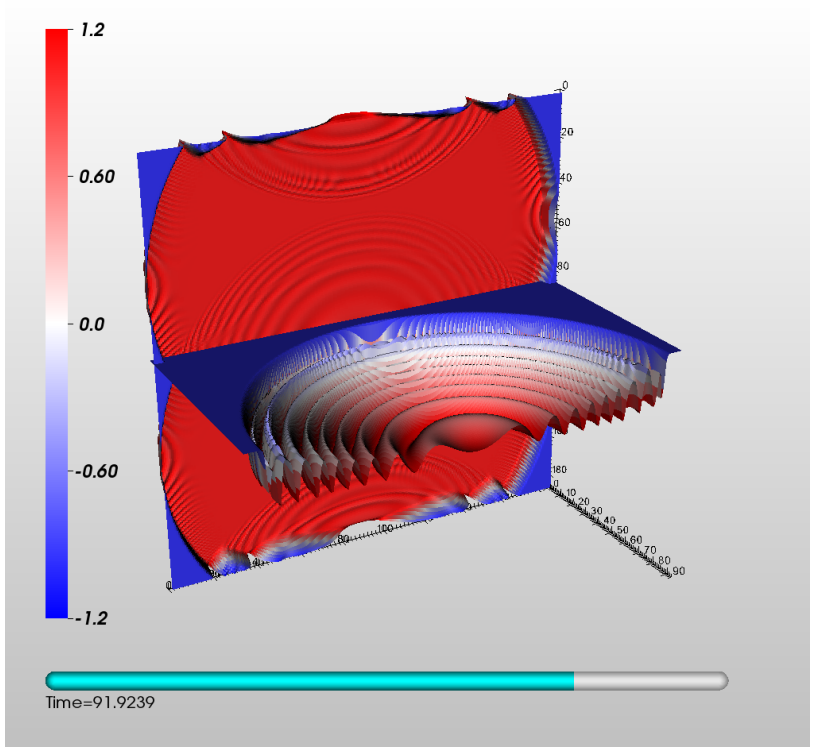

$$s^2 = t^2 - y^2 - z^2$$

# Numerical Test : Exact Instanton Initial Conditions

## SO(2,1) Invariant Initial Condition

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{bounce}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false}$$

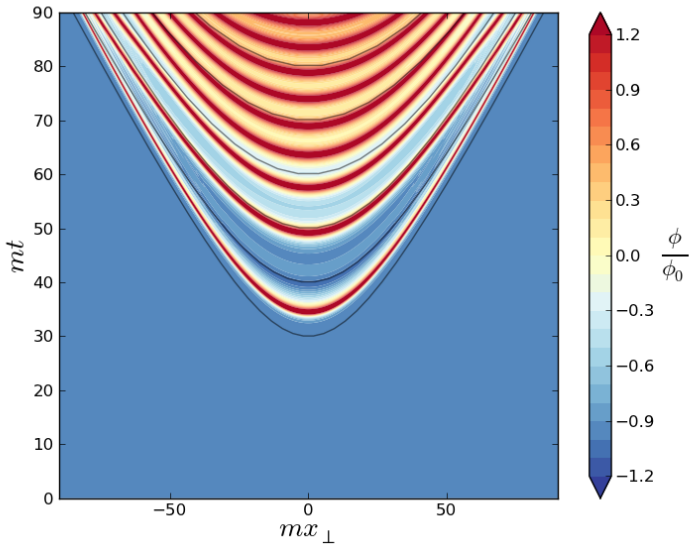


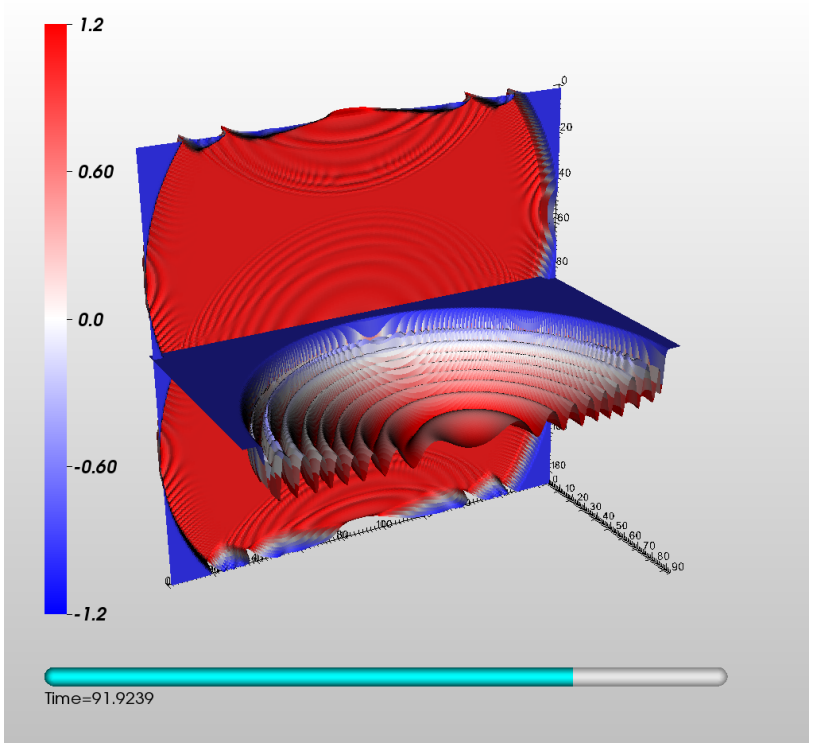



# Numerical Test : Exact Instanton Initial Conditions

## SO(2,1) Invariant Initial Condition

$$\delta\phi = 0$$

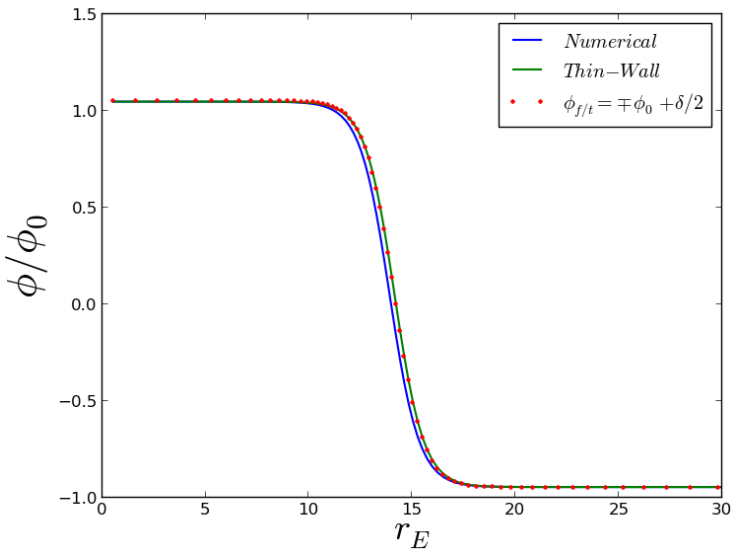


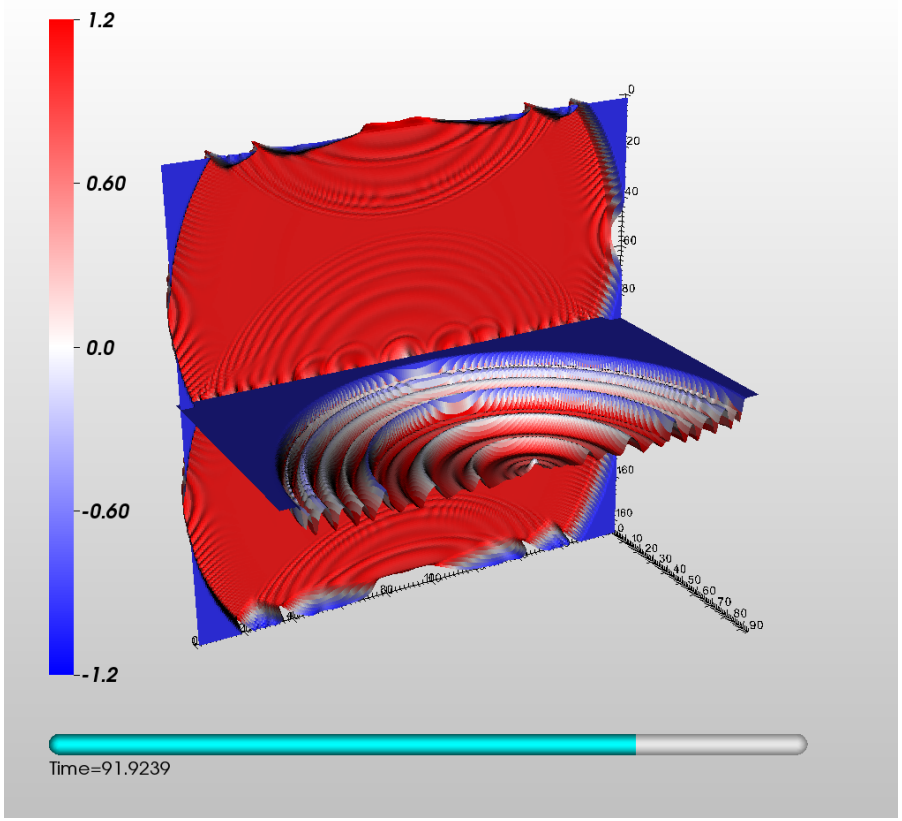



# Thin-Wall Approximation

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{tw}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false}$$

Break boost invariance of time-evolved solution



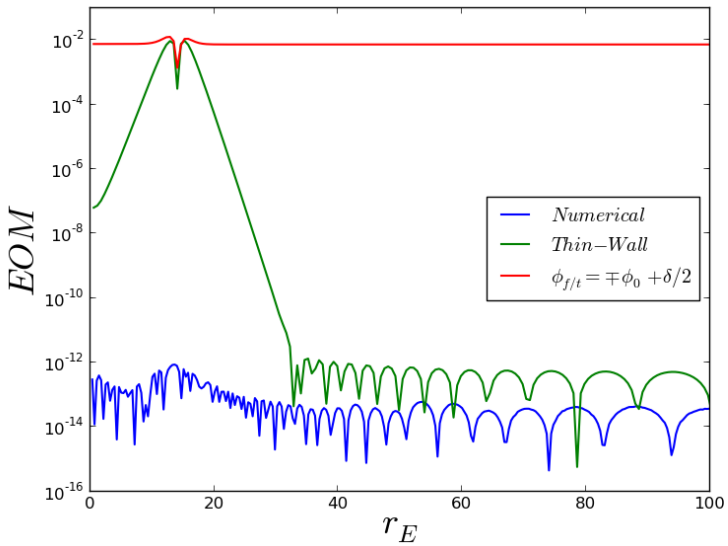



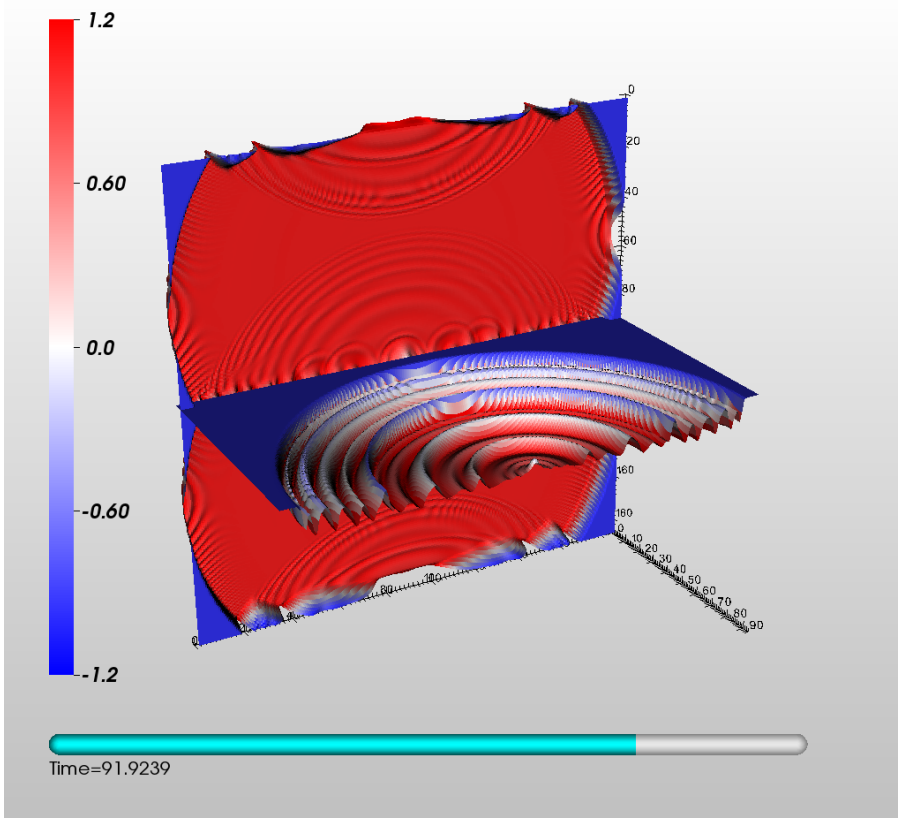

$$\phi_{tw} = \phi_0 \tanh\left(\frac{m(r - R_0)}{\sqrt{2}}\right) + \frac{\delta}{2}$$

# Thin-Wall Approximation

$$\delta\phi = \sum_{\mathbf{r_i}} \left(\phi_{bounce}(|\mathbf{x} - \mathbf{r_i}|) - \phi_{tw}(|\mathbf{x} - \mathbf{r_i}|)\right)$$

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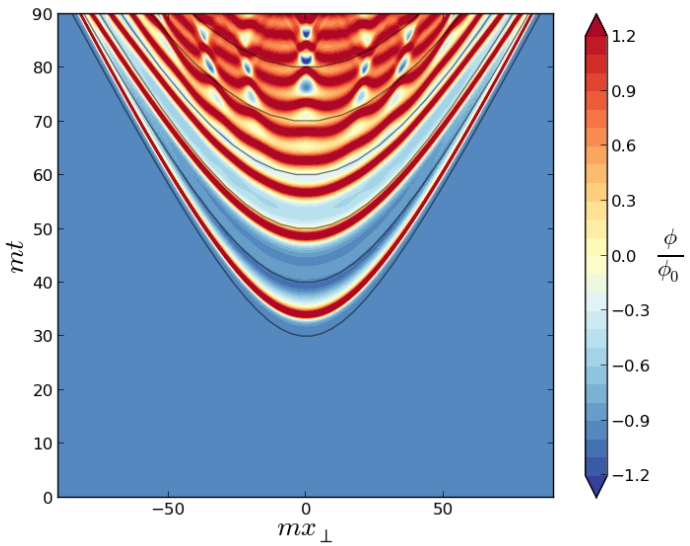



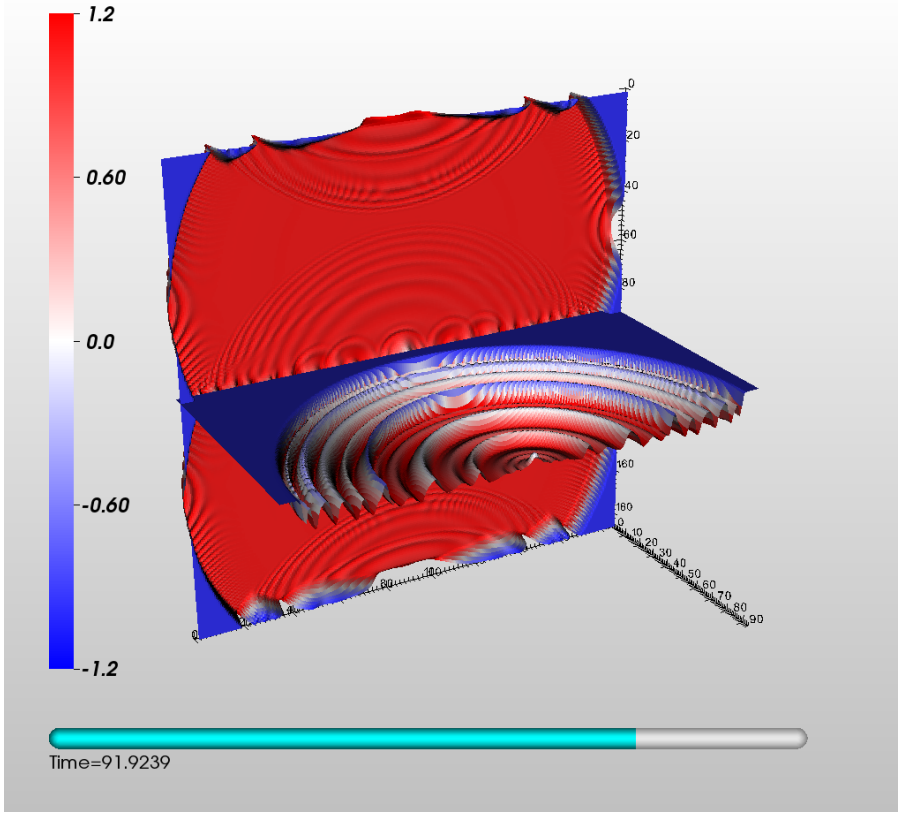

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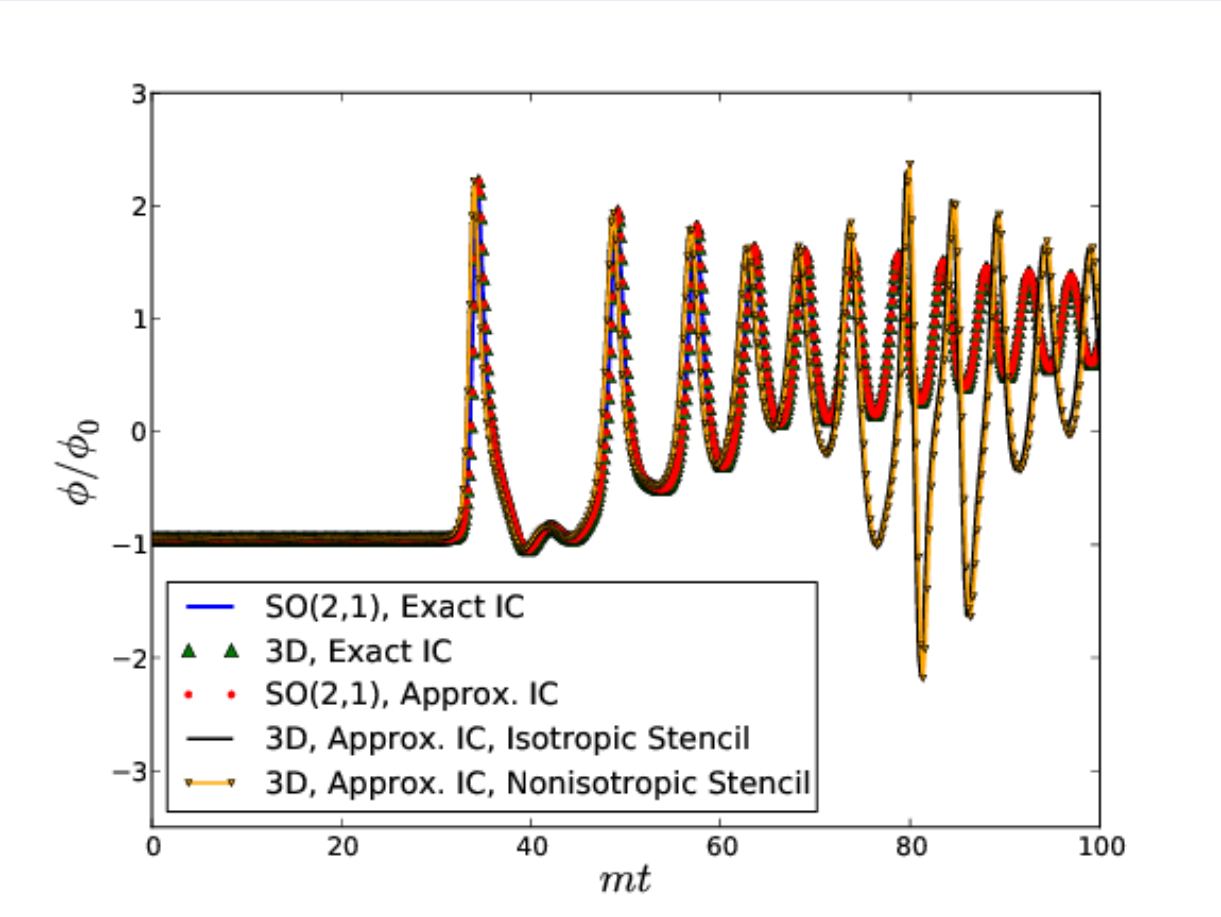



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# Comparison with One-Dimensional Simulation

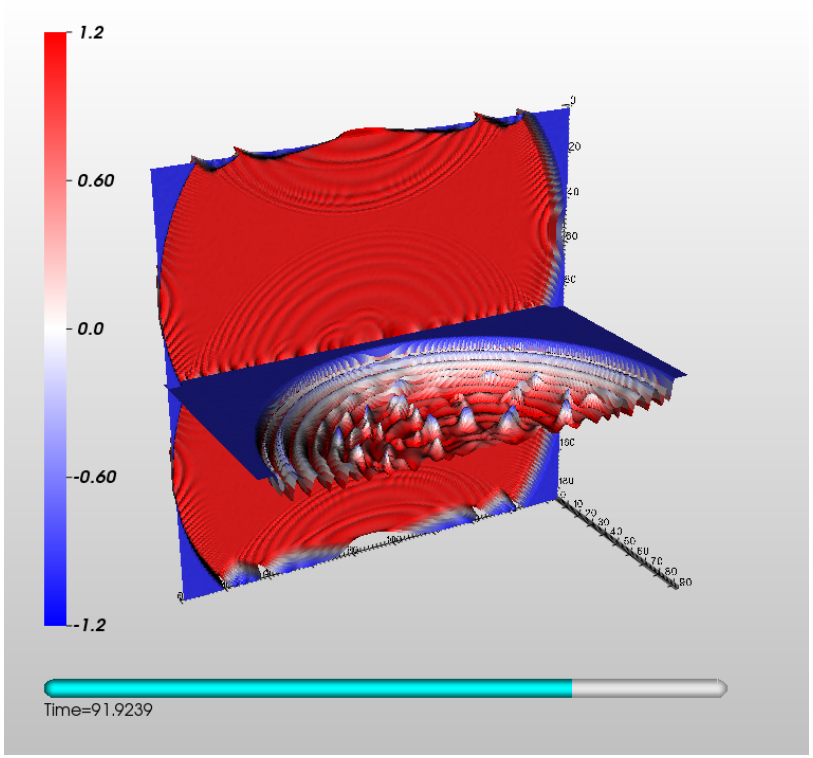
## This Effect is Missed by Assuming SO(2,1)



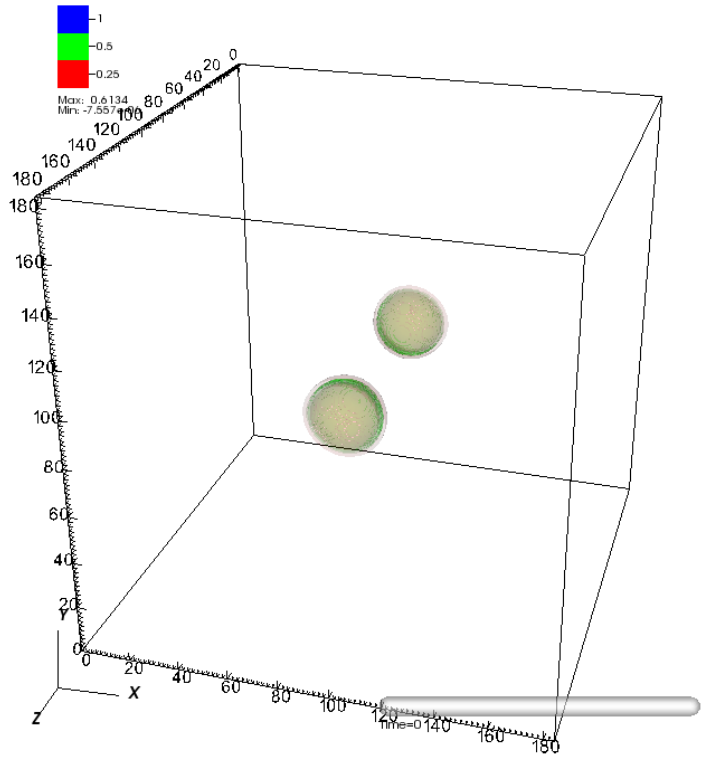
# Field Evolution with Fluctuations ...

## Bulk Vacuum Fluctuations

$$\phi_{init} = \sum_{\mathbf{r_i}} \phi_{bounce}(|\mathbf{x} - \mathbf{r_i}|) - (N_{bub} - 1)\phi_{false} + \delta\phi(x, y, z)$$




without Hubble

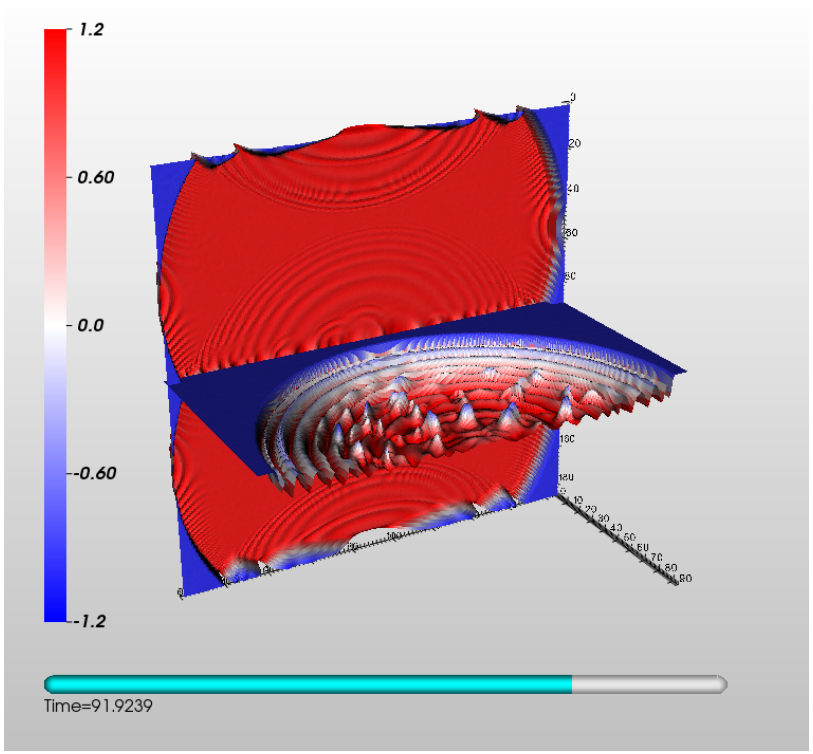



with Hubble

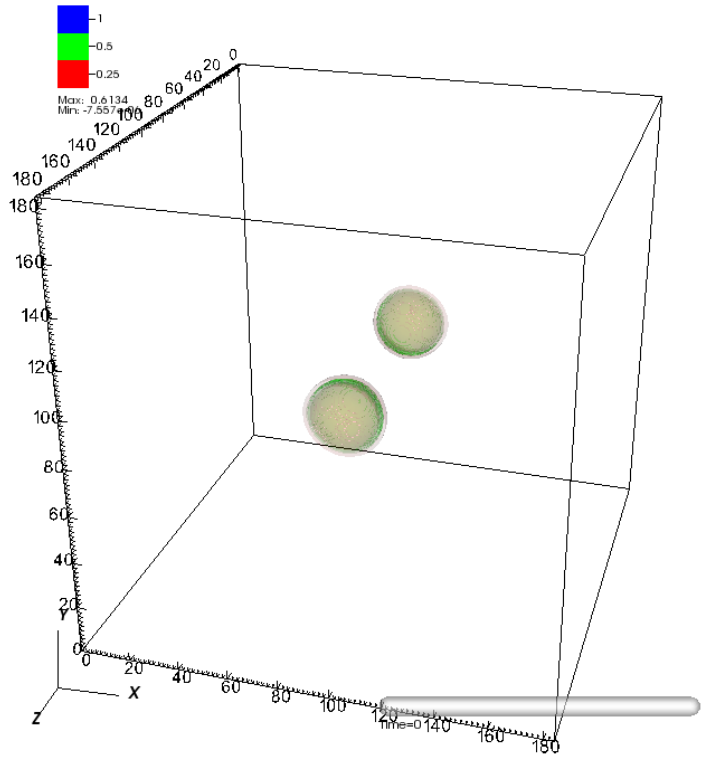
# Field Evolution with Fluctuations ...

## Bulk Vacuum Fluctuations

$$\delta\tilde{\phi}_{\mathbf{k}} \sim \frac{a_k}{\sqrt{k^2 + V''(\phi_{false})}} \qquad \delta\dot{\tilde{\phi}}_{\mathbf{k}} \sim b_k \sqrt{k^2 + V''(\phi_{false})}$$



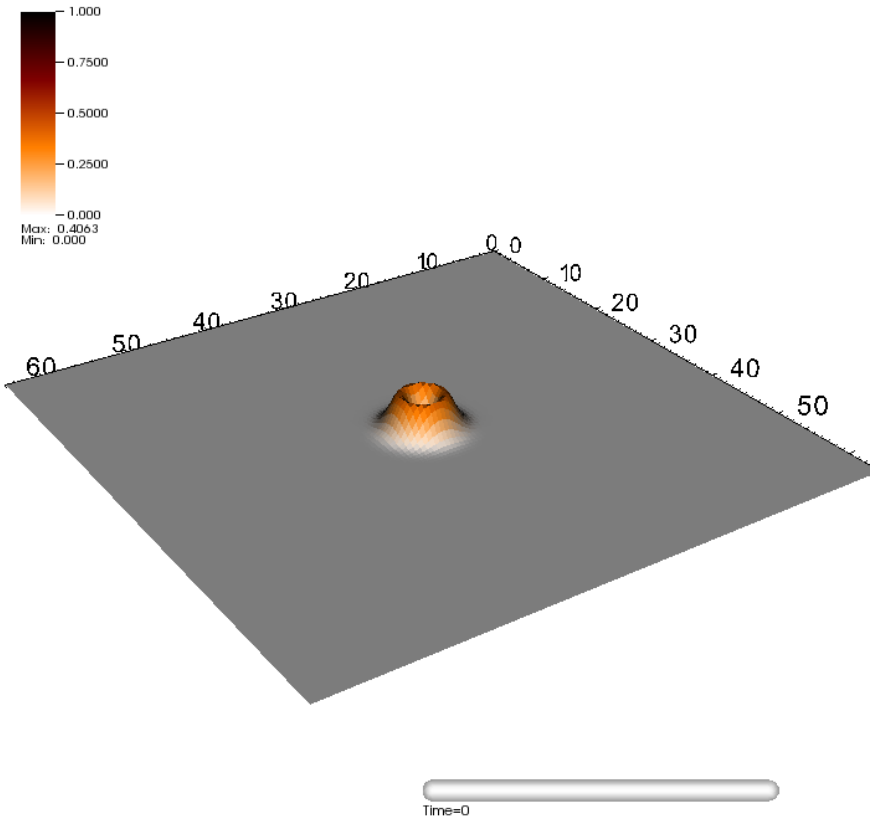

without Hubble

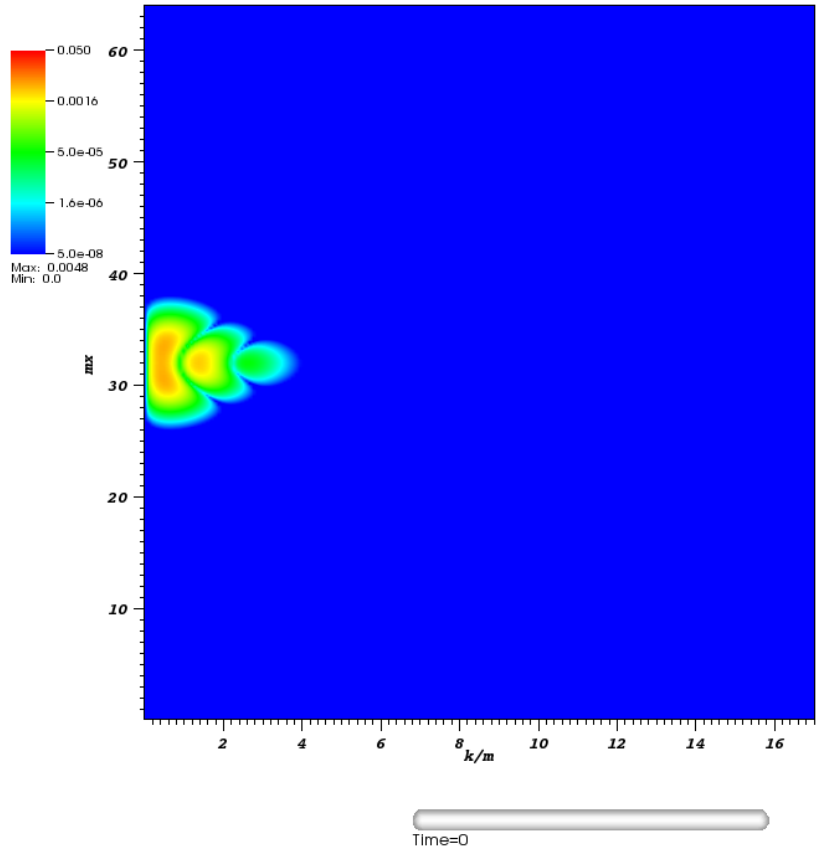



with Hubble

# ... Produces Oscillons

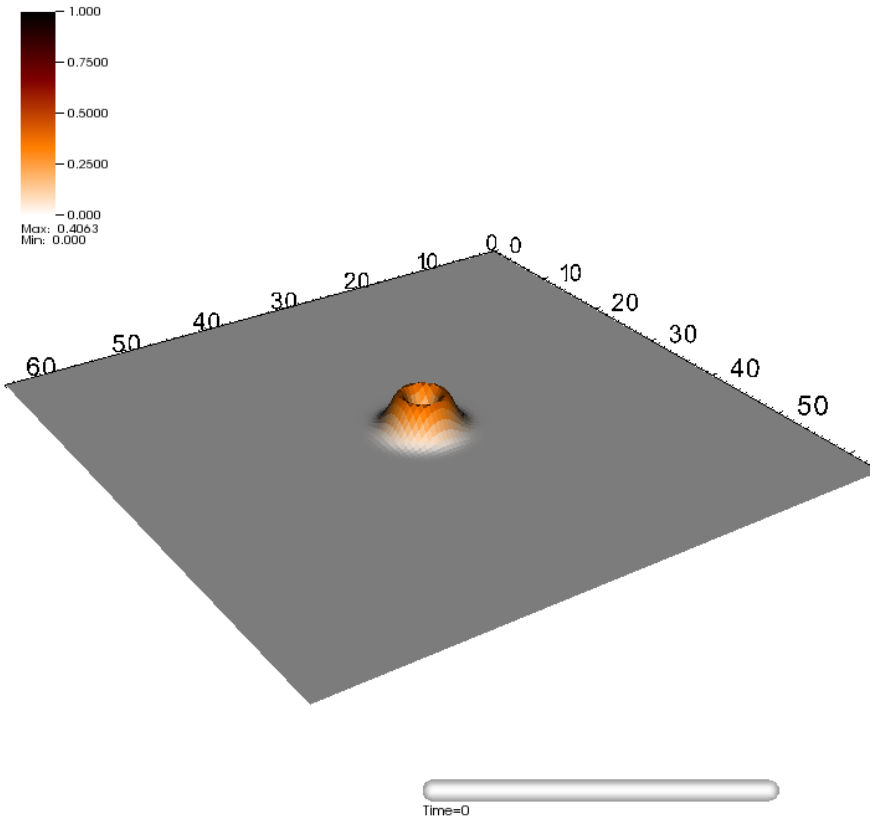
[Bogolubsky and Makhanov],[Gleiser,Copeland,et al][Guth,Farhi,et al][Amin,Easther,Finkel,Shirokoff][Hertzberg],...

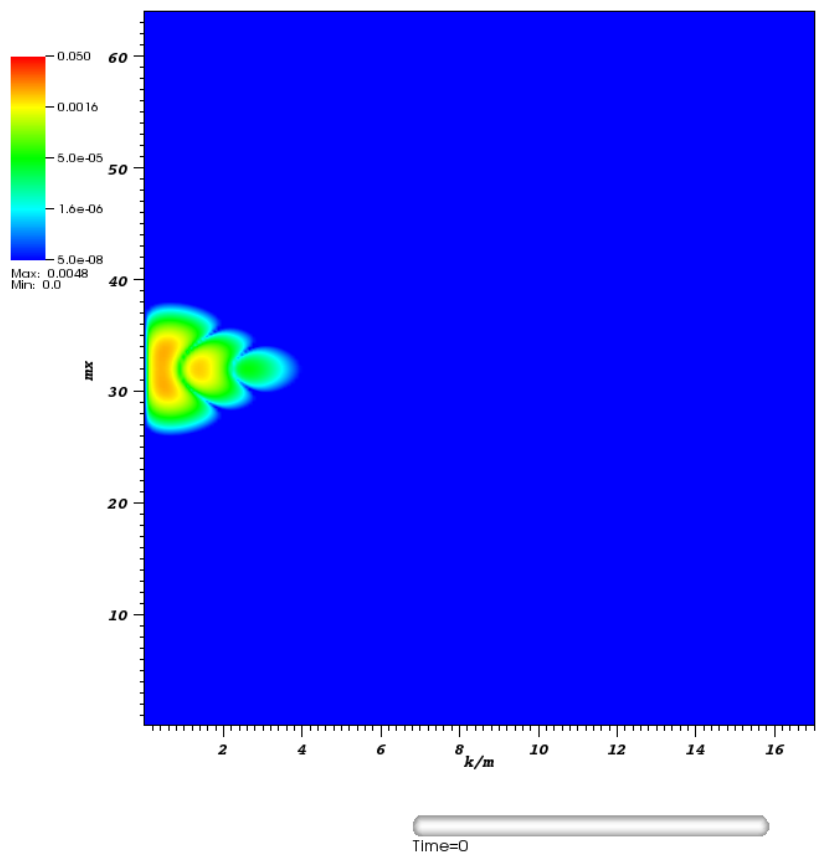


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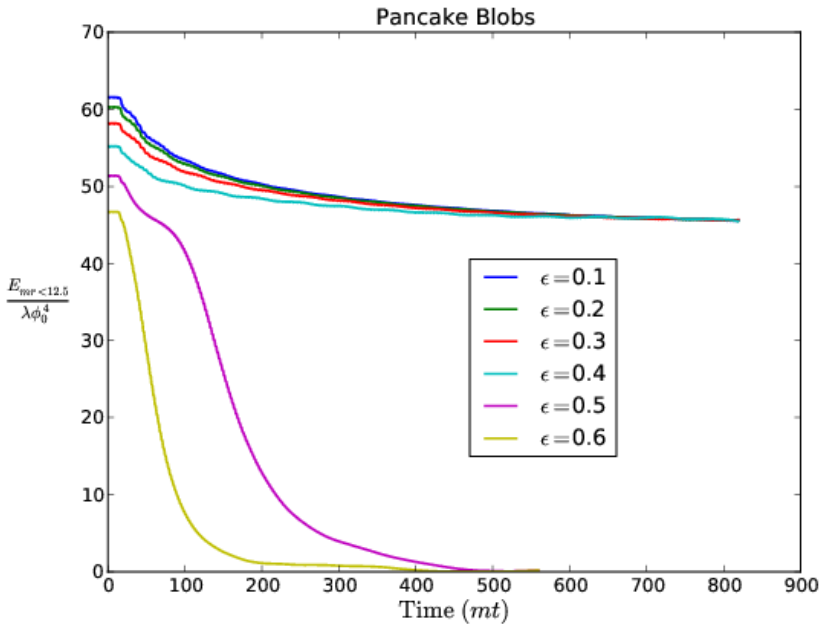


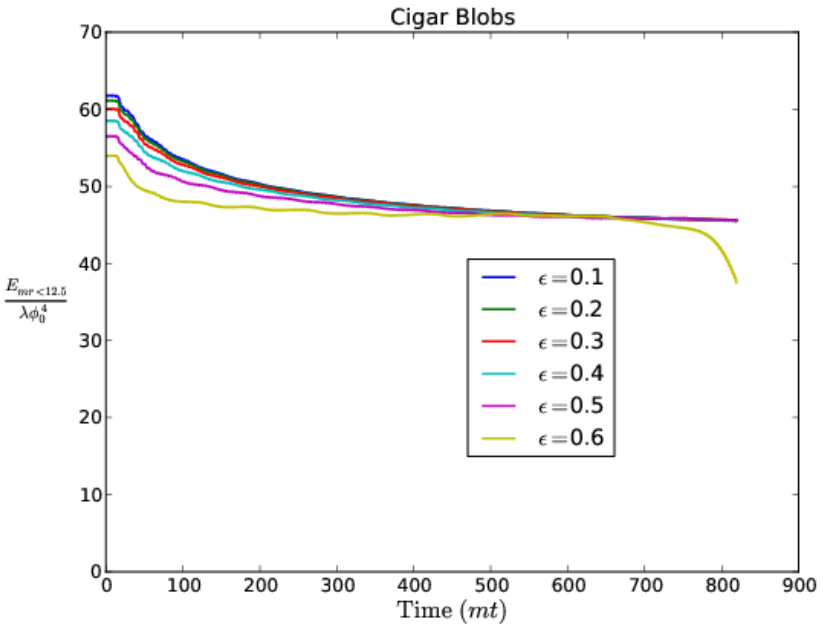
# Oscillons Form from Asymmetric Blobs

## Initial Blob of Field

$$\phi_{init} = \phi_{true} + (\phi_{false} - \phi_{true}) exp\left(-\frac{x^2}{a^2} - \frac{y^2 + z^2}{b^2}\right)$$

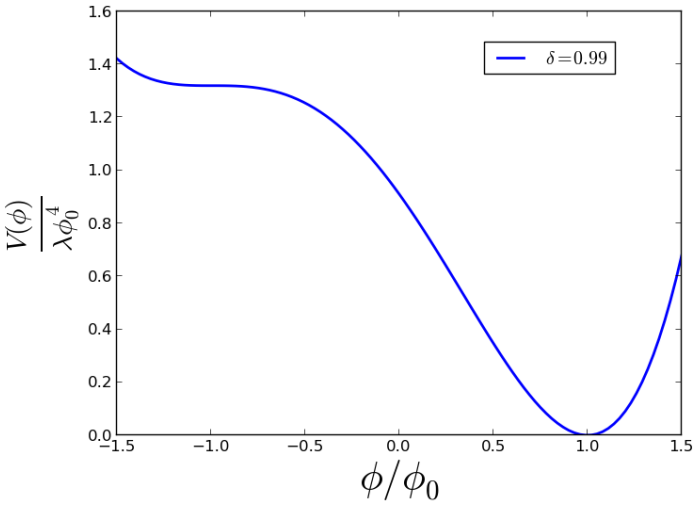


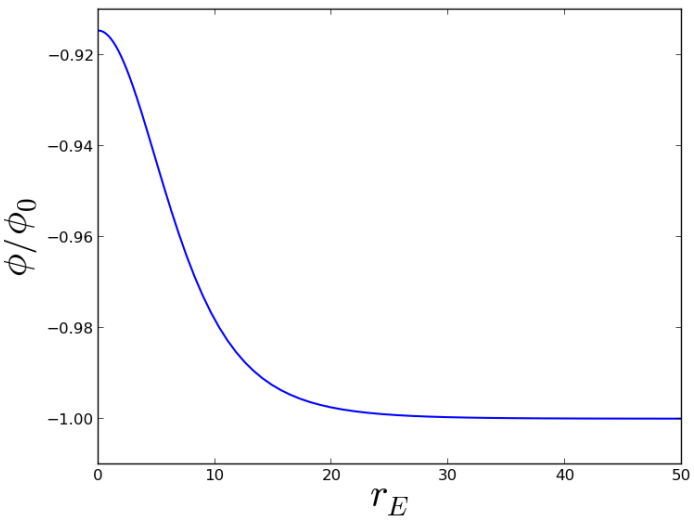

$a^2 < b^2$

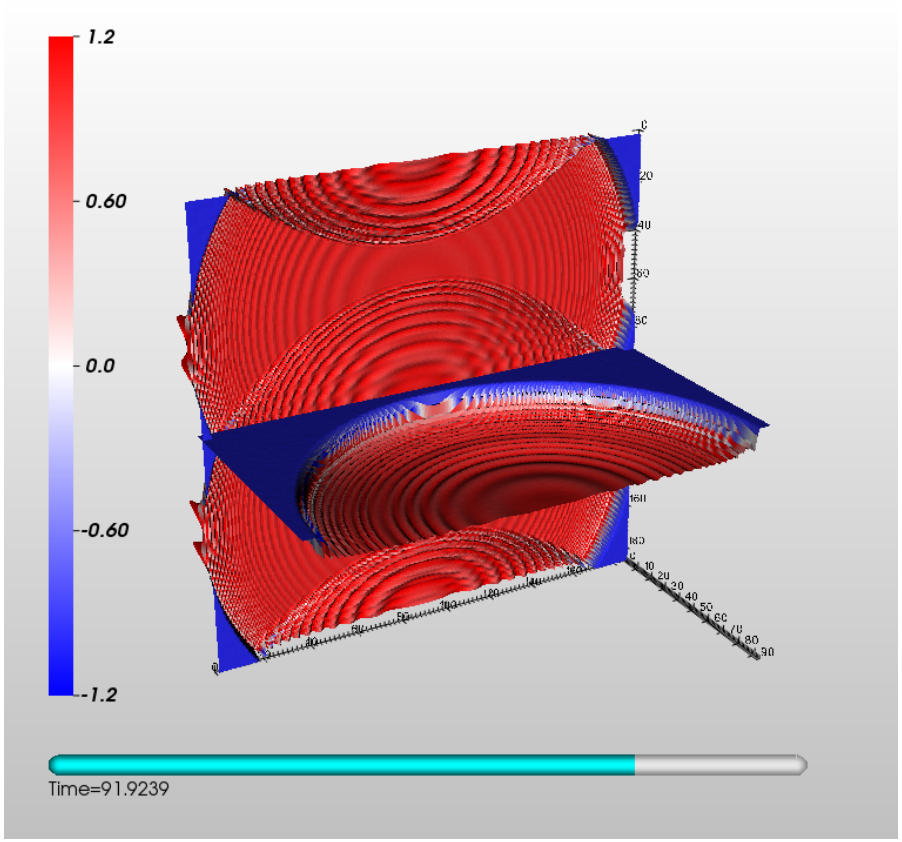



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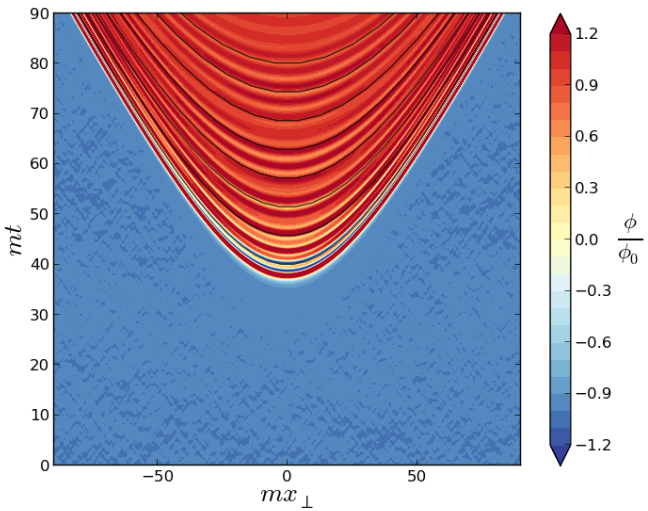
# A Model without Amplification

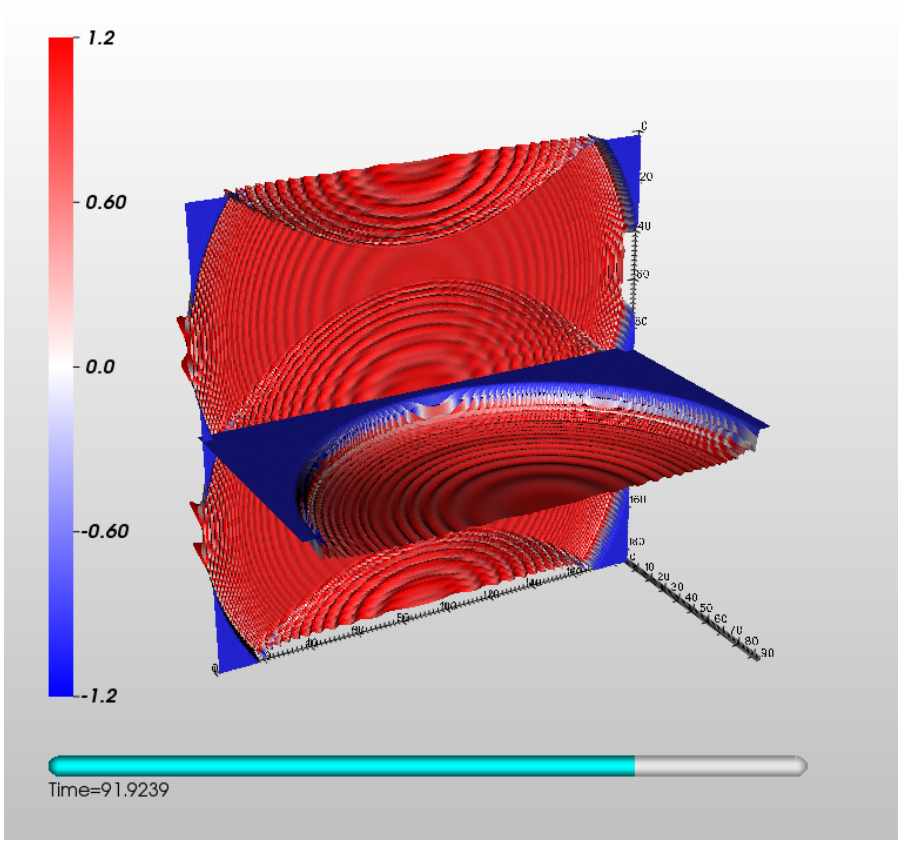


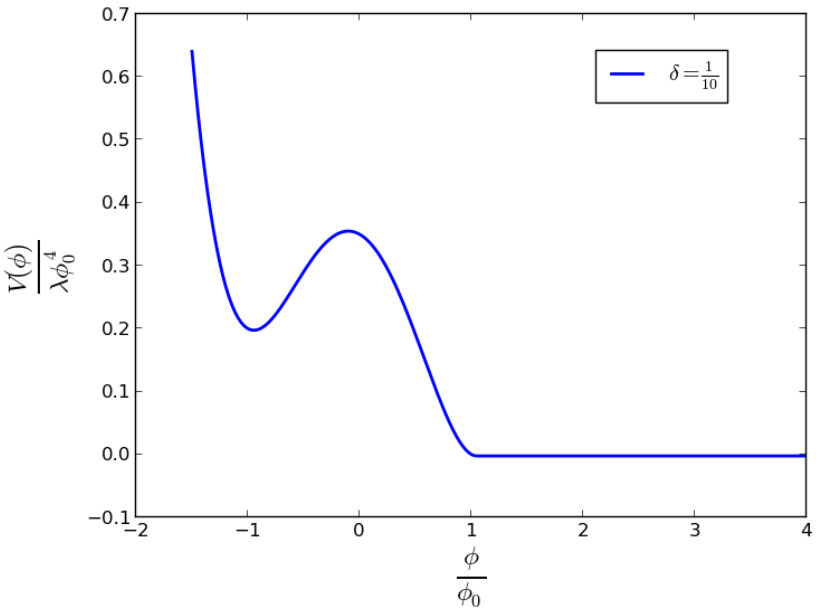



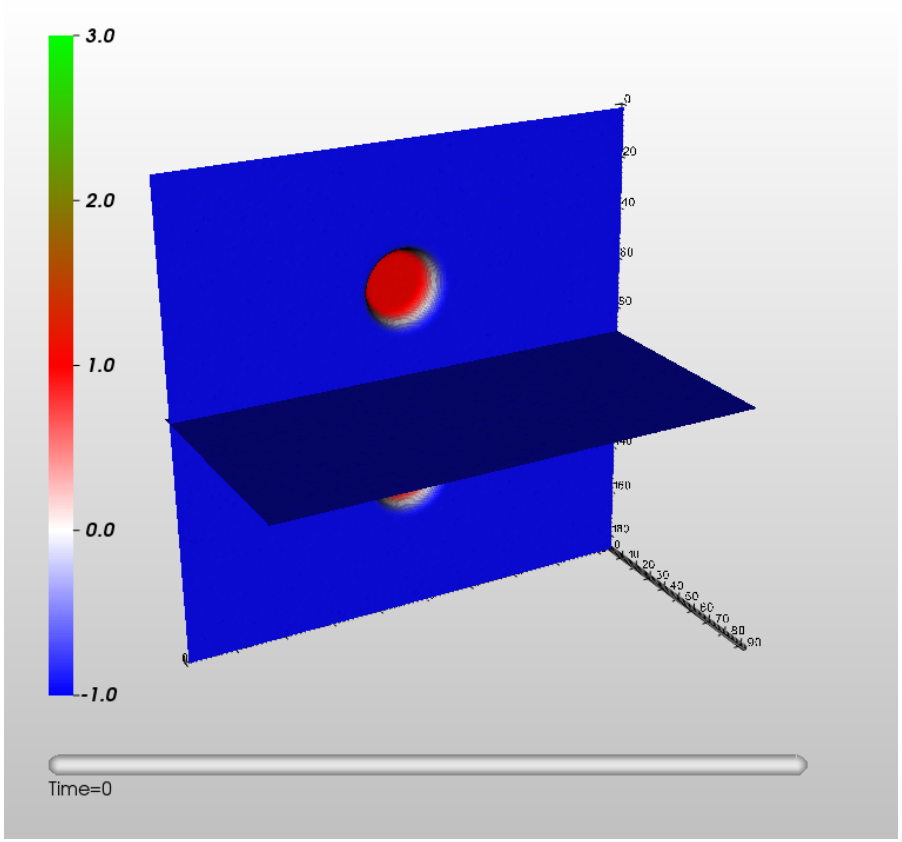
# A Model without Amplification



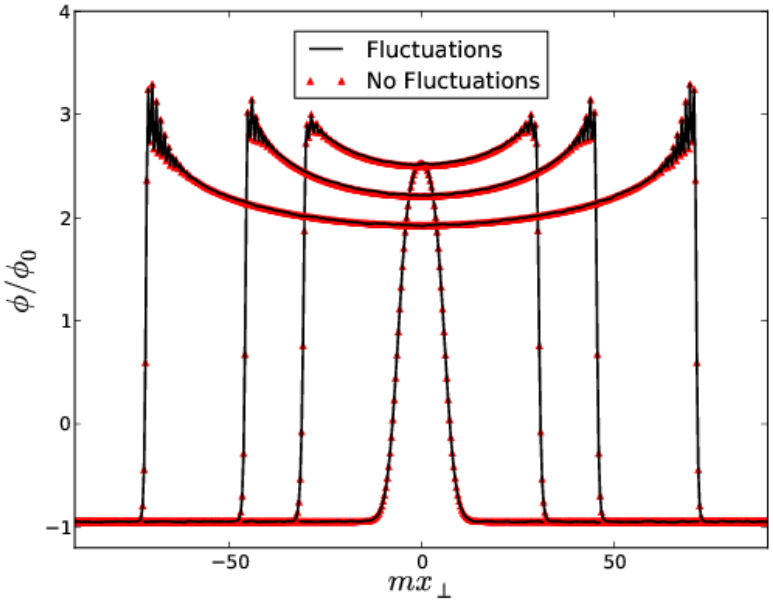



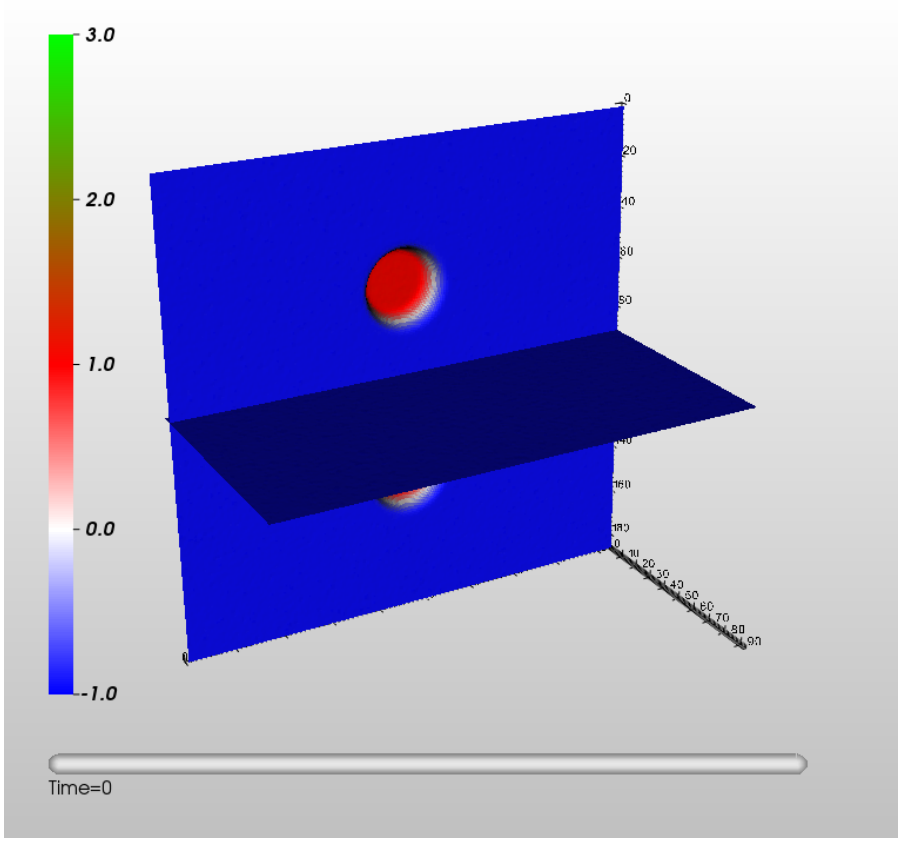
# Oscillons with Inflation?



# Oscillons with Inflation?

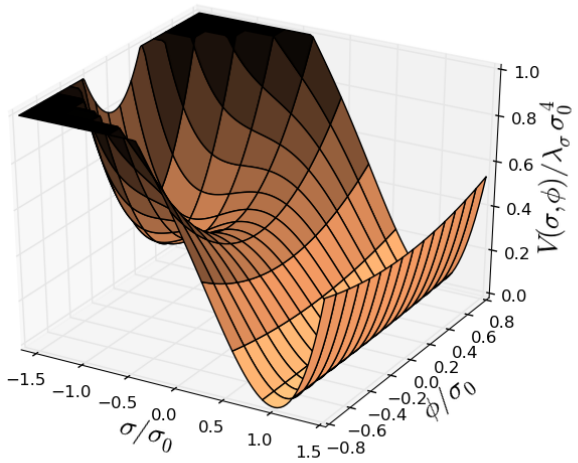


# An Inflationary Model with Oscillons

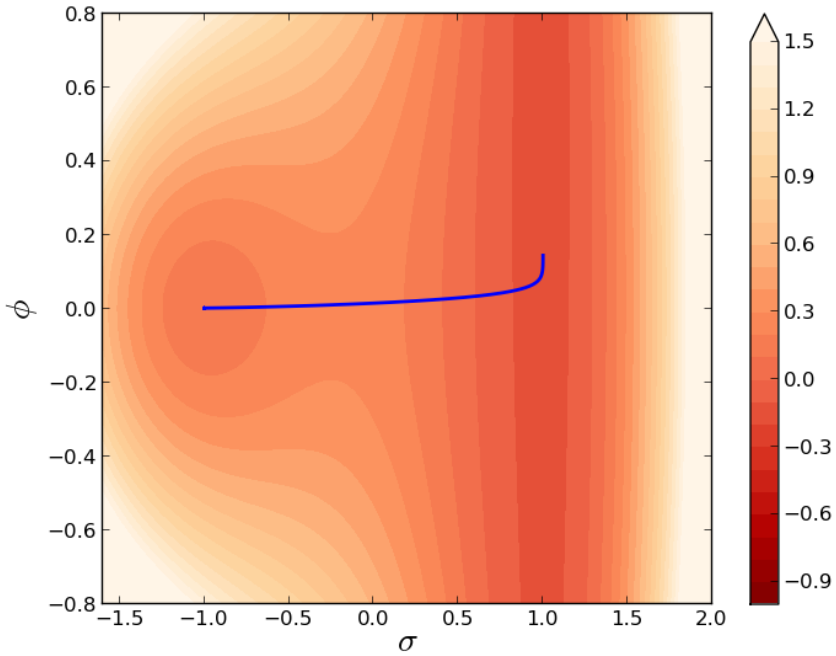
## Two Field Model

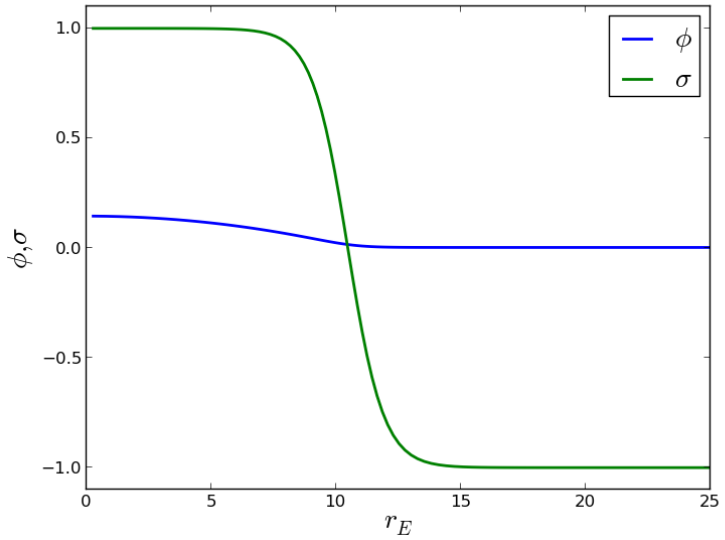
$$V(\sigma,\phi) = \lambda_\sigma \sigma_0^4 \left[ \frac{1}{4}\left(\frac{\sigma^2}{\sigma_0^2} - 1\right)^2 + \delta\left(\frac{\sigma^3}{3\sigma_0^3} - \frac{\sigma}{\sigma_0} + \frac{2}{3}\right)\right] + \frac{g^2\lambda_\sigma\sigma_0^2}{2}(\sigma-\sigma_0)^2\phi^2 + \lambda\sigma_0^3\epsilon\phi + V_0$$

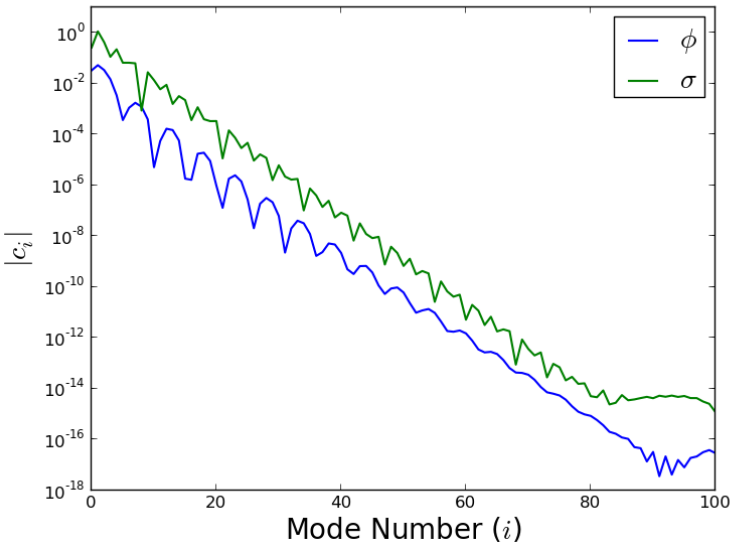



$$V(\sigma,\phi) = V_{tunnel}(\sigma) + V_{coupling}(\sigma,\phi) + V_{inflation}(\phi)$$

# Instanton in Two-Field Model



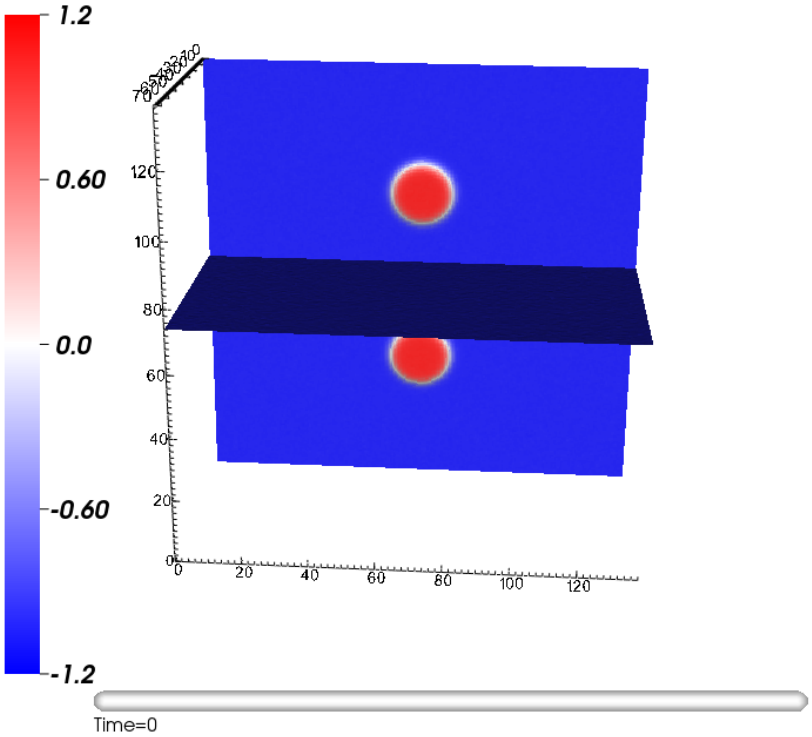



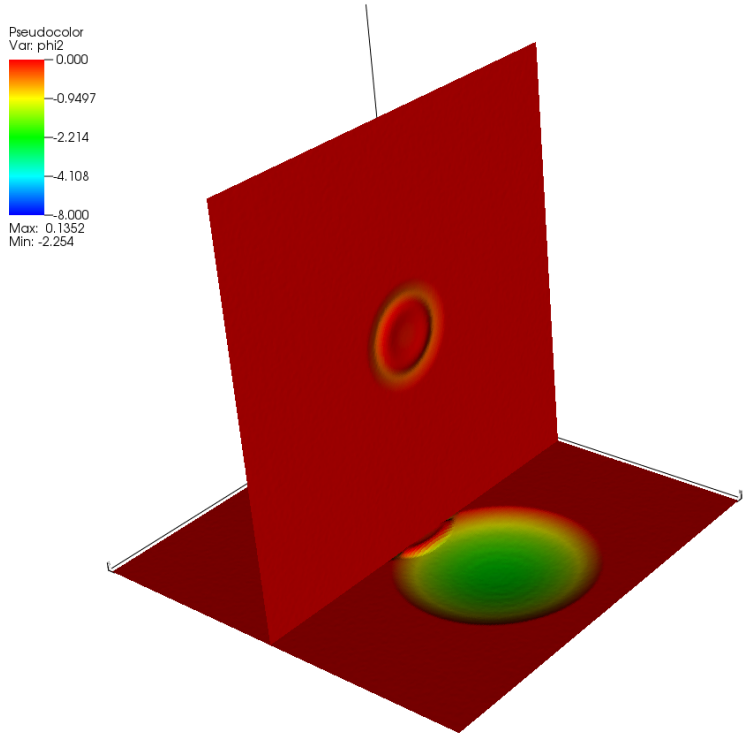

This solution has only a single negative eigenmode in the O(4) symmetric sector

# Field Evolution in Two-Field Model

## $\sigma$ Evolution

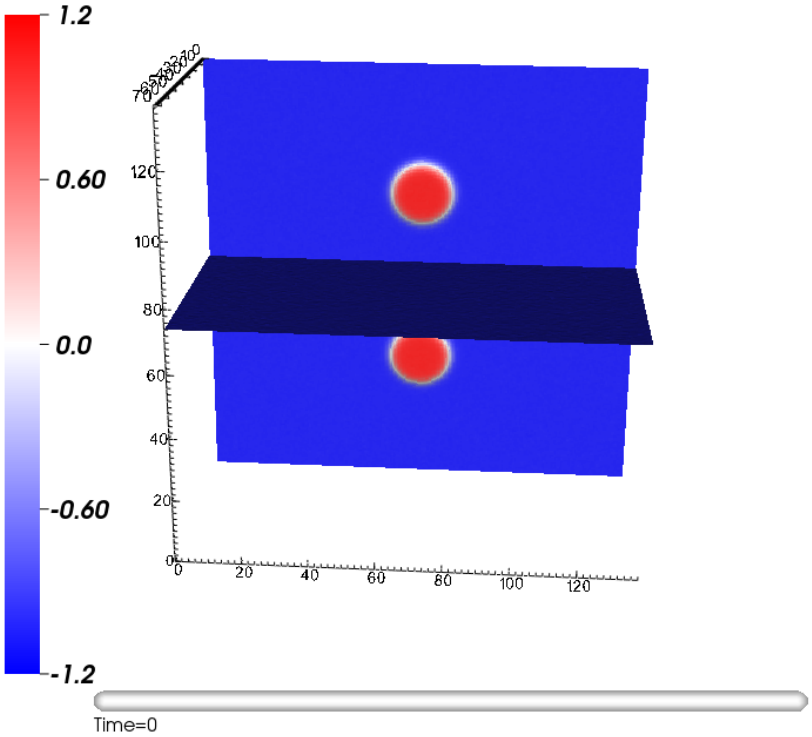



## $\phi$ Evolution

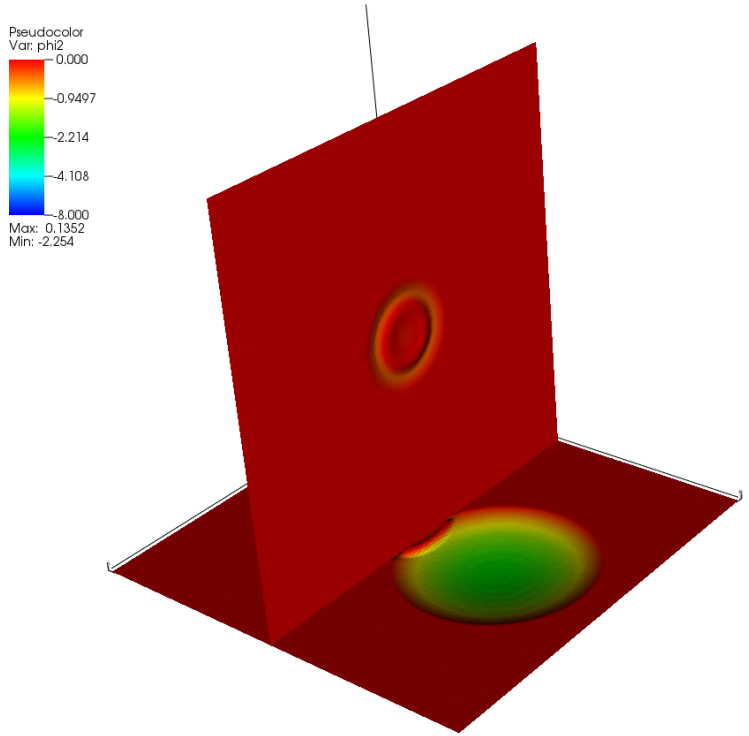


# Field Evolution in Two-Field Model

## $\sigma$ Evolution




## $\phi$ Evolution



# Parametric Resonance for Linear Fluctuations

## Linear Fluctuations Around SO(2,1) Solution

$$\phi(s,x,\psi,\theta) = \phi_{bg}(s,x) + \delta\phi(s,x,\psi,\theta)$$

$$\frac{\partial^2\phi_{bg}}{\partial s^2} + \frac{2}{s}\frac{\partial\phi_{bg}}{\partial s} - \frac{\partial^2\phi_{bg}}{\partial x^2} + V'(\phi_{bg}) = 0$$

$$\frac{\partial^2}{\partial s^2}(s\delta\phi_\kappa) - \frac{\partial^2}{\partial x^2}(s\delta\phi_\kappa) + \left(\frac{\kappa^2}{s^2} + V''(\phi_{bg})\right)(s\delta\phi_\kappa) = 0$$

# Parametric Resonance for Linear Fluctuations

## Linear Fluctuations Around Planar Solution

$$\phi(t,x,y,z) = \phi_{bg}(t,x) + \delta\phi(t,x,y,z)$$

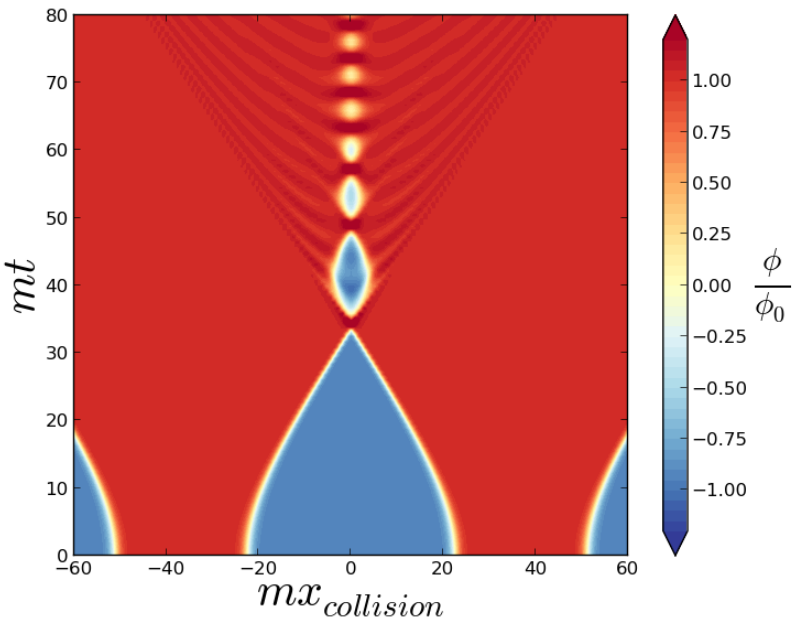
$$\frac{\partial^2 \phi_{bg}}{\partial t^2} - \frac{\partial^2 \phi_{bg}}{\partial x^2} + V'(\phi_{bg}) = 0$$

$$\frac{\partial^2}{\partial t^2}\left(\delta\phi_{k_\perp}\right) - \frac{\partial^2}{\partial x^2}\left(\delta\phi_{k_\perp}\right) + \left(k_\perp^2 + V''(\phi_{bg})\right)\left(\delta\phi_{k_\perp}\right) = 0$$

## Planar Limit

- $s \gg 1$
- Time scales much shorter than $s$
- $\kappa^2 \gg s^2$

# Form of V"

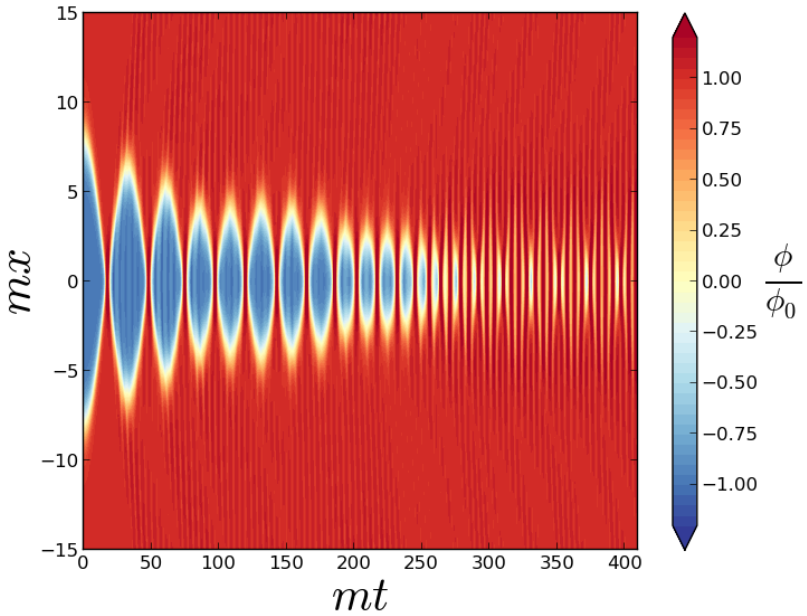


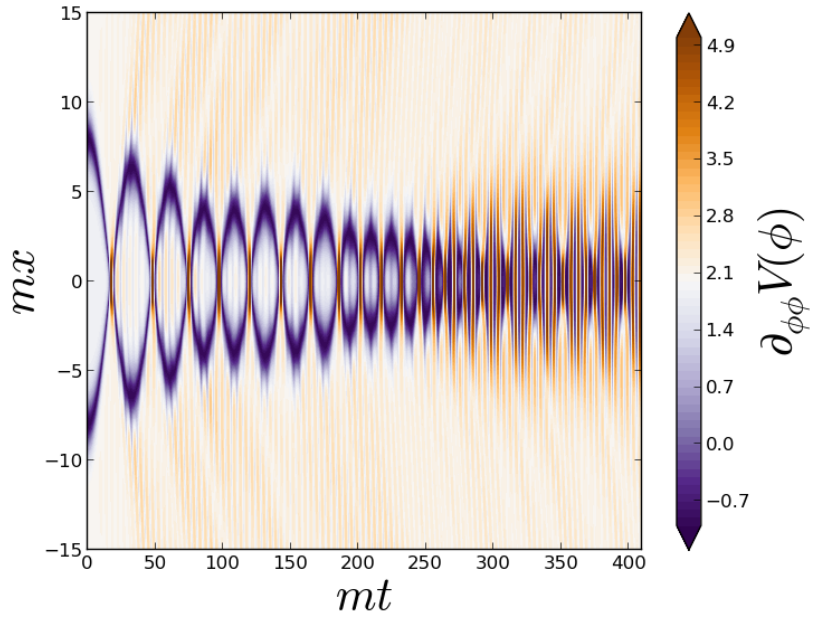

Oscillating Background $\rightarrow$ Floquet Theory

## Exactly Periodic Effective Mass

$$\delta\phi_{Floquet} = P(x,t)e^{\mu t} \qquad P(x,t+2T) = P(x,t) \qquad P \in \mathbb{R}$$

# Form of V"



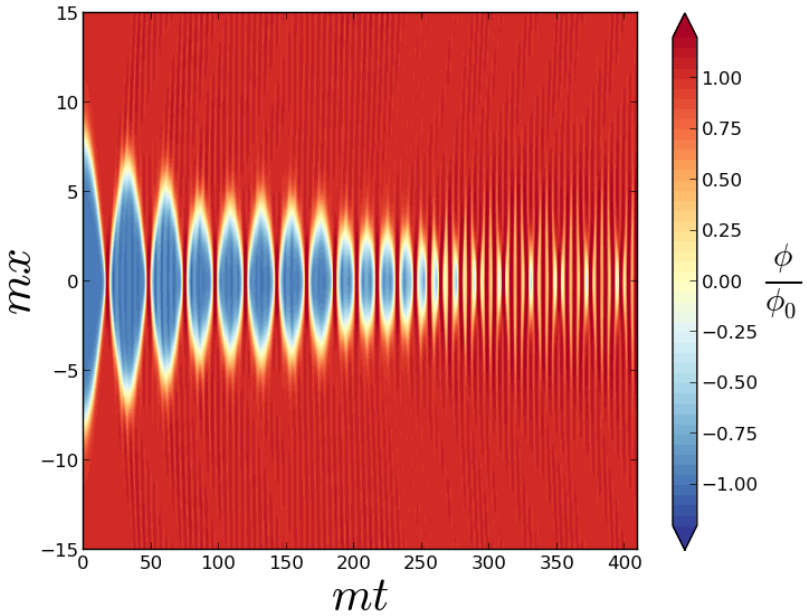

Two wells that repeatedly annihilate

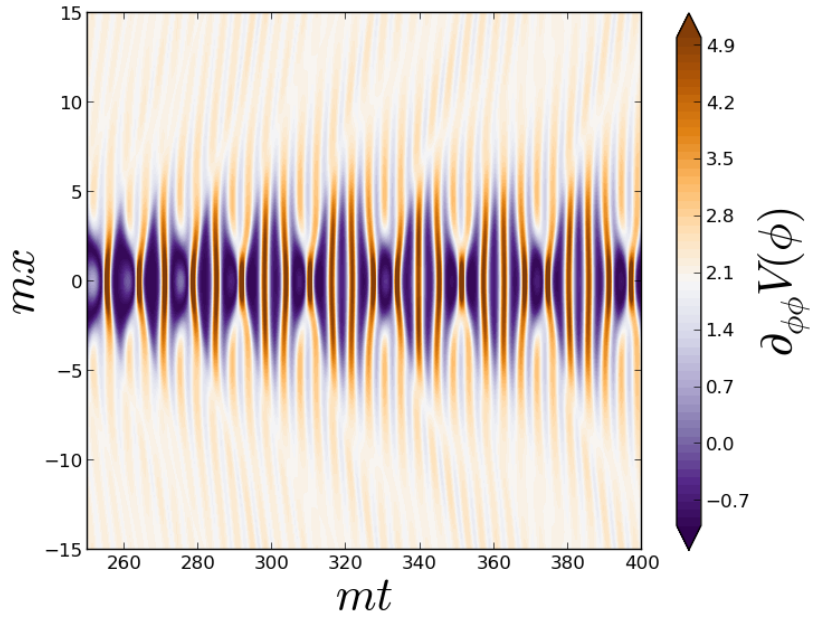
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# Form of V"




One well oscillating up and down

Oscillating Background $\rightarrow$ Floquet Theory

## Exactly Periodic Effective Mass

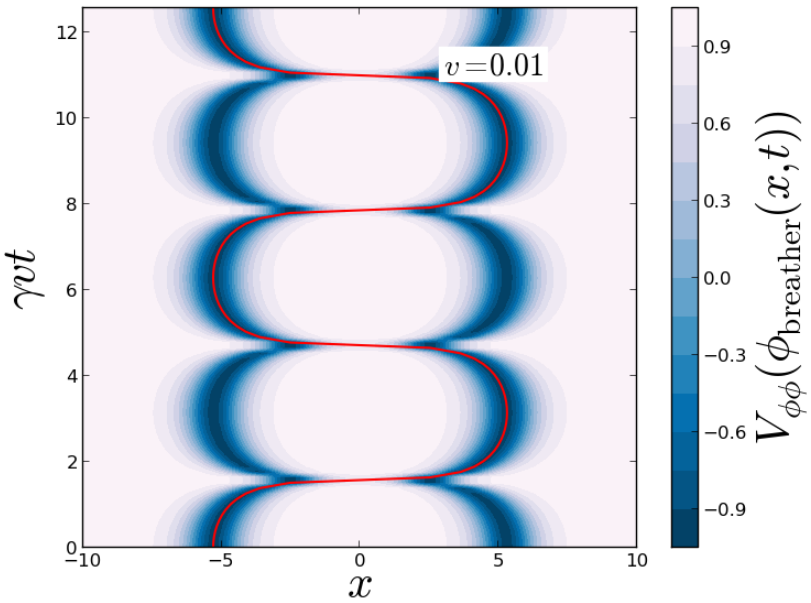
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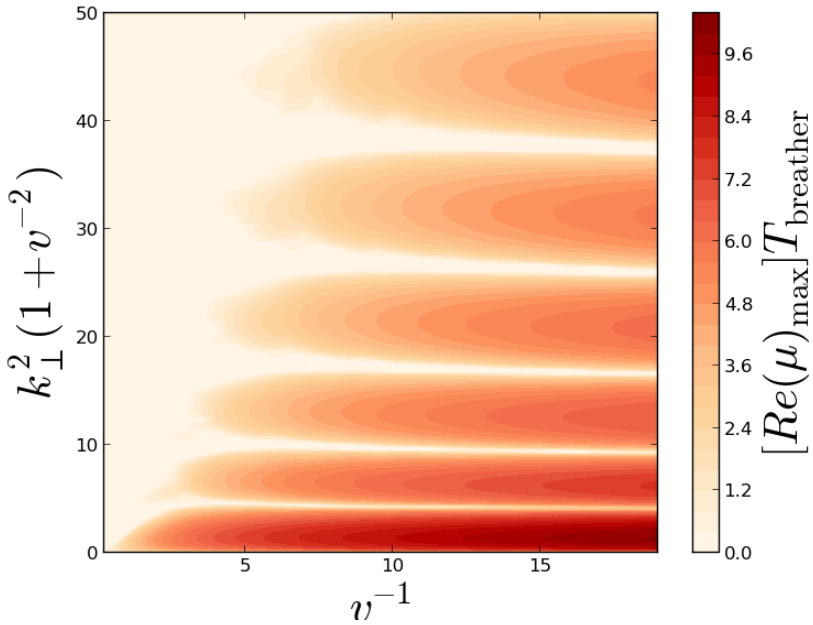
# Instability in Sine-Gordon Model

## Exactly Periodic Backgrounds

$$\phi_{breather} = 4\tan^{-1}\left(\frac{\cos(\gamma_v v t)}{v\cosh(\gamma_v x)}\right) \qquad \gamma_v \equiv (1+v^2)^{-1/2}$$

$v \ll 1$



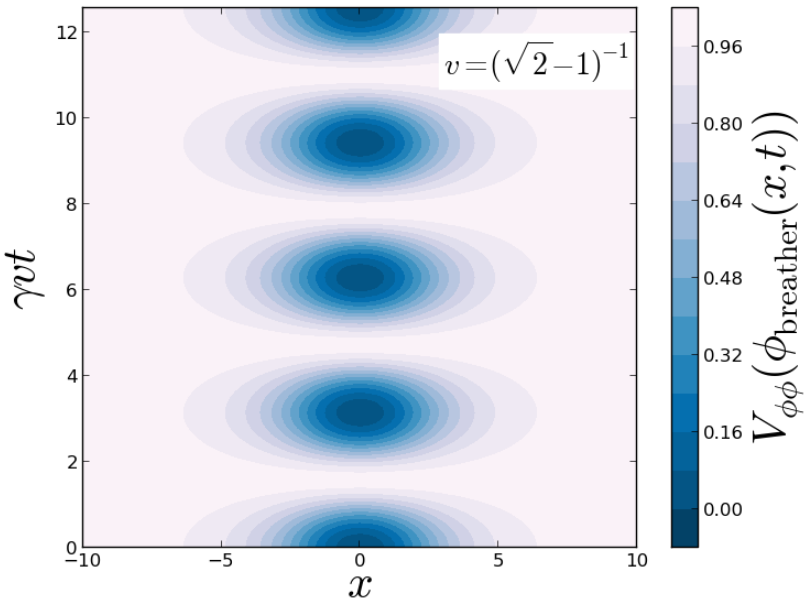



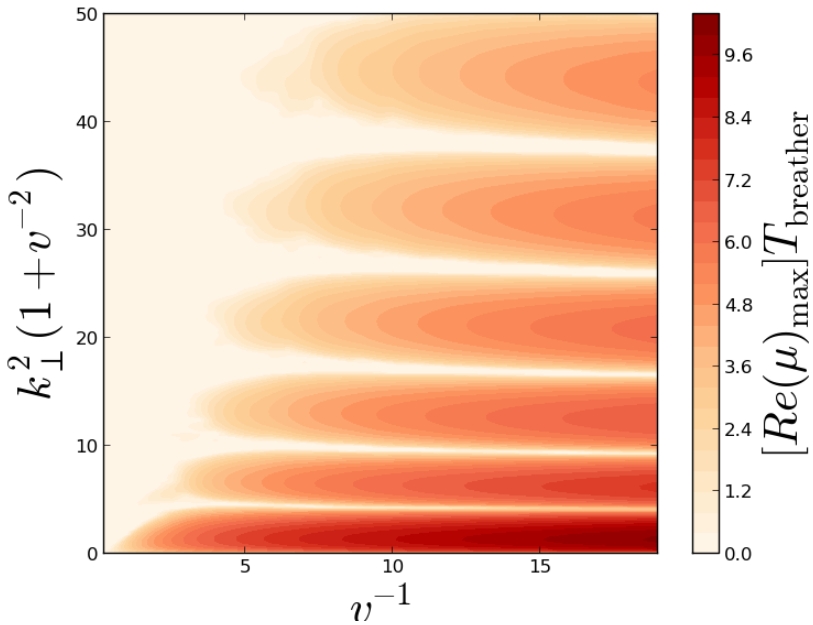
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$v \gtrsim 1$



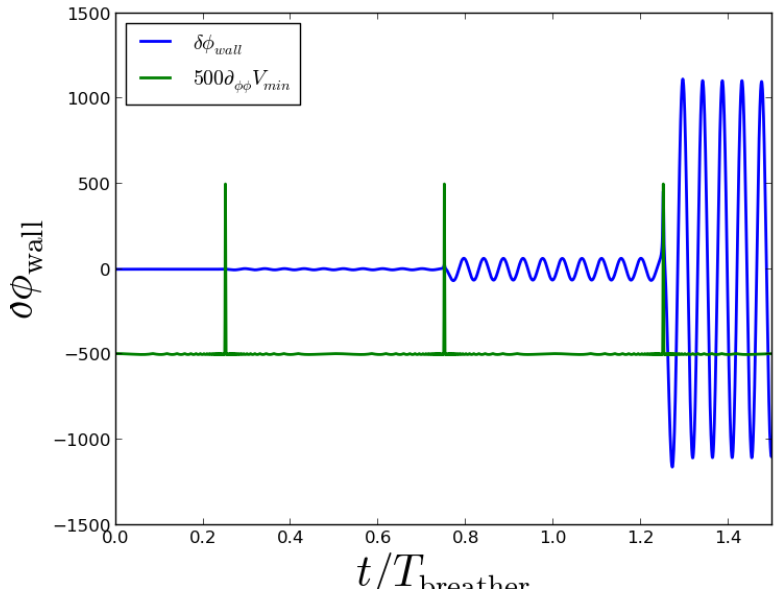



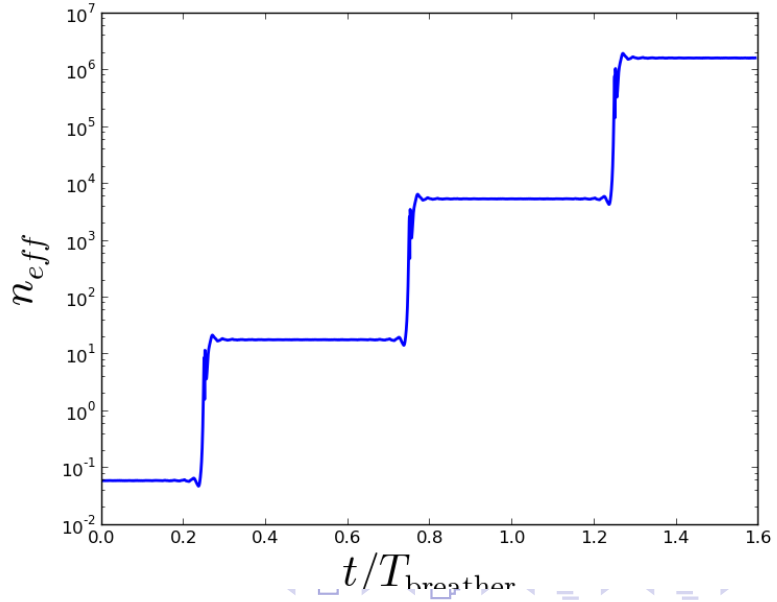
# Broad Resonance : Well-Defined Wall Collisions

## Single Wall, $\phi_{kink,SG} = 4tan^{-1}(e^x)$, $\phi_{kink,DW} = \tanh(x/\sqrt{2})$

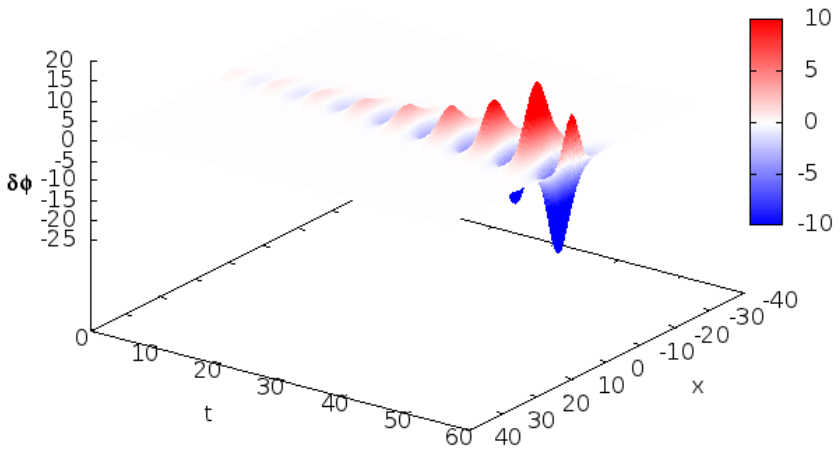
In 1d theory $\delta\phi = \partial_x \phi_{kink}(x)$ is a zero mode
In 3d theory $\rightarrow$ bound fluctuations with $\omega = k_\perp$
Very General : Goldstone for spontaneously broken translation invariance

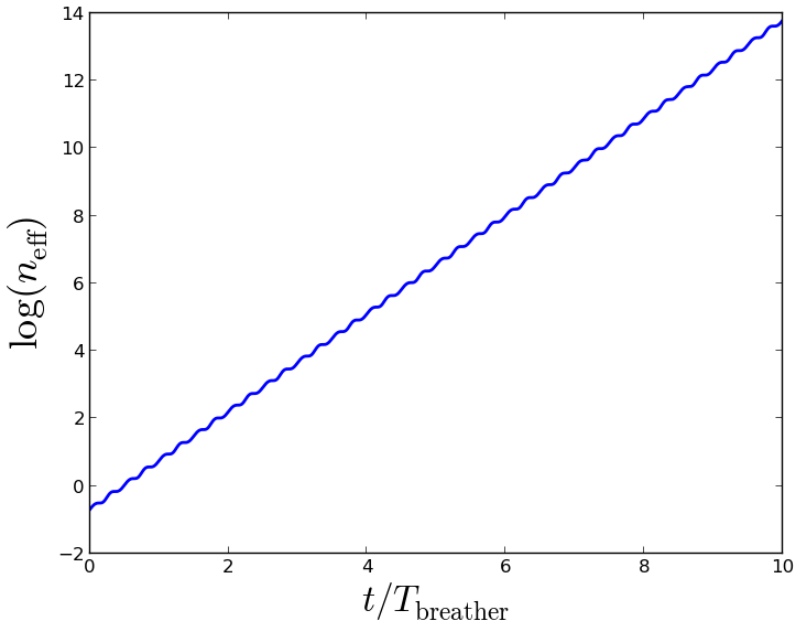
During the collision, there is a short interval when these are not eigenmodes



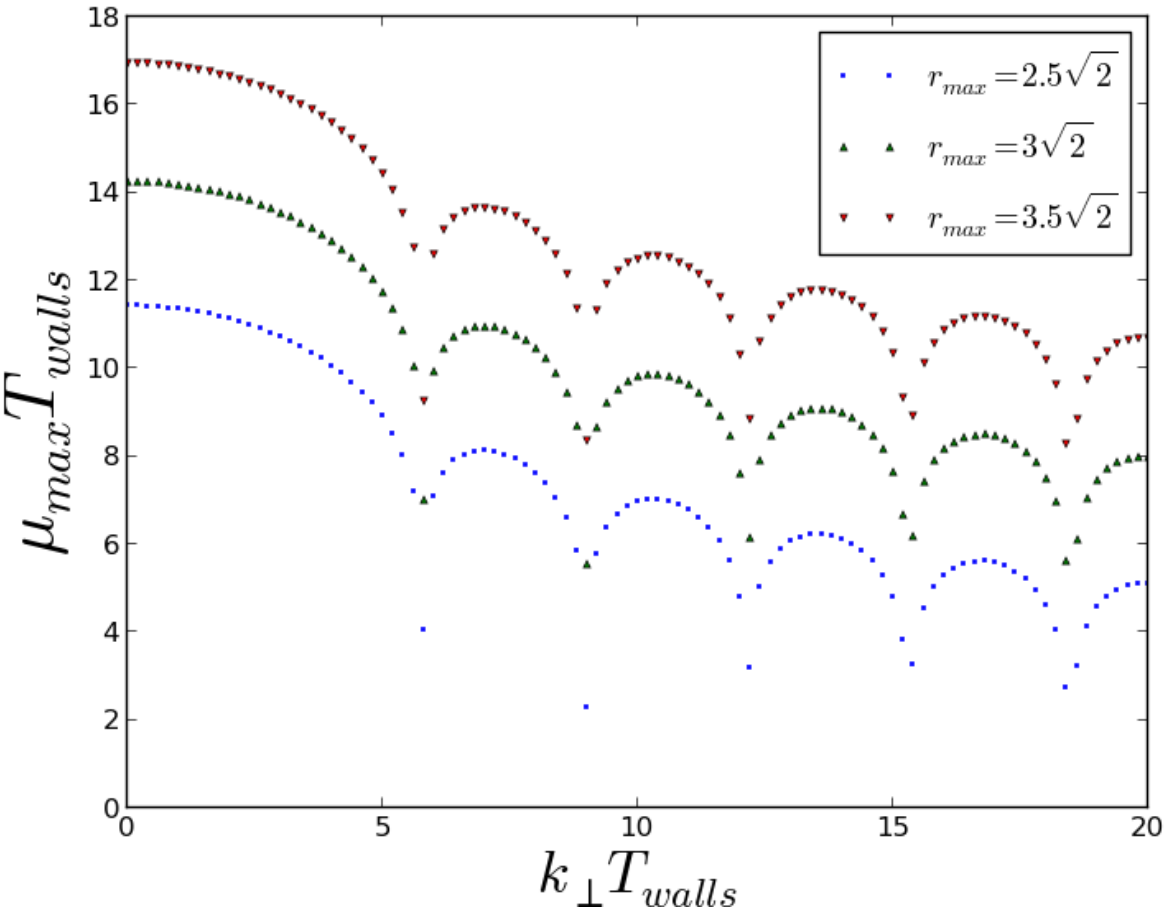



# Weak Resonance : Oscillating Blob



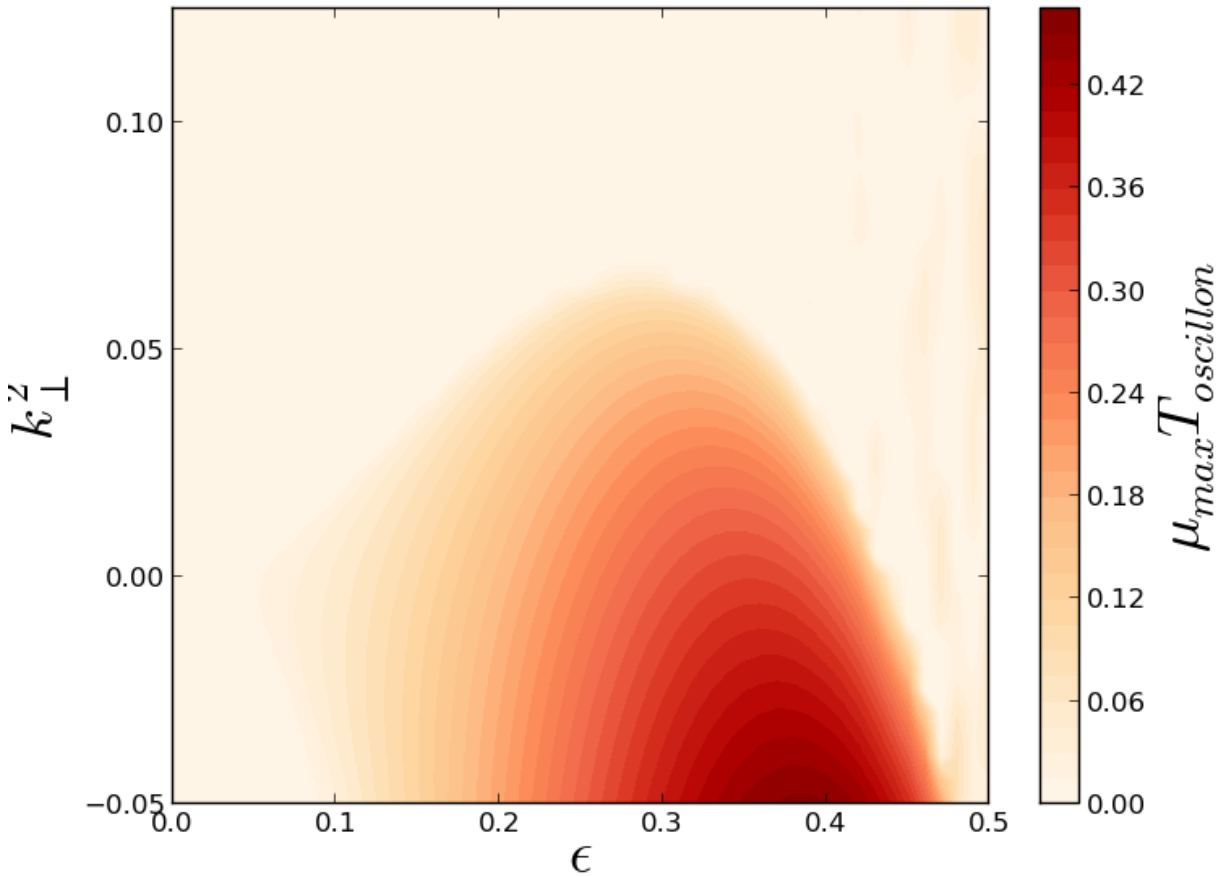
# Resonance in Double Well (with approximate backgrounds)



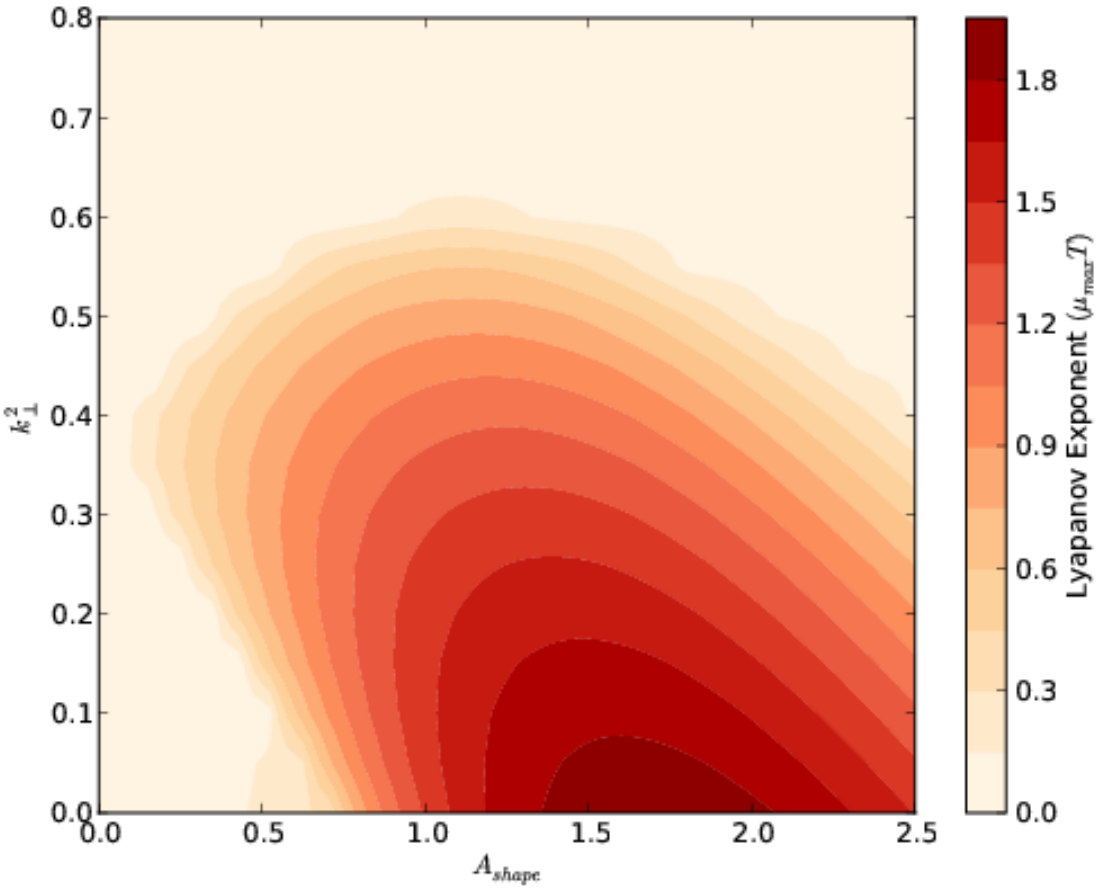

Collisions of Walls

# Resonance in Double Well (with approximate backgrounds)

Oscillating Blob

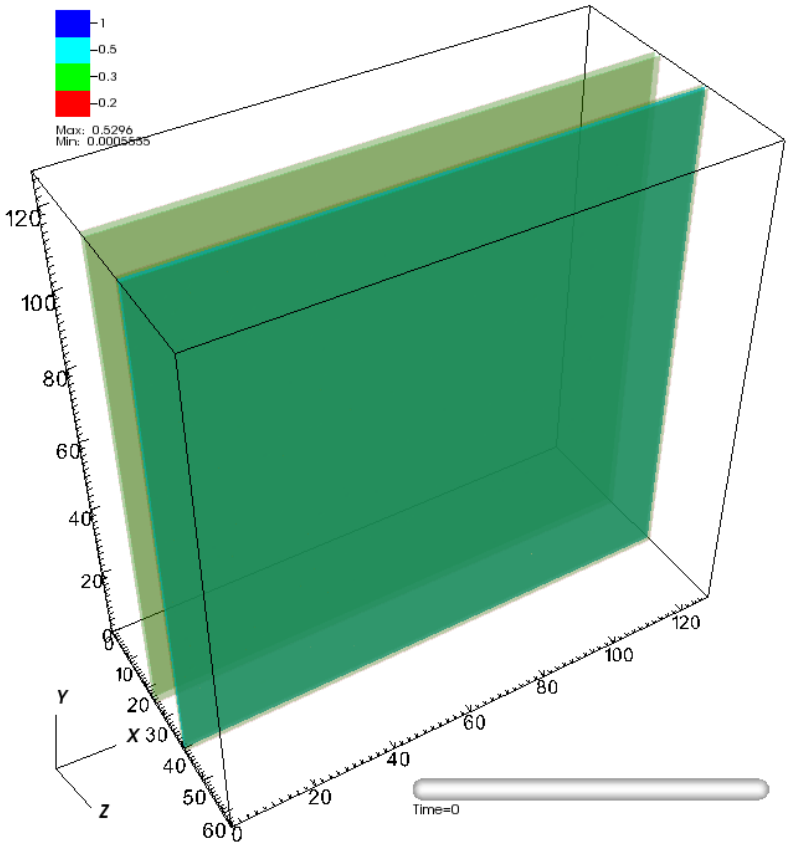


# Resonance in Double Well (with approximate backgrounds)

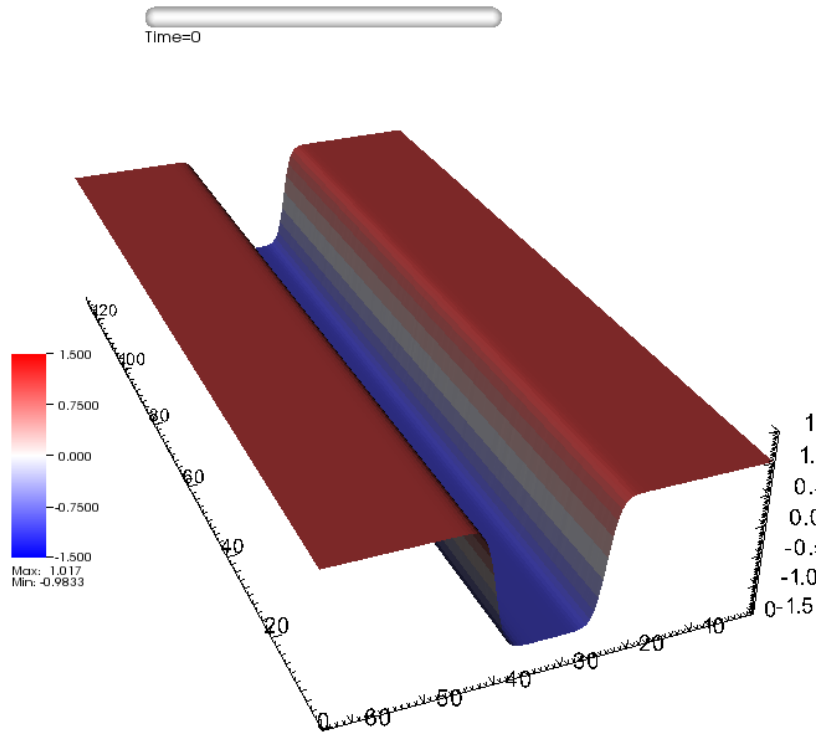



Oscillating Wall Width

# Demonstration in Full Lattice Simulation

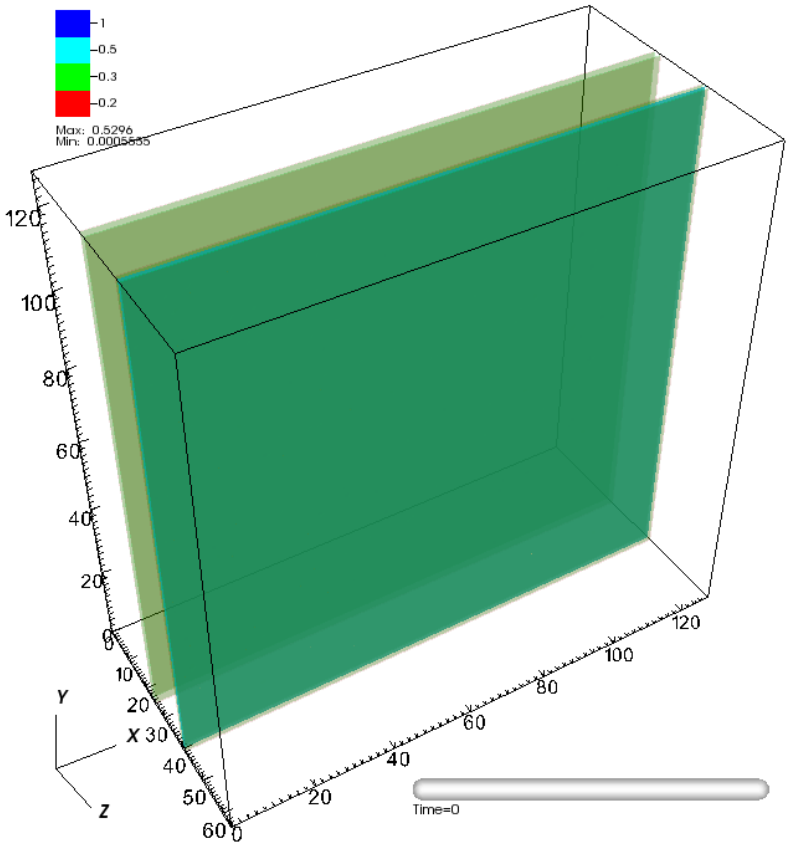



Contours of $\rho/\lambda\phi_0^4$

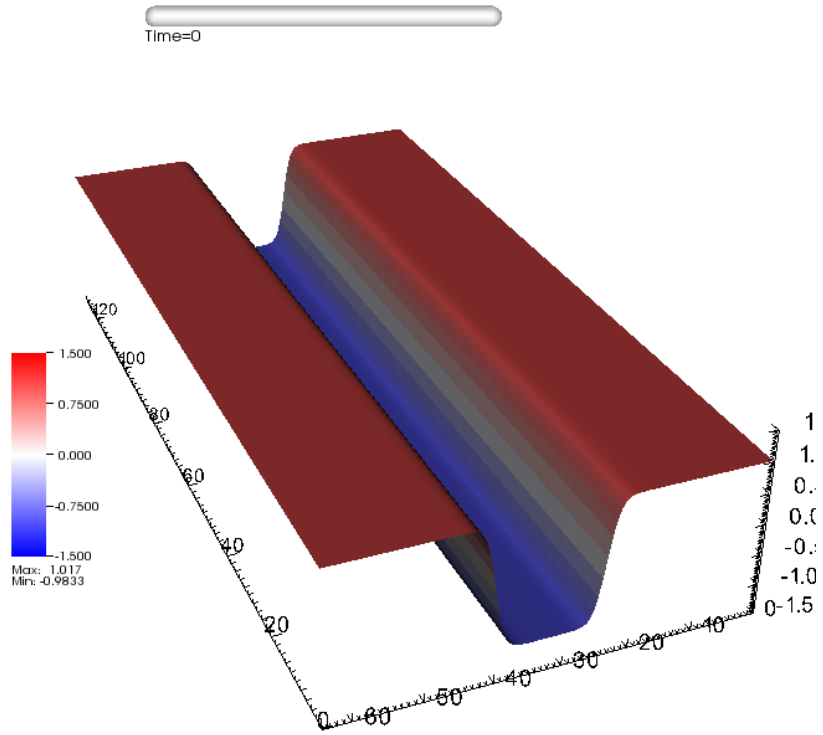



Evolution of $\phi$

# Demonstration in Full Lattice Simulation

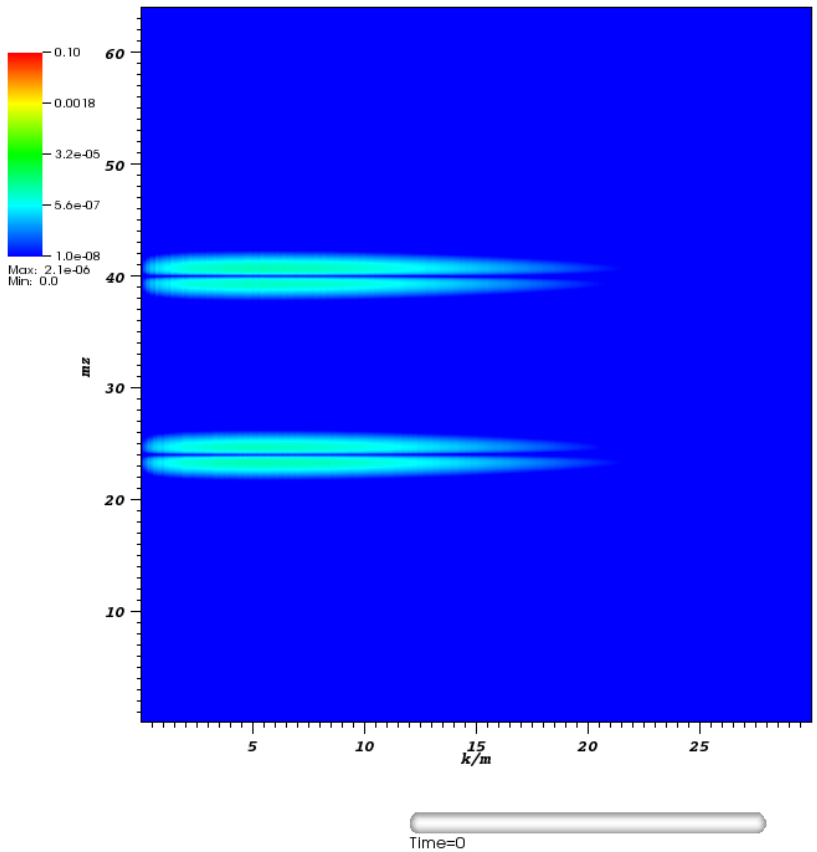



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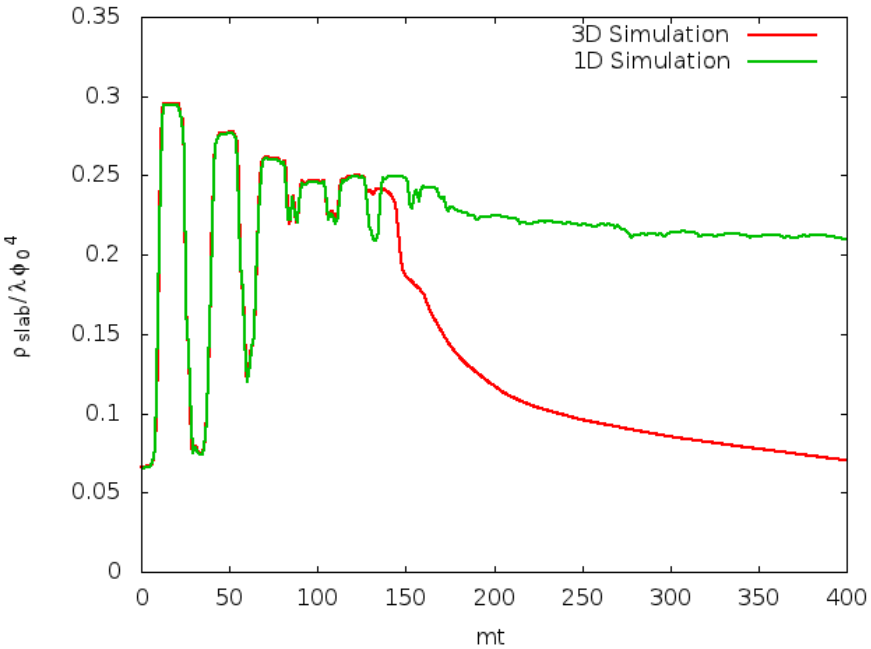



Evolution of $\phi$

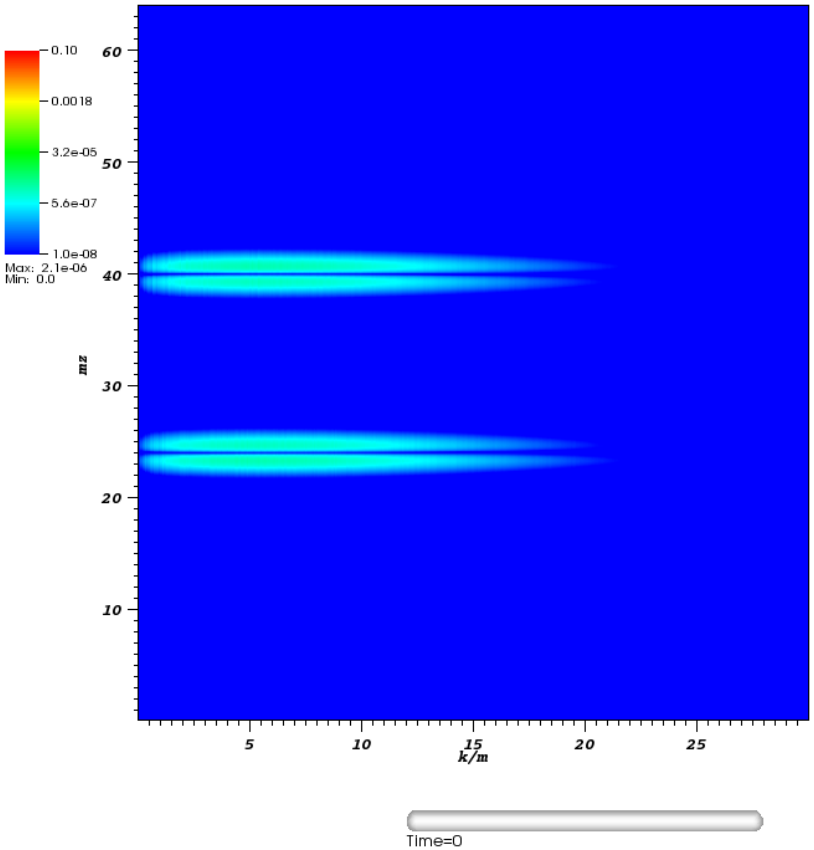
# Spectrum of Growing Instabilities



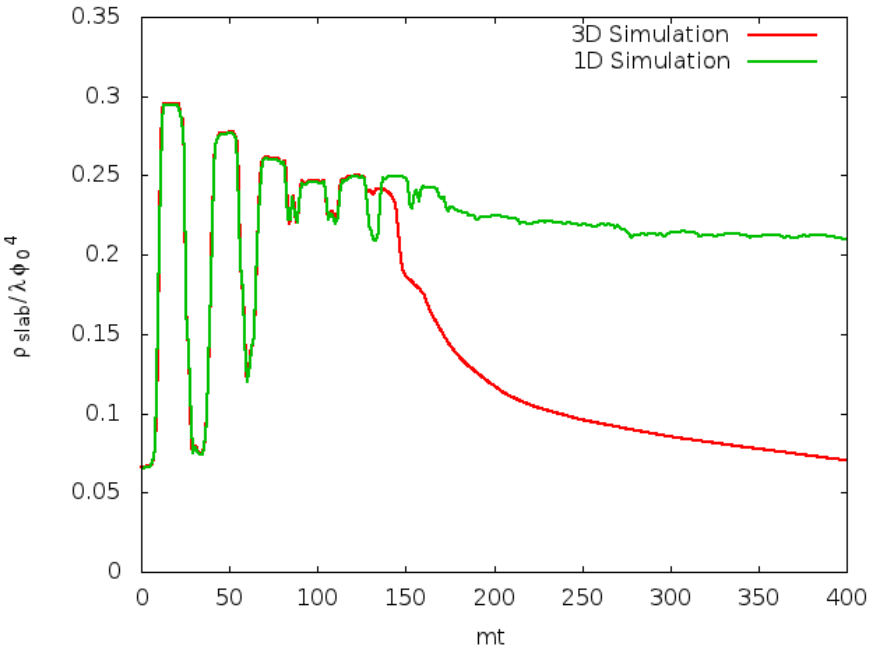

## Distribution of Energy



# Spectrum of Growing Instabilities




## Distribution of Energy



# Review of Mechanism

SO(2,1) symmetry can be badly broken by amplified quantum fluctuations

1. Initial state evolves as a piece that preserves SO(2,1) plus a small perturbation that doesn't
2. Perturbations are unstable in the evolving symmetric background
3. Grow ripples and bumps on the bubble walls
4. Large ripples and bumps lead to a random field with blobs whose characteristic size is determined by linear instability
5. Nonlinearities condense these blobs into oscillons

Can have oscillons and inflation in multifield models

# Implications

## SO(2,1) symmetry can be badly broken

Observables don't necessarily have azimuthal symmetry

- Beam smoothing versus inhomogeneity scale
- Tensor modes are produced by fracturing of walls
- Distribution of energy density is different than w/ SO(2,1)
- Hot vs. Cold Spot in one field versus two field model

Analysis doesn't depend on eternal inflation scenario

- Oscillons as nonequilibrium environment for baryogenesis?
- Oscillons dilute as $a^{-3} \rightarrow$ perturbed EOS during phase transition?
- Application to braneworlds with colliding walls

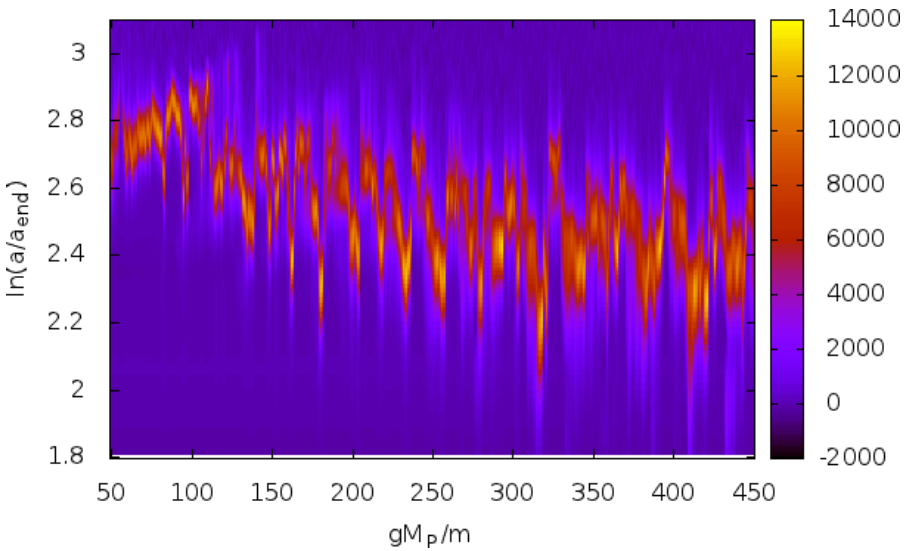
These signals are spatially **intermittent**...

# ... just like density perturbations from preheating caustics

[B$^2$FH: Bond, Braden, Frolov, Huang]

$$\zeta = \zeta_{inflaton} + F_{NL}(\chi)$$

$\chi$ : Gaussian random field $\qquad$ $F_{NL}$ : Nonlinear Function



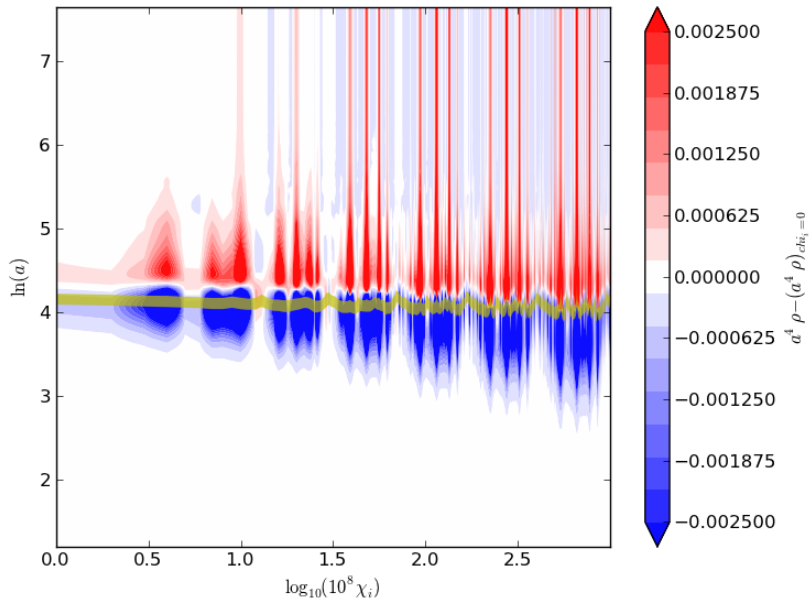

Modulated Couplings

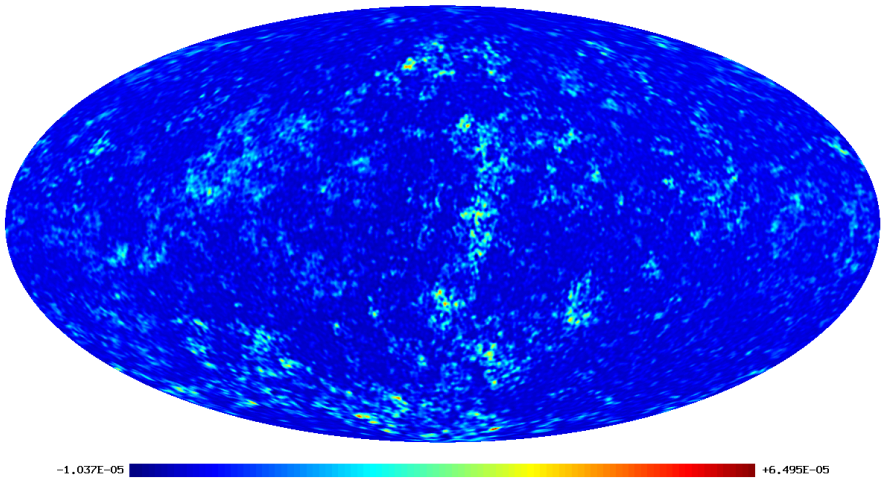
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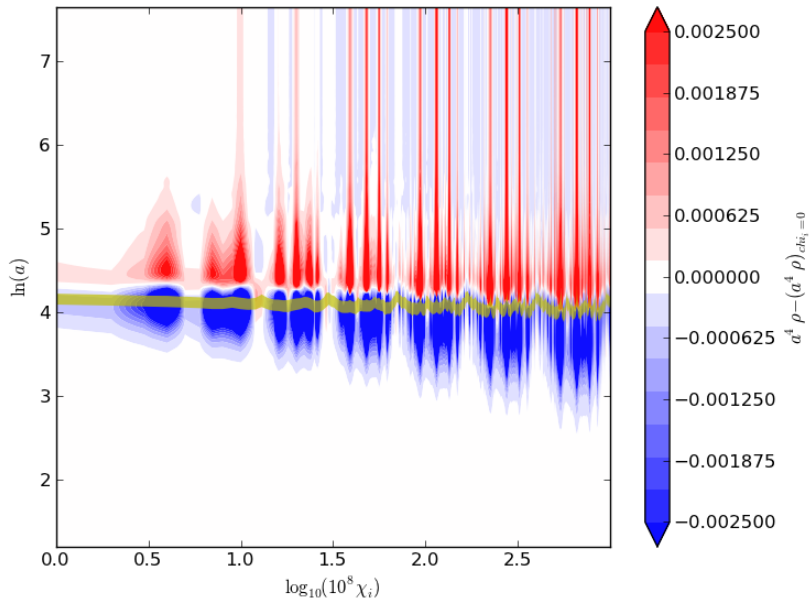

Modulated Initial Value of Decay Field

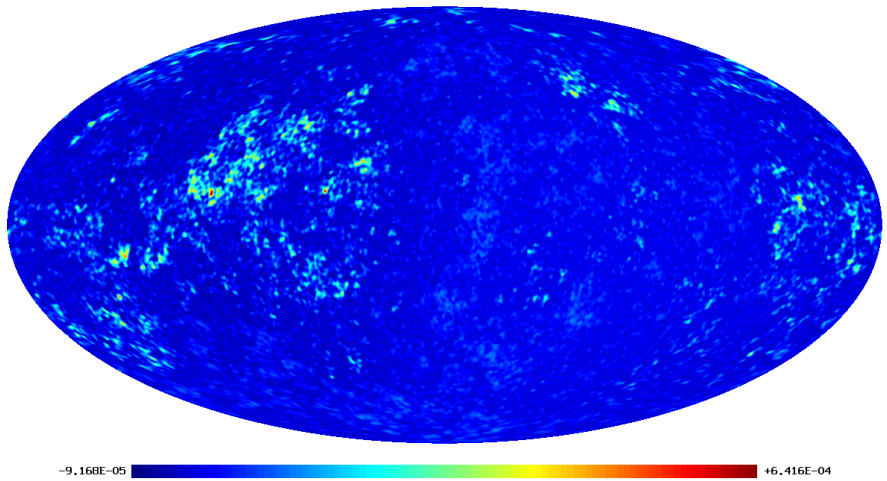
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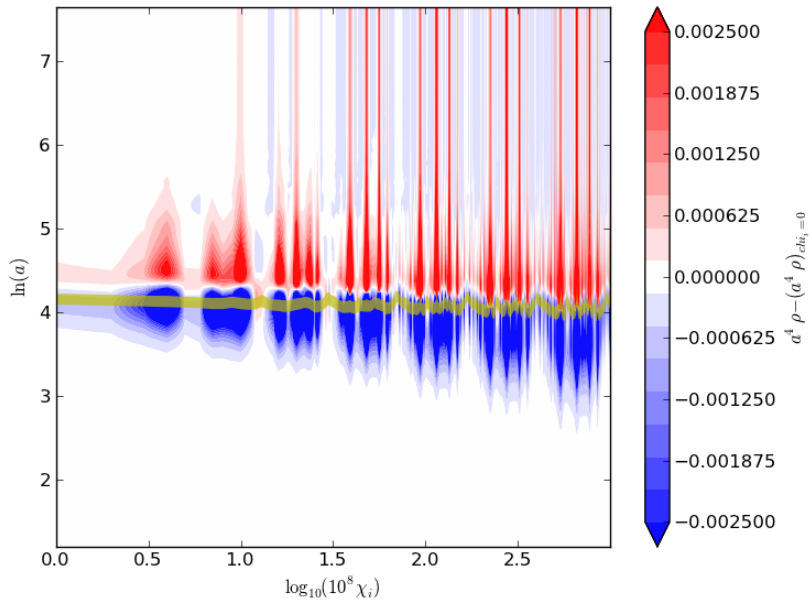

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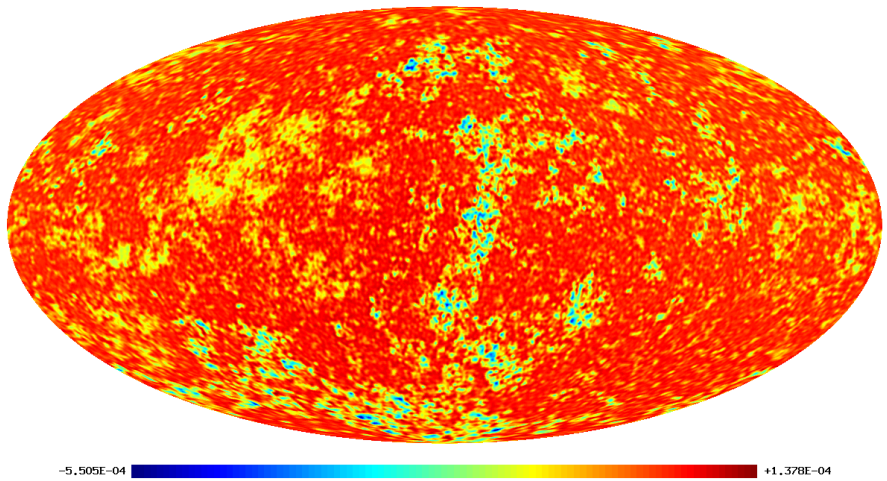
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