Collisions of Vacuum Bubbles with Quantum Fluctuations

Jonathan Braden, CITA / U. Toronto

Perimeter Institute, Waterloo, Ontario

December 17, 2013

in collaboration with Dick Bond, Laura Mersini-Houghton

1312.XXXX : Cosmic Bubbles and Domain Walls I : Parametric Amplification of Linear Fluctuations
1312.XXXX : Cosmic Bubbles and Domain Walls II : Nonlinear Fracturing of Colliding Walls
1312.XXXX : Cosmic Bubbles and Domain Walls III : The Role of Oscillons in Three-Dimensional Bubble Collisions

Videos at www.cita.utoronto.ca/~jbraden/
Outline

- Bubbly Overview and Review of SO(2,1) Framework
- Setting Initial Conditions (solution of bounce equation)
- Full Nonlinear 3D Dynamics
  1. double-well with slightly broken $Z_2$ (symmetry breaks)
  2. double-well with strongly broken $Z_2$ (symmetry remains)
  3. single-well with plateau (symmetry remains)
  4. two-field potential supporting inflation (symmetry breaks)
- Linear Fluctuation Analysis
- Implications for Observations
The Bubbly Universe

\[ N_{col} \sim \sqrt{\Omega_k} \left( \frac{H_{\text{false}}}{H_{\text{inflation}}} \right)^2 \frac{\Gamma}{\mathcal{V}} H^{-4}_{\text{false}} \]

[Aguirre,Johnson],[Freivogel,Kleban,Nicolis,Sigurdson]

\[ \frac{\Gamma}{\mathcal{V}} \sim B^2 |\text{det}(\delta^2 S)|^{-1/2} e^{-B} \]

What are the dynamics of individual collisions?
Large Body of Past Work

Single Instantons
- Coleman, deLuccia
- Hawking, Moss
- Turok
- Sasaki, Linde, Tanaka, Yamamoto
- Garriga, Vilenkin, Montes, Garcia-Bellido
- Guth, Guven
- Freese, Adams
- Susskind et al
- ...

Vacuum Bubble Collisions
- Hawking, Moss, Stewart
- Kosowski, Turner, Watkins, Kamionkowski
- Johnson, Aguirre, Tysanner, Larfors
- Chang, Kleban, Levy, Sigurdson
- Easther, Giblin, Lim, Lau (3D, but symmetric IC’s)
- Johnson, Lehner, Peiris, … (GR)
- ...

Observations
- Johnson, Peiris, Mortlock, McEwan, Feeney, ...
- Smith, Senatore, Osborne

Assume (Spacetime) Symmetries
Numerical Approach

Massively Parallel Lattice Simulation

- Solve field equation (e.g.)
  
  \[ \ddot{\phi}_i + 3\frac{\dot{a}}{a} \dot{\phi}_i - \frac{\nabla^2 \phi_i}{a^2} + V'(\phi) = 0 \]

- 10th order Gauss-Legendre integration (general) or 8th order Yoshida (nonlinear sigma models)
- Finite-difference (fully parallel) or Spectral (OpenMP, but MPI version in the works)
- Optional absorbing boundaries
- Quantum fluctuations → realization of random field

Energy conservation \( \mathcal{O}(10^{-9} - 10^{-14}) \)
Standard Framework SO(2,1) Symmetry

- Most likely bubble has SO(3,1) symmetry
- Second bubble breaks
  - Boosts along axis connecting centers
  - Rotations about any axis in plane orthogonal to axis connecting centers
- Preserve SO(2,1)
Standard Framework SO(2,1) Symmetry

\[ t = s \cosh(\psi) \]
\[ x = x \]
\[ y = s \sinh(\psi) \cos(\theta) \]
\[ z = s \sinh(\psi) \sin(\theta) \]

- *Most likely* bubble has SO(3,1) symmetry
- Second bubble breaks
  - Boosts along axis connecting centers
  - Rotations about any axis in plane orthogonal to axis connecting centers
- Preserve SO(2,1)

1+1-Dimensional Dynamics (e.g. in Minkowski)

\[ \frac{\partial^2 \phi}{\partial s^2} + \frac{2}{s} \frac{\partial \phi}{\partial s} - \frac{\partial^2 \phi}{\partial x^2} - V'(\phi) = 0 \]
Standard Framework SO(2,1) Symmetry

\[ t = s \cosh(\psi) \]
\[ x = x \]
\[ y = s \sinh(\psi) \cos(\theta) \]
\[ z = s \sinh(\psi) \sin(\theta) \]

- Most likely bubble has SO(3,1) symmetry
- Second bubble breaks
  - Boosts along axis connecting centers
  - Rotations about any axis in plane orthogonal to axis connecting centers
- Preserve SO(2,1)

1+1-Dimensional Dynamics (e.g. in Minkowski)

\[ \frac{\partial^2 \phi}{\partial s^2} + \frac{2}{s} \frac{\partial \phi}{\partial s} - \frac{\partial^2 \phi}{\partial x^2} - V'(\phi) = 0 \]

Should We Trust This When Quantum Fluctuations are Included?
Effect of Fluctuations on the Collision

Dilute Gas Initial Conditions

\[ \phi_{init} = \sum \phi_{bounce}(|x - r_i|) - (N_{bub} - 1)\phi_{false} + \delta\phi(x, y, z) \]

\(\delta\phi\) is not SO(2,1) symmetric and **must** be included in quantum theory

- The bubbles nucleate
- Inflation amplifies subhorizon fluctuations

**Is \(\delta\phi\) dynamically important?**

Need simulations with more than one spatial dimension
SO(4) Bounce Equation

\[ \frac{\partial^2 \phi}{\partial r_E^2} + \frac{3}{r_E} \frac{\partial \phi}{\partial r_E} - V'(\phi) = 0 \]

\[ \phi(r_E = \infty) = \phi_{\text{false}} \quad \frac{\partial \phi(r_E = 0)}{\partial r_E} = 0 \]

Pseudospectral Solution

\[ \phi(r_E) = \sum_i c_i B_{2i} \left( h \left( \frac{r_E}{\sqrt{r_E^2 + L^2}} \right) \right) \]

\[ h(x) \equiv \frac{1}{\pi} \tan^{-1} \left( d^{-1} \tan \left( \pi \left[ x - \frac{1}{2} \right] \right) \right) + \frac{1}{2} \]

Global Expansion \rightarrow Extremely Accurate

Mapping Parameters

\[ L : \sim \text{radius of bubble} \]
\[ d : \sim \text{width / radius} \]

Extendable to . . .

- dynamical metric
- multiple fields (shooting hard)
- PDEs (shooting breaks)
Versatile and Accurate (Even for Very Thin Walls)

Jonathan Braden, CITA / U. Toronto (Perimeter Institute, Waterloo, Ontario)

December 17, 2013
Collisions in a Double Well Potential

$$V(\phi) = \frac{\lambda}{4} (\phi^2 - \phi_0^2)^2 - \delta \lambda \phi_0^3 (\phi - \phi_0)$$

Exactly SO(2,1) Invariant Collision

$$s^2 = t^2 - y^2 - z^2$$
Numerical Test: Exact Instanton Initial Conditions

SO(2,1) Invariant Initial Condition

\[ \phi_{\text{init}} = \sum_{r_i} \phi_{\text{bounce}}(|x - r_i|) - (N_{\text{bub}} - 1) \phi_{\text{false}} \]
Numerical Test: Exact Instanton Initial Conditions

SO(2,1) Invariant Initial Condition

\[ \delta \phi = 0 \]
Thin-Wall Approximation

\[ \phi_{init} = \sum_{r_i} \phi_{tw}(|x - r_i|) - (N_{bub} - 1)\phi_{false} \]

Break boost invariance of time-evolved solution

\[ \phi_{tw} = \phi_0 \tanh \left( \frac{m(r - R_0)}{\sqrt{2}} \right) + \frac{\delta}{2} \]
Thin-Wall Approximation

\[ \delta \phi = \sum_{r_i} (\phi_{\text{bounce}}(|x - r_i|) - \phi_{\text{tw}}(|x - r_i|)) \]

Break boost invariance of time-evolved solution

\[ \phi_{\text{tw}} = \phi_0 \tanh \left( \frac{m(r - R_0)}{\sqrt{2}} \right) + \frac{\delta}{2} \]
Thin-Wall Approximation

\[ \delta \phi = \sum_{r_i} (\phi_{\text{bounce}}(|x - r_i|) - \phi_{tw}(|x - r_i|)) \]

Break boost invariance of time-evolved solution

\[ \phi_{tw} = \phi_0 \tanh \left( \frac{m(r - R_0)}{\sqrt{2}} \right) + \frac{\delta}{2} \]
Comparison with One-Dimensional Simulation

This Effect is Missed by Assuming SO(2,1)
Bulk Vacuum Fluctuations

\[ \phi_{\text{init}} = \sum_{r_i} \phi_{\text{bounce}}(|x - r_i|) - (N_{\text{bub}} - 1)\phi_{\text{false}} + \delta\phi(x, y, z) \]
Field Evolution with Fluctuations ...

Bulk Vacuum Fluctuations

\[ \delta \tilde{\phi}_k \sim \frac{a_k}{\sqrt{k^2 + V''(\phi_{false})}} \]

\[ \dot{\delta \tilde{\phi}_k} \sim b_k \sqrt{k^2 + V''(\phi_{false})} \]

without Hubble

with Hubble
... Produces Oscillons

Oscillons Form from Asymmetric Blobs

Initial Blob of Field

\[ \phi_{\text{init}} = \phi_{\text{true}} + (\phi_{\text{false}} - \phi_{\text{true}}) \exp \left( -\frac{x^2}{a^2} - \frac{y^2 + z^2}{b^2} \right) \]

\[ a^2 < b^2 \quad \text{versus} \quad a^2 > b^2 \]
A Model without Amplification
A Model without Amplification
Oscillons with Inflation?

$W(\phi) / \lambda \phi_0^4$

$\frac{\phi}{\phi_0}$
Oscillons with Inflation?

\[ \frac{\phi}{\phi_0} \]

\[ n_{\text{eff}} \]

- Fluctuations
- No Fluctuations

\[ \text{Time}=0 \]
An Inflationary Model with Oscillons

Two Field Model

\[ V(\sigma, \phi) = \lambda \sigma \sigma_0^4 \left[ \frac{1}{4} \left( \frac{\sigma^2}{\sigma_0^2} - 1 \right)^2 + \delta \left( \frac{\sigma^3}{3\sigma_0^3} - \frac{\sigma}{\sigma_0} + \frac{2}{3} \right) \right] + \frac{g^2 \lambda \sigma_0^2}{2} (\sigma - \sigma_0)^2 \phi^2 + \lambda \sigma_0^3 \epsilon \phi + V_0 \]

\[ V(\sigma, \phi) = V_{tunnel}(\sigma) + V_{coupling}(\sigma, \phi) + V_{inflation}(\phi) \]
Instanton in Two-Field Model

This solution has only a single negative eigenmode in the $O(4)$ symmetric sector
Field Evolution in Two-Field Model

σ Evolution

ϕ Evolution
Parametric Resonance for Linear Fluctuations

Linear Fluctuations Around SO(2,1) Solution

\[ \phi(s, x, \psi, \theta) = \phi_{bg}(s, x) + \delta\phi(s, x, \psi, \theta) \]

\[ \frac{\partial^2 \phi_{bg}}{\partial s^2} + \frac{2}{s} \frac{\partial \phi_{bg}}{\partial s} - \frac{\partial^2 \phi_{bg}}{\partial x^2} + V'(\phi_{bg}) = 0 \]

\[ \frac{\partial^2}{\partial s^2} (s\delta\phi_\kappa) - \frac{\partial^2}{\partial x^2} (s\delta\phi_\kappa) + \left( \frac{\kappa^2}{s^2} + V''(\phi_{bg}) \right) (s\delta\phi_\kappa) = 0 \]
Parametric Resonance for Linear Fluctuations

Linear Fluctuations Around Planar Solution

\[ \phi(t, x, y, z) = \phi_{bg}(t, x) + \delta\phi(t, x, y, z) \]

\[ \frac{\partial^2 \phi_{bg}}{\partial t^2} - \frac{\partial^2 \phi_{bg}}{\partial x^2} + V'(\phi_{bg}) = 0 \]

\[ \frac{\partial^2}{\partial t^2} (\delta \phi_{k_{\perp}}) - \frac{\partial^2}{\partial x^2} (\delta \phi_{k_{\perp}}) + (k_{\perp}^2 + V''(\phi_{bg})) (\delta \phi_{k_{\perp}}) = 0 \]

Planar Limit

- \( s \gg 1 \)
- Time scales much shorter than \( s \)
- \( \kappa^2 \gg s^2 \)
Form of $V''$

Oscillating Background $\rightarrow$ Floquet Theory

Exactly Periodic Effective Mass

$$\delta \phi_{\text{Floquet}} = P(x, t)e^{\mu t} \quad P(x, t + 2T) = P(x, t) \quad P \in \mathbb{R}$$
Form of $V''$

Two wells that repeatedly annihilate

Oscillating Background $\rightarrow$ Floquet Theory

Exactly Periodic Effective Mass

$$\delta \phi_{\text{Floquet}} = P(x, t)e^{\mu t} \quad P(x, t + 2T) = P(x, t) \quad P \in \mathbb{R}$$
Form of $V''$

One well oscillating up and down

Oscillating Background $\rightarrow$ Floquet Theory

Exactly Periodic Effective Mass

$$\delta \phi_{Floquet} = P(x, t)e^{\mu t} \quad P(x, t + 2T) = P(x, t) \quad P \in \mathbb{R}$$
Instability in Sine-Gordon Model

Exactly Periodic Backgrounds

\[ \phi_{\text{breather}} = 4 \tan^{-1} \left( \frac{\cos(\gamma_v vt)}{v \cosh(\gamma_v x)} \right) \]

\[ \gamma_v \equiv (1 + v^2)^{-1/2} \]

\[ v \ll 1 \]
Instability in Sine-Gordon Model

Exactly Periodic Backgrounds

\[ \phi_{\text{breather}} = 4 \tan^{-1} \left( \frac{\cos(\gamma v t)}{v \cosh(\gamma v x)} \right) \]

\[ \gamma_v \equiv (1 + v^2)^{-1/2} \]

\[ v \gtrsim 1 \]
Broad Resonance: Well-Defined Wall Collisions c.f. [Kofman,Linde,Starobinski] for homogeneous bg

Single Wall, $\phi_{kink,SG} = 4\tan^{-1}(e^x)$, $\phi_{kink,DW} = \tanh(x/\sqrt{2})$

In 1d theory $\delta \phi = \partial_x \phi_{kink}(x)$ is a zero mode

In 3d theory $\rightarrow$ bound fluctuations with $\omega = k_\perp$

Very General: Goldstone for spontaneously broken translation invariance

During the collision, there is a short interval when these are not eigenmodes
Broad Resonance: Well-Defined Wall Collisions c.f. [Kofman, Linde, Starobinski] for homogeneous bg

Single Wall, $\phi_{kink,SG} = 4\tan^{-1}(e^x)$, $\phi_{kink,DW} = \tanh(x/\sqrt{2})$

- In 1d theory $\delta\phi = \partial_x \phi_{kink}(x)$ is a zero mode
- In 3d theory $\rightarrow$ bound fluctuations with $\omega = k_\perp$

Very General: Goldstone for spontaneously broken translation invariance

$$n_{\text{eff}}^{\omega_{\text{bound}}} + \frac{1}{2} = \int dx \frac{1}{2k_\perp}(k_\perp^2 \delta \phi_{k_\perp}^2 + \delta \dot{\phi}_{k_\perp}^2)$$
Weak Resonance: Oscillating Blob

\[ n_{\text{eff}}^{(\omega_{\text{breather}})} + \frac{1}{2} \equiv \frac{1}{2\omega_{\text{breather}}} \int dx \left( \delta \dot{\phi}^2 + \omega_{\text{breather}}^2 \delta \phi^2 \right) \]
Resonance in Double Well (with approximate backgrounds)

Collisions of Walls

Jonathan Braden, CITA / U. Toronto (Perimeter Institute, Waterloo, Ontario)

Collisions of Vacuum Bubbles with Quantum Fluctuations

December 17, 2013 27 / 32
Resonance in Double Well (with approximate backgrounds)

Oscillating Blob

\[ \kappa^\frac{2}{1} \]

\[ \epsilon \]

\[ \mu_{\text{max}} T_{\text{oscillon}} \]
Resonance in Double Well (with approximate backgrounds)

Oscillating Wall Width
Contours of $\rho/\lambda\phi_0^4$  
Evolution of $\phi$
Spectrum of Growing Instabilities

Distribution of Energy
SO(2,1) symmetry can be badly broken by amplified quantum fluctuations

1. Initial state evolves as a piece that preserves SO(2,1) plus a small perturbation that doesn’t
2. Perturbations are unstable in the evolving symmetric background
3. Grow ripples and bumps on the bubble walls
4. Large ripples and bumps lead to a random field with blobs whose characteristic size is determined by linear instability
5. Nonlinearities condense these blobs into oscillons

Can have oscillons and inflation in multifield models
Implications

**SO(2,1) symmetry can be badly broken**

Observables don’t necessarily have azimuthal symmetry

- Beam smoothing versus inhomogeneity scale
- Tensor modes are produced by fracturing of walls
- Distribution of energy density is different than w/ SO(2,1)
- Sign of \( \zeta = \delta \ln(a) \) in one field versus two field model

Analysis doesn’t depend on eternal inflation scenario

- Oscillons as nonequilibrium environment for baryogenesis?
- Oscillons dilute as \( a^{-3} \rightarrow \) perturbed EOS during phase transition?
- Application to braneworlds with colliding walls

These signals are spatially **intermittent**...
... just like density perturbations from preheating caustics $[B^2FH: \text{Bond, Braden, Frolov, Huang}]$\textsuperscript{14XX.XXXX?}

\[ \zeta = \zeta_{\text{inflaton}} + F_{NL}(\chi) \]

\[ \chi: \text{Gaussian random field} \quad F_{NL}: \text{Nonlinear Function} \]

Modulated Couplings
... just like density perturbations from preheating caustics [B²FH: Bond, Braden, Frolov, Huang]14XX.XXXX?

\[ \zeta = \zeta_{\text{inflaton}} + F_{NL}(\chi) \]

\( \chi \): Gaussian random field \quad F_{NL} : \text{Nonlinear Function}
... just like density perturbations from preheating caustics $[B^2FH: \text{Bond, Braden, Frolov, Huang}]$ 14XX.XXXX?

\[
\zeta = \zeta_{\text{inflaton}} + F_{NL}(\chi)
\]

\(\chi\) : Gaussian random field \hspace{1cm} F_{NL} : \text{Nonlinear Function}

Modulated Initial Value of Decay Field
... just like density perturbations from preheating caustics [B^2FH: Bond, Braden, Frolov, Huang] 14XX.XXXX?

\[ \zeta = \zeta_{\text{inflaton}} + F_{\text{NL}}(\chi) \]

\[ \chi : \text{Gaussian random field} \quad F_{\text{NL}} : \text{Nonlinear Function} \]

Modulated Initial Value of Decay Field