Entropy Production During Preheating : A Shock-in-Time

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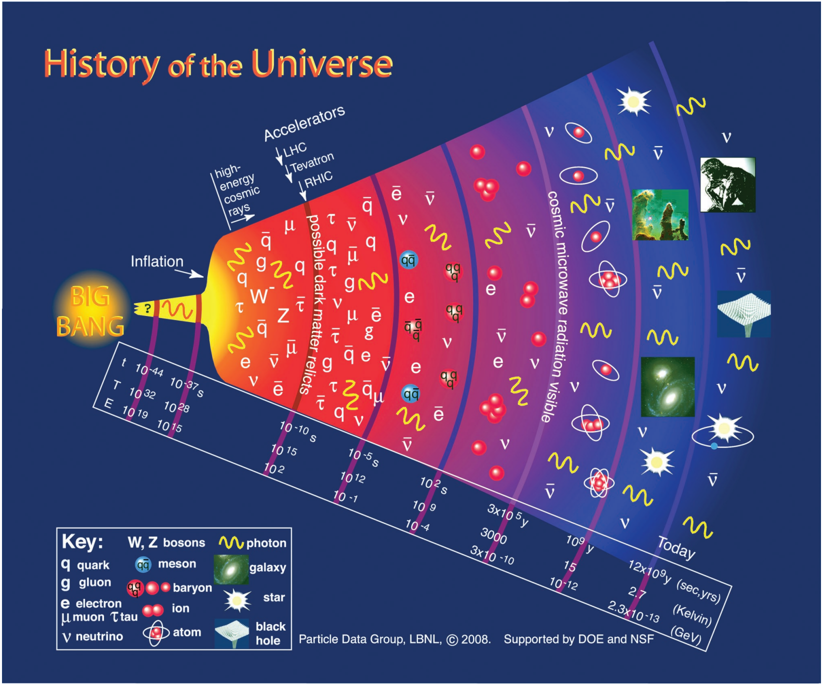
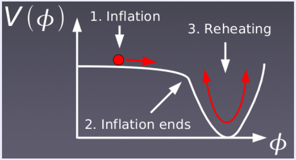

History of the Universe
BIG BANG
Inflation
high-energy cosmic rays
Accelerators
LHC
Tevatron
RHIC
possible dark matter relicts
cosmic microwave radiation visible
Today
t 10-44 10-37 s 10-10 s 10-5 s 10² s 3x10⁵ y 10⁹ y 12x10⁹ y (sec,yrs)
T 10³² 10²⁸ 10¹⁵ 10¹² 10⁹ 3000 15 2.7 (Kelvin)
E 10¹⁹ 10¹⁵ 10² 10-1 10-4 3x10-10 10-12 2.3x10-13 (GeV)
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
star
e electron
ion
μ muon τ tau
ν neutrino
atom
black hole
Particle Data Group, LBNL, © 2008. Supported by DOE and NSF

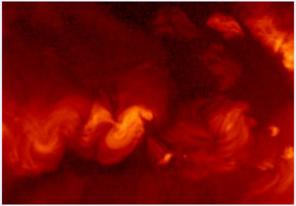
Starting the Big Bang

Inflation



- Cold ($T \sim 0$)
- Few active d.o.f.

Big Bang



- Hot ($T > MeV$)
- Many active d.o.f.

Huge entropy production

But how does it happen?

(Toy) Model for the Transition

Basic Requirements

- Inflation must end
- Must produce Standard Model particles $\rightarrow$ couplings to other particles

Model

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

Background Evolution

During inflation, $H \gg m$

$$\phi \sim \phi_0 - \sqrt{\frac{3}{2}}mt \rightarrow \sim \text{constant}$$

After inflation $H \leq m$

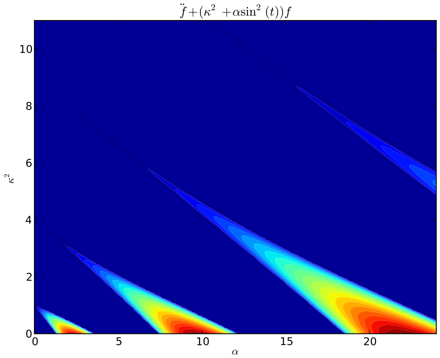
$$\phi \sim \phi_{end}\frac{\sin(mt)}{a^{3/2}} \rightarrow \text{oscillating}$$

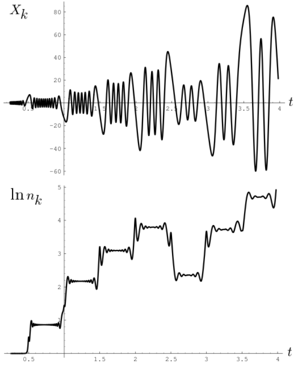
Preheating: Linear Resonance

[Kofman, Linde, Starobinski '94, '97]

Linear Fluctuations

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{m^2 a^2} + \frac{\phi_{end}^2 g^2}{m^2 a^3}\sin^2(mt)\right)\chi_k = 0$$





Existence of Parametric Instability

- Dynamical field with oscillating mass
- Background field to create this oscillating mass

This is **not** special to this model

Model Requirments

- Inflation must end $\rightarrow$ inflaton oscillates about minimum
- Inflaton must decay $\rightarrow$ time-dependent parameters for coupled fields

Rich zoo of instabilities

- tachyonic preheating $M_{eff}^2 = k^2 + V''(\bar{\phi}) < 0$ (e.g. hybrid inflation)
- tachyonic resonance ($\sim \phi\chi^2$, $\sim \phi F\tilde{F}$)
- resonance persists with multiple inflatons
- stringy effects

Linear growth of fluctuations must end

- Expansion weakens resonance

or

- Nonlinear effects and backreaction become important

Focus on second

Effective particle occupation number

During linear regime

$$n_k = \frac{1}{2\omega_k}\left(\omega_k^2|\phi_k|^2 + |\dot{\phi}_k|^2\right) - \frac{1}{2} \sim \frac{E_k}{\omega_k}$$

$$\omega_k^2 = \frac{k^2}{a^2} + g^2\bar{\phi}^2$$

$a^3 n_k \gg 1 \to$ classical wave regime.

We can perform **classical** numerical simulations.

Assumed metric and Lagrangian

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$$

$$\mathcal{L}_{mat} = -\sum_i \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - V(\vec{\phi})$$

Equations of Motion (derived from action)

$$\ddot{\phi}_i + 3H\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + \frac{\partial V}{\partial\phi_i} = 0 \qquad H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\langle\rho\rangle}{3M_{pl}^2}$$

$$\rho = -T^0_0 = \sum_i \frac{\dot{\phi}_i^{\,2}}{2} + \frac{(\nabla\phi_i)^2}{2a^2} + V(\vec{\phi})$$

Initial Spectrum

Gaussian Random Field: $P_\phi(k) = \frac{1}{2\omega_k}$

Evolution of $\ln(\rho/\bar{\rho})$

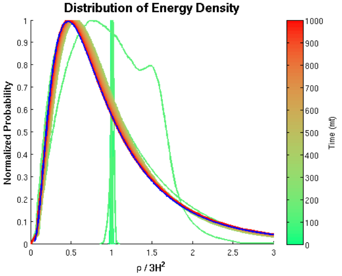
MPI-enabled DEFROST [Frolov] with symplectic integrator[Frolov, Huang] (up to $O(dt^7)$)

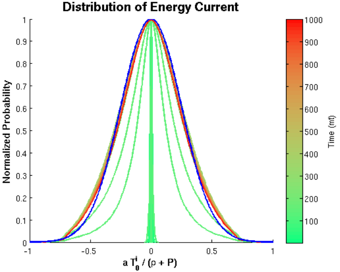
This all looks complicated

But it has some amazing simplicity

Fluid variables have simple PDF's after initial transient

$\rho/\bar{\rho}$





$$v_x = \frac{aT^x_0}{\rho + P}$$

How can we quantify this transition?

Nonequilibrium Entropy in Field Theory

Shannon (*c.f.* Von Neumann) Entropy

$$S_{shannon} = -\mathrm{Tr} P[f] \ln P[f]$$

$P[f]$: Probability density functional

$$P[f] = ?$$

Coarse-Graining Procedure

- Take all of our knowledge of the system
- Maximize S subject to constraints imposed by this knowledge

A coarse-graining is already implicit in taking our system to be probabilistic (c.f. difference between mixed state and pure state)

Application to Preheating

Constraint = Measured Power Spectrum

Power Spectrum $P(k) \to$ Covariance matrix C (homogeneous field)
$S_{shannon}$ maximized by multivariate-Gaussian with spectrum $P(k)$

$$\begin{aligned} \frac{S}{N} &= \frac{1}{2} + \frac{1}{2}\ln(2\pi) + \frac{1}{2N}\ln\det C \\ &= \frac{1}{2} + \frac{1}{2}\ln(2\pi) + \frac{1}{2N}\sum_{\mathbf{k}}\ln(P(k)) \end{aligned}$$

N : number of modes (or volume in case of continuum field)

Dynamical Field

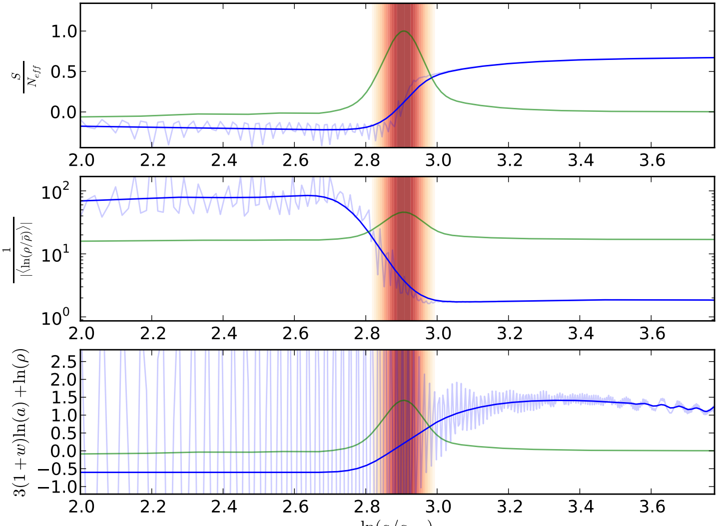
Choose : $f = \ln(\rho(\mathbf{x})/\bar{\rho}) \sim$ phonons (sound waves)

- Measurable quantity (regardless of model choice)
- ρ cannot be Gaussian

Power Spectrum Evolution ...

... leads to a **Shock-in-Time**

[Bond, JB (in preparation)]



The Analogy

Spatial Shock

- $v_{bulk}^2 > c_s^2 \rightarrow v_{bulk}^2 < c_s^2$
- Characteristic spatial scale
- Mediated by viscosity or collisionless dynamics
- Randomizing : shock front ΔS
- Post-shock evolution towards thermalization
- Jump in conserved quantities
- Timelike surface

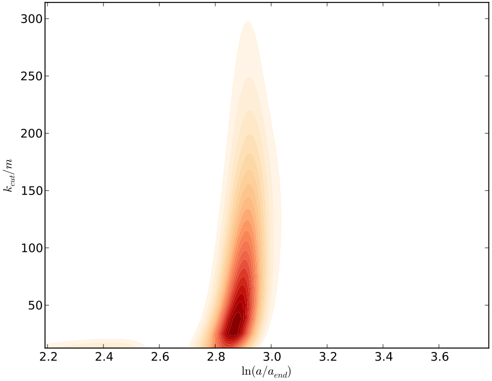
Shock-in-Time

- $ln(\frac{\rho}{\bar{\rho}})^{-1} \gg 1 \rightarrow \ln(\frac{\rho}{\bar{\rho}})^{-1} \sim 1$
- Characteristic time scale
- Mediated by gradients and nonlinearities
- Randomizing : cascade/part. production ΔS
- Slow post-shock evolution
- Jump in $a^{3(1+w)}\rho$
- Can be spacelike surface

Scale Dependence of the Entropy

$$\frac{\mathrm{d}(S/N)}{\mathrm{d}t} \propto \frac{1}{2N}\sum_{k\leq k_{cut}} \frac{\mathrm{d}}{\mathrm{d}t}\ln(P(k))$$

Normalized Entropy Production Rate



The shock is robust to our choice of momentum cutoff.

The shock in real space

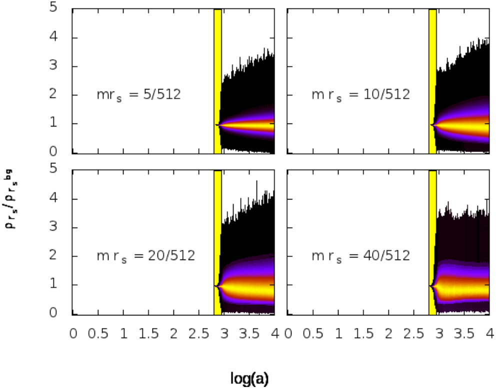
Loss of coherence $\leftrightarrow$ shock-in-time

Scaling Properites of the Post-Shock State

$$\rho_{s+1} = \sum_{s+(i,j,k)} \rho_s$$

$$0 \leq i, j, k \leq 1$$

Hierarchy of fluctuations around local background.



And in Fourier Space ...

Evolution of excess kurtosis for fourier amplitudes.

...the PDF's also become Gaussian

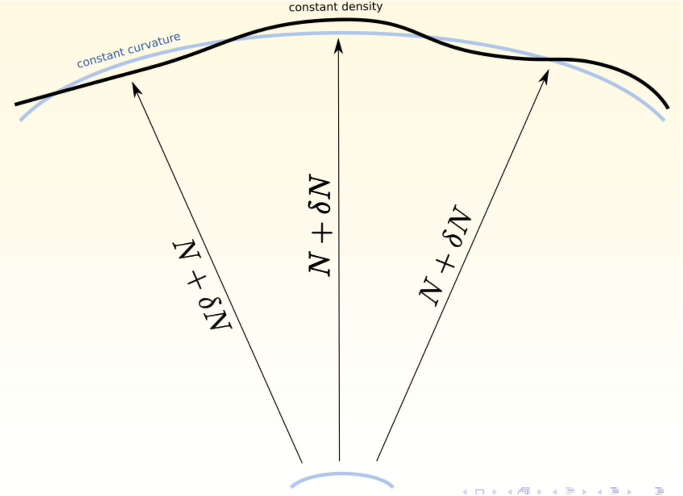
$$0 < \frac{k}{m} < 20$$

...the PDF's also become Gaussian

$$20 < \frac{k}{m} < 40$$

Density Perturbations from Preheating

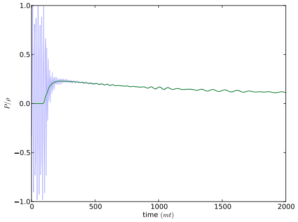
Density Perturbations from Preheating



$$\zeta = \delta \ln(a)|_H = \delta \ln(a)|_\rho$$

Spatially modulate expansion history $\rightarrow$ curvature perturbations

Modulated Preheating



Homogeneous Universe

$$\rho e^{3(\ln(a)+(\int w d ln(a)))} = \mathrm{const.}$$

$$w \text{ determines } \delta \ln(a)|_\rho$$

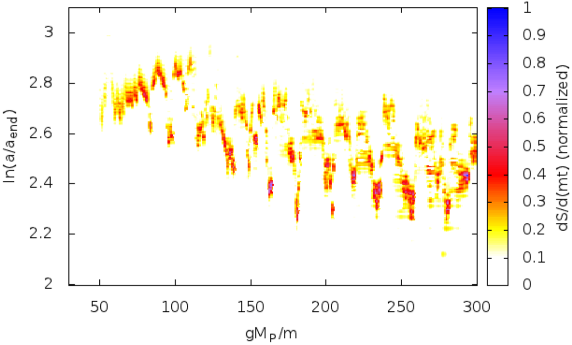
What can we modulate

- The timing of the shock (yes)
- The in-shock evolution (yes)
- The post-shock evolution (probably)

Timing: Modulated Preheating

[Bond, JB (in preparation)]

Allow couplings between ϕ,χ to depend on other light fields.



Differences in shock timing lead to perturbations in $\delta \ln(a)$.

But...

- $w_{post-shock} \neq \frac{1}{3}$, not instantaneous decay
- $g^2(\sigma + \delta\sigma)$ is a complicated function, can't Taylor expand

What about the post-shock evolution?

$$\langle P \rangle = \frac{1}{3}\langle \rho \rangle + H\langle \phi_i \dot{\phi}_i \rangle + \frac{1}{3}\langle V_{|\phi_i}\phi_i \rangle - \frac{4}{3}\langle V \rangle + \frac{1}{6}\langle \frac{d^2\phi_i^2}{dt^2} \rangle \qquad (1)$$

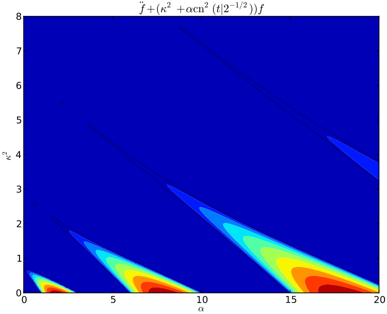
Deviations from radiation EOS are determined by various field moments.

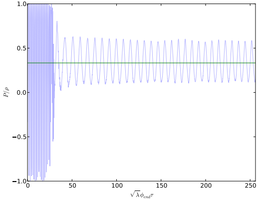
Chaotic Billiards: In-Shock Modulation

[Bond, Frolov, Huang, Kofman '09][Bond, JB, Frolov, Huang '12]

Model

$$V(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$





$$(a\chi_k)'' + \left(\frac{k^2}{\lambda\phi_{end}^2} + \frac{g^2}{\lambda}\mathrm{cn}^2(\tau|2^{-1/2})\right)(a\chi_k) = 0$$

$$1 < \frac{g^2}{\lambda} < 3 : \ \chi_{k=0} \text{ unstable}$$

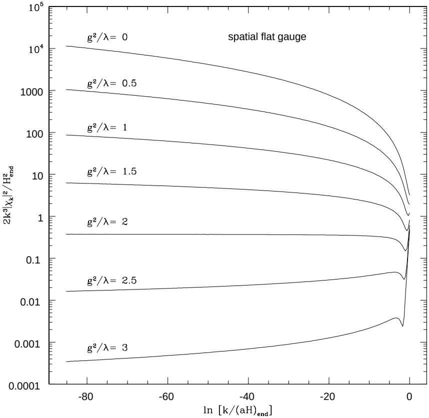
Evolution of $\ln(\rho/\bar{\rho})$

Isocurvature Perturbations ...

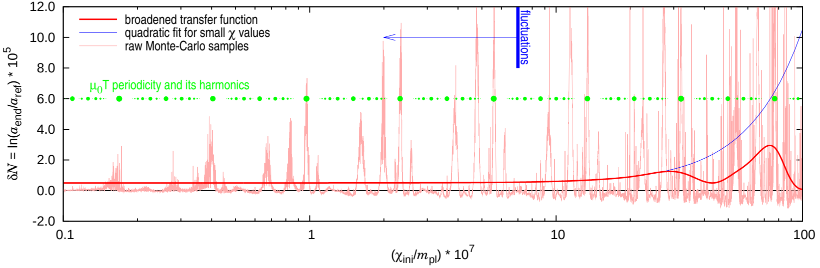
$$\frac{m^2_{\chi,\text{eff}}}{H^2} = 12\frac{g^2}{\lambda\bar{\phi}^2}$$

$$\frac{m^2_{\phi,\text{eff}}}{H^2} = \frac{36}{\bar{\phi}^2}$$

χ fluctuations are light during inflation $\rightarrow$ isocurvature modes



... Create Density Perturbations

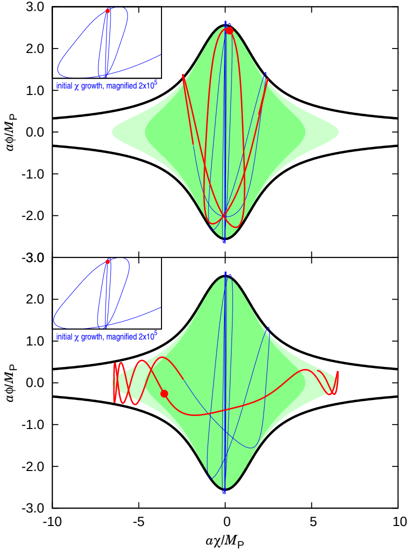


$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + F_{NL}(\chi_G)$$

This is not the usual parameterization

$$\Phi = \Phi_G + f_{nl}(\Phi_G^2 - \langle \Phi_G^2 \rangle)$$

Where do the spikes come from?



$$V_{eff} = \frac{\lambda}{4}\phi^4 + \frac{1}{2}(\phi^2 + \langle\delta\phi^2\rangle)\chi^2$$

In arm and out-of-arm trajectories have different expansion histories

$$\chi(t+T) = e^{\mu_0 T}\chi(t)$$

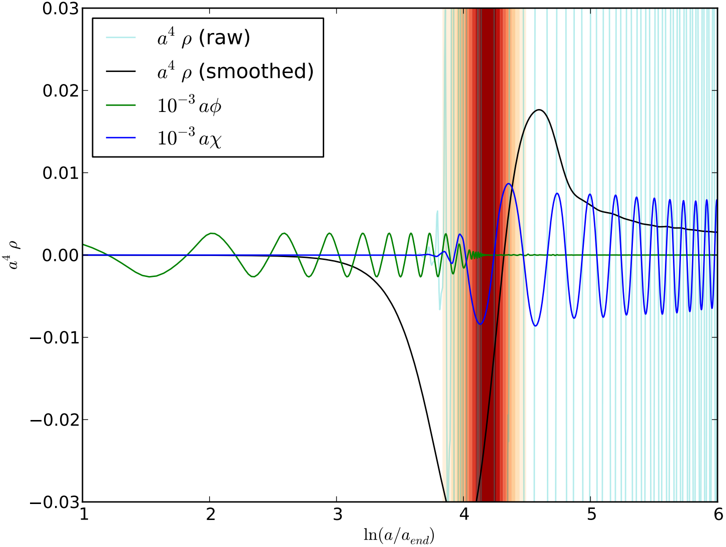
Evolution on a Non-Spike Trajectory

[Video courtesy of Andrei Frolov]

Evolution on a Spike Trajectory

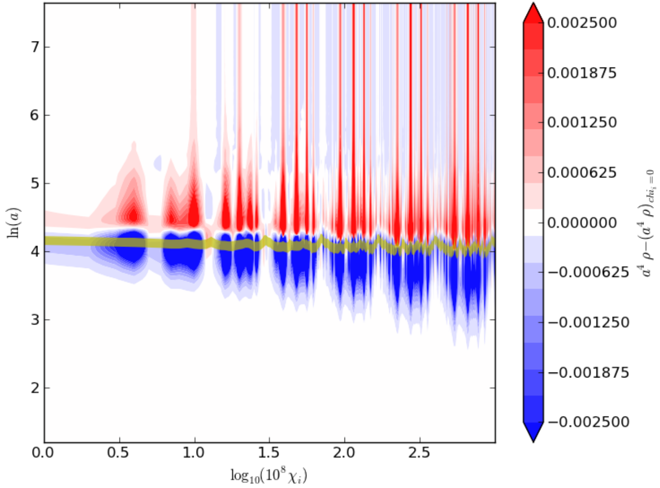
[Video courtesy of Andrei Frolov]

The view on a single spike



How is this related to the shock?

$\delta ln(a)$ structure set at the shock



Conclusions

- New framework with which to discuss preheating using entropy
  - Shock-in-time connects linear regime to nonlinear (and nonequilibrium) post-shock regime
  - Most of the entropy is produced at the shock (most violent part of preheating)
  - Post-shock state evolves slowly,
- Preheating can leave nonGaussian density fluctuations
  - Requires a source of modulation in preheating dynamics (new light fields or long-wavelengths of preheat fields)
  - The final perturbation is largely set at the time of the shock
- Future Directions
  - What happens when we incorporate more fields (e.g. Standard Model)
  - Classify potentials that allow spiky nongaussianity