Bubbles to Blobs: Breakup of Bubbles and Domain Walls via Parametric Resonance

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Gravity and Cosmology 2012, YITP

November 22, 2012

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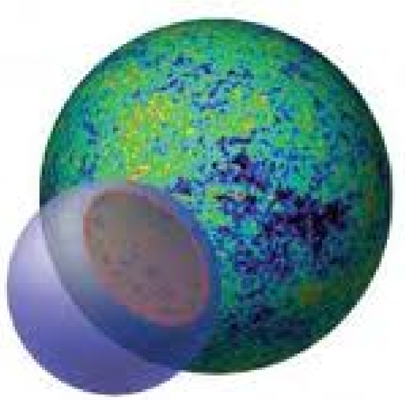
Motivation

- Vacuum Bubble Collisions
  - Test Usual Symmetry Assumptions
  - Computational Capabilities allow 3-D simulations
- More Generally, Domain Wall Collisions



Lattice simulations are a powerful tool to explore nonequilibrium/nonperturbative phenomena.

Past Investigations

Vacuum Bubble Collisions

- Hawking, Moss, Stewart
- Kosowski, Turner, Watkins, Kamionkowski
- Chang, Kleban, Levy, Sigurdson
- Johnson, Lehner, Tysanner, Aguirre
- Easther, Giblin, Lim, Lau (3D, but symmetric IC's)
- ...

All Invoke (Spacetime) Symmetry Assumptions

What about fluctuations (c.f. single bubble?):

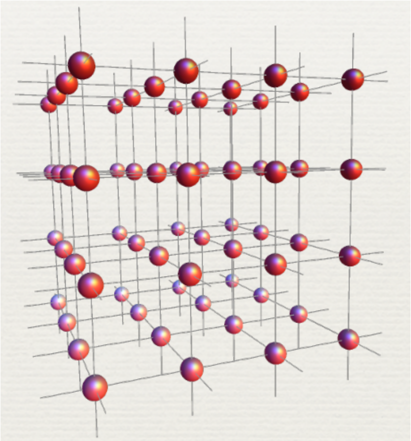
- Sasaki, Linde, Tanaka, Yamamoto
- Garriga, Vilenken, Montes, Garcia-Bellido
- Guven
- Freese and Adams
- ...

Numerical Approach

- Solve field equation

$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{\nabla^2\phi_i}{a^2} + V'(\vec{\phi}) = 0$$

- Massively parallel lattice simulation
- Absorbing boundary conditions
- Arbitrary (homogeneous) background evolution
- High order symplectic time evolution
- Quantum fluctuations → realization of random field

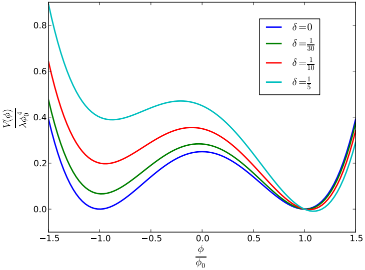


Model

Double Well Potential

$$V(\phi) = \frac{\lambda\phi_0^4}{4}\left(\frac{\phi^2}{\phi_0^2} - 1\right)^2 - \delta\lambda\phi_0^4\left(\frac{\phi}{\phi_0} - 1\right) + V_0$$

Express dimensionful quantities in units of $m \equiv \sqrt{\lambda}\phi_0$.



- Very simple potential
- Already displays complex and novel phenomena
- Extensions to more complicated theories relatively unexplored

Highly Symmetric Defect Solutions

- 1D kinks solution → planar wall in higher dimensions

$$\phi_{kink} \approx \phi_0 \tanh\left(\frac{m(x - x_0)}{\sqrt{2}}\right) + \frac{\delta}{2}$$

- Bubble Solutions in higher dimensions

$$\phi_{\mathrm{thinwall}} \approx \phi_0 \tanh\left(m\frac{\sqrt{(\mathbf{r} - \mathbf{r_0})^2 - t^2} - R_0}{\sqrt{2}}\right) + \frac{\delta}{2}$$

What happens when the kinks/bubbles collide?

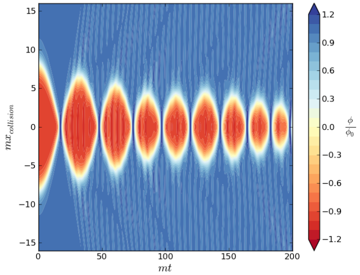
Planar Domain Walls

Symmetry Reduced Dynamics : Planar Walls

Kink Solutions

$$\phi_{kink} \approx \phi_0 \tanh\left(\frac{m(x-x_0)}{\sqrt{2}}\right) + \frac{\delta}{2}$$

$$\delta = \frac{1}{30}$$



What About Fluctuations?

$$\phi = \phi_{bg}(x_{col}, t) + \delta\phi \qquad \langle\delta\phi\rangle_{2\mathrm{d}} = 0$$

Fourier transform perturbations in directions perpendicular to collision.

Linearized Fluctuation Equation

$$\ddot{\widetilde{\delta\phi}}_{\mathbf{k_2}} - \frac{\partial\widetilde{\delta\phi}_{\mathbf{k_2}}}{\partial x^2} + \left(k_2^2 + V''(\phi_{bg}(x,t))\right)\widetilde{\delta\phi}_{\mathbf{k_2}} = 0$$

Time and space dependent mass for the fluctuations

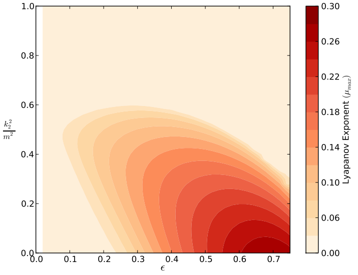
- Outgoing radiation (removed with boundary conditions $\partial_x\delta\phi = \pm\partial_t\delta\phi$)
- **Can we get localized exponentially growing instabilities (a la preheating)?**

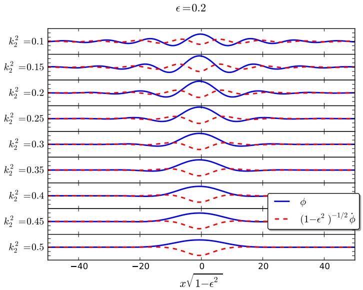
YES...

Approximate BG Solution

$$\phi_{bg} \approx \phi_0 + \frac{\delta}{2} + \frac{4\epsilon}{\sqrt{\gamma}} \frac{\cos(\sqrt{1-\epsilon^2}t)}{\cosh(\epsilon x)} + \mathcal{O}(\epsilon^2, \delta^2)$$

Look for solutions of the form $\delta\phi(x, T) = \delta\phi(x, 0)e^{\mu T}$, with T the period.





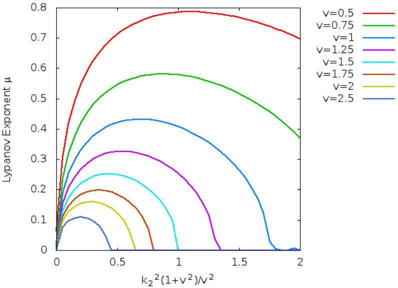
Sine-Gordon Model : Resonance is Real

Check: The instability isn't due to a poor approximation of the background

$$V(\phi) = 1 - \cos(\phi)$$

Breather Solution (Exact)

$$\phi_{breather} = 4\tan^{-1}\left(\frac{\sin(\gamma_v vt)}{v\cosh(\gamma_v x)}\right)$$



$$\ddot{\widetilde{\delta\phi}}_{\mathbf{k_2}} - \widetilde{\delta\phi}_{\mathbf{k_2,xx}} + \left[k_2^2 + \cos\left(4\tan^{-1}\left(\frac{\sin(\gamma_v v)t}{v\cosh(\gamma_v x)}\right)\right)\right]\widetilde{\delta\phi}_{\mathbf{k_2}} = 0$$

Full Three-Dimensional Dynamics

Lowest Mass Excitations

$$\phi_{wall} = \phi_{kink}((x - x_0) + \delta x_1(y, z), 0) \qquad \langle|\widetilde{\delta x}_{\mathbf{k_2}}|^2\rangle \sim k^{-1}$$

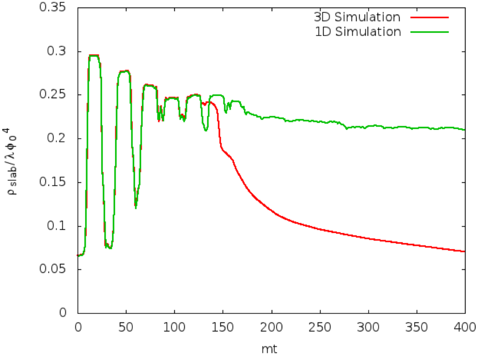
What happens when the fluctuations become large?

Spectrum of Excited Fluctuations

ϕ sliced parallel to collision axis

$$P_\rho(k_2, x) \equiv k^2 \langle |\tilde{\rho}_{\mathbf{k_2}}(x)|^2 \rangle_{2d}$$

Energy Release vs. Planar Symmetric Ansatz



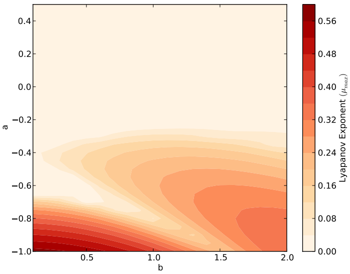
Released energy is absorbed by our boundary conditions.

- Initially planar radiation emitted
- Breakup of Walls $\rightarrow$ formation of clumps of energy
- $\sim$10% of energy remains trapped in collision region

Observers on the Wall (Shape Mode Excitations)

1D kink has a bound state excitation with oscillation frequency $\omega_1^2 = 3/2$. Is generically excited/deexcited in 1-d collisions.

$$\phi_{bg} \approx \tanh(\zeta) + A_0 \cos(\omega_1 t) \frac{\sinh(\zeta)}{\cosh^2(\zeta)} \qquad \zeta \equiv \frac{m(x - x_0)}{\sqrt{2}}$$



Evolution of Excited Shape Mode

Bubble Collisions

Symmetry Reduced Dynamics : Bubbles

Instanton Equation (in Minkowski)

$$\phi'' + \frac{d}{\rho}\phi' - V'(\phi) = 0$$

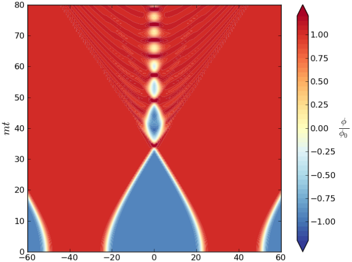
Thin-Wall Limit

$$\phi_{CdL} = \phi_0 \tanh\left(\frac{m(\sqrt{(\mathbf{r}-\mathbf{r_c})^2 - t^2} - R_0)}{\sqrt{2}}\right) + \frac{\delta}{2} \qquad mR_0 = \frac{\sqrt{2}}{\delta}$$

Assume $SO(2,1)$ symmetry $\implies$

$$\phi = \phi(s, x)$$

$$s^2 = t^2 - y^2 - z^2$$



Full 3D Collision w/out Fluctuations

3D Collision w/ Fluctuations

Bulk fluctuations corresponding to $\lambda = 10^{-4}$.

Collision Products (Oscillons)

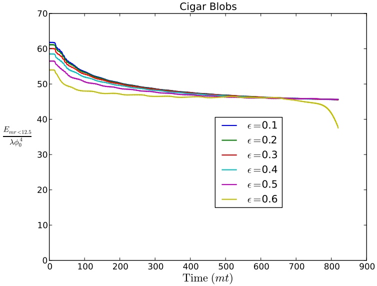
Collision Products : Oscillons

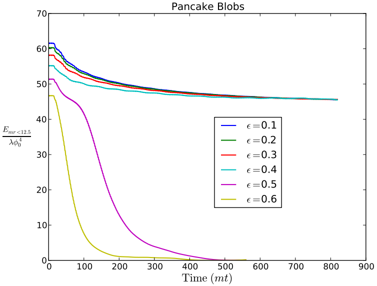
Collapse of Asymmetric Blobs

$$\phi_{init} = \phi_{true} + (\phi_{false} - \phi_{true}) \exp\left(-\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2}\right)\right)$$

$a^2 > b^2$: cigar

$a^2 < b^2$: pancake





$$\epsilon^2 = 1 - \left(\frac{min(a,b)}{max(a,b)}\right)^2$$

Possible Implications

- Symmetric ansatz fails for bubble collisions in double well potentials $\implies$ imprint on sky would not have cylindrical symmetry
  - Scale of inhomogeneity vs. experimental beam smoothing scale?
  - Rate of energy injection into bubble interior is very different than symmetric case
- Cosmological Signatures of Oscillons?
  - Embed in a theory with inflation in the bubble
  - Need $m \gg H$, so ϕ cannot be canonical inflaton
- Key feature is bouncing during wall collisions
  - Applications to braneworld models with colliding domain walls
- Modified equation of state during first order phase transition?
  - Oscillons dilute as $a^{-3} \implies$ perturbed EOS
  - Oscillons may act as source of nonequilibrium for baryogenesis

Summary / Future Work

Summary

- Usual spacetime symmetry assumptions in bubble and domain wall collisions must be checked
- Even simple models can produce interesting long-lived relics

Future Work

- Include inhomogeneous gravity $\rightarrow$ different scale factors inside and outside bubbles
  - Challenging numerical problem
- Embedding into a more realistic cosmological model
  - Hopefully can stimulate discussion at this workshop