Nonlinear Dynamics in the Early Universe : Intermittent Density Perturbations from Preheating Caustics and the Shock-in-Time

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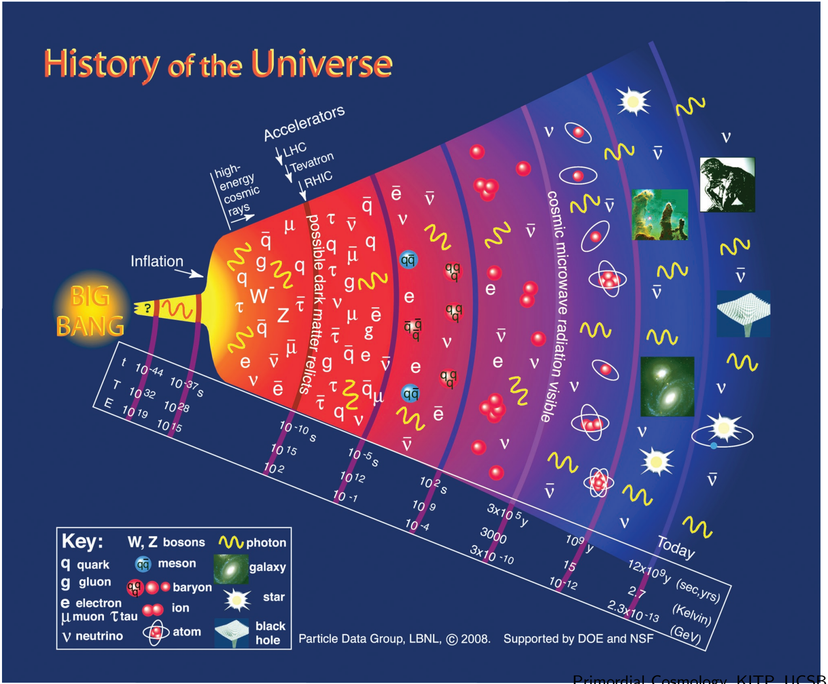
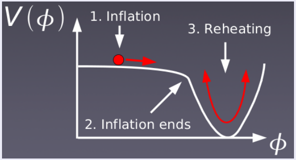
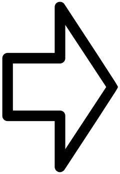

History of the Universe
Accelerators
LHC
Tevatron
RHIC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10 -44 10 -37 s
T 10 32 10 28
E 10 19 10 15
10 -10 s
10 15
10 2
10 -5 s
10 12
10 -1
10 2 s
10 9
10 -4
3x10 5 y
3000
3x10 -10
10 9 y
15
10 -12
Today
12x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10 -13 (GeV)
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
star
e electron
ion
μ muon τ tau
black hole
ν neutrino
atom
Particle Data Group, LBNL, © 2008. Supported by DOE and NSF

Starting the Big Bang

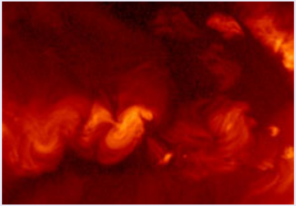
Inflation



- Cold ($T \sim 0$)
- Few active d.o.f.



Big Bang



- Hot ($T > MeV$)
- Many active d.o.f.

Huge entropy production

- But how does it happen?
- Does it leave observable signatures?

(Toy) Model for the Transition

Basic Requirements

- Inflation must end
- Must produce Standard Model particles $\rightarrow$ couplings to other particles

Model

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

Background Evolution

During inflation, $H \gg m$

$$\phi \sim \phi_0 - \sqrt{\frac{3}{2}}mt \rightarrow \sim \text{constant}$$

After inflation $H \leq m$

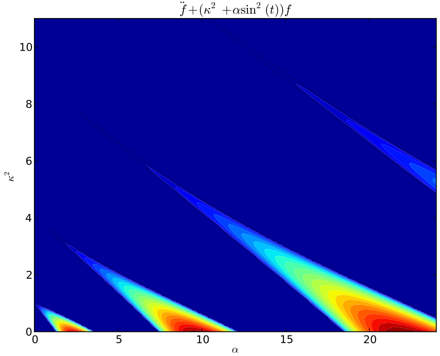
$$\phi \sim \phi_{end}\frac{\sin(mt)}{a^{3/2}} \rightarrow \text{oscillating}$$

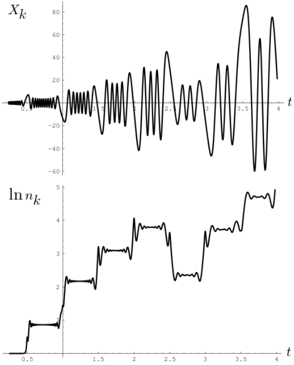
Preheating: Linear Resonance

[Kofman, Linde, Starobinski '94, '97]

Linear Fluctuations

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{m^2a^2} + \frac{\phi_{end}^2 g^2}{m^2a^3}\sin^2(mt)\right)\chi_k = 0$$





Existence of Parametric Instability

- Dynamical field with oscillating mass
- Background field to create this oscillating mass

This is **not** special to this model

Model Requirments

- Inflation must end $\rightarrow$ inflaton oscillates about minimum
- Inflaton must decay $\rightarrow$ time-dependent parameters for coupled fields

Rich zoo of instabilities

- tachyonic preheating $M_{eff}^2 = k^2 + V''(\bar{\phi}) < 0$ (e.g. hybrid inflation)
- tachyonic resonance ($\sim \phi\chi^2$, $\sim \phi F\tilde{F}$)
- resonance persists with multiple inflatons

Linear growth of fluctuations must end

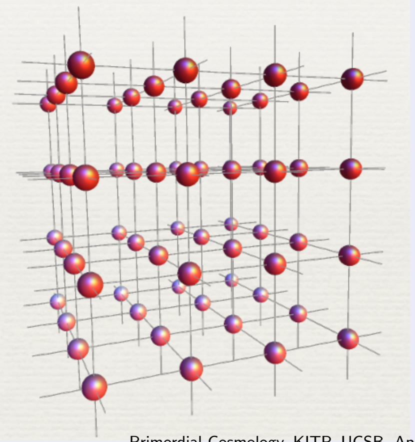
- Expansion weakens resonance

or

- Nonlinear effects and backreaction become important

Numerical Simulations

- Massively parallel
- High order (up to $\mathcal{O}(dt^9)$) symplectic integrator
  - Preserve Hubble constraint
- Quantum fluctuations $\rightarrow$ realization of random field



Assumed metric and Lagrangian

$$ds^2 = -a(t)^2(d\tau^2 + d\mathbf{x}^2)$$

$$\mathcal{L}_{mat} = -\sum_i \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - V(\vec{\phi})$$

Equations of Motion Follow from Hamiltonian

$$\partial_\tau\phi_i = \Pi_i/a^2$$

$$\partial_\tau\Pi_i = a^2\nabla^2\phi_i - a^4\partial_i V$$

$$\partial_\tau a = -\Pi_a/6.$$

$$\partial_\tau\Pi_a = a^3\left\langle \sum_i\left(\frac{\Pi_i^2}{a^6} + \frac{(\phi_i\nabla^2\phi_i)}{2a^2}\right) - 4V\right\rangle$$

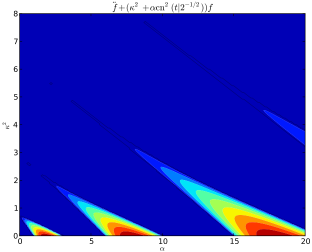
$$\rho = -T_0^0 = \sum_i \frac{\dot{\phi}_i^2}{2} + \frac{(\nabla\phi_i)^2}{2a^2} + V(\vec{\phi})$$

Initial Spectrum

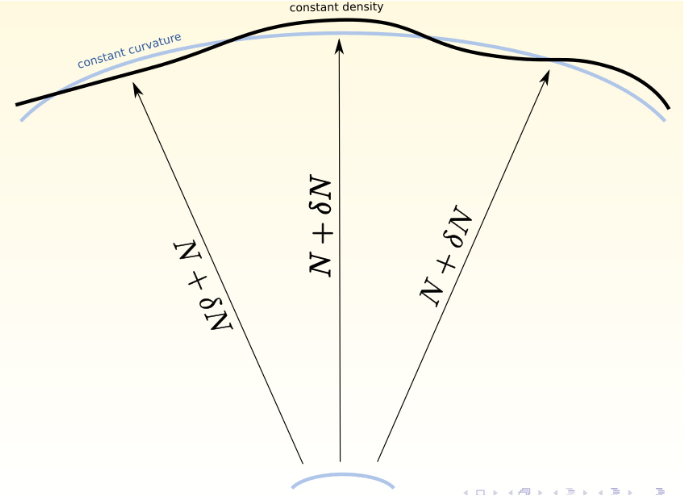
Gaussian Random Field: $P_\phi(k) = \frac{1}{2\omega_k}$, $P_{\dot{\phi}}(k) = \frac{\omega_k}{2}$

Evolution of $\ln(\rho/\bar{\rho})$

$$V(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\chi^2$$



Can this Leave Density Perturbations Behind?



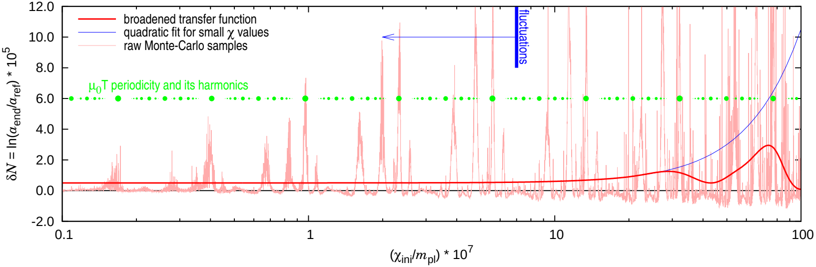
$$\zeta = \delta \ln(a)|_H = \delta \ln(a)|_\rho$$

Spatially modulate expansion history $\rightarrow$ curvature perturbations

Example I : $V(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\chi^2$

[Bond, Frolov, Huang, Kofman '09][Bond, JB, Frolov, Huang '13]

Superhorizon isocurvature fluctuations in χ are converted into adiabatic fluctuations.

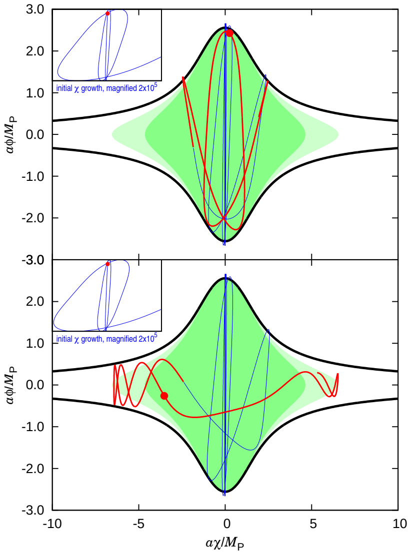


Obtained from many highly accurate symplectic lattice simulations.

$$\zeta = \zeta_G + F_{NL}(\chi)$$

χ : decay field for inflaton (or field modulating couplings)

Where do the spikes come from?



$$V_{eff} = \frac{\lambda}{4}\phi^4 + \frac{1}{2}(\phi^2 + \langle\delta\phi^2\rangle)\chi^2$$

In arm and out-of-arm trajectories have different expansion histories

Trajectories in arms pile-up $\rightarrow$ focusing and caustic formation

$$\chi(t+T) = e^{\mu_0 T}\chi(t)$$

Ballistic Dynamics After Inflation

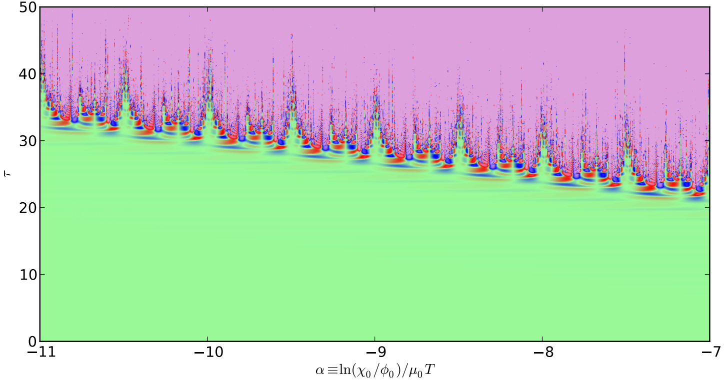
Post-Inflation δN

We can effectively extend the independent trajectory approach past $\epsilon = 1$ until site-to-site coupling (ie. gradients) are important.

Following bundles of phase-space trajectories $\rightarrow$ focussing/defocussing leads to caustics

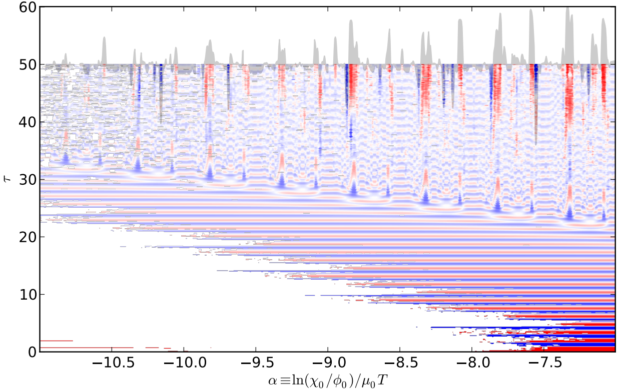
Caustics and Perturbations

For this model, $\langle a^2 H\rangle_{\rm time} = $ const. $\rightarrow$ encodes $\zeta = \delta \ln a|_H$.



$$\frac{\partial(a^2 H)}{\partial \chi_{init}}$$

Spikes from Caustics



- Phase space string of IC's (more generally a sheet)
- Treat *homogeneous* evolution along each trajectory
- Horizon scale evolution obtained by ensemble averaging trajectories

Development of Caustics

End of Ballistics : Entropy Production and the Shock-in-Time

(Nonequilibrium) Shannon Entropy

$$S_{shannon} = -\mathrm{Tr} P[f] \ln P[f]$$

$P[f]$: Probability density functional

$P[f] = ?$

Coarse-Graining Procedure

- Take all of our knowledge of the system
- Maximize S subject to constraints imposed by this knowledge

Application to Preheating

Constraint = Measured Power Spectrum

Power Spectrum $P(k) \rightarrow$ Covariance matrix C (homogeneous field)
$S_{shannon}$ maximized by multivariate-Gaussian with spectrum $P(k)$

$$\begin{aligned}\frac{S}{N} &= \frac{1}{2} + \frac{1}{2}\ln(2\pi) + \frac{1}{2N}\ln\det C \\ &= \frac{1}{2} + \frac{1}{2}\ln(2\pi) + \frac{1}{2N}\sum_{\mathbf{k}}\ln(P(k))\end{aligned}$$

N : number of modes (or volume in case of continuum field)

Dynamical Field

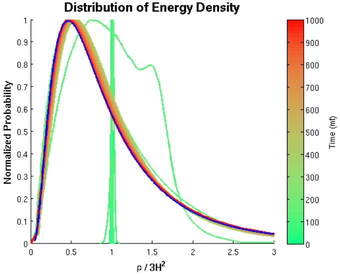
Choose : $f = \ln(\rho(\mathbf{x})/\bar{\rho}) \sim$ phonons (sound waves)

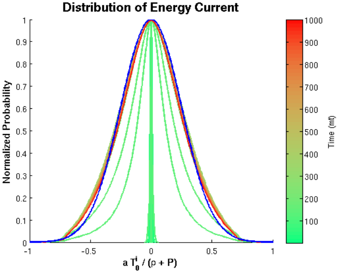
- Measurable quantity (regardless of model choice)
- ρ cannot be Gaussian

Why $\ln(\rho/3H^2)$: Gaussian in Position Space

Fluid variables have simple PDF's after initial transient

$$\rho/\bar{\rho}$$





$$v_x = \frac{aT^x_0}{\rho + P}$$

...and in Fourier Space...

Evolution of excess kurtosis for Fourier amplitudes.

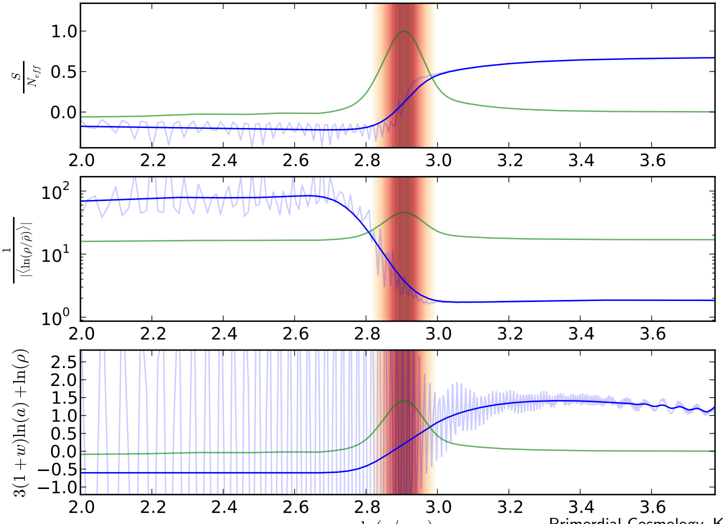
...the PDF's are also Gaussian

$$0 < \frac{k}{m} < 20$$

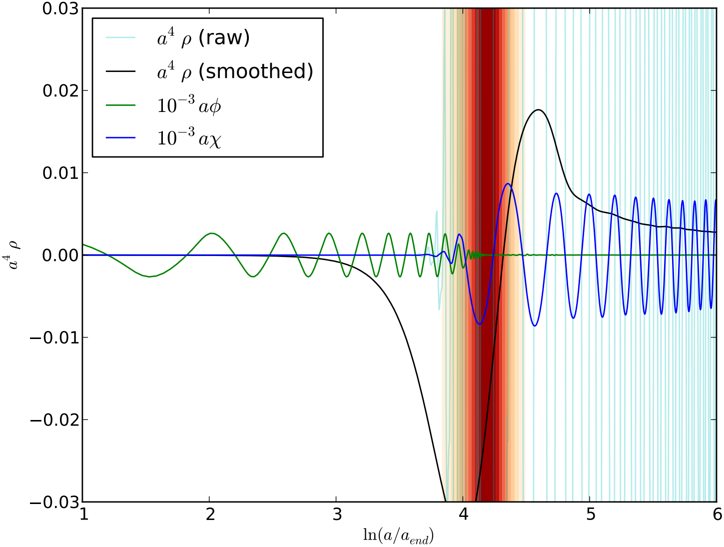
Power Spectrum Evolution ...

... leads to a **Shock-in-Time**

[Bond, JB (in preparation)]

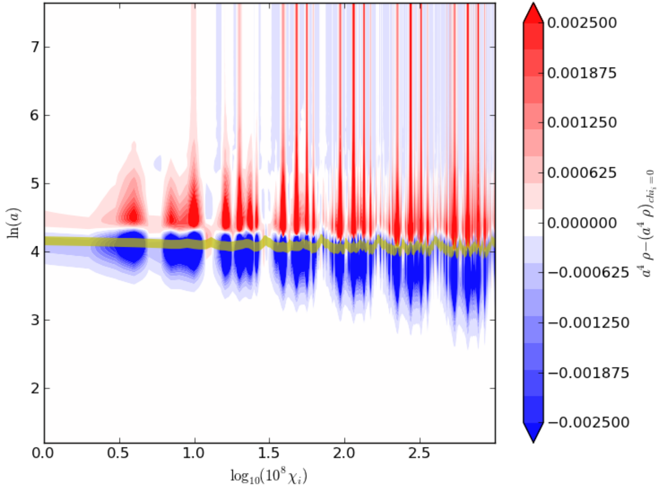


View on a Single Spike

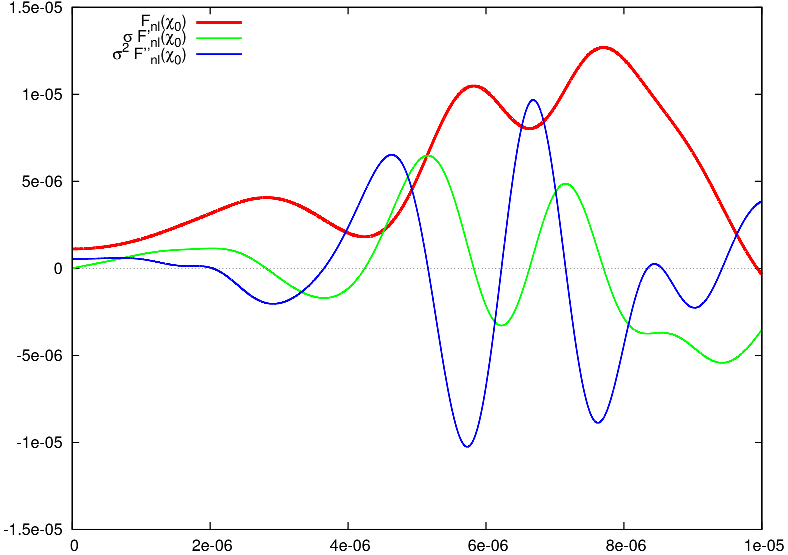


The Shock and Caustics

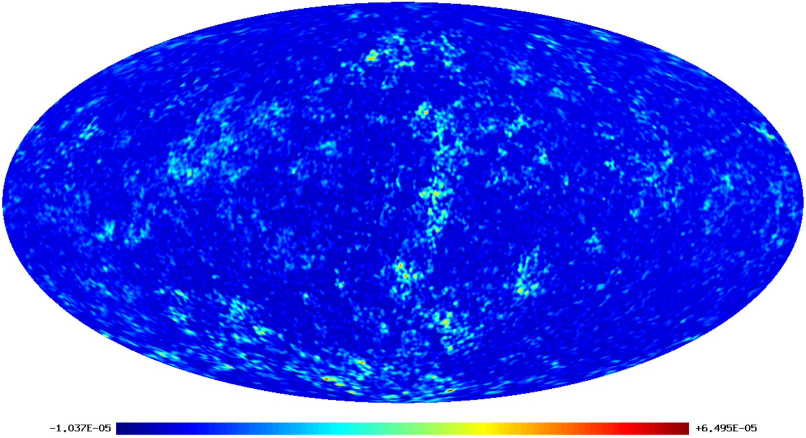
$\delta ln(a)$ structure set at the shock



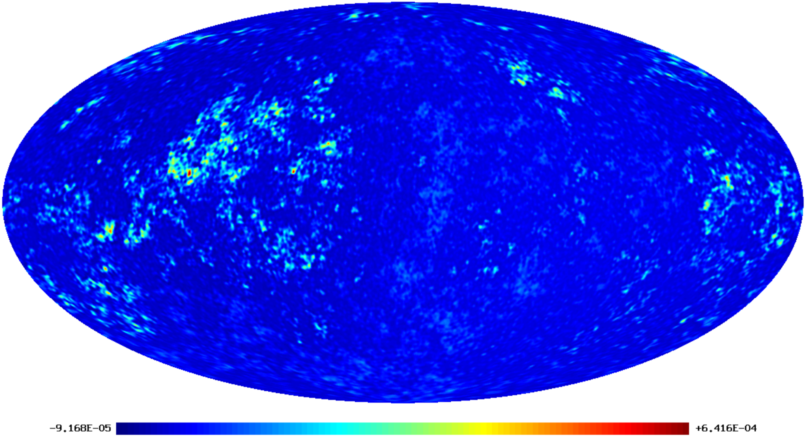
F_{NL} on the Sky



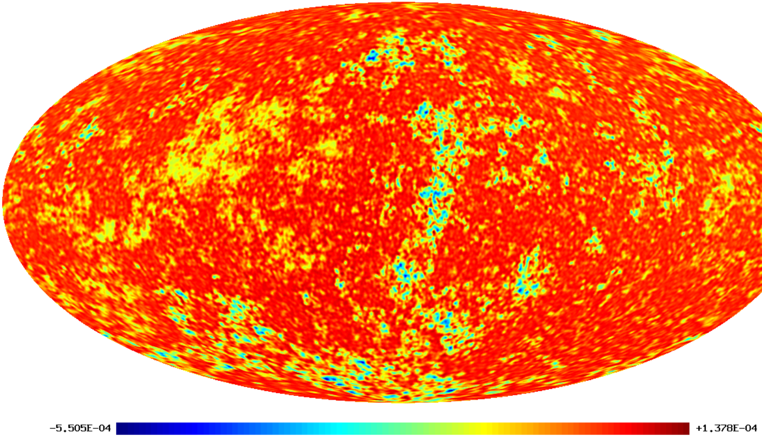
F_{NL} on the Sky



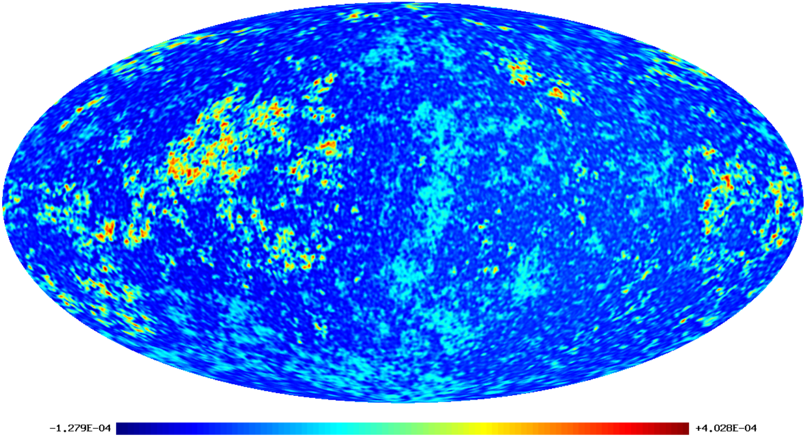
F_{NL} on the Sky



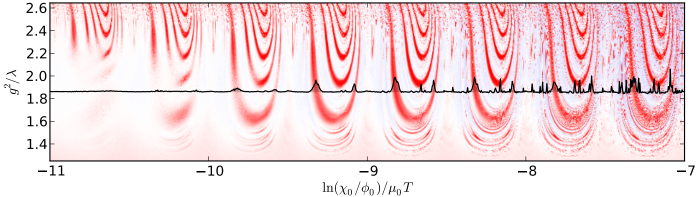
F_{NL} on the Sky



F_{NL} on the Sky

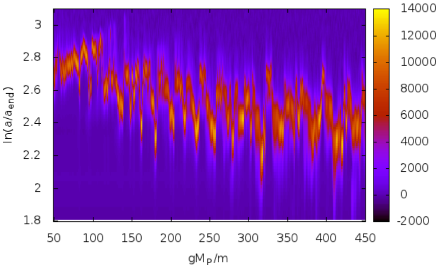


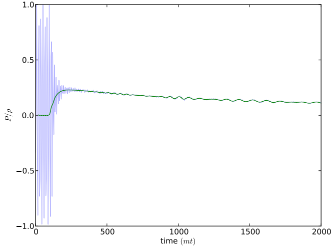
Modulation of Couplings

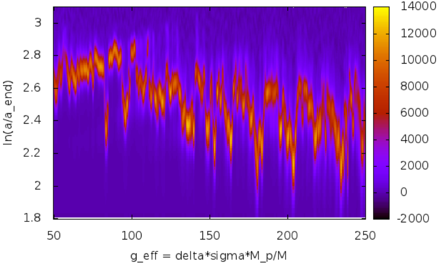


$$V(\phi,\chi) = \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$

Modulation via Shock Timing





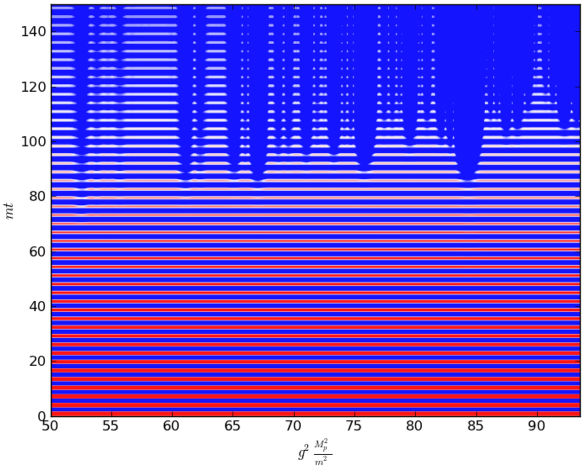


- $w_{preshock} \neq w_{postshock}$
- $\delta \ln(a_{\mathrm{shock}}) \to \zeta$

but...

- $w_{postshock} \neq \frac{1}{3}$
- $w_{postshock}$ depends on g^2

Modulation in the Ballistic Regime



Conclusions

- Local Nonlinear Conversion of Isocurvature to Adiabatic Fluctuations
  - Presence of random field in addition to inflaton
  - Caustic formation seems to be quite general for motion in the bottom of a potential
  - Can also be mediated by shock-in-time surface directly
- Can create localized features on the sky
- Information theoretic measures of field entropy
  - Shock-in-Time surface with rapid production of entropy