

Constraining the Ultra-Large Scale Structure of the Universe Using Numerical Relativity

Jonathan Braden

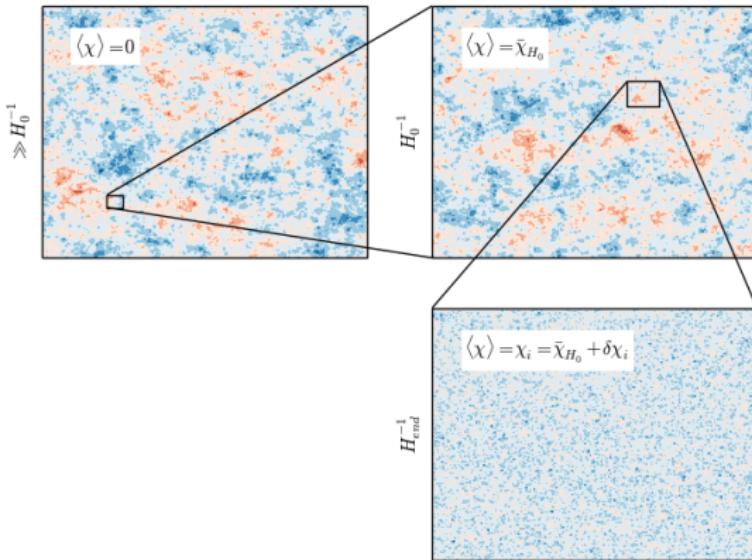
University College London

COSMO2016, University of Michigan, August 8, 2016

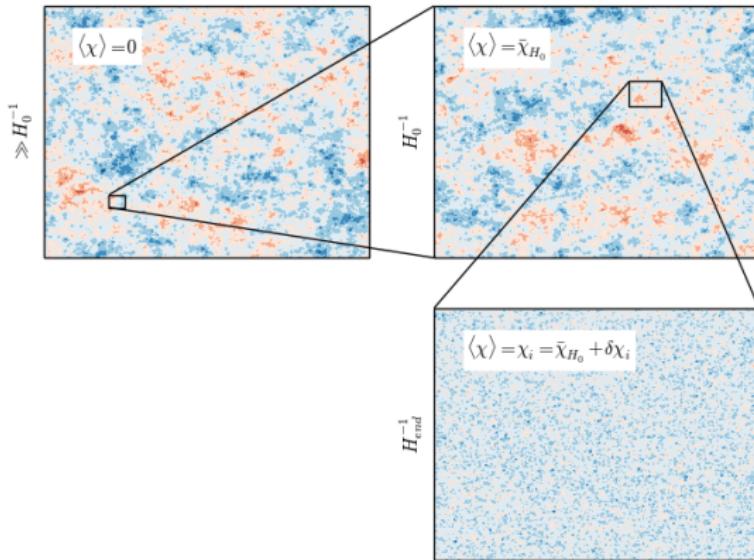
w/ Hiranya Peiris, Matthew Johnson, and Anthony Aguirre
based on arXiv:1604.04001 and *in progress*



Ultra-Large Scale Structure

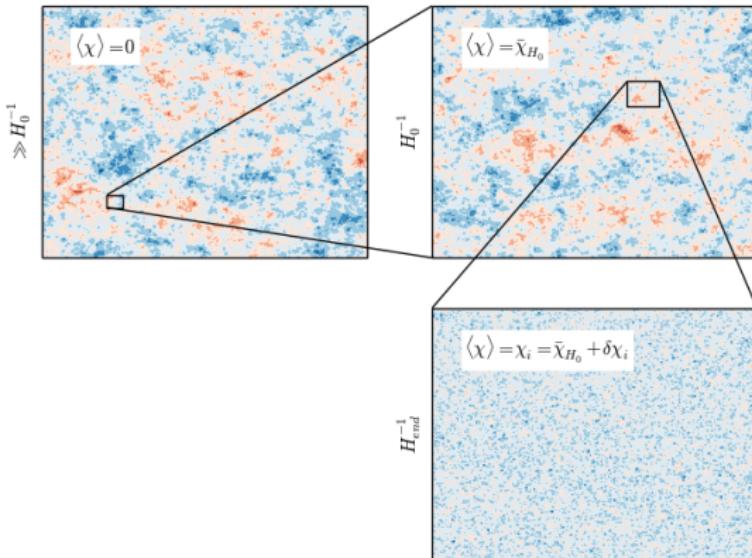


Ultra-Large Scale Structure



Local Remnants of Ultra-Large Scale Structure?

Ultra-Large Scale Structure



Local Remnants of Ultra-Large Scale Structure?

- ▶ Structure present at start of inflation
- ▶ Conversion of structure during or after inflation

Modelling Initial Conditions

Monte Carlo Sampling: Planar Symmetry

$$ds^2 = -d\tau^2 + a_{\parallel}^2(x, \tau)dx^2 + a_{\perp}^2(x, \tau)(dy^2 + dz^2)$$

Inflaton on $a_{\parallel}(\tau = 0) = 1 = a_{\perp}(\tau = 0)$

$$\phi(x) = \bar{\phi} + \delta\hat{\phi}$$

$$\bar{\phi} \text{ gives } \mathcal{N} \text{ e-folds} \quad 3H_I^2 \equiv V(\bar{\phi})$$

Field Fluctuations

$$\delta\hat{\phi}(x_i) = A_{\phi} \sum_{n=1} \hat{G} e^{ik_n x_i} \sqrt{P(k_n)} \quad \hat{G} = \sqrt{-2 \ln \hat{\beta}} e^{2\pi i \hat{\alpha}}$$

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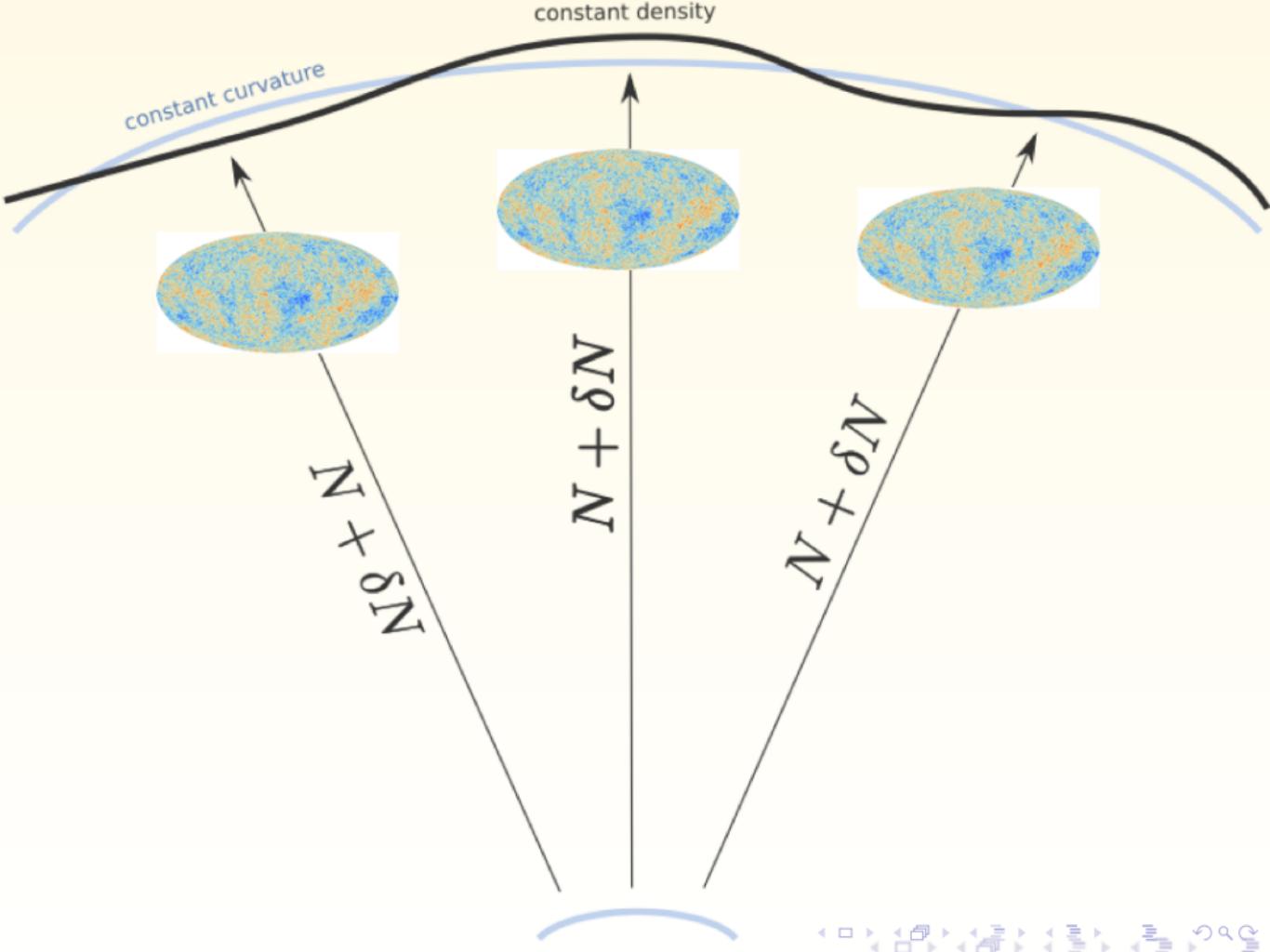
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$$P(k) = \Theta(k_{\max} - k) \quad H_{\text{I}}^{-1} k_{\max} = 2\pi\sqrt{3}$$



Dawn of Cosmological Numerical Relativity

- ▶ Bentivegna, Bruni
- ▶ Mertens, Giblin, Starkman
- ▶ Kleban, Linde, Senatore, West
- ▶ Peiris, Johnson, Feeney, Aguirre, Wainwright (symmetry reduced bubbles)
- ▶ Adamek, Daverio, Durrer, Kunz (weak field subhorizon)
- ▶ Clough, Figueras, Finkel, Kunesch, Lim, Tunyasuvanakool

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Connection of Initial Conditions with Observables

- ▶ Requires Monte Carlo sampling of Initial Conditions
- ▶ Highly accurate integrators to achieve machine precision

Solving Einstein's Equations for a Self-Gravitating Inflaton

Machine precision accuracy

- ▶ Gauss-Legendre time-integrator ($\mathcal{O}(dt^{10})$, symplectic)
- ▶ Fourier pseudospectral discretisation (exponential convergence)

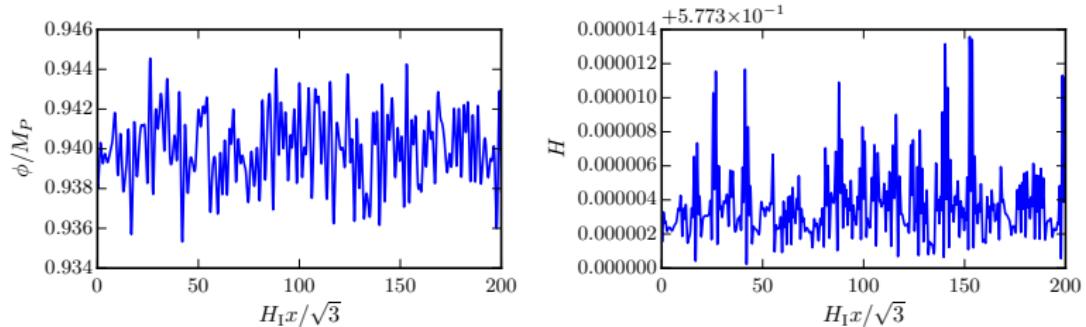
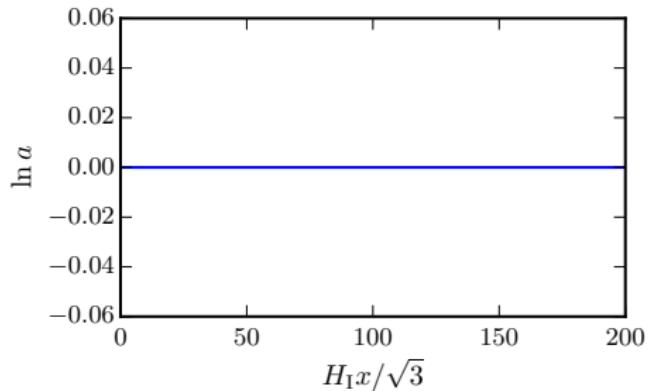
Fast to allow sampling

- ▶ Adaptive time-stepping
- ▶ Adaptive grid spacing

$\mathcal{O}(1s - 10s)$ to evolve through 60 e-folds of inflation

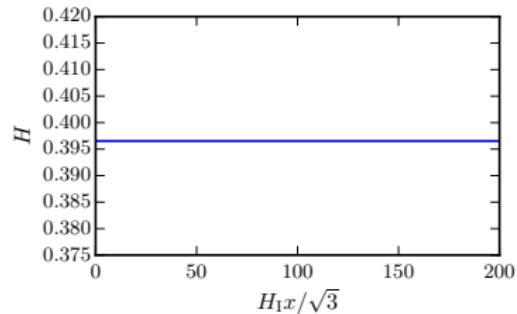
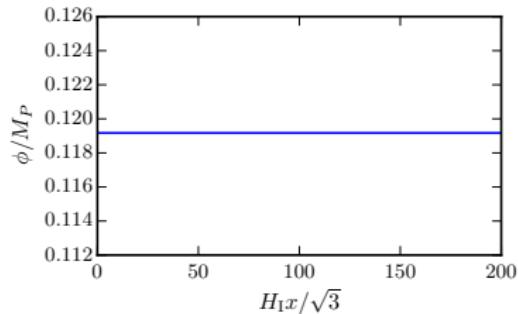
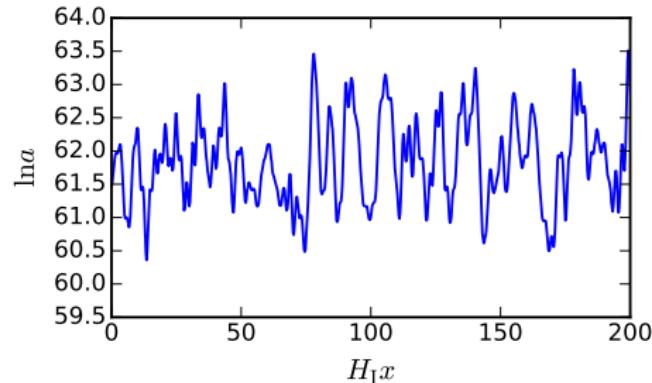
Machine precision convergence and constraint preservations

Required Evolution



Initial Conditions ($\tau = 0$)

Required Evolution



End of Inflation ($\epsilon_H = -d \ln H / d \ln a = 1$)

Observational Constraints

$$\Pr(A_\phi, H_I L_{\text{obs}} | C_2^{\text{obs}}, \dots) \propto \mathcal{L}(A_\phi, H_I L_{\text{obs}}) \Pr(A_\phi, H_I L_{\text{obs}} | \dots)$$
$$\mathcal{L} = \Pr(C_2^{\text{obs}} | A_\phi, H_I L_{\text{obs}}, \dots)$$

- ▶ A_ϕ : Fluctuation Amplitude $P(k) \propto A_\phi^2$
- ▶ $H_I L_{\text{obs}}$: Uncertain post-inflation expansion history
- ▶ ... : $V(\phi)$, spectrum shape, IC hypersurface, $C_2^{\text{high-}\ell}$, etc.

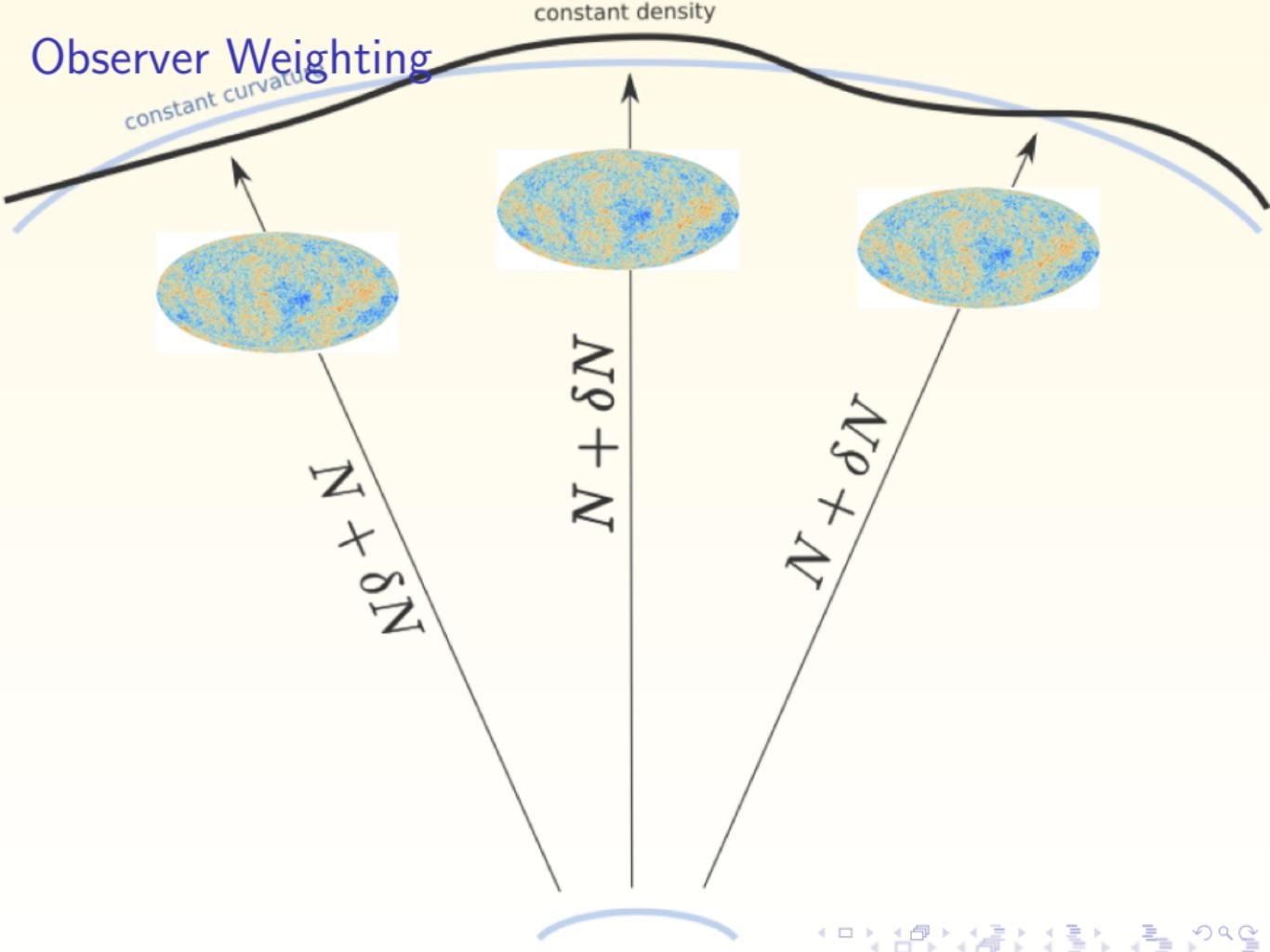
Planck measured C_ℓ

$$C_2^{\text{obs}} = 253.6 \mu K^2 \quad C_2^{\text{high-}\ell} = 1124.1 \mu K^2$$

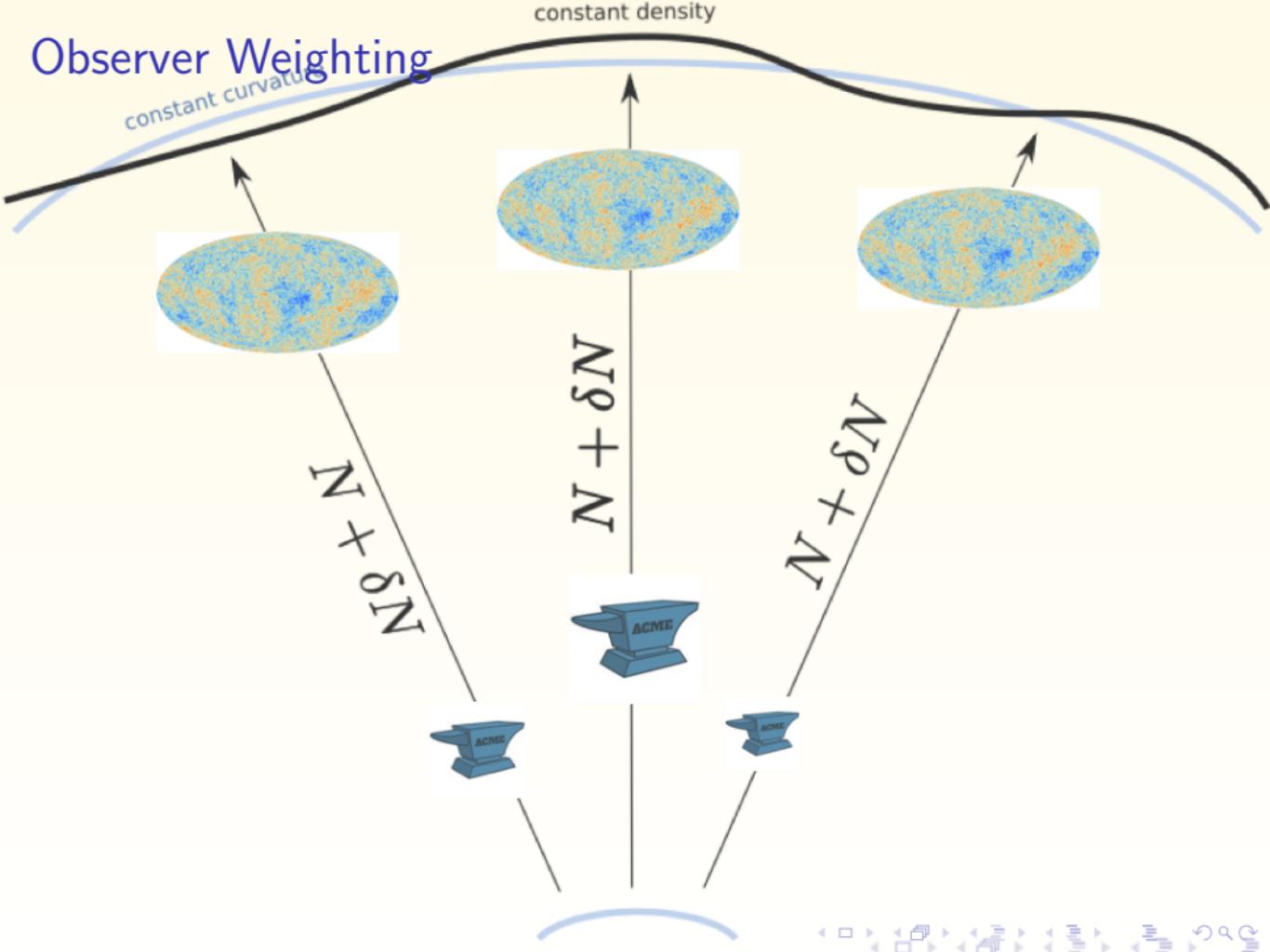
Numerical GR Input

$$\Pr(\hat{C}_2 | A_\phi, H_I L_{\text{obs}}, \dots)$$

Observer Weighting



Observer Weighting



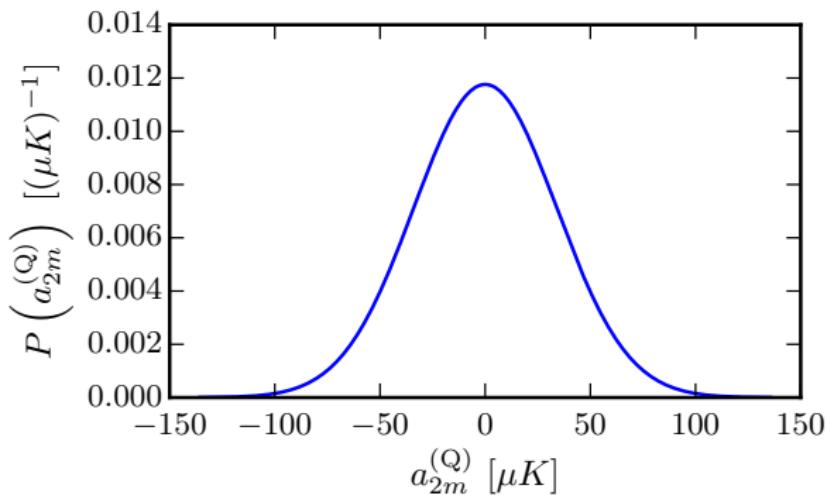
Evaluation of CMB Quadrupole

$$\hat{C}_2 = \frac{1}{5} \left[\left(a_{20}^{(\text{UL})} + a_{20}^{(Q)} \right)^2 + \sum_{m=-2, m \neq 0}^{m=2} \left(a_{2m}^{(Q)} \right)^2 \right]$$

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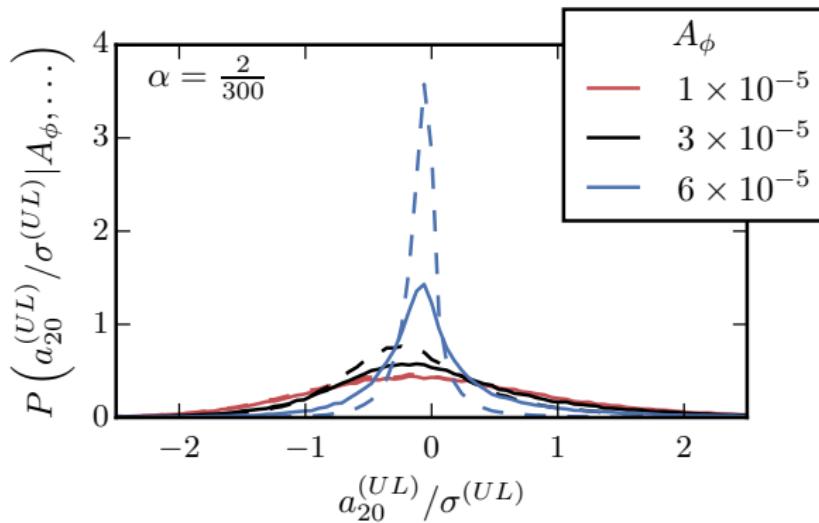
$a_{2m}^{(\text{Q})}$: Gaussian with $\langle (a_{2m}^{(\text{Q})})^2 \rangle = 1124.1 \mu K^2$



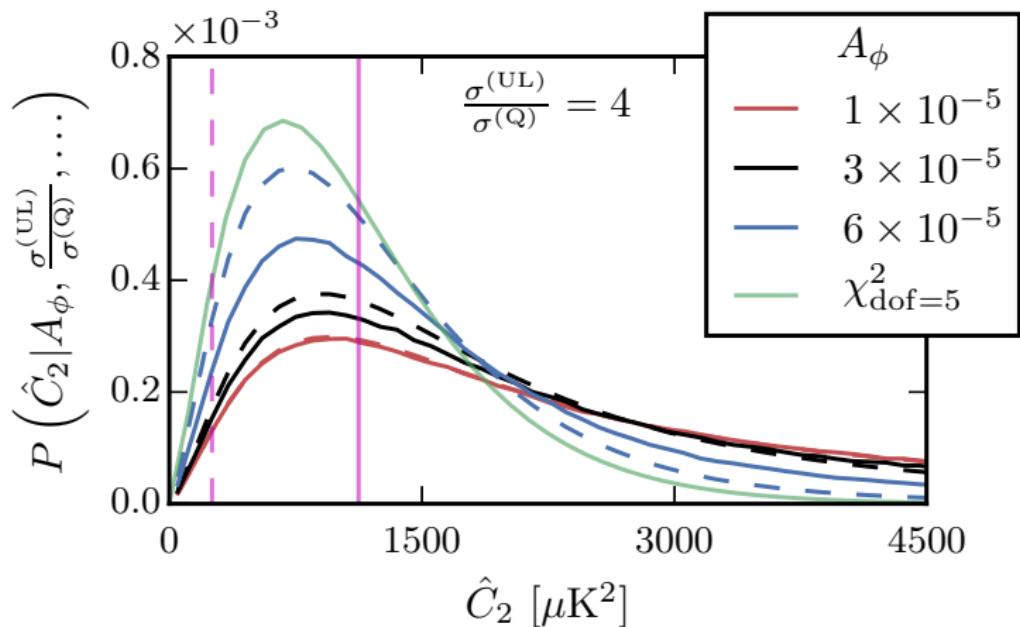
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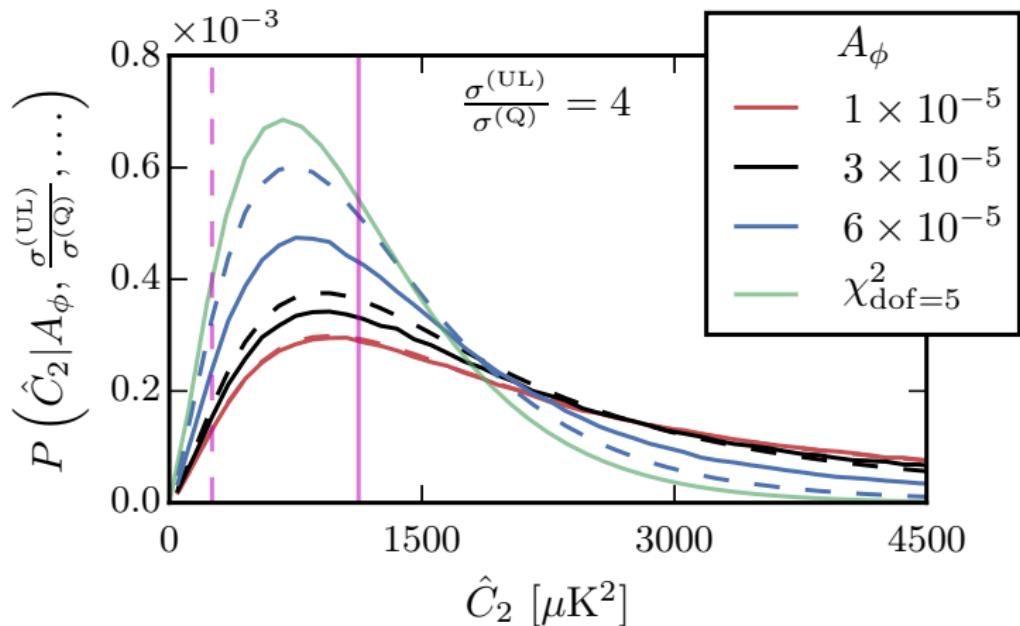
$a_{20}^{(\text{UL})}$: NR simulations



Dependence of \hat{C}_2 on Model Parameters

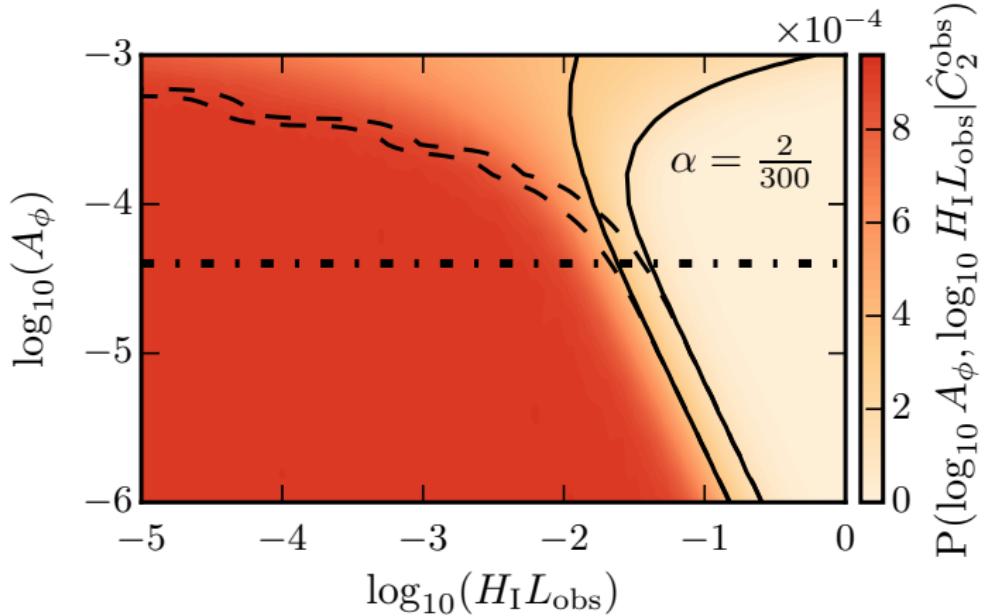


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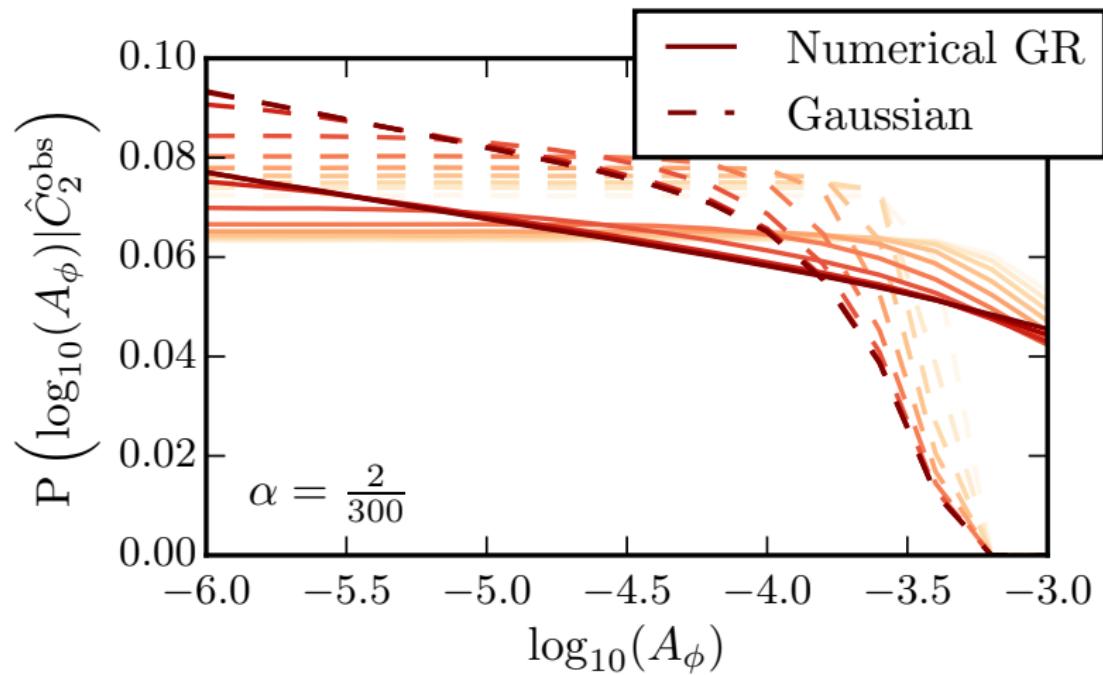
$$a_{20}^{(\text{UL})} \approx F (H_I L_{\text{obs}})^2 \partial_{x_p x_p} \zeta \approx F (H_I L_{\text{obs}})^2 \frac{1}{a_{\parallel}} \frac{\partial}{\partial x} \left(\frac{1}{a_{\parallel}} \partial_x \zeta \right)$$

Final Posterior



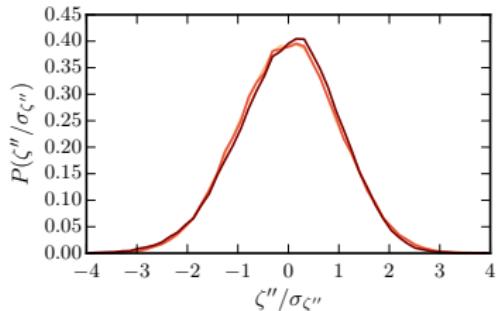
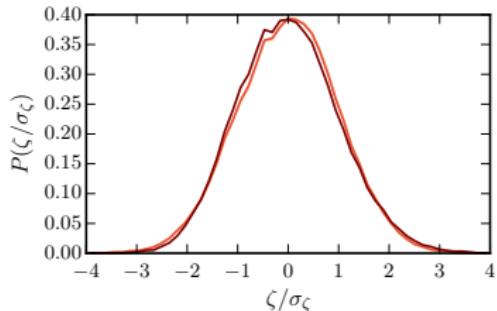
Significant deviations from Gaussian approximation

Marginalised Constraints on Model Parameters: Amplitude



Analytic Approximation

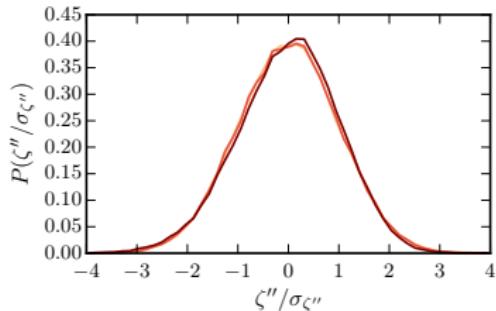
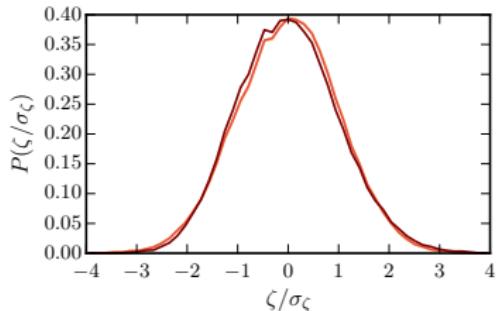
ζ and comoving derivatives nearly Gaussian



Treat as Gaussian random field

Analytic Approximation

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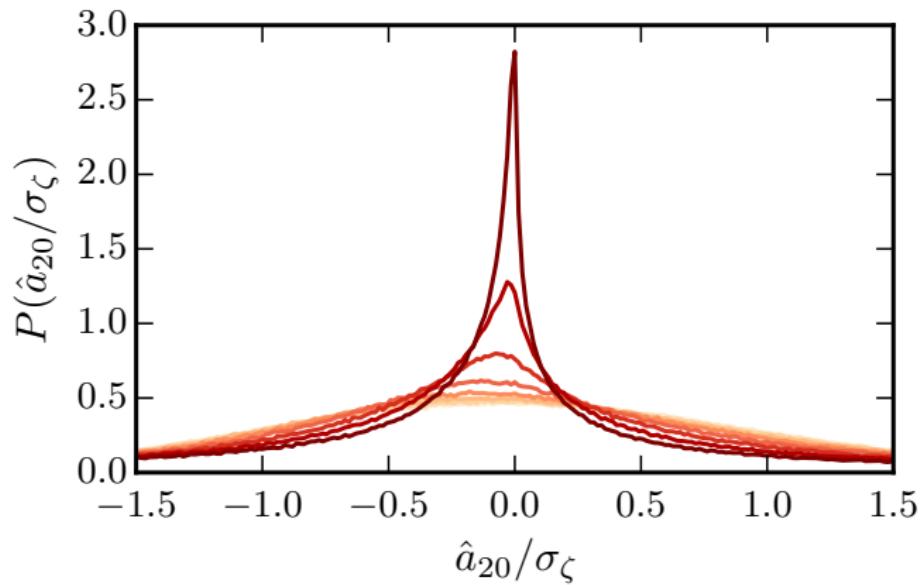


Treat as Gaussian random field

Large-Scale Approximation for a_{20}

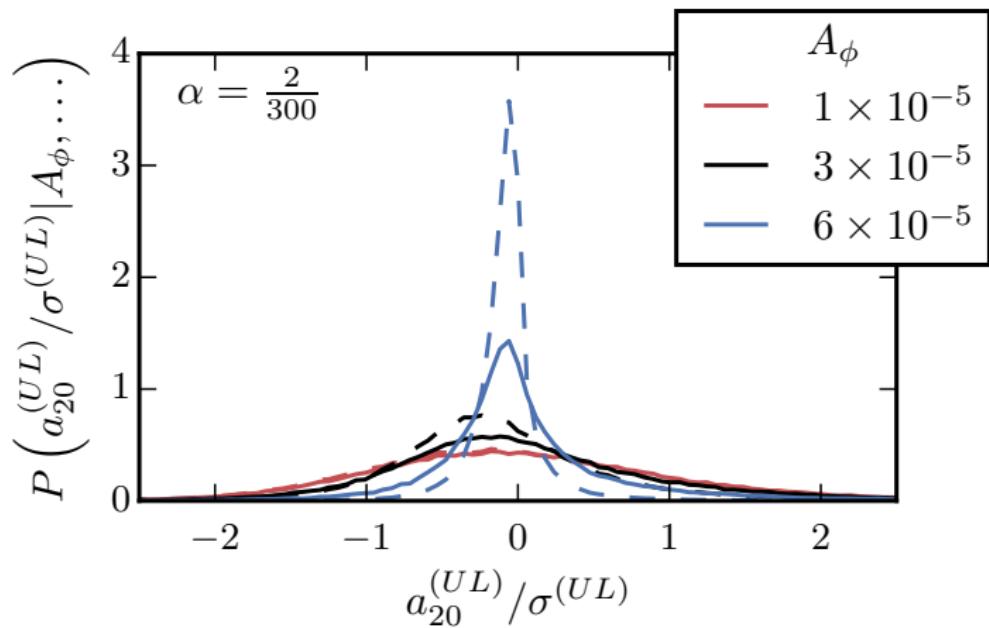
$$a_{20}(x_0) \approx \mathcal{A} e^{-2\zeta(x_0)} (\zeta''(x_0) - \zeta'(x_0)^2)$$

Anaytic a_{20} Distributions



Vary σ_ζ at fixed $\sigma_{\zeta^{(p)}}/\sigma_\zeta$

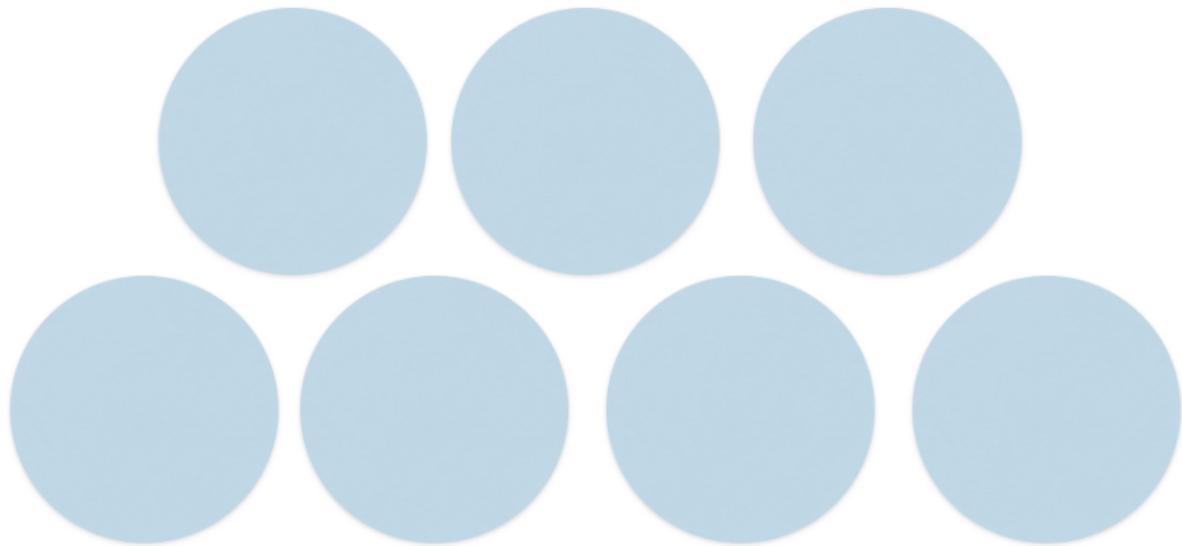
Recall The Numerical Result



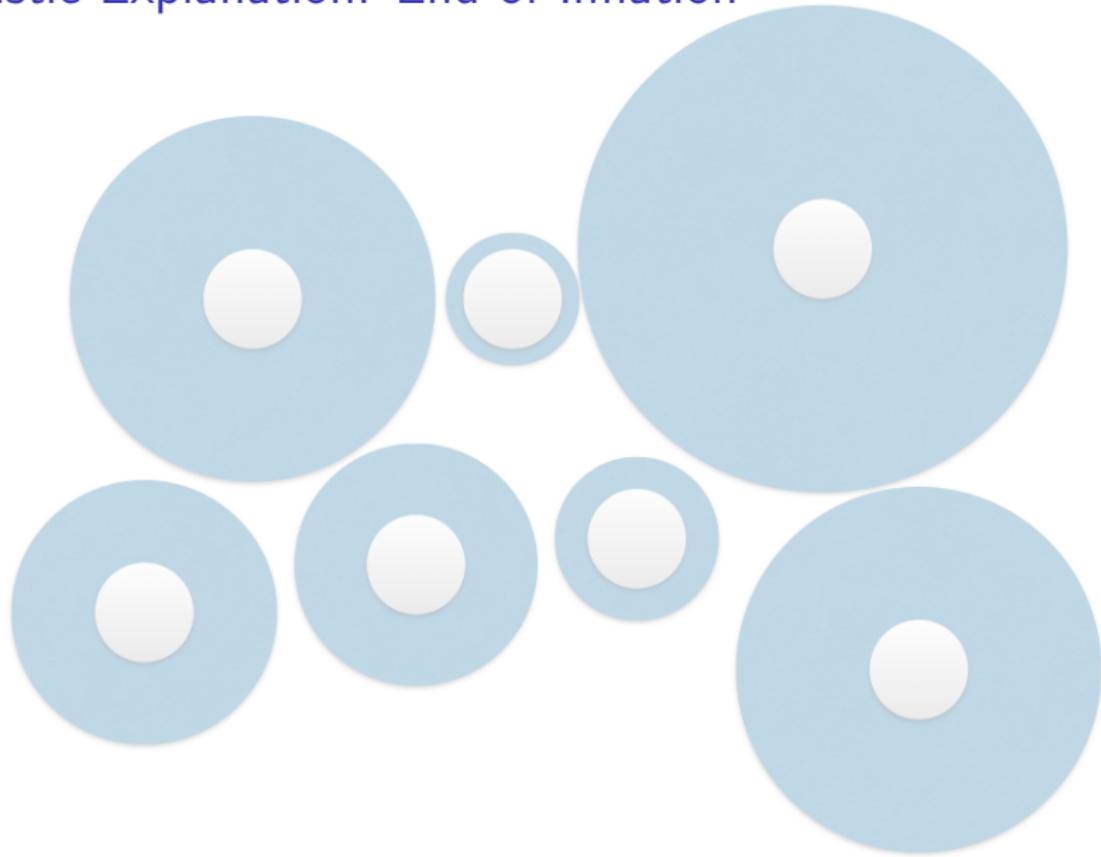
Conclusions

- ▶ Numerical relativity is a useful framework for making cosmological predictions
 - ▶ Sometimes it is a *necessary* tool (deviations from Gaussianity)
- ▶ Robust qualitative conclusions over a variety of inflationary models
- ▶ Constraining large amplitude superhorizon structure is hard even in the most optimistic case
- ▶ Inflation is effective at hiding large amplitude initial fluctuations
- ▶ Gaussianity of ζ in comoving coordinates suggests analytic approach in 3D

Heuristic Explanation: Initial Conditions



Heuristic Explanation: End-of-Inflation

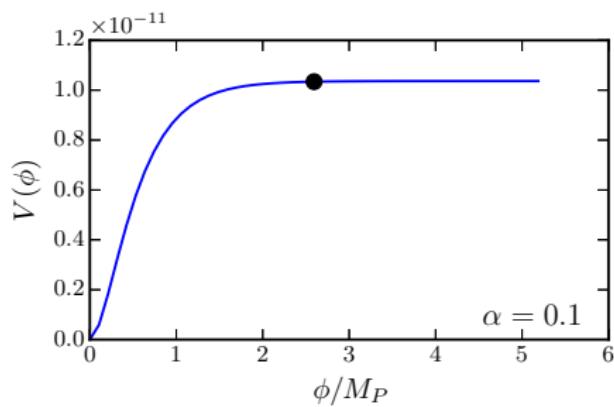


Model Choices

$$V(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_P}}\right)^2$$

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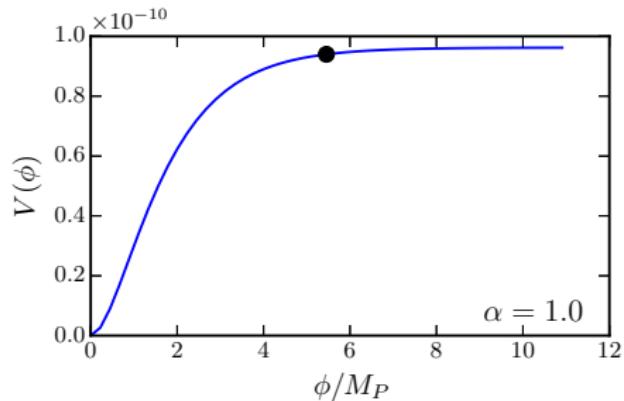
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- $\alpha \ll 1 \rightarrow$ small-field model

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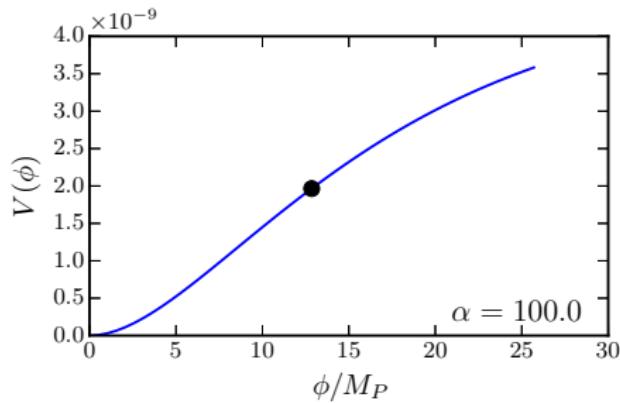
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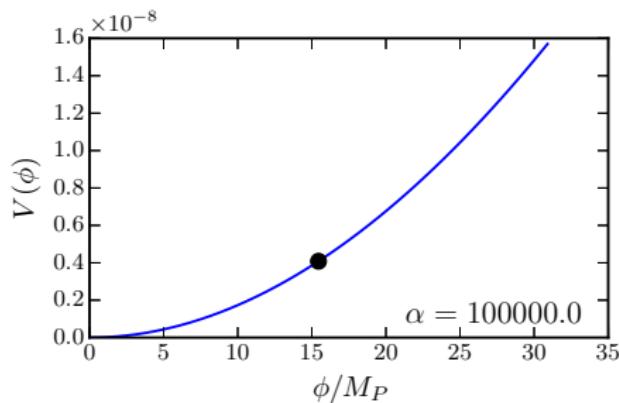
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- ▶ $\alpha \gg 1 \rightarrow m^2 \phi^2 / 2$

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