Cosmological Imprints of the Ultra-Large Scale Universe

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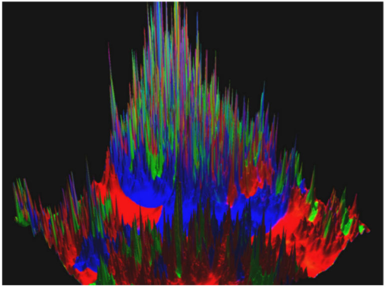
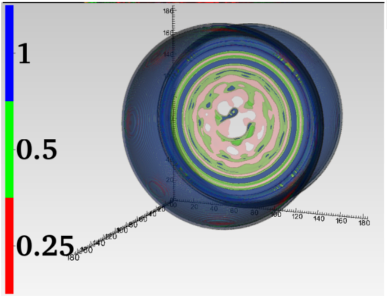
Imperial College London, November 18, 2016

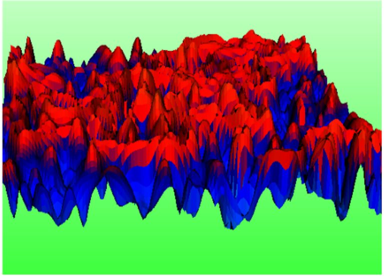
w/ Hiranya Peiris, Matthew Johnson and Anthony Aguirre

based on arXiv:1604.04001 and *in progress*

Inhomogeneous Nonlinear Ultra-Large Scale Cosmology







… A Mosaic of Interesting Dynamics

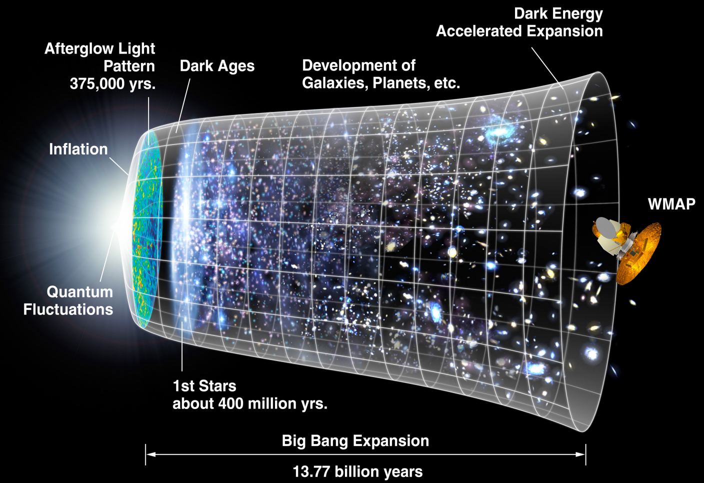
- **Strongly inhomogeneous and nonlinear cosmological initial conditions** (this talk) [JB, Peiris, Johnson, Aguirre]
- Isocurvature mode conversion into intermittent density perturbations [JB, Bond, Frolov, Huang]
  - Caustic formation in chaotic long wavelength dynamics
  - Generalised form of local nonGaussianity
- First order phase transitions [JB, Bond, Mersini-Houghton]
- Entropy production in highly inhomogeneous nonlinear field theories [JB, Bond]

Spatially intermittent dynamics, fundamental issues in QFT, novel and poorly constrained observables

Outline

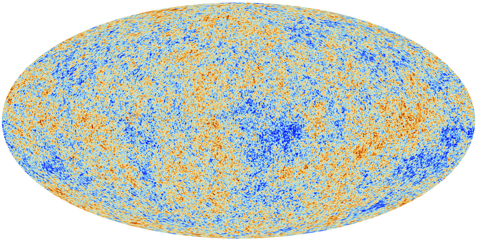
- Brief Review of Cosmology and Role of Inflation
- Ultra-Large Scale Structure and the Superhorizon Universe
- Application of Numerical Relativity to Constrain Ultra-Large Scale Structure
- Constraints on Inflationary Initial Conditions from the CMB
- (Time Permitting) Intermittent NonGaussianity from Preheating (End-of-Inflation)

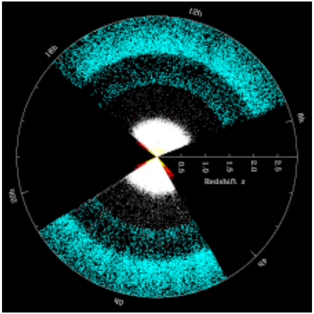
History of the Universe



NASA/WMAP Science Team

The CMB and LSS



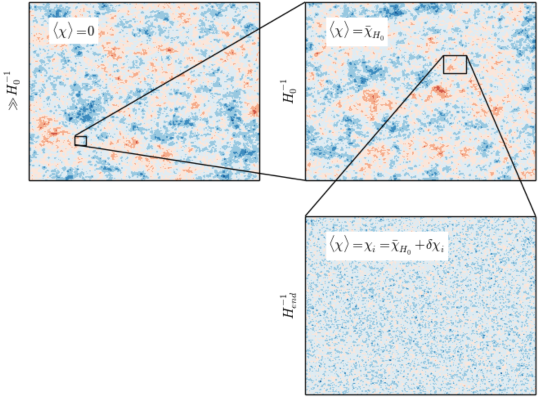


Inflation: Only a few parameters

$$P_\zeta(k) = A k^{n_s - 1} \qquad r = 16\epsilon$$

- Nature of Inflaton?
- Initial Conditions?

Ultra-Large Scale Structure



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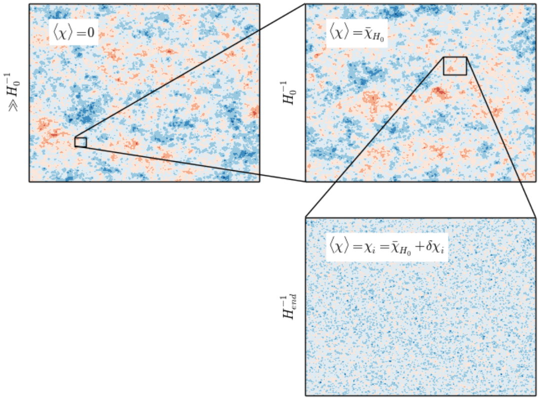
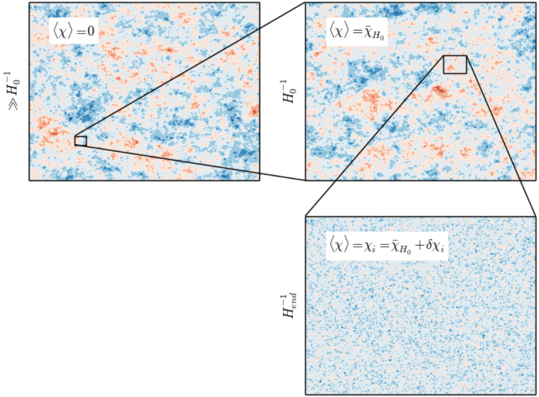


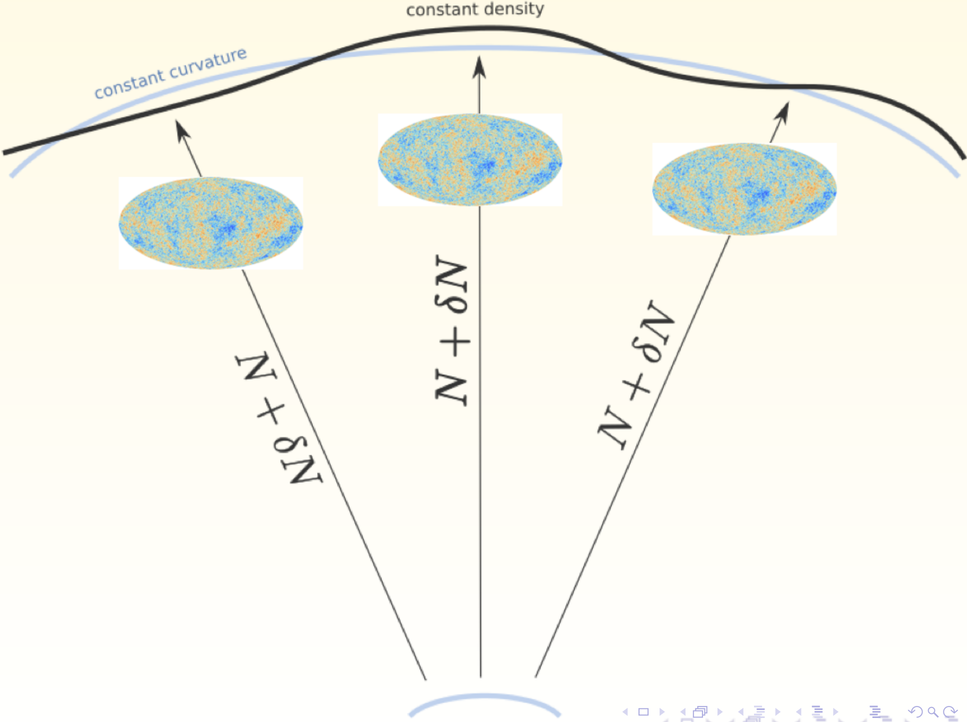
Local Remnants of Ultra-Large Scale Structure?

Ultra-Large Scale Structure



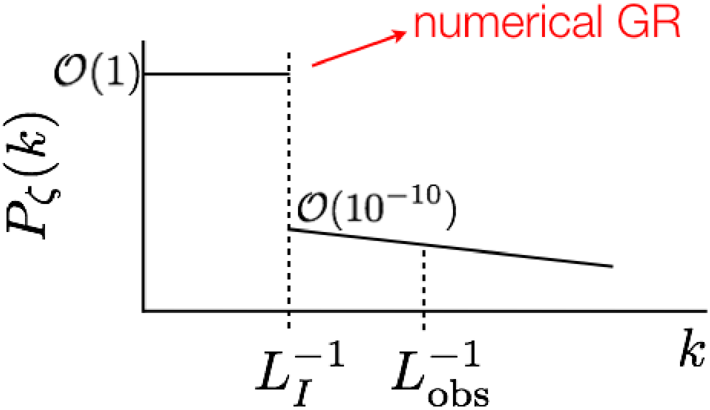
Local Remnants of Ultra-Large Scale Structure?

- **Structure present at start of inflation**
- Conversion of structure during or after inflation



constant density
constant curvature
N + δN
N + δN
N + δN

What About Ultra-Large Scales



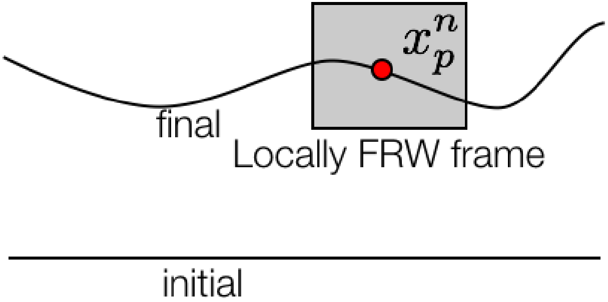
Evolve long wavelength modes dynamically

CMB scales see locally homogeneous background

Ultra Large Scale Structure

$$\zeta(x_p) \simeq \underline{\zeta(x_p^n)} + \underline{\partial_{x_p}\zeta(x_p^n)\left(x_p - x_p^n\right)} + \underline{\partial^2_{x_p^n}\zeta(x_p^n)\left(x_p - x_p^n\right)^2/2} + \dots$$

log(local scale factor) — gauge mode (not observable) — Maps to CMB quadrupole



Dawn of Numerical Relativity in Cosmology

- Bentivegna, Bruni
- Mertens, Giblin, Starkman
- Kleban, Linde, Senatore, West
- Peiris, Johnson, Feeney, Aguirre, Wainwright (symmetry reduced bubbles)
- Adamek, Daverio, Durrer, Kunz (weak field subhorizon)
- Clough, Lim, DiNunno, Fischler, Flauger, Paban

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Connection of Initial Conditions with Observables

- Requires Monte Carlo sampling of Initial Conditions
- Highly accurate integrators to achieve machine precision

Observational Constraints

$$\Pr(A_\phi, H_I L_{\rm obs} | C_2^{\rm obs}, \dots) \propto \mathcal{L}(A_\phi, H_I L_{\rm obs}) \Pr(A_\phi, H_I L_{\rm obs} | \dots)$$

- A_ϕ : Fluctuation Amplitude $P(k) \propto A_\phi^2$
- $H_I L_{\rm obs}$: Uncertain post-inflation expansion history
- $\dots$: $V(\phi)$, spectrum shape, IC hypersurface, $C_2^{\text{high}-\ell}$, etc.

Planck measured C_ℓ : low ℓ observed and high ℓ "theory"

$$C_2^{\rm obs} = 253.6 \mu K^2 \qquad C_2^{\text{high}-\ell} = 1124.1 \mu K^2$$

Numerical GR Input

$$\mathcal{L}(A_\phi, H_I L_{\rm obs}) = \Pr\left(\hat{C}_2 | A_\phi, H_I L_{\rm obs}, \dots\right)$$

Not equivalent to $\langle C_2 \rangle$ from integrating spectrum

Dimensional Reduction: Planar Symmetry

Needed to Monte Carlo over initial field realisations

Synchronous Gauge

$$ds^2 = -d\tau^2 + a_{\parallel}^2(x,\tau)dx^2 + a_{\perp}^2(x,\tau)(dy^2 + dz^2)$$

Residual gauge freedom : $a_{\parallel}(x,\tau = 0)$, $a_{\perp}(x,\tau = 0)$

Isotropised Expansion

$$a \equiv \det(\gamma_{ij})^{1/6} = \left(a_{\parallel}a_{\perp}^2\right)^{1/3}$$
$$H \equiv -\frac{1}{3}\gamma^{ij}K_{ij} = -\frac{1}{3}\left(K^x{}_x + 2K^y{}_y\right)$$

Initial Conditions: Field

Work in Spatially Flat Gauge $a_{\|}(\tau = 0) = 1 = a_{\perp}(\tau = 0)$

$$\phi(x) = \bar{\phi} + \delta\hat{\phi}$$

$\bar{\phi}$ gives desired $\mathcal{N}$ e-folds of inflation in homogeneous limit

$$3H_{\mathrm{I}}^2 \equiv V(\bar{\phi})$$

Field Fluctuations

$$\delta\hat{\phi}(x_i) = A_\phi \sum_{n=1} \hat{G} e^{ik_n x_i} \sqrt{P(k_n)} \qquad \hat{G} = \sqrt{-2\ln\hat{\beta}}\, e^{2\pi i\hat{\alpha}}$$

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$$P(k) = \Theta(k_{\rm max} - k) \qquad H_{\rm I}^{-1} k_{\rm max} = 2\pi\sqrt{3}$$

Initial Conditions: Metric

Work in Spatially Flat Gauge $a_{\|}(\tau = 0) = 1 = a_{\perp}(\tau = 0)$

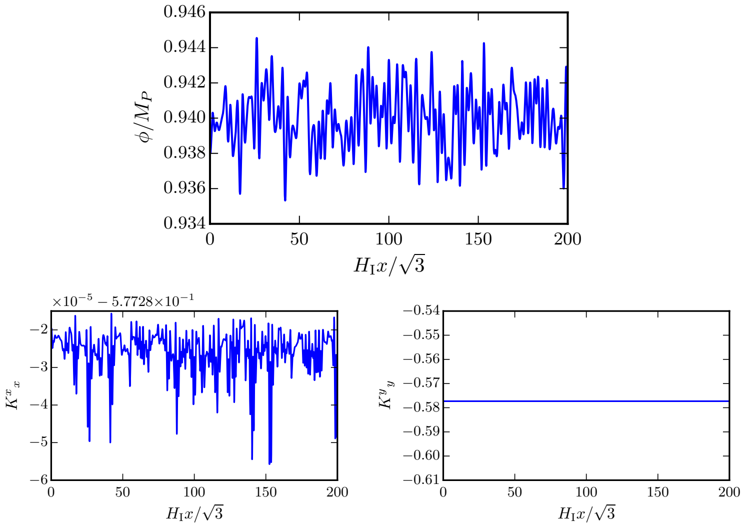
Momentum Constraint Sews Neighbouring Grid Sites Together

$0 = \mathcal{P} = {K^y}_y{}' - \frac{a'_\perp}{a_\perp}\left({K^x}_x - {K^y}_y\right) - \frac{\phi' \Pi_\phi}{2a_\| M_P^2}$

Hamiltonian Constraint Enforces Energy Conservation

$$0 = \mathcal{H} = \frac{2a_\perp a'_\| a'_\perp - a_\| a'^2_\perp - 2a_\| a_\perp a''_\perp}{a^3_\| a^2_\perp} + 2{K^x}_x {K^y}_y + {K^y}_y{}^2 - M_P^{-2}\left(\frac{\phi'^2 + \Pi^2_\phi}{2a^2_\|} + V\right) = 0$$

Distribution of Initial Conditions



What's the response in the curvature perturbation?

Time Evolution

$$\dot{a_\parallel} = -a_\parallel K^x{}_x, \qquad \dot{a_\perp} = -a_\perp K^y{}_y,$$

$$\dot{K^x{}_x} = \frac{a_\perp'^2}{a_\perp^2 a_\parallel^2} + K^x{}_x{}^2 - K^y{}_y{}^2 + \frac{\left(\Pi_\phi^2 - \phi'^2\right)}{2a_\parallel^2 M_P^2},$$

$$\dot{K^y{}_y} = -\frac{a_\perp'^2}{2a_\perp^2 a_\parallel^2} + \frac{3}{2}K^y{}_y{}^2 - \frac{V(\phi)}{2M_P^2} + \frac{\left(\Pi_\phi^2 + \phi'^2\right)}{4a_\parallel^2 M_P^2},$$

$$\dot{\Pi}_\phi = 2K^y{}_y \Pi_\phi + \frac{1}{a_\parallel}\phi'' + \left(\frac{2a_\perp'}{a_\parallel a_\perp} - \frac{a_\parallel'}{a_\perp^2}\right)\phi' - a_\perp \partial_\phi V(\phi),$$

$$\dot{\phi} = \frac{\Pi_\phi}{a_\parallel}$$

Model Choices

$$V(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_P}}\right)^2$$

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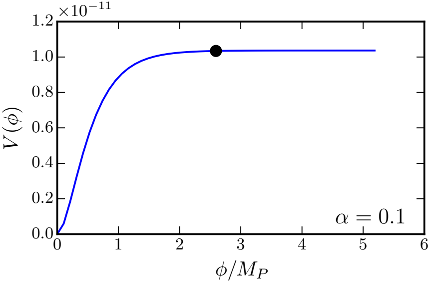

- $\alpha \ll 1 \rightarrow$ small-field model

Model Choices

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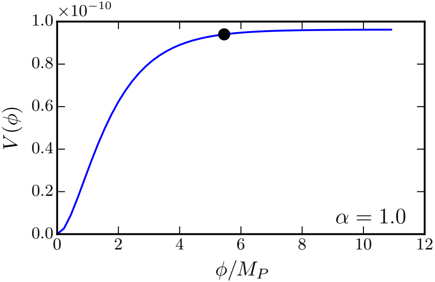

- $\alpha \ll 1 \rightarrow$ small-field model
- $\alpha = 1 \rightarrow$ Starobinski

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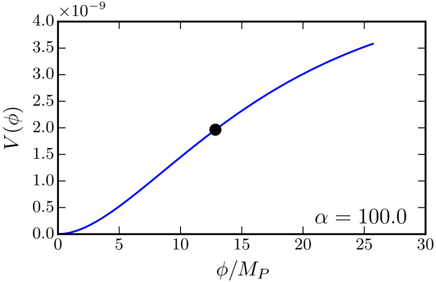

- $\alpha \ll 1 \rightarrow$ small-field model
- $\alpha = 1 \rightarrow$ Starobinski
- $\alpha \gg 1 \rightarrow m^2\phi^2/2$

Model Choices

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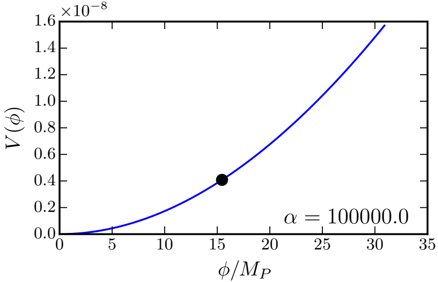

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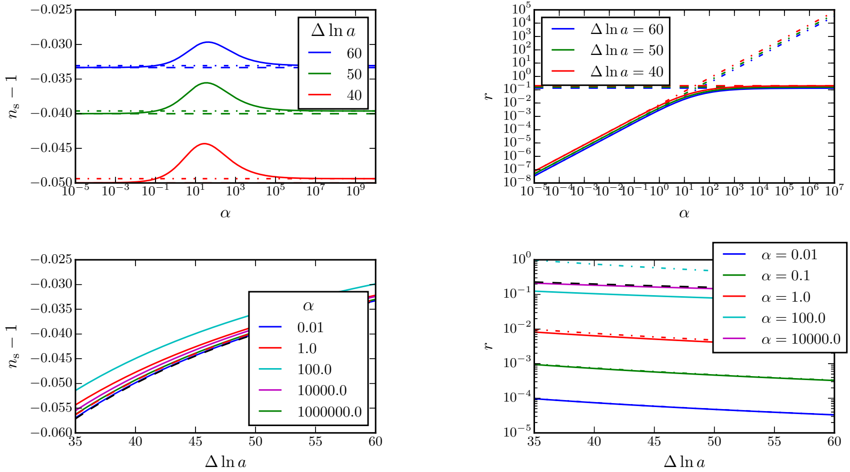
CMB Parameter Predictions



Numerical Approach

Machine precision accuracy

- Gauss-Legendre time-integrator ($\mathcal{O}(dt^{10})$, symplectic)
- Fourier pseudospectral discretisation (exponential convergence)

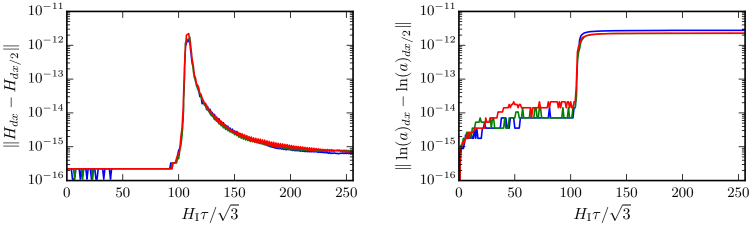
Fast to allow sampling

- Adaptive time-stepping
- Adaptive grid spacing

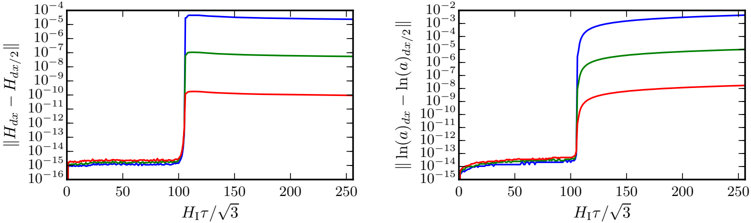
$\mathcal{O}(1s - 10s)$ **to evolve through 60 e-folds of inflation**

Convergence Testing : Dynamical Variables

Vary grid spacing dx at fixed dx/dt

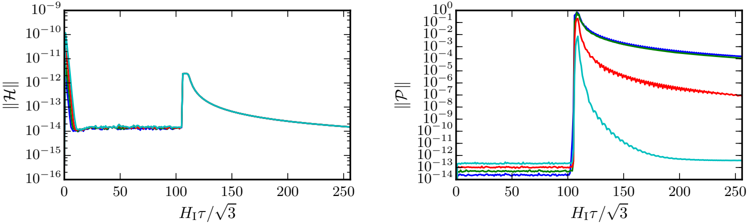


Vary time-step dt at fixed dx

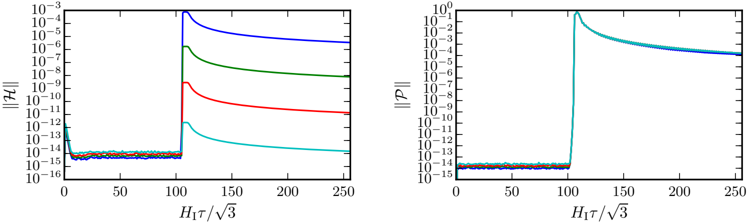


Convergence Testing : Constraint Preservation

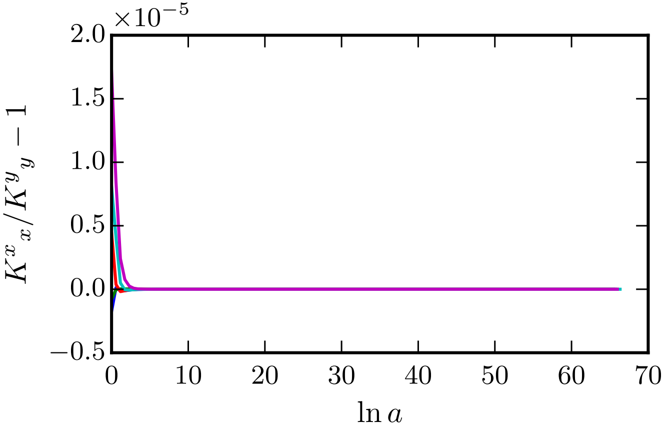
Vary grid spacing dx at fixed dx/dt



Vary time-step dt at fixed dx

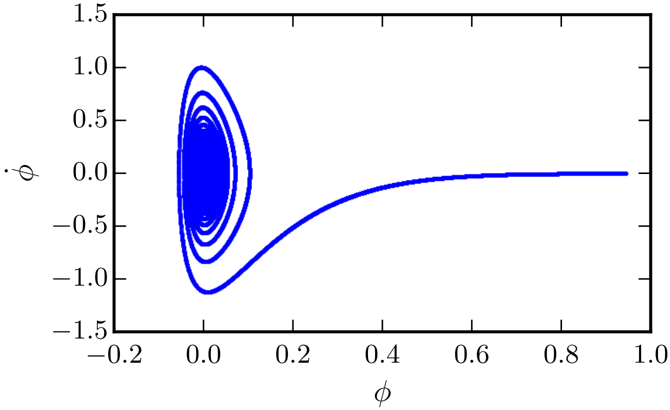


Self-Gravitating Dynamics: Isotropisation of Expansion

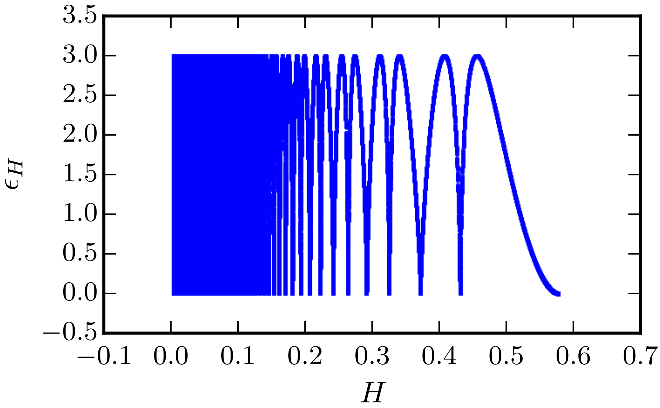


Expansion quickly isotropises itself

Self-Gravitating Dynamics: Attractor Behaviour



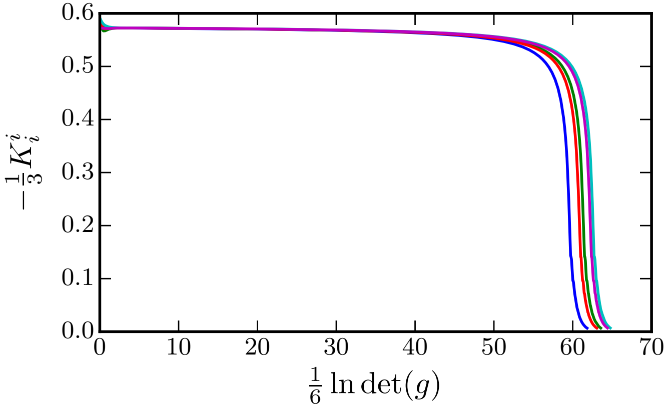
Self-Gravitating Dynamics: Geometry Attractor



Individual Lattice Sites Evolve Along Attractor

Fixed H, ϵ_H are equivalent

Self-Gravitating Dynamics: Spatially Dependent Expansion History



Observations: Relation to Local Multipoles

$$\zeta(H) \equiv \delta \ln a|^{H}_{a_{\parallel}=1=a_{\perp}}$$

Large-scale perturbations $\rightarrow$
evaluate ζ on last-scattering surface around a point x_0

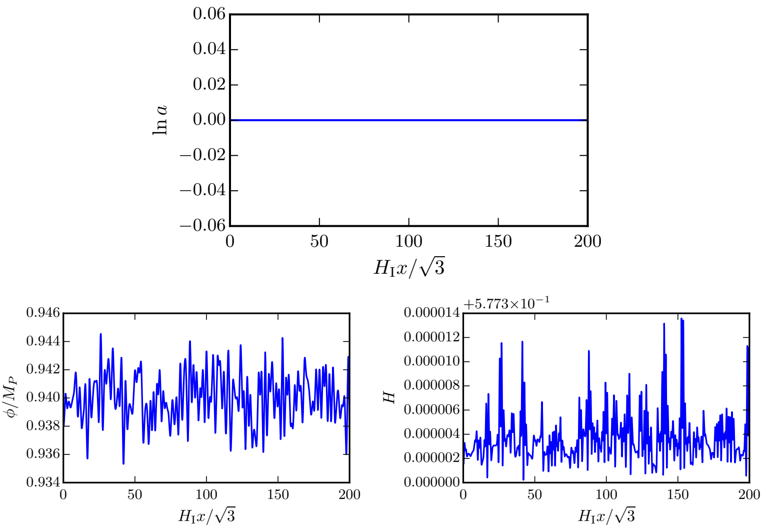
Expand for Large-Scale Fluctuations

$$\zeta(x_0+r_{\rm ls}) \approx \zeta(x_0)+(H_I r_{ls})\frac{\partial\zeta}{\partial(H_I x_p)}(x_0)+\frac{(H_I r_{ls})^2}{2}\frac{\partial^2\zeta}{\partial(H_I x_p)^2}(x_0)+\dots$$

Matches onto Multipoles

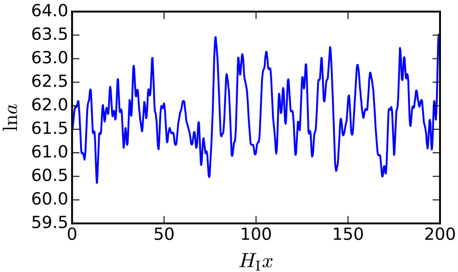
$$a_{20}^{(UL)} \approx F(L_{\rm obs}H_{\rm I})^2\partial_{x_p}^2\zeta \simeq F(L_{\rm obs}H_{\rm I})^2\frac{1}{a_{\parallel}}\partial_x\left[\frac{\partial_x\zeta}{a_{\parallel}}\right]$$

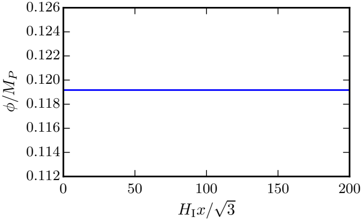
Required Evolution

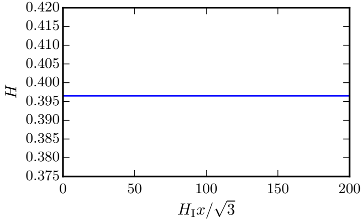


Initial Conditions ($\tau = 0$)

Required Evolution

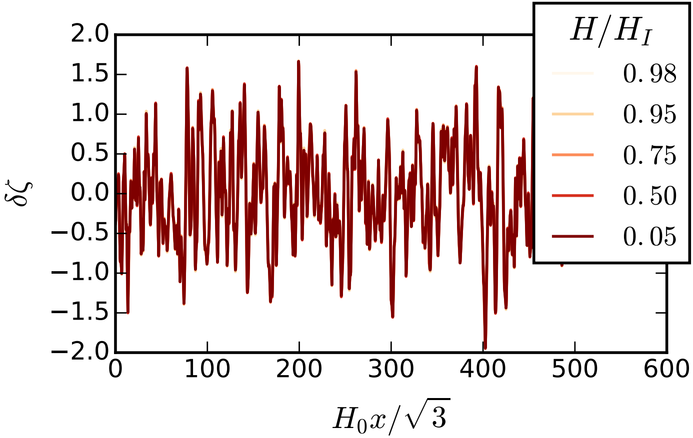






End of Inflation ($\epsilon_H = -d\ln H/d\ln a = 1$)

Freeze-in of Superhorizon Perturbations



Initial transient modifies result from separate universe approximation

Observational Constraints

$$\Pr(A_\phi, H_I L_{\rm obs} | C_2^{\rm obs}, \ldots) \propto \mathcal{L}(A_\phi, H_I L_{\rm obs}) \Pr(A_\phi, H_I L_{\rm obs} | \ldots)$$

$$\mathcal{L} = \Pr(C_2^{\rm obs} | A_\phi, H_I L_{\rm obs}, \ldots)$$

- A_ϕ : Fluctuation Amplitude $P(k) \propto A_\phi^2$
- $H_I L_{\rm obs}$: Uncertain post-inflation expansion history
- $\ldots$: $V(\phi)$, spectrum shape, IC hypersurface, $C_2^{{\rm high}-\ell}$, etc.

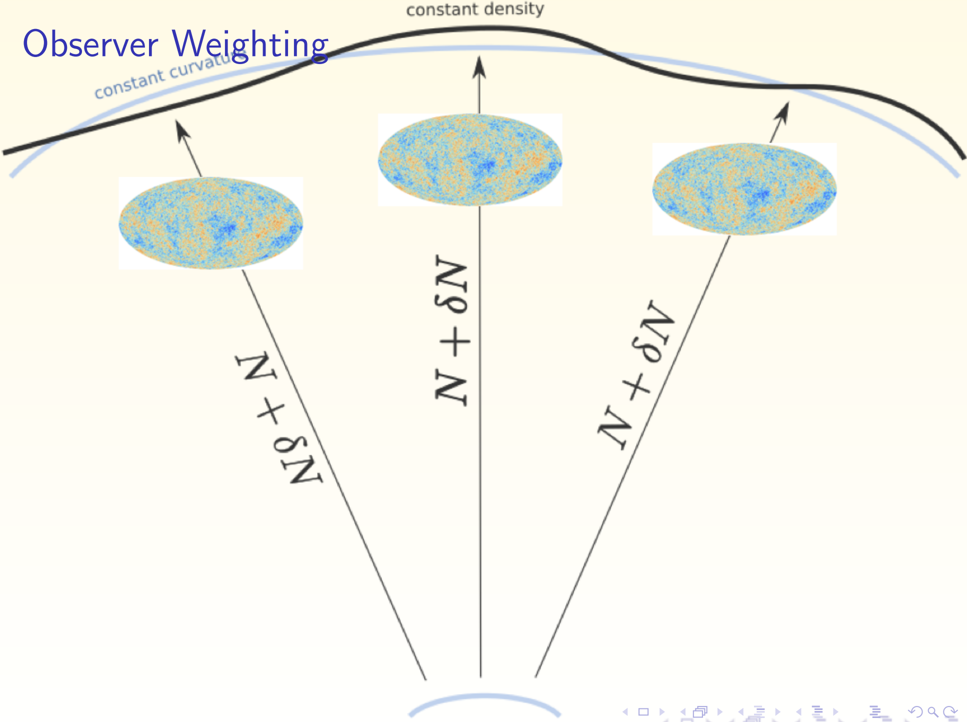
Planck measured C_ℓ : low ℓ observed and high ℓ "theory"

$$C_2^{\rm obs} = 253.6 \mu K^2 \qquad C_2^{{\rm high}-\ell} = 1124.1 \mu K^2$$

Numerical GR Input

$$\Pr\left(\hat{C}_2 | A_\phi, H_I L_{\rm obs}, \ldots\right)$$

Observer Weighting



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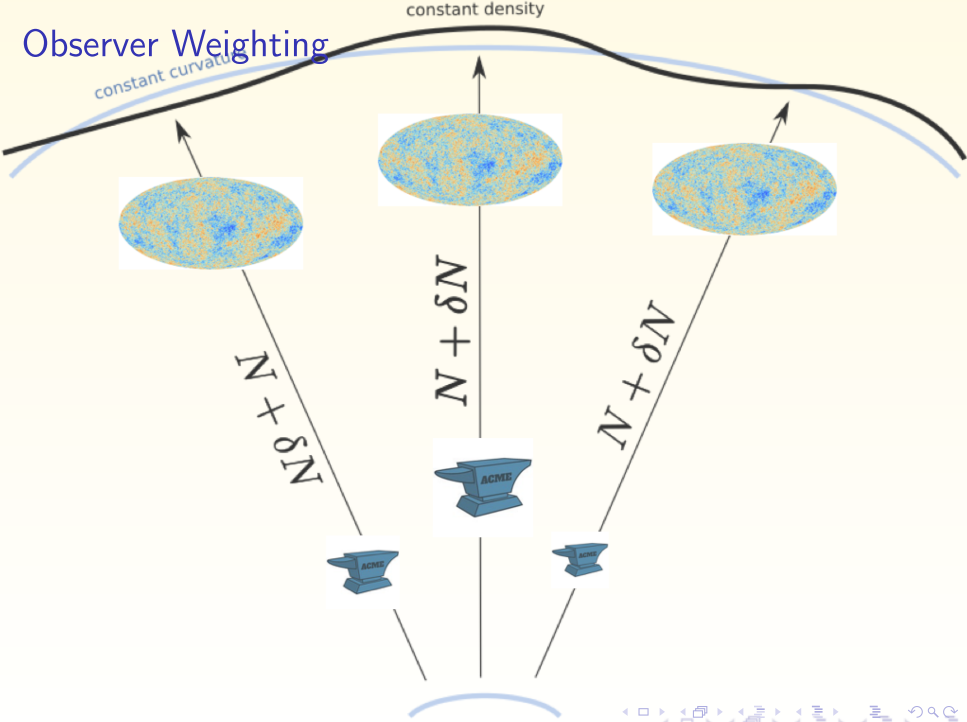

Evaluation of CMB Quadrupole

$$\hat{C}_2 = \frac{1}{5}\left[\left(a_{20}^{(\mathrm{UL})} + a_{20}^{(\mathrm{Q})}\right)^2 + \sum_{m=-2,m\neq 0}^{m=2}\left(a_{2m}^{(\mathrm{Q})}\right)^2\right]$$

Evaluation of CMB Quadrupole

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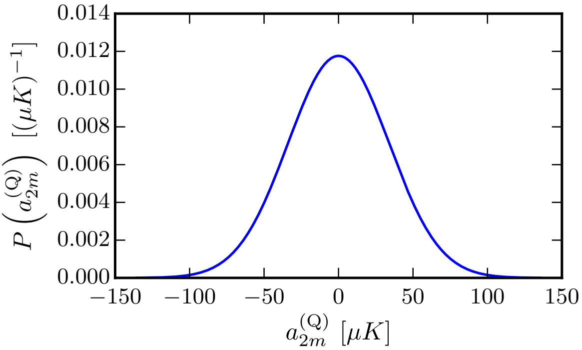
$a_{2m}^{(\mathrm{Q})}$: Gaussian with $\langle (a_{2m}^{(\mathrm{Q})})^2 \rangle = 1124.1 \mu K^2$



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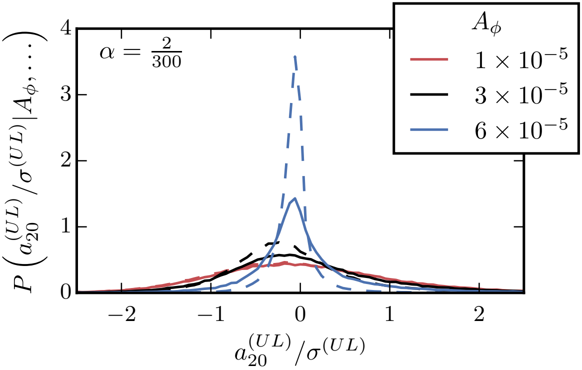
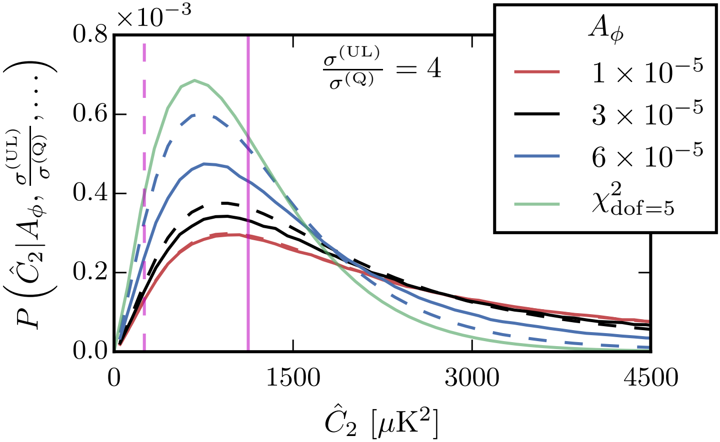
$a_{20}^{(\mathrm{UL})}$: NR simulations



Distribution of CMB quadrupole

$$\hat{C}_2 = \frac{1}{5}\left[\left(a_{20}^{(\mathrm{UL})} + a_{20}^{(\mathrm{Q})}\right)^2 + \sum_{m=-2,\ m\neq 0}^{2} \left(a_{2m}^{(\mathrm{Q})}\right)^2\right]$$

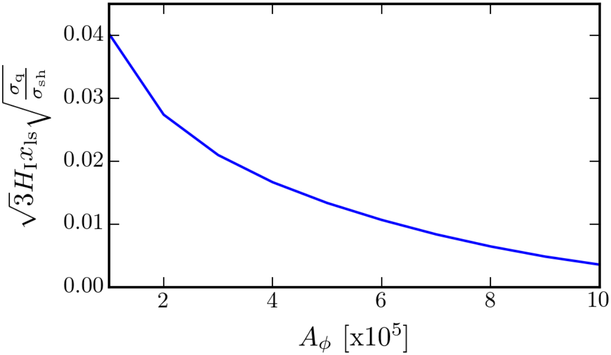


$$a_{20}^{(UL)} \approx F(H_{\rm I} L_{\rm obs})^2 \partial_{x_p}^2 \zeta \simeq F(L_{\rm obs} H_{\rm I})^2 \frac{1}{a_\parallel} \partial_x \left[\frac{\partial_x \zeta}{a_\parallel} \right]$$

For a given $V(\phi)$

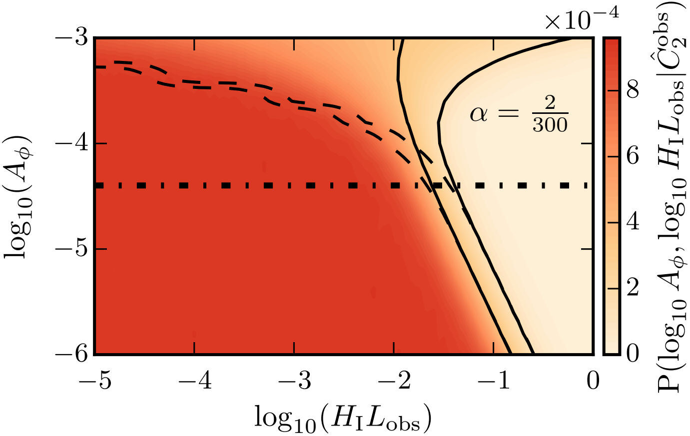
- F : nearly independent of A_ϕ and $H_I L_{\rm obs}$
- $H_{\rm I} L_{\rm obs}$: Unkown free parameter
- $\left\langle \left(\frac{1}{a_\parallel} \partial_x \left[\frac{\partial_x \zeta}{a_\parallel} \right] \right)^2 \right\rangle$: A_ϕ dependent only

Relative RMS of superhorizon contribution depends of distance to last-scattering surface



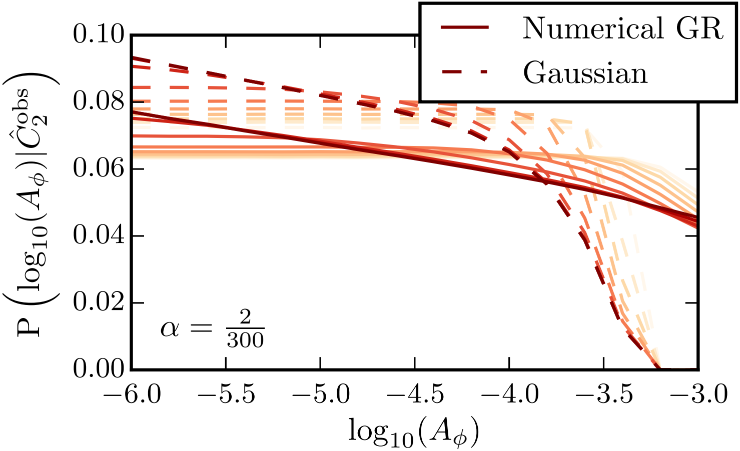
$$a_{20}^{(UL)} \approx F(L_{\rm obs}H_{\rm I})^2\partial_{x_p}^2\zeta \simeq F(L_{\rm obs}H_{\rm I})^2\frac{1}{a_\parallel}\partial_x\left[\frac{\partial_x\zeta}{a_\parallel}\right]$$

Constraints from the CMB Quadrupole



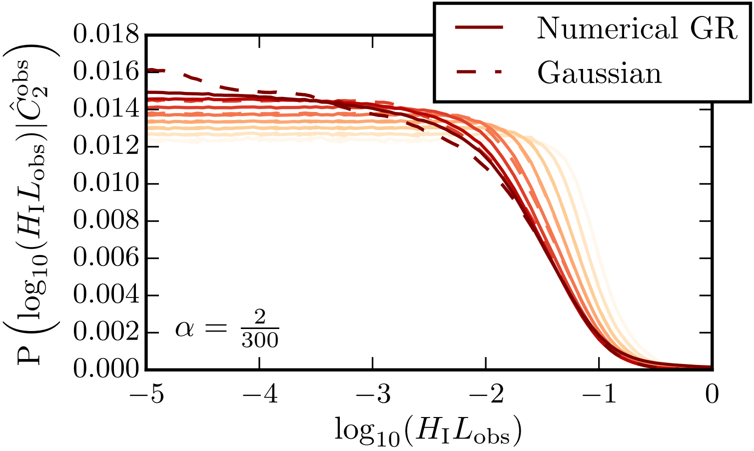
Strong Modification from Gaussian Ansatz at Large A_ϕ

Marginalised Constraints : A_ϕ



Constraint of A_ϕ as we marginalise over L_{obs}

Marginalised Constraints : $H_I L_{\rm obs}$



Constraint of $L_{\rm obs}$ as we marginalise over A_ϕ

Heuristic Picture of Large Amplitude Effects

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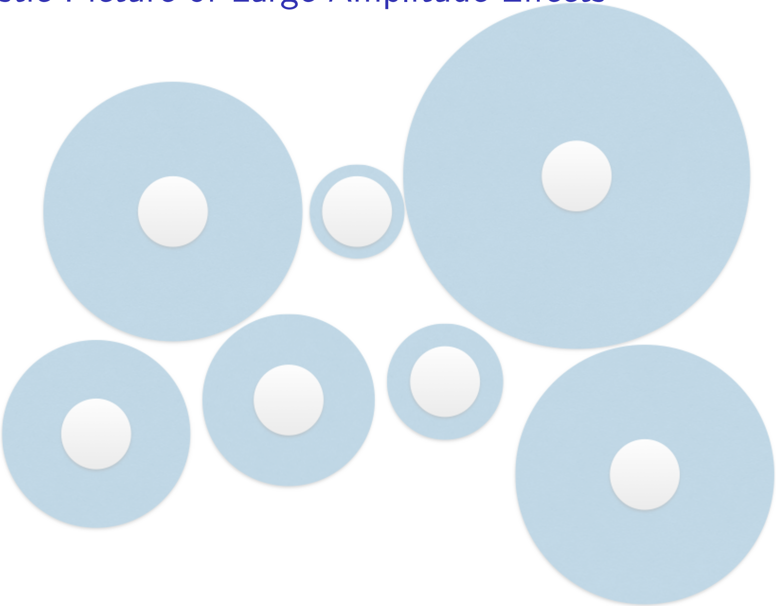
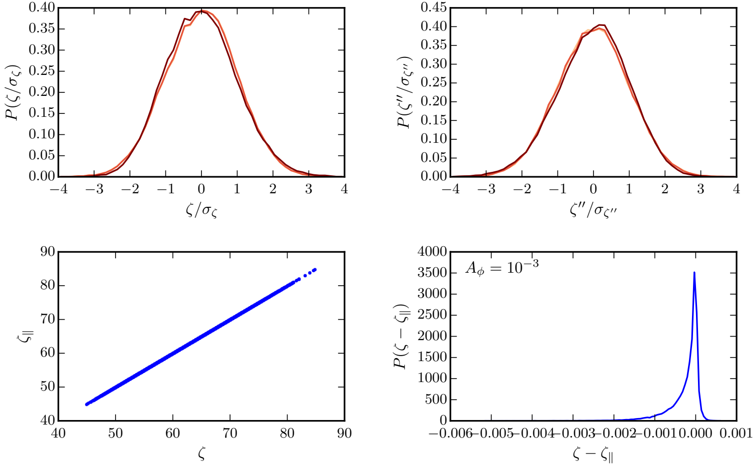


Statistics of ζ

ζ and comoving derivatives nearly Gaussian



Analytical Modelling

$$\zeta(H, x_{\mathrm{com}}) \sim \mathrm{GRF}$$

Large-Scale Approximation for a_{20}

$$a_{20}(x_0) \approx \mathcal{A} e^{-2\zeta_{\parallel}(x_0)} \left(\zeta''(x_0) - \zeta'_{\parallel}(x_0)\zeta'(x_0) \right)$$

ζ, ζ', ζ'' are correlated Gaussian random deviates, with covariance

$$C_\zeta = \begin{bmatrix} \sigma_0^2 & 0 & -\sigma_1^2 \\ 0 & \sigma_1^2 & 0 \\ -\sigma_1^2 & 0 & \sigma_2^2 \end{bmatrix}$$

$$\sigma_i = \int dk k^{2i} \left\langle \left| \tilde{\zeta}_k \right|^2 \right\rangle$$

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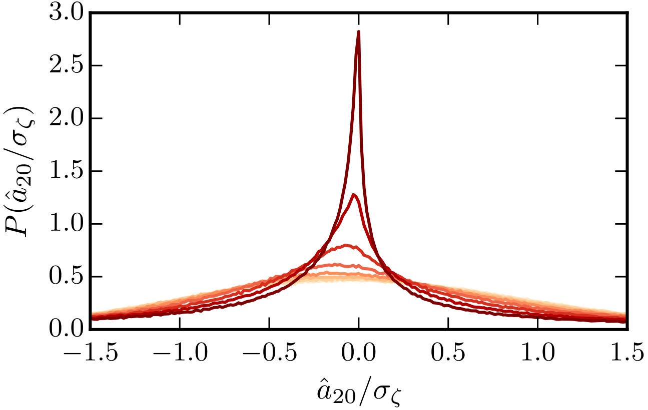
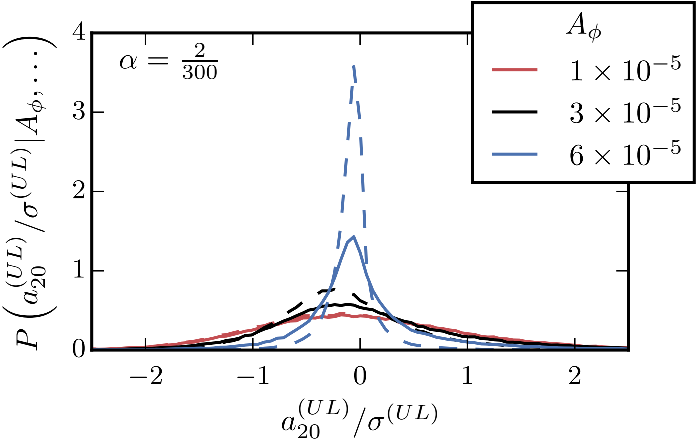
Anaytic a_{20} Distributions

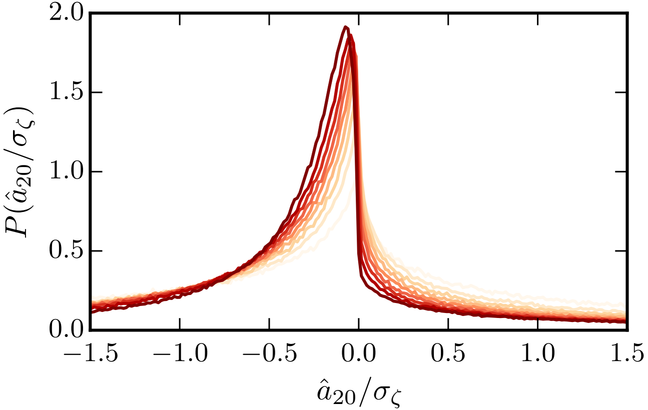


Vary σ_ζ at fixed $\sigma_{\zeta^{(p)}}/\sigma_\zeta$

Recall The Numerical Result



Anaytic a_{20} Distributions



Vary $\sigma_{\zeta'}/\sigma_\zeta$ at fixed σ_ζ and $\sigma_{\zeta''}/\sigma_\zeta$

Future Steps

- Three-dimensional simulations (in progress)
- Multi-field models (additional isocurvature modes, non-attractor, etc.)
- Inclusion of stochastic effects from subhorizon fluctuations (technical challenge)
- Correlated anomalies

NonGaussian Perturbations from Preheating Caustics

... A Mosaic of Interesting Dynamics

- **Strongly inhomogeneous and nonlinear cosmological initial conditions** (this talk) [JB, Peiris, Johnson, Aguirre]
- Isocurvature mode conversion into intermittent density perturbations [JB, Bond, Frolov, Huang]
  - Caustic formation in chaotic long wavelength dynamics
  - Generalised form of local nonGaussianity
- First order phase transitions [JB, Bond, Mersini-Houghton]
- Entropy production in highly inhomogeneous nonlinear field theories [JB, Bond]

Spatially intermittent dynamics, fundamental issues in QFT, novel and poorly constrained observables

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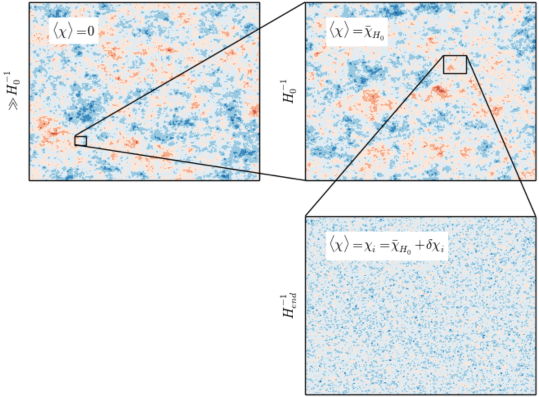
Spatially intermittent dynamics, fundamental issues in QFT, novel and poorly constrained observables

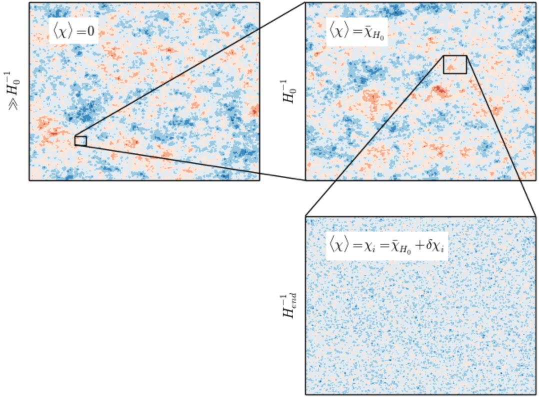
Ultra-Large Scale Structure



Local Remnants of Ultra-Large Scale Structure?

- **Structure present at start of inflation**
- Conversion of structure during or after inflation

Ultra-Large Scale Structure



Local Remnants of Ultra-Large Scale Structure?

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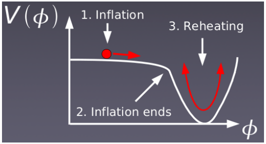
- **Conversion of structure during or after inflation**

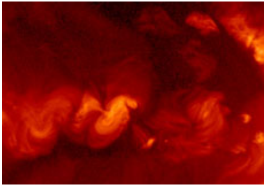
Starting the Big Bang

Inflation



- Cold $(T \sim 0)$, $\frac{S}{V} \approx 0$
- Few active d.o.f.

Hot Big Bang

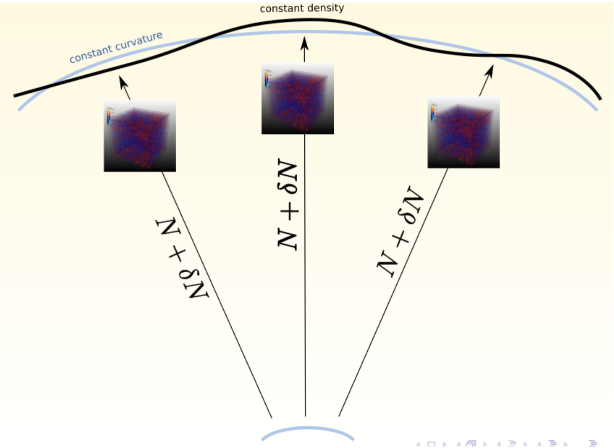


- Hot $(T > MeV)$, $\frac{S}{V} \propto g_{eff}(T) T^3$
- Many active d.o.f.

Huge entropy production

Does this leave any imprints?

Density Perturbations from Lattice Simulations c.f. [Rajantie and Chambers, Bond, Frolov, Huang, and Kofman]



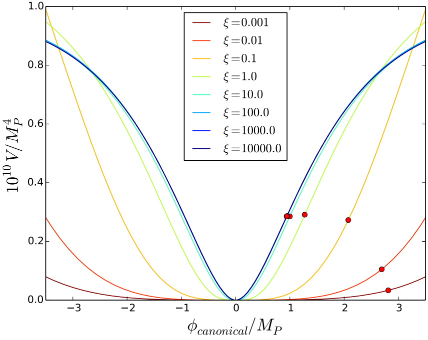
$$\zeta = \delta \ln(a)|_H = \delta \ln(a)|_\rho$$

Spatially modulate expansion history → curvature perturbations

Example : "Higgs" Inflation [Bond,JB,Frolov,Huang]

Jordan Frame

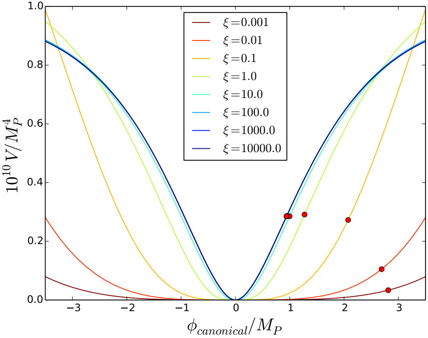
$$\frac{\mathcal{L}}{\sqrt{|g|}} = \frac{M_P^2}{2}(1+\xi\phi^2)\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi$$
$$- \frac{\lambda}{4}\phi^4 - \frac{g^2}{2}\phi^2\chi^2$$



Example : "Higgs" Inflation [Bond,JB,Frolov,Huang]

Einstein Frame

$$\frac{\mathcal{L}}{\sqrt{|g|}} = \frac{M_P^2}{2}\mathcal{R} - \frac{1}{2}\frac{1+\xi(1+6\xi)\phi^2}{(1+\xi\phi^2)^2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\frac{\partial_\mu\chi\partial^\mu\chi}{1+\xi\phi^2} - \frac{\lambda}{4}\frac{\phi^4}{(1+\xi\phi^2)^2} - \frac{g^2}{2}\frac{\phi^2\chi^2}{(1+\xi\phi^2)^2}$$



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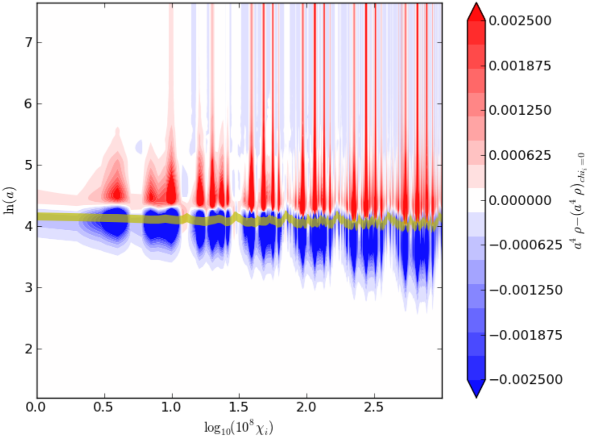
- For certain choices of $\frac{g^2}{\lambda}$, $\chi_{k=0}$ mode is unstable
- $\langle\chi\rangle \equiv \chi_i$ has superhorizon fluctuations from inflation
- Chaotic billiards in a potential $\rightarrow$ caustics

Evolution of $\ln(\rho/\bar{\rho})$

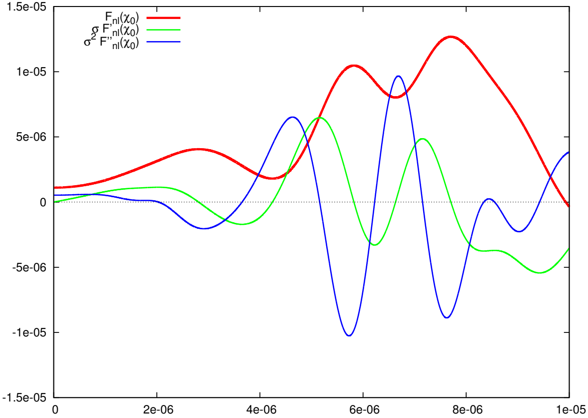
Creation of Superhorizon Density Perturbations in Time

$\delta \ln(a)$ structure set prior to onset of mode-mode coupling



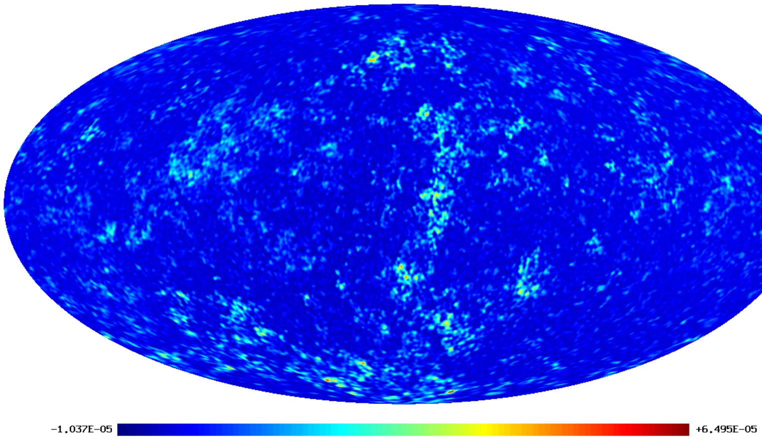
Effective Modulating Function of Density Perturbations

$$\zeta = \zeta_{\rm inf} + F_{NL}(\chi)$$



$F_{NL}(\chi)$ on the CMB Sky

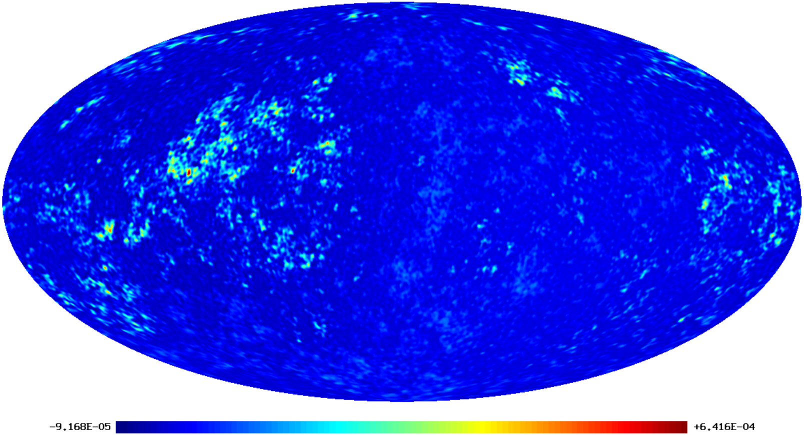
$$\zeta = F_{NL}(\chi)$$



$$\chi_i = 10^{-6}$$

$F_{NL}(\chi)$ on the CMB Sky

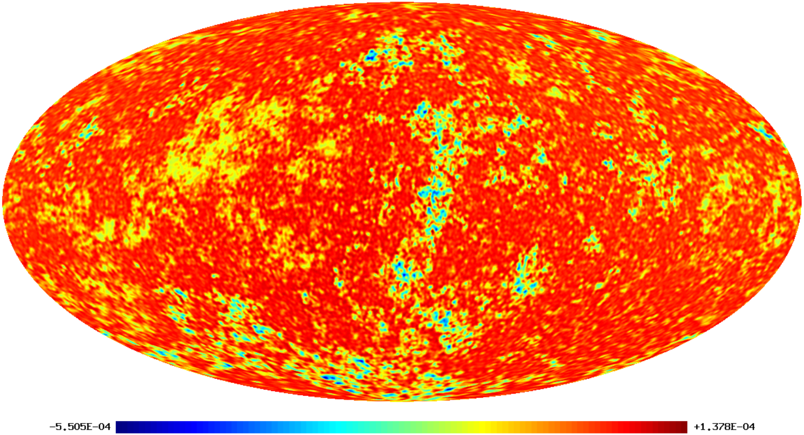
$$\zeta = F_{NL}(\chi)$$



$$\chi_i = 4.2 \times 10^{-6}$$

$F_{NL}(\chi)$ on the CMB Sky

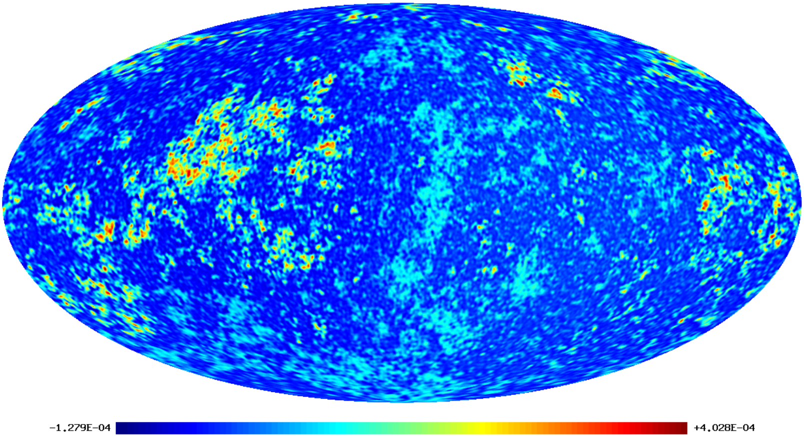
$$\zeta = F_{NL}(\chi)$$



$$\chi_i = 5.8 \times 10^{-6}$$

$F_{NL}(\chi)$ on the CMB Sky

$$\zeta = F_{NL}(\chi)$$



$$\chi_i = 6.6 \times 10^{-6}$$

Initial Evolution on a Phase String

Ballistic Approach

Reduced Phase Space

$$\Pi(x,t) = (\phi(x,t), \Pi_\phi(x,t), a(t), H(t)) \rightarrow q(t) = (\phi(t), \Pi(t), a(t), H(t))$$

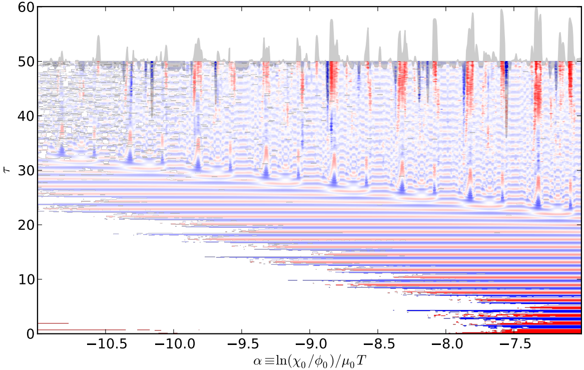
Lattice Averages from Ballistics

$$\langle \mathcal{O}(q) \rangle_{\text{lattice}} \rightarrow \int dq_{\text{init}} \mathcal{O}(q(t|q_{\text{init}})) P(q(t_{\text{init}}))$$

Order or magnitudes gain in computational efficiency!

Conceptual simplicity

Comparison: Billiards vs. Lattice



Density Perturbations from Caustic Formation

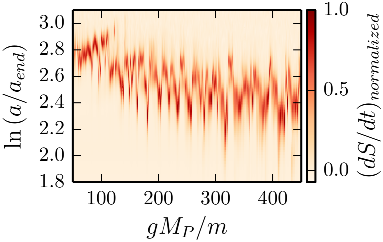
Basic Ingredients

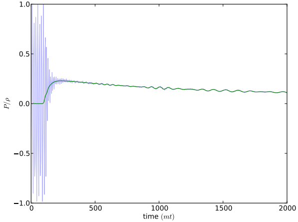
- Large-scale fluctuations in a modulating (i.e. isocurvature) field
- Zero-mode unstable (for billiard approximation)
- Trajectory dependent expansion history (such as flat directions)

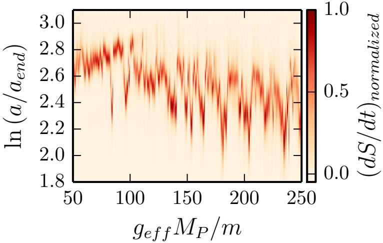
These appear generically in theoretically motivated scenarios

... and some of them can be relaxed

An Alternative Mechanism: Modulation via Shock Timing







- $w_{\text{preshock}} \neq w_{\text{postshock}}$
- $\delta \ln(a_{\text{shock}}) \to \zeta$

but...

- $w_{\text{postshock}} \neq \frac{1}{3}$
- $w_{\text{postshock}}$ depends on g^2

Summary and Conclusions

Inflationary Initial Conditions

- Generation of dynamically generated ultra-large scale structure using numerical relativity
- Constraints obtained from CMB quadrupole based on full Bayesian analysis
- Gravitational nonlinearities extremely important in determining final distributions of observables
- Very large amplitude initial inhomogeneities more poorly constrained than intermediate amplitudes
- Comoving curvature perturbation ζ approximately GRF

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Preheating Caustics

- Generalised form of local nonGaussianity from preheating
- Novel interpretation in terms of decoupled trajectories
- Good agreement between ballistic and full lattice approach