



















Modelling BEC Dynamics

$$\hat{\Psi}_i = \psi_i + \Delta \hat{\psi}_i$$

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij} \frac{\hbar^2}{2m_i} \nabla^2 + \cancel{V(\mathbf{x})} + g_{ij} |\psi_j|^2 \right) \psi_i - \nu_{ij} \psi_j$$

Diagram illustrating the components of the BEC dynamics equation and their relationships:

- KE** (Kinetic Energy) is associated with the term $-\delta_{ij} \frac{\hbar^2}{2m_i} \nabla^2$.
- PE** (Potential Energy) is associated with the term $\cancel{V(\mathbf{x})}$.
- S-wave** is associated with the term $g_{ij} |\psi_j|^2$.
- Conversion** is associated with the term $-\nu_{ij} \psi_j$.

Blue arrows indicate the mapping from the terms in the equation to the labels below. A red scribble is present over the $V(\mathbf{x})$ term.

$$\mathcal{H} = \frac{\hbar^2}{2m_i} |\nabla \psi_i|^2 + V(x) |\psi_i|^2 + \frac{g_{ij}}{2} |\psi_i|^2 |\psi_j|^2 + \frac{\nu_{ij}}{2} (\psi_i \psi_j^* + \psi_j \psi_i^*)$$

BECs and Relativity

$$\psi_i = \sqrt{\rho_i} e^{-i\phi_i}$$

Can. Momentum
(Particle density)

Can. Position
(Complex phase)

Assumptions

$$\rho_i(x, t) = n_i + \delta\rho_i(x, t)$$

Useful limit $\tilde{\nu} \equiv \frac{\nu}{g\bar{n}} \ll 1$

1) Homogeneous

2) Small

First Hint: Relativistic Dispersion for $k < k_{\text{heal}}$

$$\hbar^2 \omega^2 = m^2 + c^2 k^2 \left(1 + \frac{k^2}{k_{\text{heal}}^2} \right)$$

$$k_{\text{heal}}^2 = \frac{4mg\bar{n}}{\hbar^2} = 4 \frac{m}{g\bar{n}} \frac{g^2 \bar{n}^2}{\hbar^2}$$