

# Small Density Fluctuations

A convenient variable is  $\varphi = \phi_2 - \phi_1$

Integrate out fluctuations in number  $Z_{\text{eff}} \propto \int d\phi e^{i\mathcal{L}_{\text{eff}}}$

Relative phase  $\varphi$  is described by sine-Gordon model

$$\mathcal{L}_{\text{eff}} = \frac{v}{2} (\partial_t \varphi)^2 + \nu \Lambda \cos \varphi + \dots$$

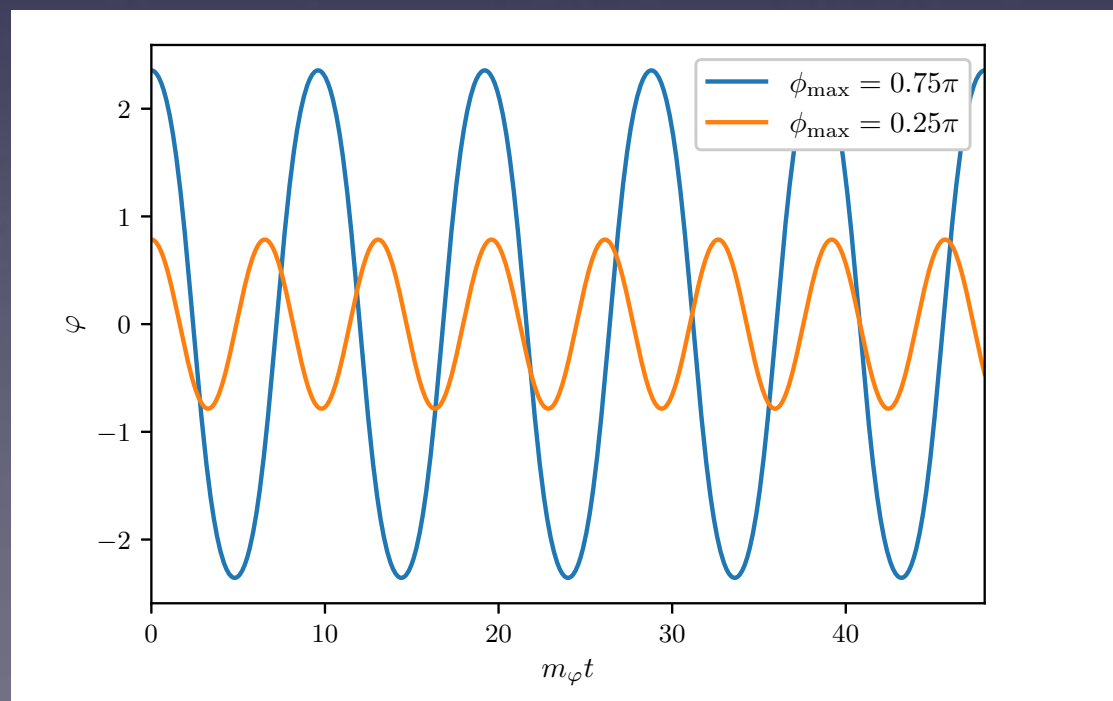
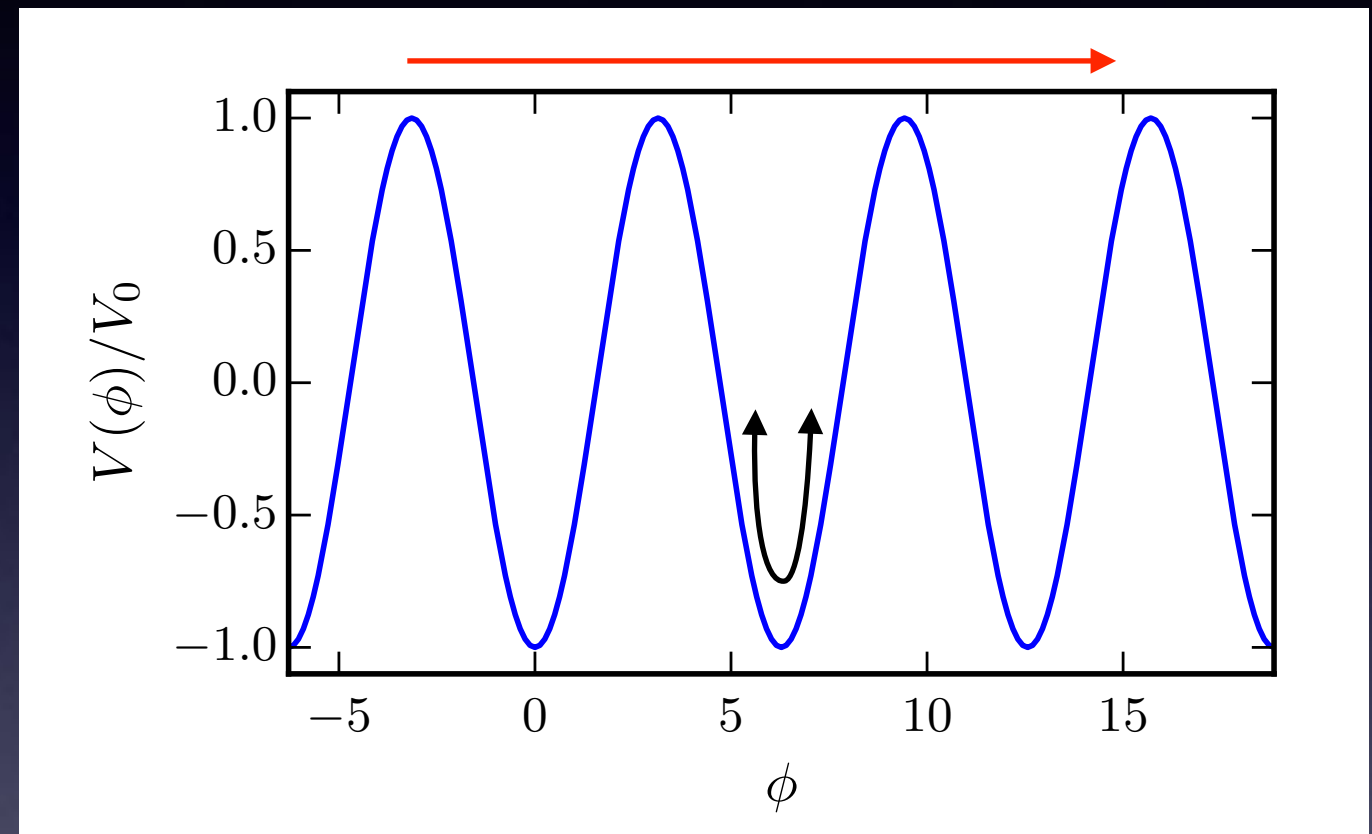
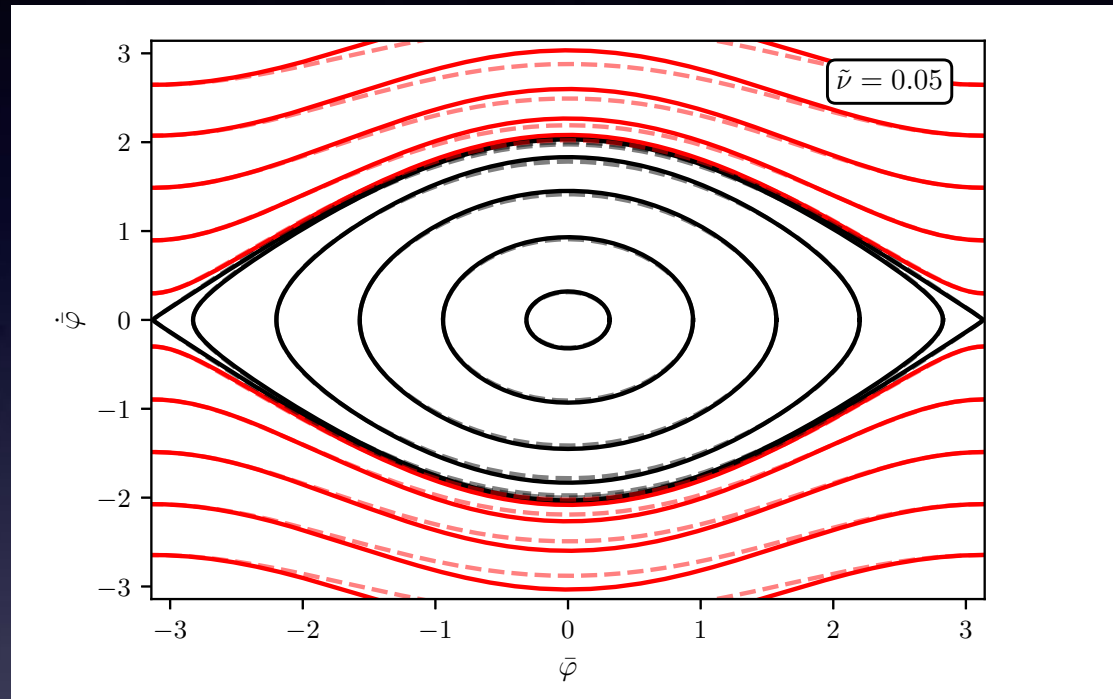
$$c_s^2 \approx \frac{g'}{m}$$

$$m_\varphi \approx \sqrt{\tilde{\nu}} \frac{2g\bar{n}}{\hbar}$$

$$L_\varphi = \frac{c_s}{m_\varphi} \sim \frac{L_{\text{heal}}}{\sqrt{\tilde{\nu}}}$$

**Sine-Gordon Model**

# Homogeneous Background Evolution



Two periodic motion types  
**Black** - single minimum  
**Red** - scan minima