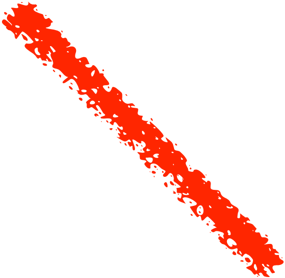

# Modelling BEC Dynamics

$$\hat{\Psi}_i = \psi_i + \Delta\hat{\psi}_i$$

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + \cancel{V(\mathbf{x})} + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

KE PE S-wave Conversion

$$\mathcal{H} = \frac{\hbar^2}{2m_i}|\nabla\psi_i|^2 + V(x)|\psi_i|^2 + \frac{g_{ij}}{2}|\psi_i|^2|\psi_j|^2 + \frac{\nu_{ij}}{2}\left(\psi_i\psi_j^* + \psi_j\psi_i^*\right)$$

# BECs and Relativity

$$\psi_i = \sqrt{\rho_i} e^{-i\phi_i}$$

Can. Momentum
(Particle density)

Can. Position
(Complex phase)

Assumptions

$$\rho_i(x,t) = n_i + \delta\rho_i(x,t)$$

1) Homogeneous

2) Small

Useful limit $\tilde{\nu} \equiv \dfrac{\nu}{g\bar{n}} \ll 1$

**First Hint**: Relativistic Dispersion for $k < k_{\text{heal}}$

$$\hbar^2\omega^2 = m^2 + c^2k^2\left(1 + \frac{k^2}{k_{\text{heal}}^2}\right)$$

$$k_{\text{heal}}^2 = \frac{4mg\bar{n}}{\hbar^2} = 4\frac{m}{g\bar{n}}\frac{g^2\bar{n}^2}{\hbar^2}$$