BECs and Relativity

$$\psi_i = \sqrt{\rho_i} e^{-i\phi_i}$$

Can. Momentum
(Particle density)

Can. Position
(Complex phase)

Assumptions

$$\rho_i(x,t) = n_i + \delta\rho_i(x,t)$$

1) Homogeneous

2) Small

Useful limit $\tilde{\nu} \equiv \frac{\nu}{g\bar{n}} \ll 1$

First Hint: Relativistic Dispersion for $k < k_{\text{heal}}$

$$\hbar^2\omega^2 = m^2 + c^2k^2\left(1 + \frac{k^2}{k_{\text{heal}}^2}\right)$$

$$k_{\text{heal}}^2 = \frac{4mg\bar{n}}{\hbar^2} = 4\frac{m}{g\bar{n}}\frac{g^2\bar{n}^2}{\hbar^2}$$

Small Density Fluctuations

A convenient variable is $\varphi = \phi_2 - \phi_1$

Integrate out fluctuations in number density $Z_{\mathrm{eff}} \propto \int d\phi e^{i\mathcal{L}_{\mathrm{eff}}}$

Relative phase governed by sine-Gordon model

$$\mathscr{L}_{\mathrm{eff}} \sim \frac{\dot{\varphi}^2}{2} - c_s^2 \frac{(\nabla \varphi)^2}{2} + \nu \Lambda \cos \varphi + \dots$$

$$c_s^2 \approx \frac{g\bar{n}}{m} \qquad m_\varphi \approx \sqrt{\tilde{\nu}} \frac{2g\bar{n}}{\hbar} \qquad L_\varphi = \frac{c_s}{m_\varphi} \sim \frac{L_{\mathrm{heal}}}{\sqrt{\tilde{\nu}}}$$