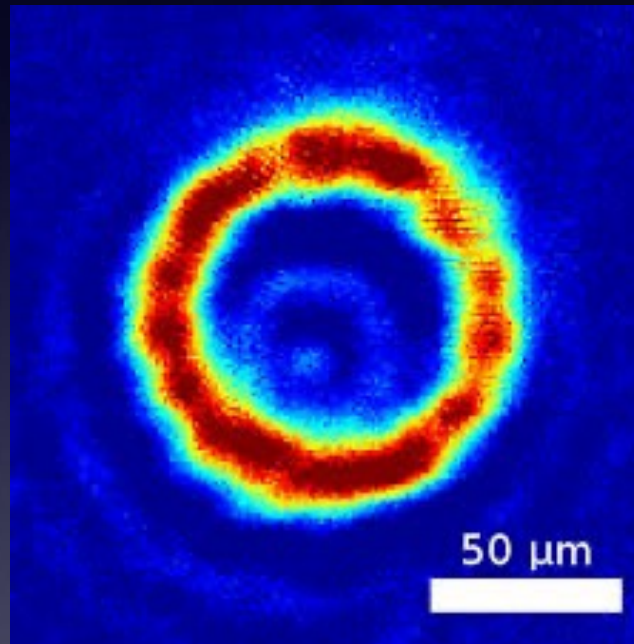




# Dimensional Reduction

**Idea** : Integrate out trapped directions



Pancake Trap

Cigar Trap

Ring Trap

$$i\hbar\dot{\psi}_i = \left( -\delta_{ij} \frac{\hbar^2}{2m_i} \nabla^2 + V(\mathbf{x}) + g_{ij} |\psi_j|^2 \right) \psi_i - \nu_{ij} \psi_j$$

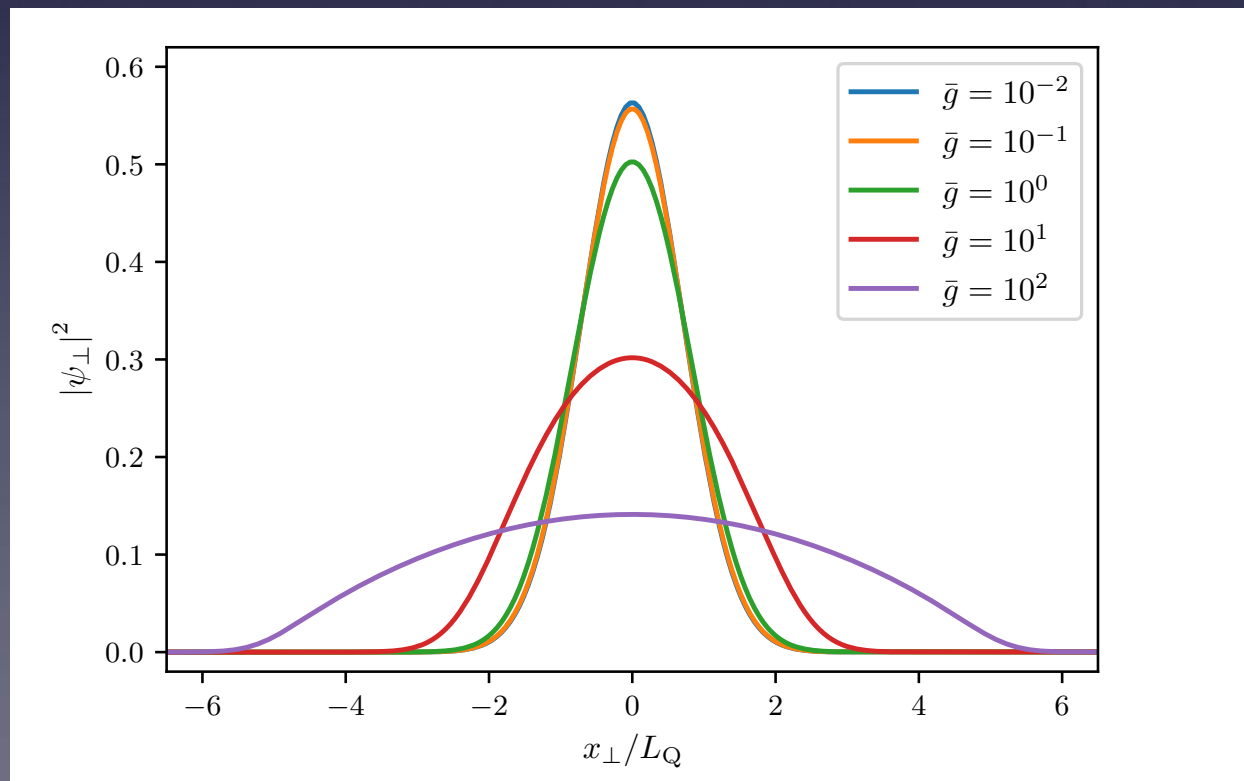
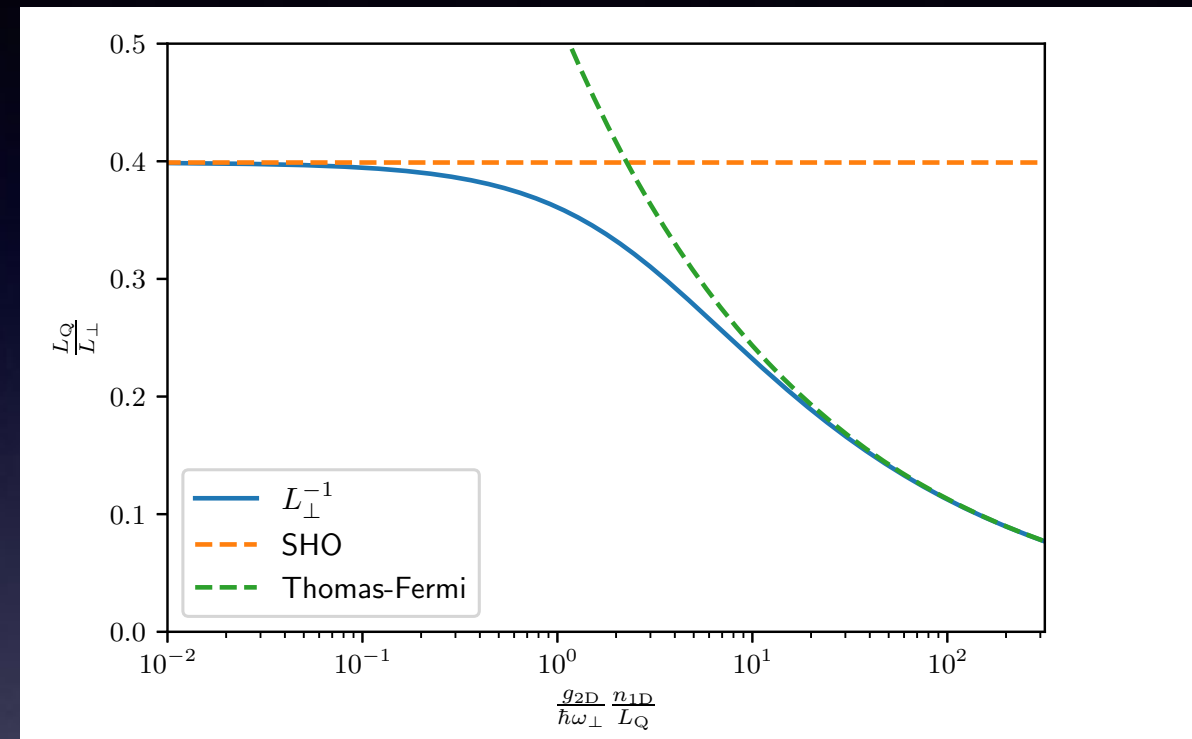
$$\frac{g^{D-1}}{g^D} = \frac{1}{L_\perp} = \frac{\int dx_\perp n^2}{\int dx_\perp n} = \frac{\int dx_\perp |\psi_\perp|^4}{\int dx_\perp |\psi_\perp|^2}$$



# Harmonic Trap

$$V(x_{\parallel}, x_{\perp}) = \frac{1}{2} m \omega_{\perp}^2 x_{\perp}^2 \quad L_Q^2 = \frac{\hbar}{m \omega_{\perp}}$$

$$\mu \psi_{\perp} = \left[ -\frac{\hbar^2}{2m} \nabla_{\perp}^2 + V_{\text{trap}}(x_{\perp}) + g^D |\psi_{\perp}|^2 \right] \psi_{\perp}$$



$$g_{\parallel} \sim \frac{L_Q^2}{L_{\text{heal}}^2} \sim \frac{1}{\tilde{v}} \frac{m_{\varphi}^2 L_Q^2}{c_s^2}$$