$$
\begin{aligned}
\frac{\mathrm{d}\delta\Pi_\vartheta}{\mathrm{d}\tilde{t}} =&\kappa^2\left(\delta\vartheta+\sqrt{\tilde{\nu}}\bar{\Pi}\delta\varphi\right)\\
\frac{\mathrm{d}\delta\vartheta}{\mathrm{d}\tilde{t}} =&-\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}\kappa^2\left(\delta\Pi_\vartheta-\sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\varphi\right)-\delta\Pi_\vartheta\\
&-\sqrt{\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}}\sin\bar{\varphi}\delta\varphi+\left(\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}\right)^{3/2}\bar{\Pi}\cos\bar{\varphi}\left(\delta\Pi_\varphi-\sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\vartheta\right)\\
\frac{\mathrm{d}\Pi_\varphi}{\mathrm{d}\tilde{t}} =&\kappa^2\left(\delta\varphi+\sqrt{\tilde{\nu}}\bar{\Pi}\delta\vartheta\right)\\
&+\cos\bar{\varphi}\sqrt{1-\tilde{\nu}\bar{\Pi}^2}\delta\varphi+\frac{\sqrt{\tilde{\nu}}\sin\bar{\varphi}}{\sqrt{1-\tilde{\nu}\bar{\Pi}^2}}\left(\delta\Pi_\vartheta-\sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\varphi\right)\\
\frac{\mathrm{d}\delta\varphi}{\mathrm{d}\tilde{t}} =&-\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}\kappa^2\left(\delta\Pi_\varphi-\sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\vartheta\right)-\delta\Pi_\varphi\\
&+\frac{\tilde{\nu}\bar{\Pi}_\varphi\sin\bar{\varphi}}{\sqrt{1-\tilde{\nu}\bar{\Pi}^2}}\delta\varphi-\frac{\tilde{\nu}\cos\bar{\varphi}}{(1-\tilde{\nu}\bar{\Pi}^2)^{3/2}}\left(\delta\Pi_\varphi-\sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\vartheta\right)\,.
\end{aligned}
$$

Perturbation Equations

$$\vartheta = \theta_1 + \theta_2 \qquad \varphi = \phi_1 - \phi_2 \qquad \Pi_\vartheta = \frac{\rho_1 + \rho_2}{2\bar{n}\sqrt{\tilde{\nu}}} \qquad \Pi_\varphi = \frac{\rho_2 - \rho_1}{2\bar{n}\sqrt{\tilde{\nu}}}$$

$$\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}t} = L(t)\mathbf{y}$$

$$\tilde{\nu} \ll 1 \implies \delta\ddot{\varphi} + (k^2 + m_\varphi^2 \cos\bar{\varphi})\delta\phi = 0$$

$$L(t+T) = L(t)$$

Our old friend the sine-Gordon model

Positive real part
=> exponential growth

Periodic $\bar{\varphi}$ and $\bar{\Pi}_\varphi \Longrightarrow$ solutions $\delta\mathbf{y} = \mathbf{P}(t)e^{\mu t}$

Linear Instability

$(\phi_{\max} = 0.25\pi)$