

Dynamical Trajectories for Inflation then & now

Inflation Then $\varepsilon(k) = (1+q)(a)$ $\sim r/16$ $0 < \varepsilon < 1$

= multi-parameter expansion in $(\ln H a \sim \ln k)$

~ 10 good e-folds ($\sim 10^{-4}$ Mpc $^{-1}$ to ~ 1 Mpc $^{-1}$ LSS) $\sim 10+$ parameters?

$r(k)$ is very prior dependent. Large (uniform), Small (monotonic). Tiny (roulette inflation of moduli).

Inflation Now $1+w(a) = \gamma f(a/a_{\Lambda eq})$ to $3(1+q)/2$

~ 1 good e-fold. Only ~ 2 parameters

Zhiqi Huang, Bond & Kofman 07

Cosmic Probes Now CFHTLS SNe (192), WL (Apr07), CMB, BAO, LSS

$$w(a) \equiv \frac{p(a)}{\rho(a)}$$

Uses latest April'07

SNe, BAO, WL, LSS, CMB data

Some Models

➤ **Cosmological Constant ($w=-1$)**

➤ **Quintessence**

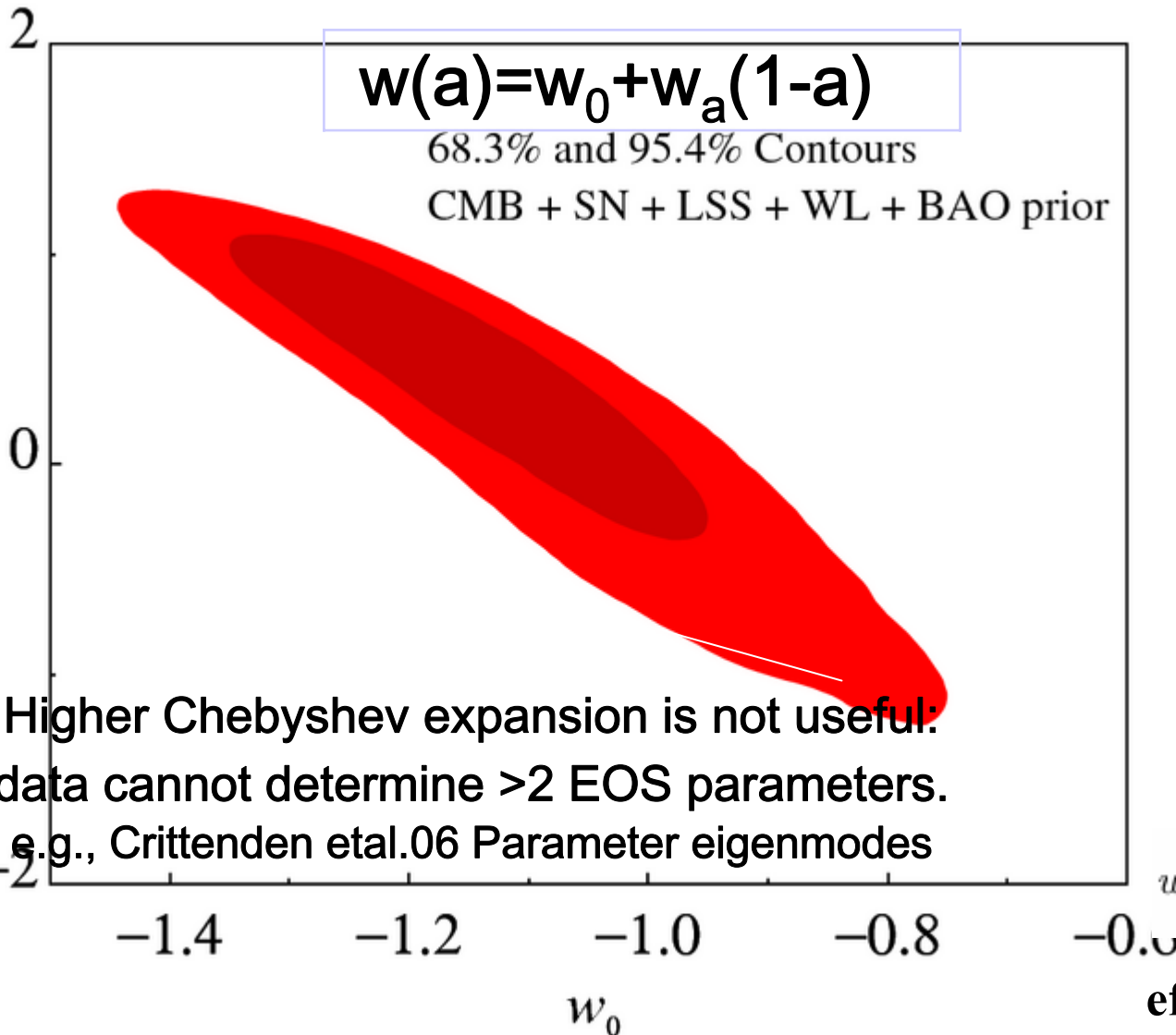
($-1 \leq w \leq 1$)

➤ **Phantom field ($w \leq -1$)**

➤ **Tachyon fields ($-1 \leq w \leq 0$)**

➤ **K-essence**

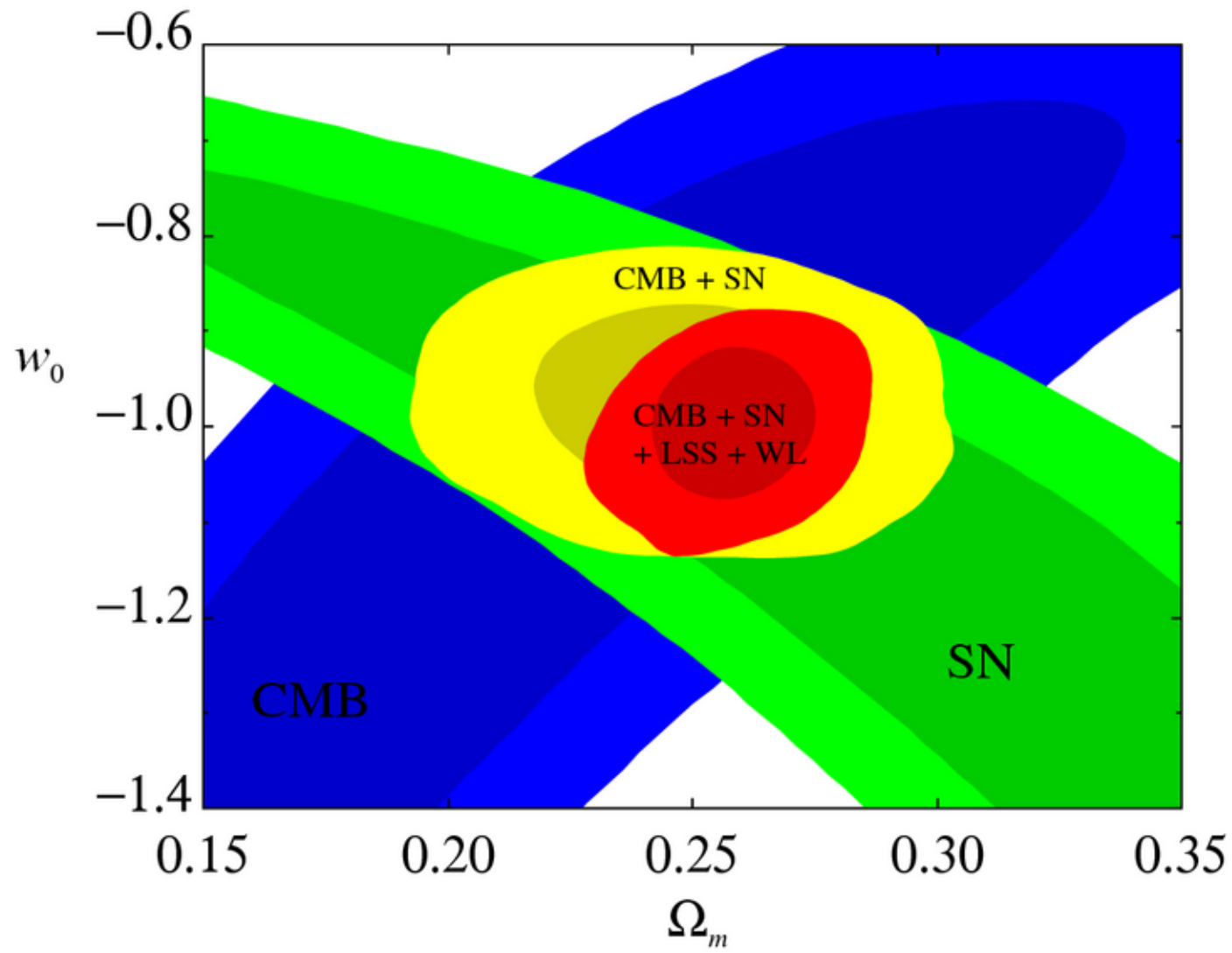
(no prior on w)



$$w_0 + w_a \left(0.264 + \frac{0.132}{\Omega_{m0}} \right) = -1$$

effective constraint eq.

Measuring constant w (SNe+CMB+WL+LSS)



Approximating Quintessence for Phenomenology

Zhiqi Huang, Bond & Kofman 07

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad + \quad \text{Friedmann Equations}$$

$$\rightarrow \begin{cases} d\theta/dN = \sqrt{\frac{3\Omega_\phi}{2}}\lambda \cos\theta - \frac{3}{2}\sin 2\theta, \\ d\Omega_\phi/dN = 3\Omega_\phi(1 - \Omega_\phi) \cos 2\theta, \\ d\lambda/dN = -\sqrt{6}\lambda^2(\Gamma - 1)\sqrt{\Omega_\phi} \sin\theta. \end{cases}$$

$$1+w=2\sin^2 \theta$$

$$\theta \equiv \sin^{-1} \frac{\dot{\phi}}{\sqrt{2\rho_\phi}}, \quad \Omega_\phi \equiv \frac{\rho_\phi}{3H^2 m_p^2}$$

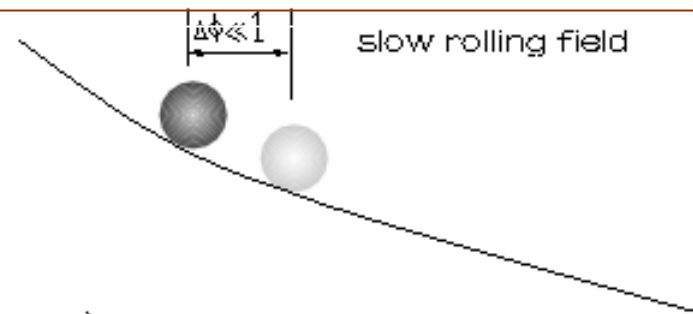
$$\gamma = \lambda^2 \quad \lambda \equiv -m_p \frac{V'}{V}, \quad \Gamma \equiv \frac{VV''}{V'^2}$$

slow-to-moderate roll conditions

$$\begin{cases} \frac{1}{2}\dot{\phi}^2 \ll V(\phi), \\ |V'/V| \lesssim O(1)m_p^{-1}, \\ |V''/V| \lesssim O(1)m_p^{-2}. \end{cases} \quad \text{at } 0 < z < 2$$

$1+w < 0.3$ (for $0 < z < 2$) and $\gamma \sim \text{const}$ give a 2-parameter model: $\gamma = \lambda^2$ & a_{ex}

$$w(a) = -1 + \frac{1}{3} \left\{ \left(\frac{a_{\text{ex}}}{a} \right)^3 + \lambda \left[\sqrt{1 + \left(\frac{a_{\text{eq}}}{a} \right)^3} - \left(\frac{a_{\text{eq}}}{a} \right)^3 \ln \left(\left(\frac{a}{a_{\text{eq}}} \right)^{\frac{3}{2}} + \sqrt{1 + \left(\frac{a}{a_{\text{eq}}} \right)^3} \right) \right] \right\}^2.$$



$$a_{\text{eq}} \equiv \left(\frac{\Omega_{m0}}{\Omega_{\Lambda 0}} \right)^{\frac{1}{3}} \sim 0.7.$$

λ varies slowly not because V is exponential-like, but because ϕ is varying slowly.

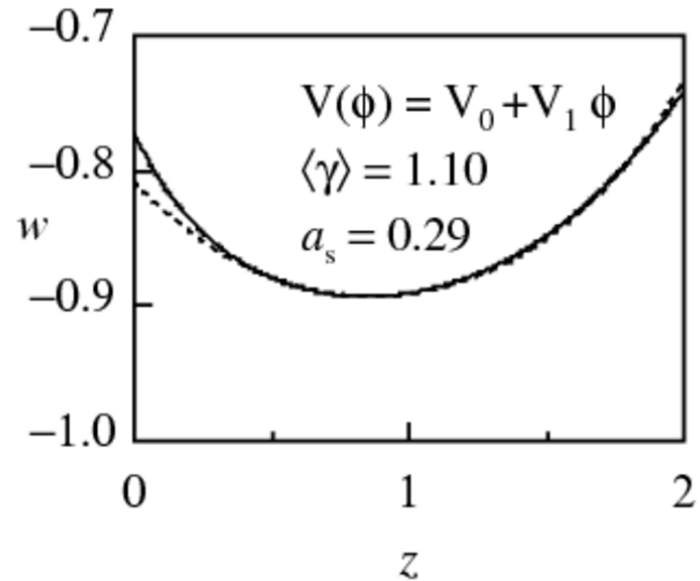
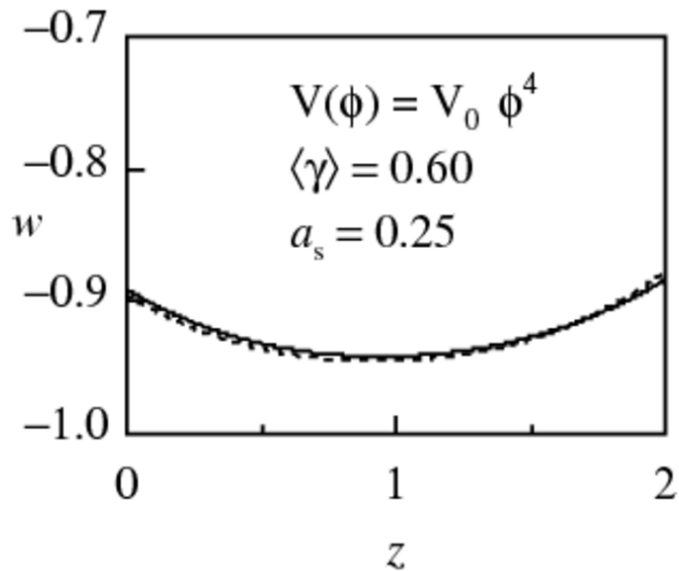
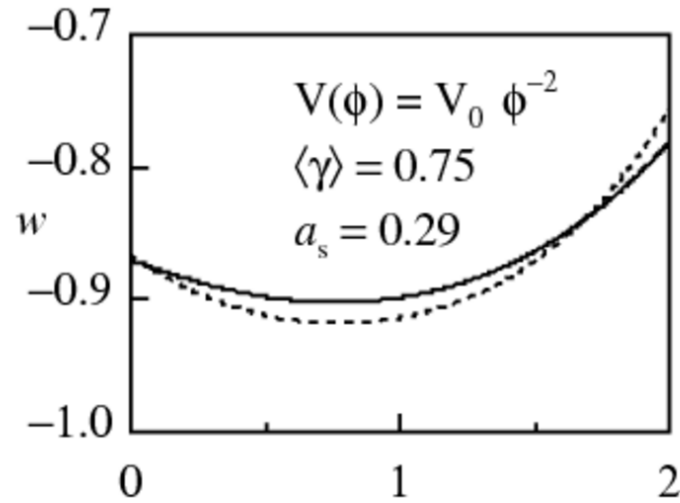
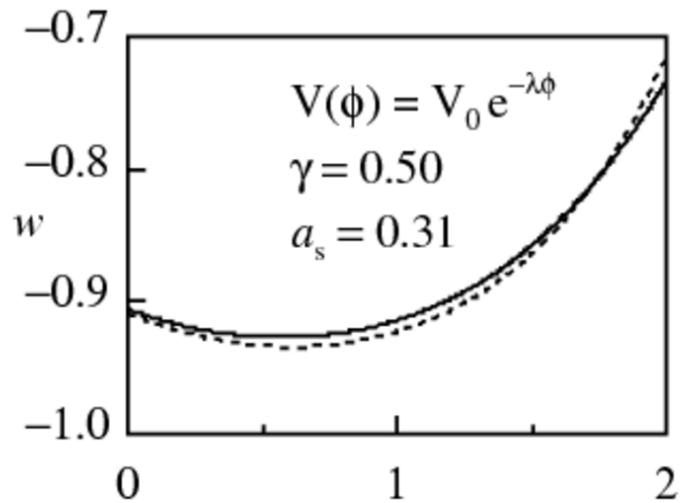
Early-Exit Scenario: scaling regime info is lost by Hubble damping, i.e. small a_{ex}

$1+w < 0.2$ (for $0 < z < 10$) and $\gamma \sim \text{const}$ give a 1-parameter model:

$$w(a) = -1 + \frac{\lambda^2}{3} \left\{ \sqrt{1 + \left(\frac{a_{\text{eq}}}{a} \right)^3} - \left(\frac{a_{\text{eq}}}{a} \right)^3 \ln \left(\left(\frac{a}{a_{\text{eq}}} \right)^{\frac{3}{2}} + \sqrt{1 + \left(\frac{a}{a_{\text{eq}}} \right)^3} \right) \right\}^2$$

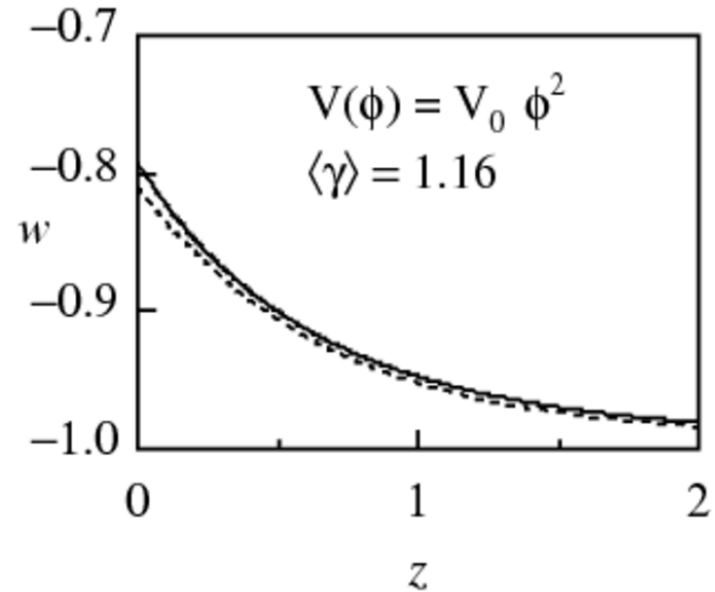
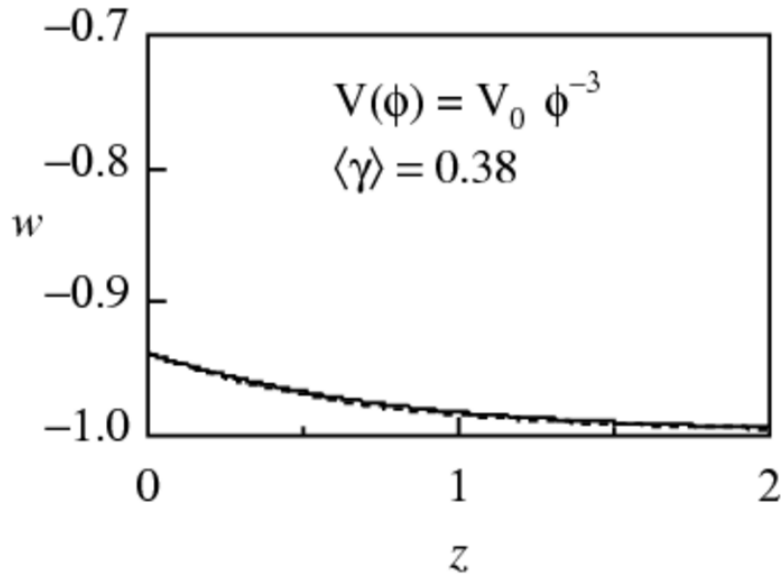
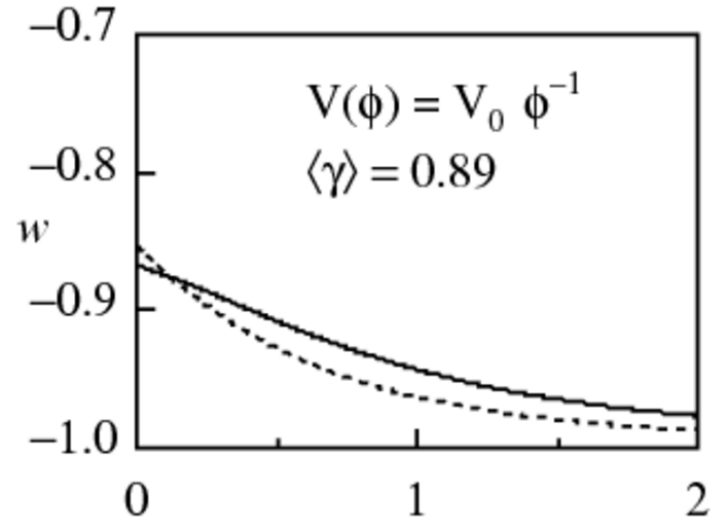
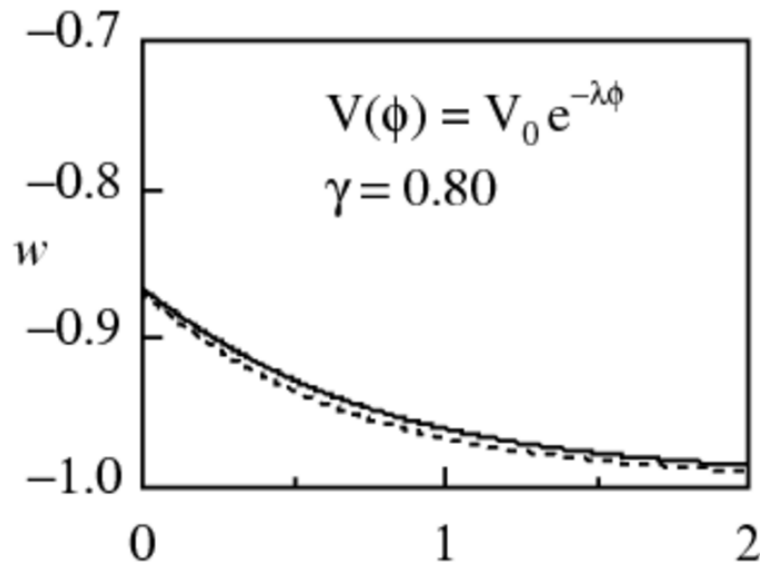
w-trajectories cf. the 2-parameter model

the field exits scaling regime at $a \sim a_{\text{ex}}$ $\gamma = (V'/V)^2$ (a) a-averaged at low z



w-trajectories cf. the 1-parameter model

ignore a_{ex} $\gamma = (V'/V)^2$ (a) a-averaged at low z



Include a $w < -1$ phantom field, via a negative kinetic energy term

$$\varphi \rightarrow i\varphi \quad \rightarrow \quad \gamma = \lambda^2 < 0$$

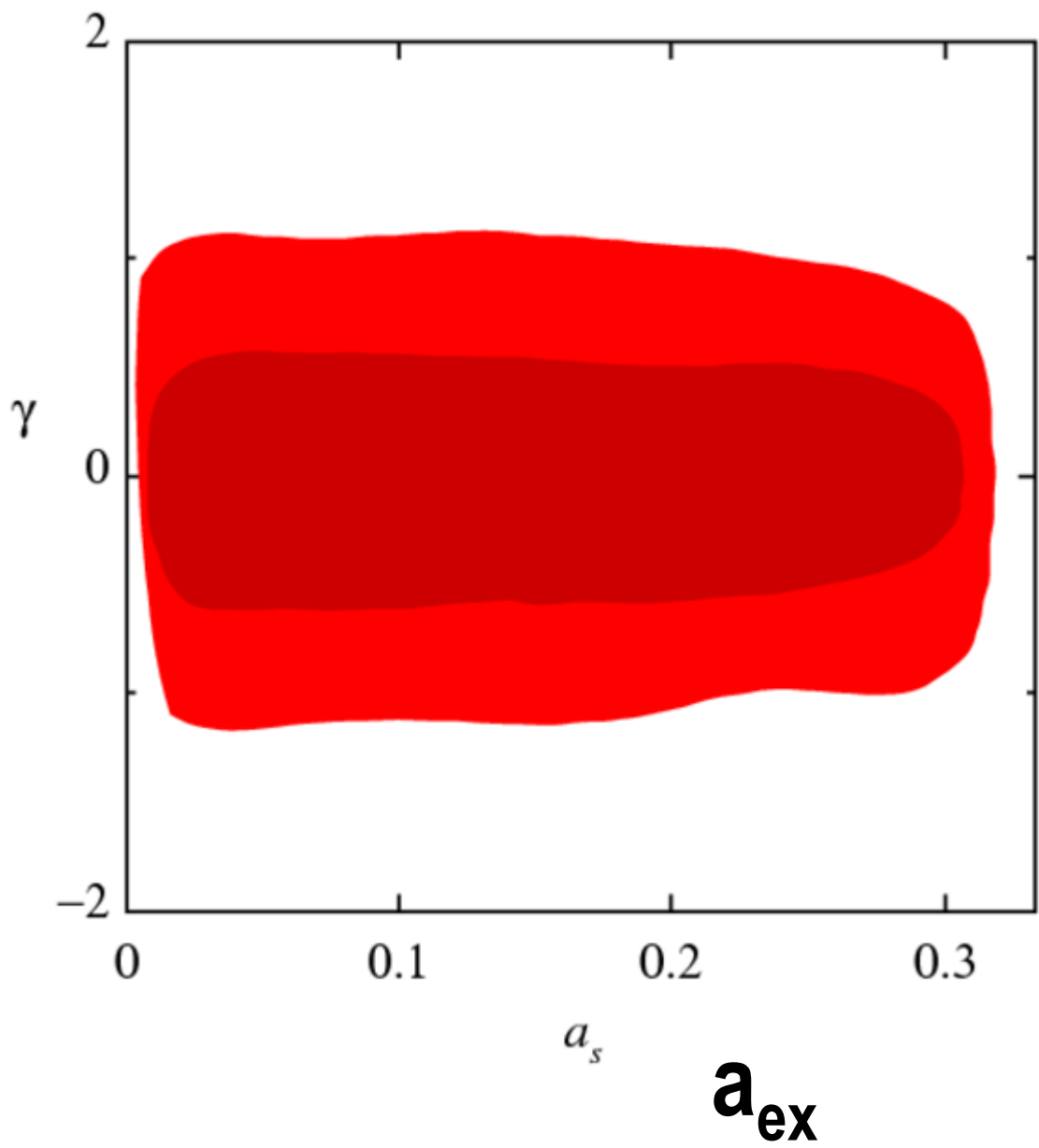
$$w(a) = -1 + \frac{\lambda^2}{3} \left\{ \sqrt{1 + \left(\frac{a_{eq}}{a}\right)^3} - \left(\frac{a_{eq}}{a}\right)^3 \ln\left(\left(\frac{a}{a_{eq}}\right)^{\frac{3}{2}} + \sqrt{1 + \left(\frac{a}{a_{eq}}\right)^3}\right) \right\}^2$$

$\gamma > 0 \rightarrow$ quintessence

$\gamma = 0 \rightarrow$ cosmological constant

$\gamma < 0 \rightarrow$ phantom field

Measuring $\gamma = \lambda^2$ (SNe+CMB+WL+LSS)



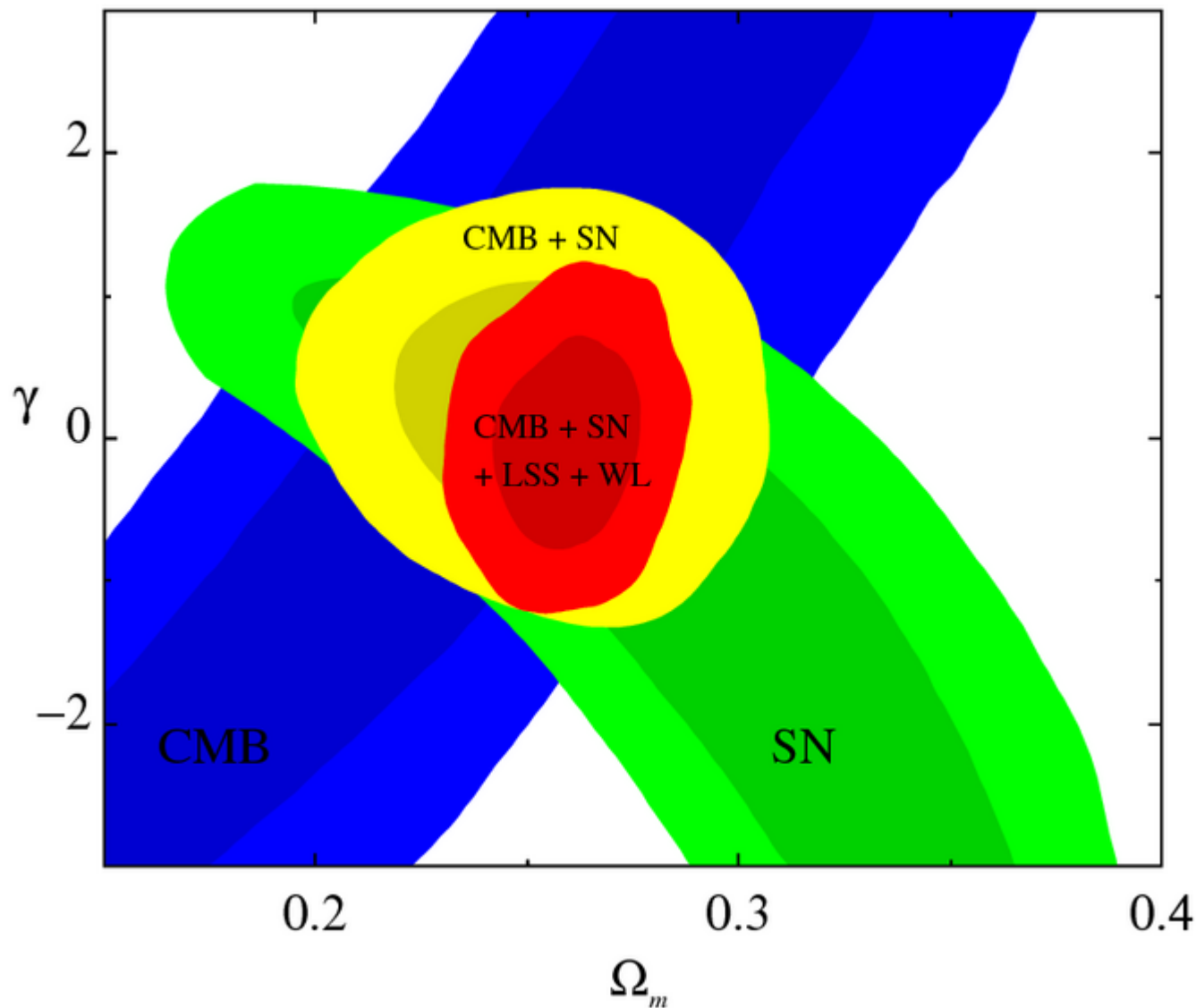
Well determined

γ

undetermined

a_{ex}

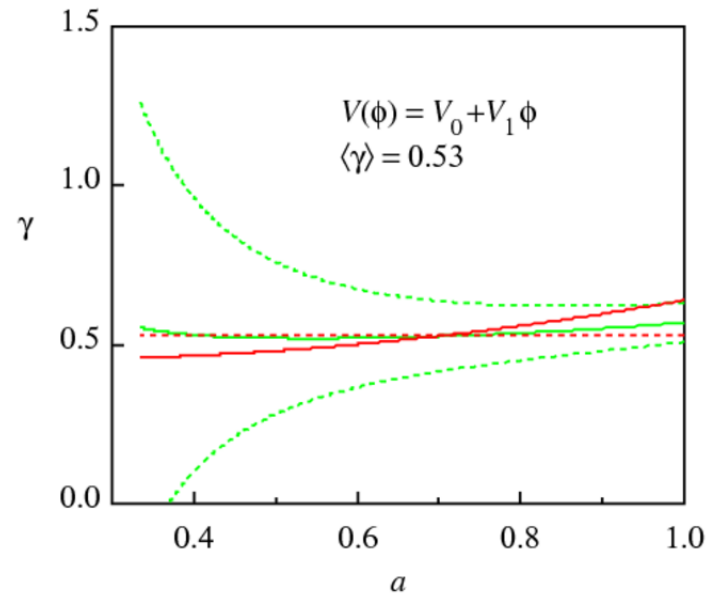
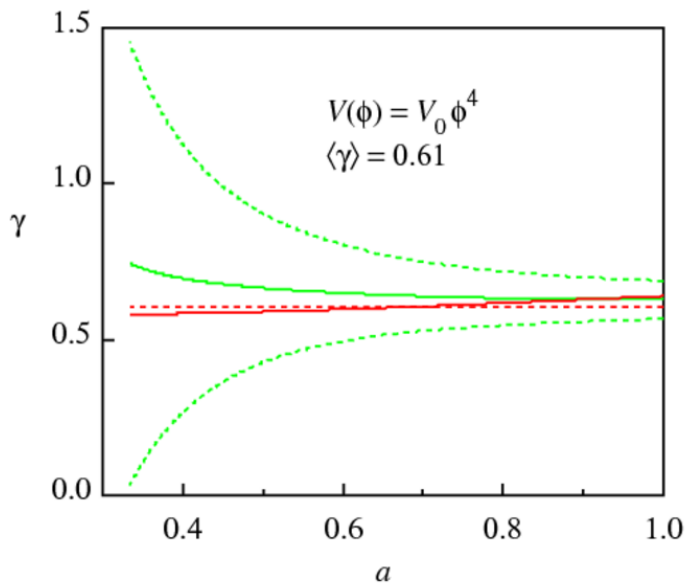
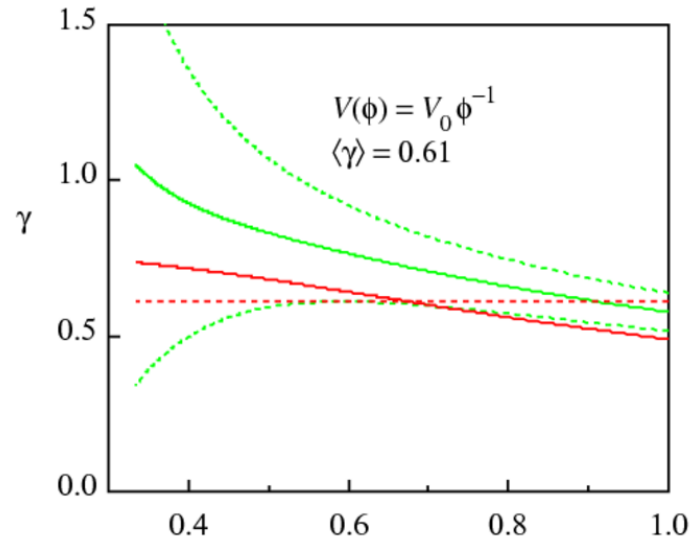
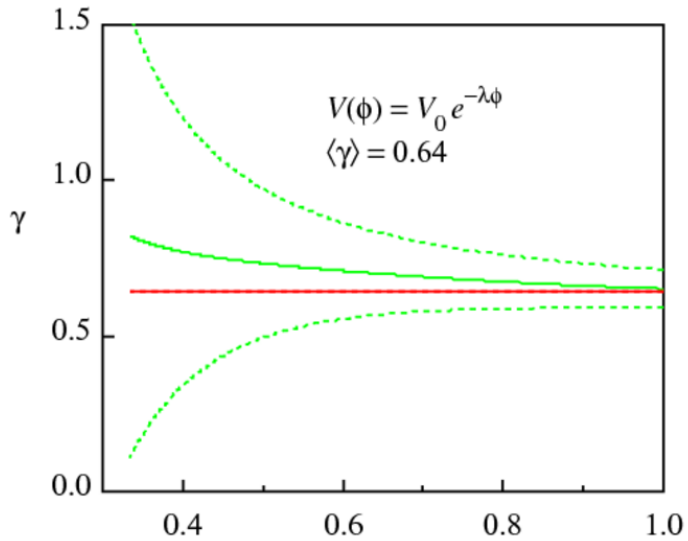
Measuring $\gamma = \lambda^2$ (SNe+CMB+WL+LSS)



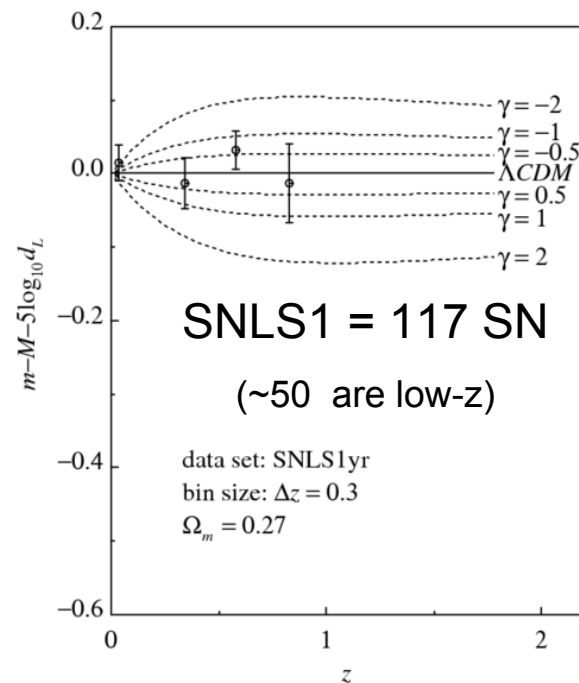
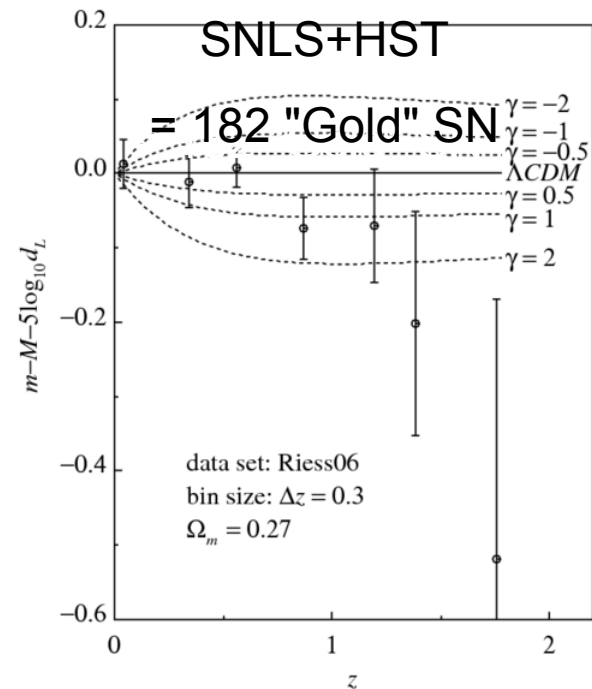
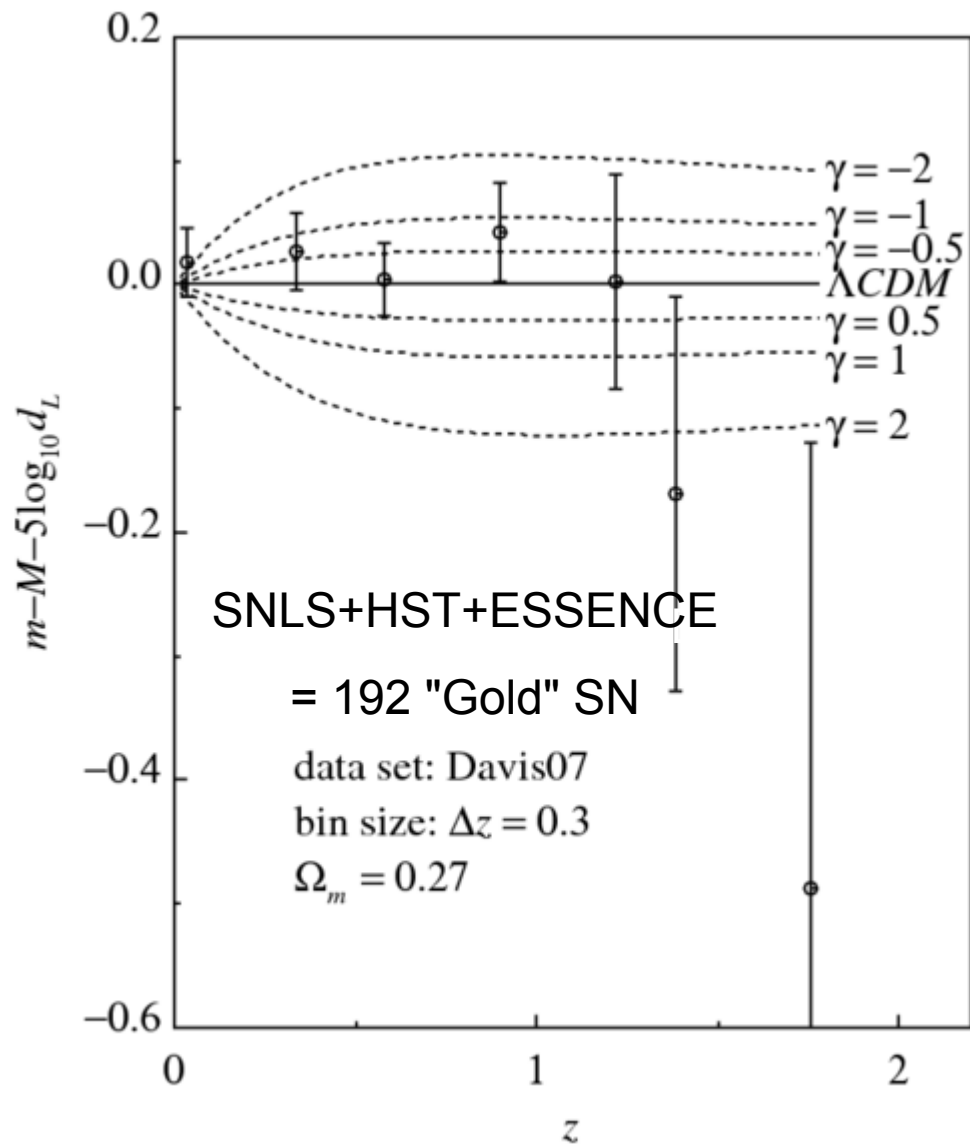
Modified
CosmoMC
with Weak
Lensing
and time-
varying w
models

γ -trajectories cf. the 1-parameter model

$\gamma = (1+w)(a)/f(a)$ cf. $(V'/V)^2(a)$



45 low-z SN + ESSENCE SN + SNLS 1st year SN
+ Riess high-z SN, all fit with MLCS



Inflation now summary

- The data cannot determine more than 2 w -parameters (+ c_{sound} ?). general higher order Chebyshev expansion in $1+w$ as for “inflation-then” $\varepsilon=(1+q)$ is not that useful – cf. Roger B & co.
- The $w(a)=w_0+w_a(1-a)$ expansion requires baroque potentials
- For general slow-to-moderate rolling one needs 2 “dynamical parameters” (a_{ex}, γ) to describe w to a few %
(cf. for a given Q -potential, IC, amp, shape to define a w -trajectory)
- In the early-exit scenario, the information stored in a_{ex} is erased by Hubble friction, w can be described by a single parameter γ . a_{ex} is not determined by the current data
- phantom ($\gamma < 0$), cosmological constant ($\gamma=0$), and quintessence ($\gamma > 0$) are all allowed with current observations $\gamma = 0.0 \pm 0.5$
- Aside: detailed results depend upon the SN data set used. Best available used here (192 SN), but this summer CFHT SNLS will deliver ~ 300 SN to add to the ~ 100 non-CFHTLS and will put all on the same analysis footing – very important.
- Lensing data is important to narrow the range over just CMB and SN

Inflation then summary

the basic 6 parameter model with no GW allowed fits all of the data OK

Usual GW limits come from adding r with a fixed GW spectrum and no consistency criterion (7 params). Adding minimal consistency does not make that much difference (7 params)

r (<.28 95%) limit come from relating high k region of σ_8 to low k region of GW C_L

Uniform priors in $\varepsilon(k) \sim r(k)$: the scalar power downturns ($\varepsilon(k)$ goes up) at low L if there is freedom in the mode expansion to do this. Adds GW to compensate, **breaks old r limit.** $T/S(k)$ can cross unity. **But monotonic prior in ε drives to low energy inflation and low r .**

Complexity of trajectories could come out of many-moduli string models. Roulette example: 4-cycle complex Kahler moduli in Type IIB string theory TINY $r \sim 10^{-10}$
a general argument that the normalized inflaton cannot change by more than unity over ~ 50 e-folds gives $r < 10^{-3}$

Prior probabilities on the inflation trajectories are crucial and cannot be decided at this time. Philosophy: be as wide open and least prejudiced as possible

Even with low energy inflation, the prospects are good with Spider and even Planck to either detect the GW-induced B-mode of polarization or set a powerful upper limit against nearly uniform acceleration. Both have strong Cdn roles.