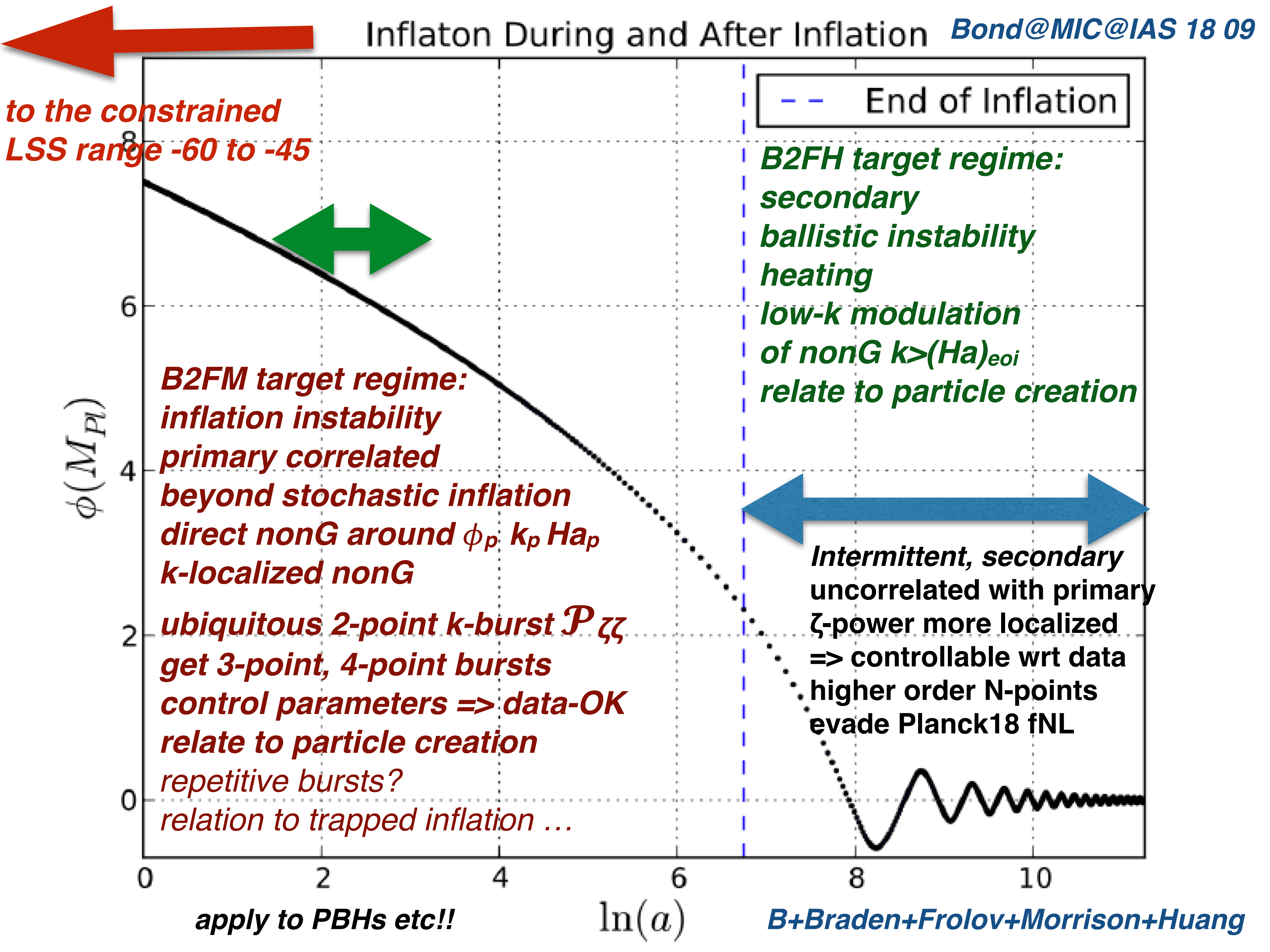


Inflaton During and After Inflation
Bond@MIC@IAS 18 09
to the constrained
LSS range -60 to -45
End of Inflation
B2FH target regime:
secondary
ballistic instability
heating
low-k modulation
of nonG k>$(Ha)_{eoi}$
relate to particle creation
B2FM target regime:
inflation instability
primary correlated
beyond stochastic inflation
direct nonG around ϕ_p k_p Ha_p
k-localized nonG
ubiquitous 2-point k-burst $\mathcal{P}_{\zeta\zeta}$
get 3-point, 4-point bursts
control parameters => data-OK
relate to particle creation
repetitive bursts?
relation to trapped inflation …
Intermittent, secondary
uncorrelated with primary
ζ-power more localized
=> controllable wrt data
higher order N-points
evade Planck18 fNL
$\phi(M_{Pl})$
$\ln(a)$
0
2
4
6
8
10
apply to PBHs etc!!
B+Braden+Frolov+Morrison+Huang

the true quadratic ζ-Websky of the ζ-scape

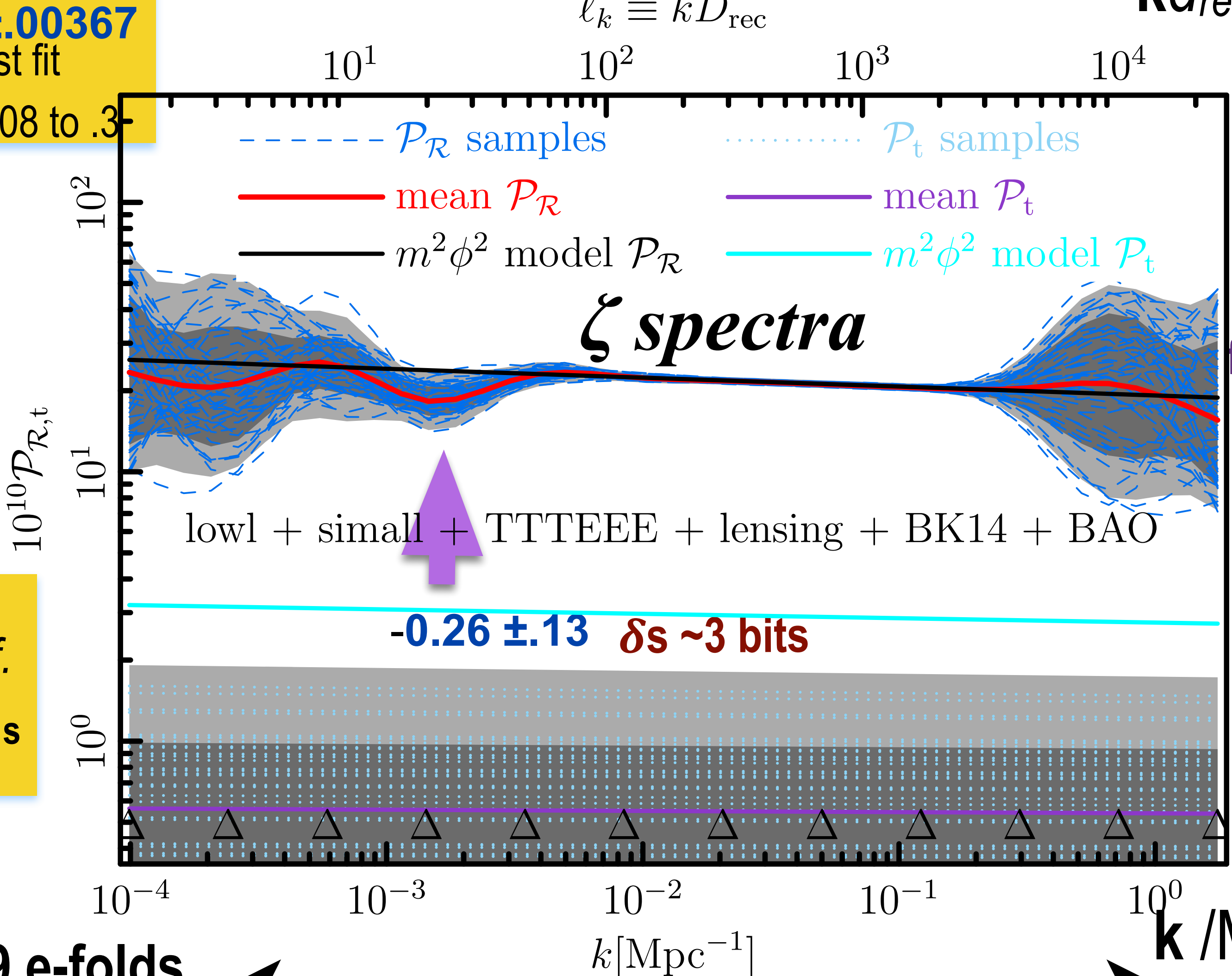
Planck 2018 inflation: TTTEEE lowL Epol + CMBlens + BK14 BB + *BAO*

CMB TT power L~ 20-30 dip => ζ-Spectrum k-dip; *includes CMB lensing, parameter marginalization*

uniform n_s=0.9669±.00367
P18+BK14 LSS best fit
superb 12-knot fit k~ .008 to .3

r<.084 95%CL *cf.*
r<0.068 uniform n_s

$kd_{rec} \gtrsim L$



=> lots of room for large ζ =>

=> V(ϕ)

k /Mpc

9 e-folds

general issue: ***role of "classical evolution/instabilities" in "particle creation" in heating, and during inflation. role in non-Gaussianity generation***

tool: **classical lattice/grid field calculations of fields**,
quantum fluctuations as classical random field sources. *Frolov, Braden*

$$\phi^A, \Pi_A\, H, \alpha$$

gravity approximation - governed by $\mathbf{H}_{cg}(X_c,t)$, $\boldsymbol{\alpha}_{cg}(X_c,t)$ usually computed on uniform $\boldsymbol{\alpha}_{cg}(X_c,t)$ hypersurfaces calculated on the box-scale. cf. comoving cg *horizon* scale $\mathbf{R_c \sim 1/Ha}$.
spatial gradient energies are included.

what is missing is the sub-$\mathbf{R_c}$ highly fluctuating contribution. this can be "enhanced" by measuring fluctuations on a poorly chosen time-hypersurface. the $\boldsymbol{\alpha}_{cg}(X_c,t)$, better $\mathbf{H}\boldsymbol{\alpha}_{cg}(X_c,t)$ is good. adequate

Huang: lattice code with quadratic nonlinearities included. full GR codes possible. much TBD.

post-inflation heating. ***oscillate + instability => nonlinear mode-mode coupling aka coarse-grained non-equilibrium Shannon entropy generation in a "shock-in-time"***.
Kolomogorov-Sinai entropy rate as a precursor to understand Shannon entropy generation in the ballistic regime, i.e. for each X_c: regular chaos => dramatic caustics in field space

what about particles? tie to entropy (see below) old way: $n_{Ak} \sim \rho_{Ak}/\hbar\omega_{Ak}(t)$

— Andrei Frolov's talk on B2FH, B2F work: modulated $\zeta_{NL}[\chi_{eoi}(X_c, t_{eoi})]$

quantum fluctuations are there, but ***only as seeds for instabilities*** in which trajectories diverge: *little probability Gaussian blobs stretch into highly deformed elongated, crossing surfaces. "phase strings" in 2D, 3D projections*

small fluctuations can develop rare coherences to tunnel. Jonathan Braden talk

map ballistic extreme parameter strain view of (pre-) heating and inflating regime i.e., role of classical instabilities during inflation, need to play off quantum fluctuations but largely classical "particle creation"

history (for me). sbb87-89 $\langle \delta \mathscr{P}_{\phi^A \phi^A}(k) \,|\, \delta V, \delta m^2_{eff} \rangle, \;\; \langle \delta \mathscr{P}_{\zeta\zeta}(k) \,|\, \delta V(\phi, \chi) \text{ controls} \rangle$

multifield hybrid, mountains/valleys of extra power. role in non-Gaussianity. role of Higgs etal.

tool: full linearized k evolution, from inside to outside

build on vilenkin/starobinsky stochastic inflation: sb90, 91

used the attractor, aka reduced Hamilton principal function, $\mathbf{S_{reduced} \sim -2M_P^2\, H}$ gives $\boldsymbol{\pi}_A$

B91 more general Langevin network leads to P(ϕA, $\boldsymbol{\pi}_A$,H|α) or better P(ϕA, $\boldsymbol{\pi}_A$,H|Ha)

=> nice expression in terms of quantum diffusion velocity/current

today:

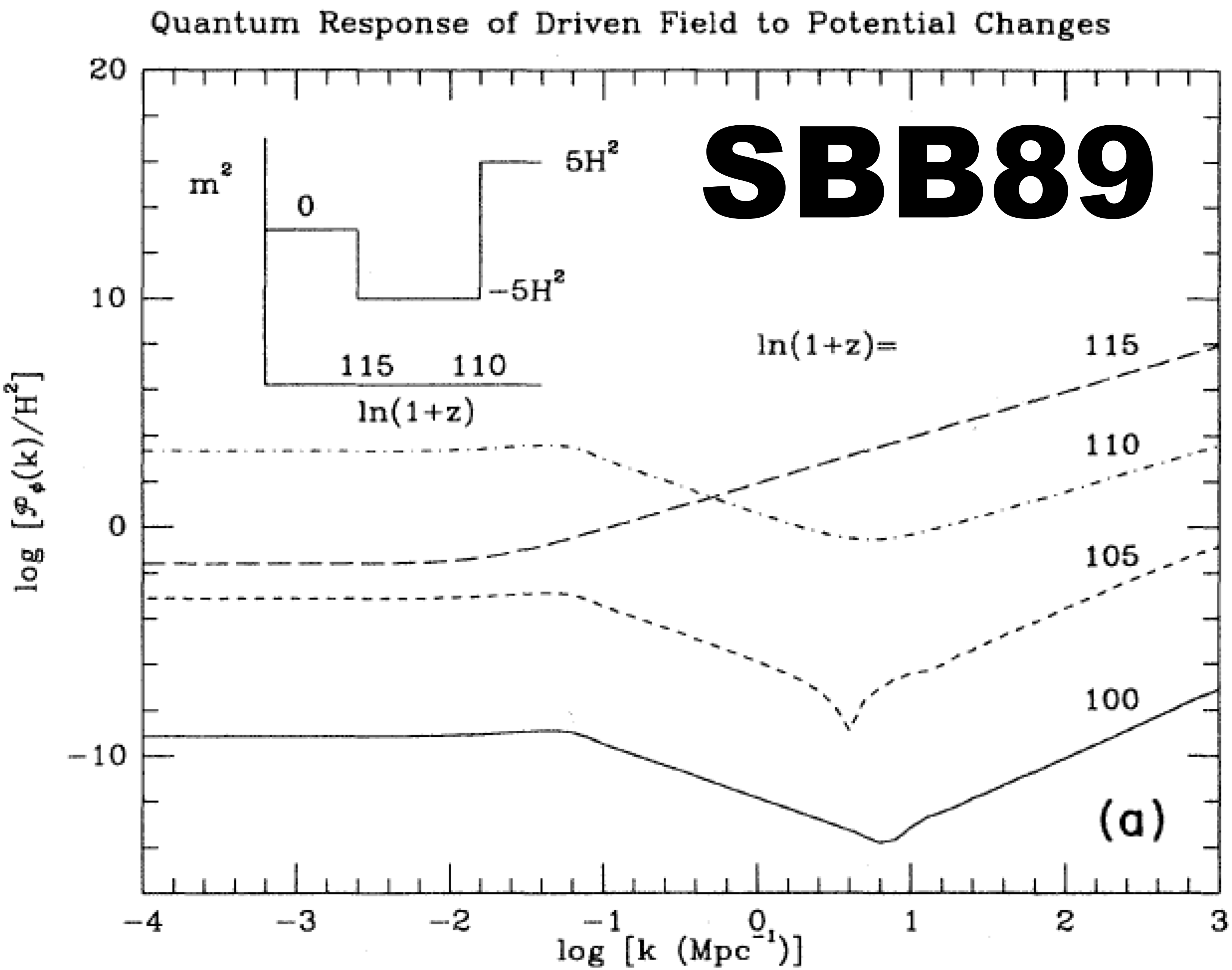
dilemma 1: missing terms in crossing horizon (coarse grain / fine grain flow)

dilemma 2: outside horizon is a condensate. how to express this

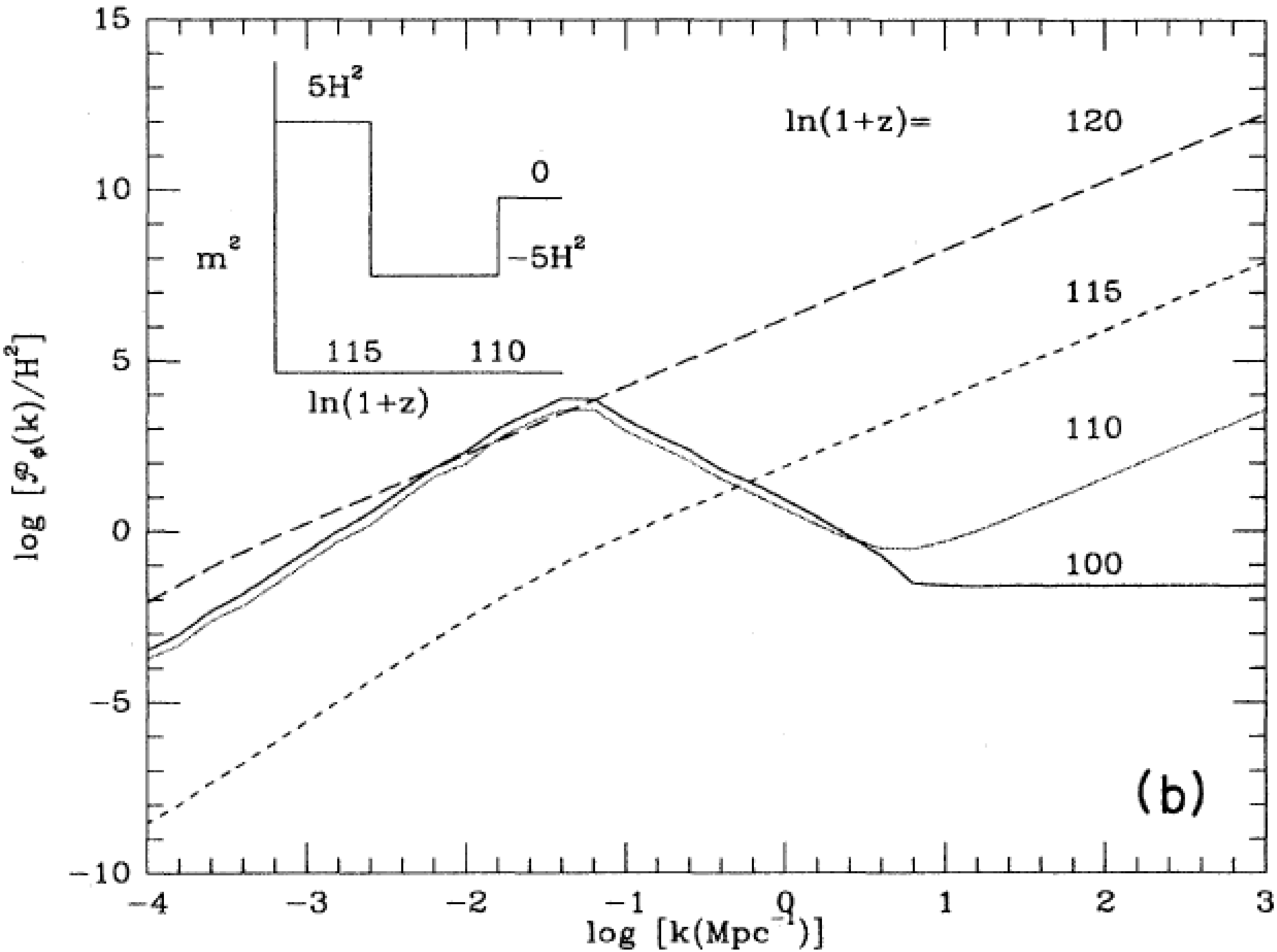
dilemma 3: phase coherence as in classical realizations are useful

- ***incorporate via coherent state condensates and fluctuations upon them??***

instabilities with quantum fluctuations the tiny seeds

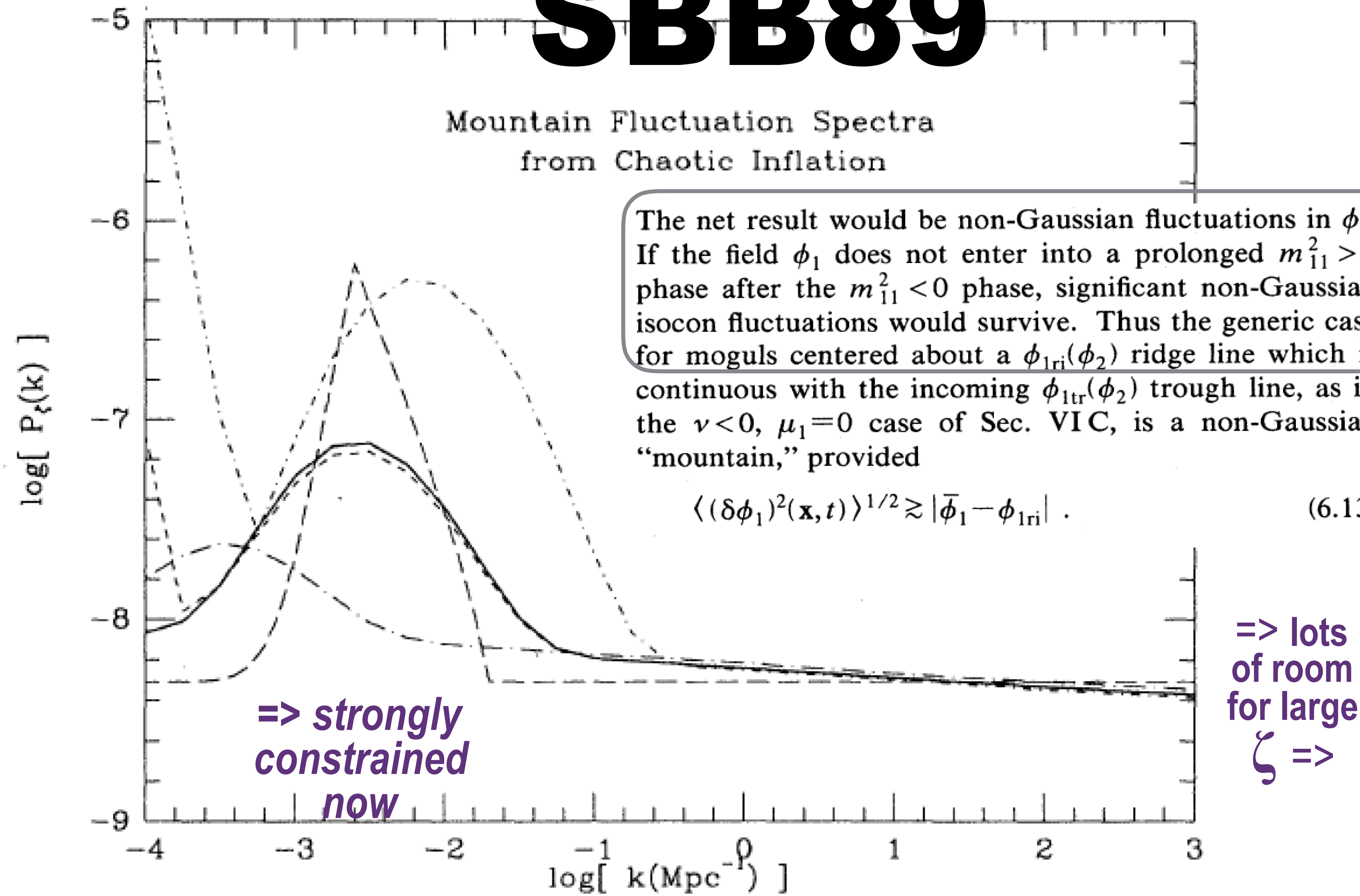


Quantum Response of Driven Field to Potential Changes
SBB89
m^2
0
$5H^2$
$-5H^2$
115
110
ln(1+z)
ln(1+z)=
115
110
105
100
(a)
20
10
0
−10
log [$\mathcal{P}_\phi(k)/H^2$]
−4
−3
−2
−1
0
1
2
3
log [k (Mpc^{-1})]



5H²
0
−5H²
m²
115
110
ln(1+z)
ln(1+z)=
120
115
110
100
(b)
log [𝒫_φ(k)/H²]
log [k(Mpc⁻¹)]
15
10
5
0
−5
−10
−4
−3
−2
−1
0
1
2
3

SBB89



The net result would be non-Gaussian fluctuations in ϕ_2. If the field ϕ_1 does not enter into a prolonged $m_{11}^2>0$ phase after the $m_{11}^2<0$ phase, significant non-Gaussian isocon fluctuations would survive. Thus the generic case for moguls centered about a $\phi_{1\text{ri}}(\phi_2)$ ridge line which is continuous with the incoming $\phi_{1\text{tr}}(\phi_2)$ trough line, as in the $\nu<0$, $\mu_1=0$ case of Sec. VI C, is a non-Gaussian "mountain," provided

$$\langle(\delta\phi_1)^2(\mathbf{x},t)\rangle^{1/2} \gtrsim |\bar{\phi}_1-\phi_{1\text{ri}}| \ . \tag{6.13}$$

stochastic inflation *Vilenken, Starobinski* Salopek+B 90/91 & B91 ..

extend heat ideas to inflation - gentle short-stretch chaos & nonG?
Stochastic inflation: *insert a moving bipartite uniform k-boundary into the full field equations:* cg/fg split at $k_c(t)$~lnHa
=> coarse-grain condensate + fine-grain quantum fluctuations

$dq_c^A = V_c^A dT + K^A_\nu \sqrt{dT}\, \eta^\nu$(GRD) via gradient expansion of Einstein+field eqs
time **T=lnHa** (breaks down at eoi, but best hypersurface for wave fronts)
diffusion tensor $D^{AB}=(KK^\dagger)^{AB}/2$
+(linearized) fluctuation equations fo r q_f^A k-modes. slow X_c variation

(constrained) Fokker-Planck equation for Shannon entropy $s(q)=-\ln\mathcal{P}(q)\sqrt{g}$
$\partial s/\partial T + (V_c + V_D)^A \partial s/\partial q^A - \partial (V_c + V_D)^A / \partial q^A = 0$ *or [ds/dT (fg->cg)]*
$\sqrt{g}$ = parameter-volume deformation
cf. QM $s=2s_I$ conserved, s_R hit by s_I quantum diffusion aka V_D^A

KS entropy *rate* **~-D**ln$\mathcal{P}$**(q)/dt ~ Trace shear** *(positive eigenvalue sum)*

diffusion velocity $V_D^A = D^{AB}\partial s/\partial q^A$ & current $J_D^A = e^{-s} V_D^A$
trajectory divergence via shear = 1/2 d/dt Trace ln g = $\partial V^A/\partial q^B$

aside: momentum kicked off the attractor is quickly damped down to the attractor => attractor approx $V_c^A \sim \partial S(q_c^A)/\partial q_c^A$ for field momentum

back to preheating:
through eoi $\mathbf{D^{AB}}$ is small, ballistic $\mathbf{dq_c^A = V_c^A dT}$ but chaotic if shear eigenvalues are positive (Kolmogorov-Sinai "entropy rate" >0)
until nonlinear couplings (shock-in-time)
often t_scramble well-separated from t_dissipation *in the MSS sense*
examples: correlated perturbative nonG cf. uncorrelated nonG subdominant to inflaton zeta fNL spike chaotic billiards

trajectory approach to nonG post-inflation:

$\mathbf{d\langle\zeta|\, \chi_{eoi}\rangle = Response(\chi_{eoi})\, d\chi_{eoi}}$ aka $\mathcal{E}(\zeta \mid \chi_{eoi})$ integrates to $\mathbf{\langle\zeta_{NL}|\, \chi_{eoi}\rangle}$

general: $\langle\zeta_{NL}|\, \chi_{eoi}, g, ..\rangle\, (x)$ via marginalization over UV (to k~1 Mpc^{-1}) and constrain in IR k <H_0 for LSS/CMB applications
complication/joy: ζ is conserved in the ballistic phase, sudden generation by fluctuation generation. but Trace shear is non-zero, the KS entropy => Shannon entropy story again

tools: fast lattice codes defrost++ and spectral code.
very fast dynamical systems theory calculations of various potentials, with conformal parameters, modified kinetic pieces in Lagrangian

condensate/fluctuation framework, classical-like approach with hbar + Bogoliubov transformations for fluctuations as condensate evolves => particle creation & fluctuation freeze-out into new condensate

stochastic inflation | q_c ; q_f>
Langevin network evolution step: $q_c(X,T+dT) = q_c(X,T) + V_c dT + \delta q_f$

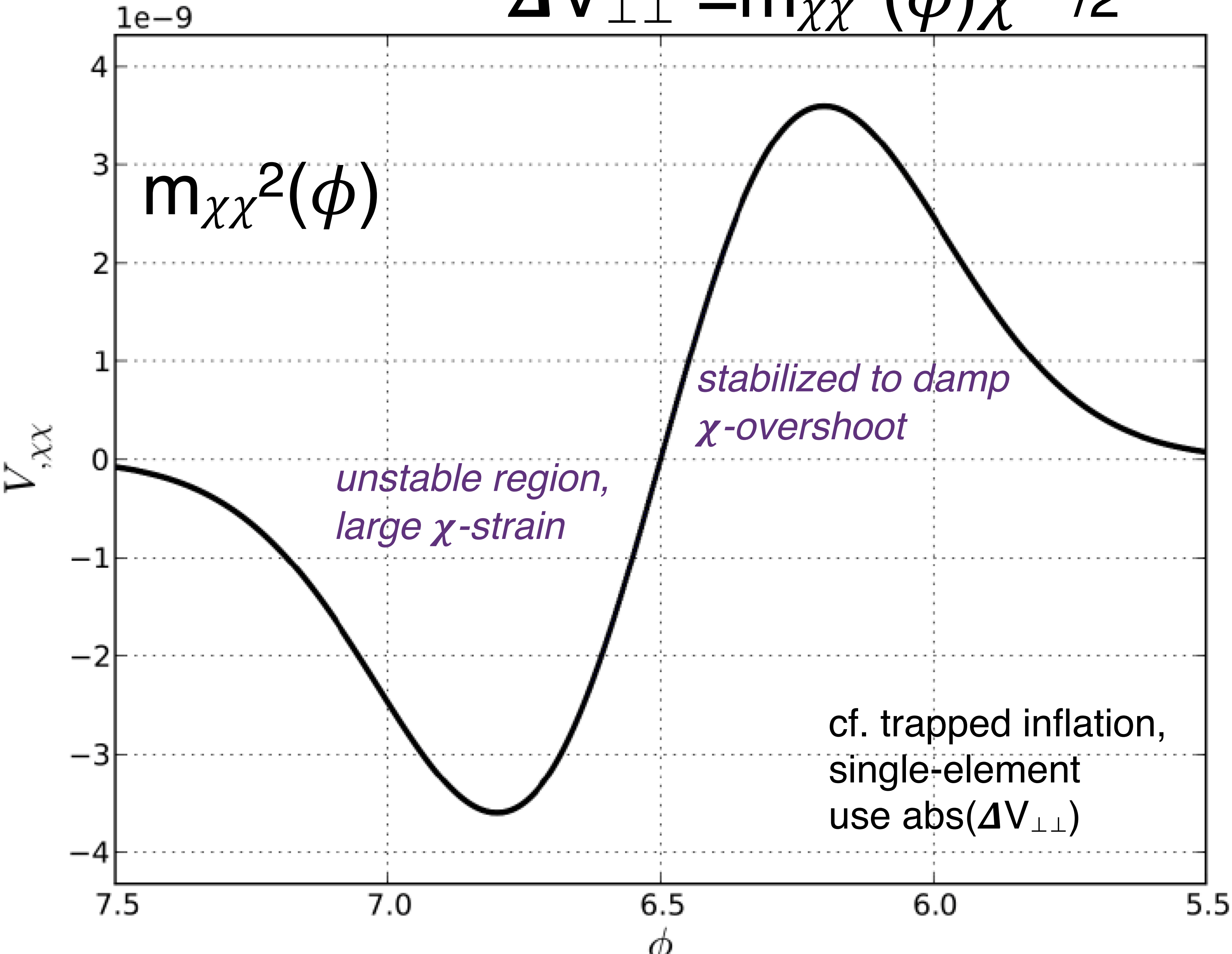
cf. **condensate evolution** step $|q_c(T+dT)\rangle = \exp(V_c dT) \exp(\delta q_f) |q_c(T)\rangle$
schematic $\delta q_f(x,T) = \sum_{k\text{-band}} (Q_k^*(x,T) a_k^T - Q_k(x,T) a_k)$
$V_c dT = V_c(x,T) dT (a_x^T - a_x)$
annihilation/creation operators in position and momentum a_x, a_x^T a_k, a_k^T

fluctuating part $|q_f\rangle \sim \exp(\sum \delta q_f) | 0\rangle$ *a coherent state description?*
what is the relation to the usual $q_{f,op} = \sum_k (Q_k^*(x,T) a_k^T + Q_k(x,T) a_k)$
operator linear in Bunch Davies vacuum operators a_k, a_k^T (sign difference)

it is an **overcomplete basis representation**, **but it conforms to a classical lattice simulation of inflation (no bipartite split).**
still use the gradient expansion for $|q_c(T)\rangle$
& mixed operators $V_c dT$ and $\delta q_f(x,T)$ **-** promising approach?

trajectory bundles & particle creation during inflation. **fast instabilities eg transverse + nonlinearity** or else **ζ-conservation** with no generation.
eg in state enters “chaotic unstable V-region” leaves as out state

$$\Delta V_{\perp\perp} = m_{\chi\chi}{}^2(\phi)\chi^2 \;/2$$

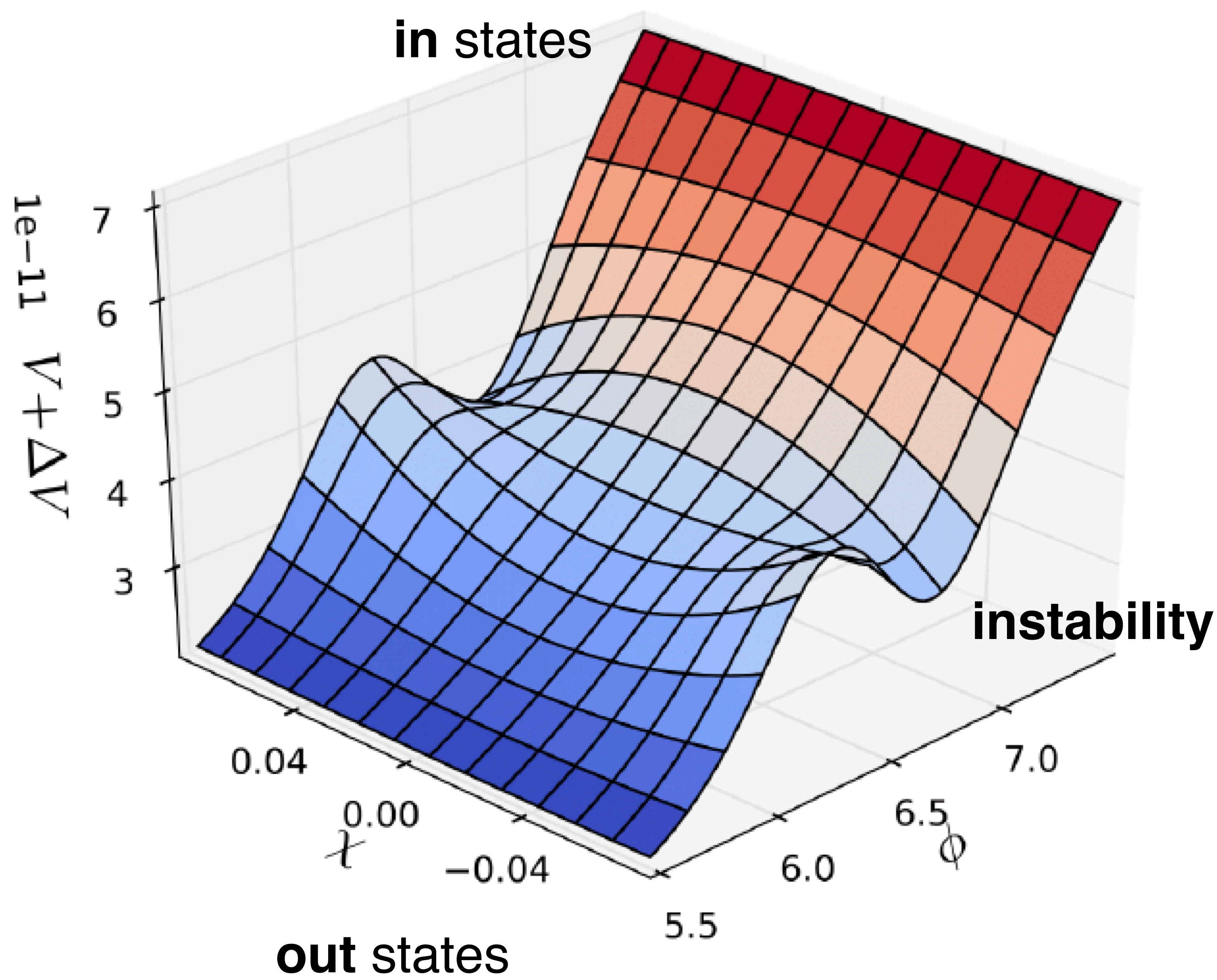


$$V(\phi,\chi) = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_\chi}{4}\chi^4 + \Delta V(\phi,\chi)$$

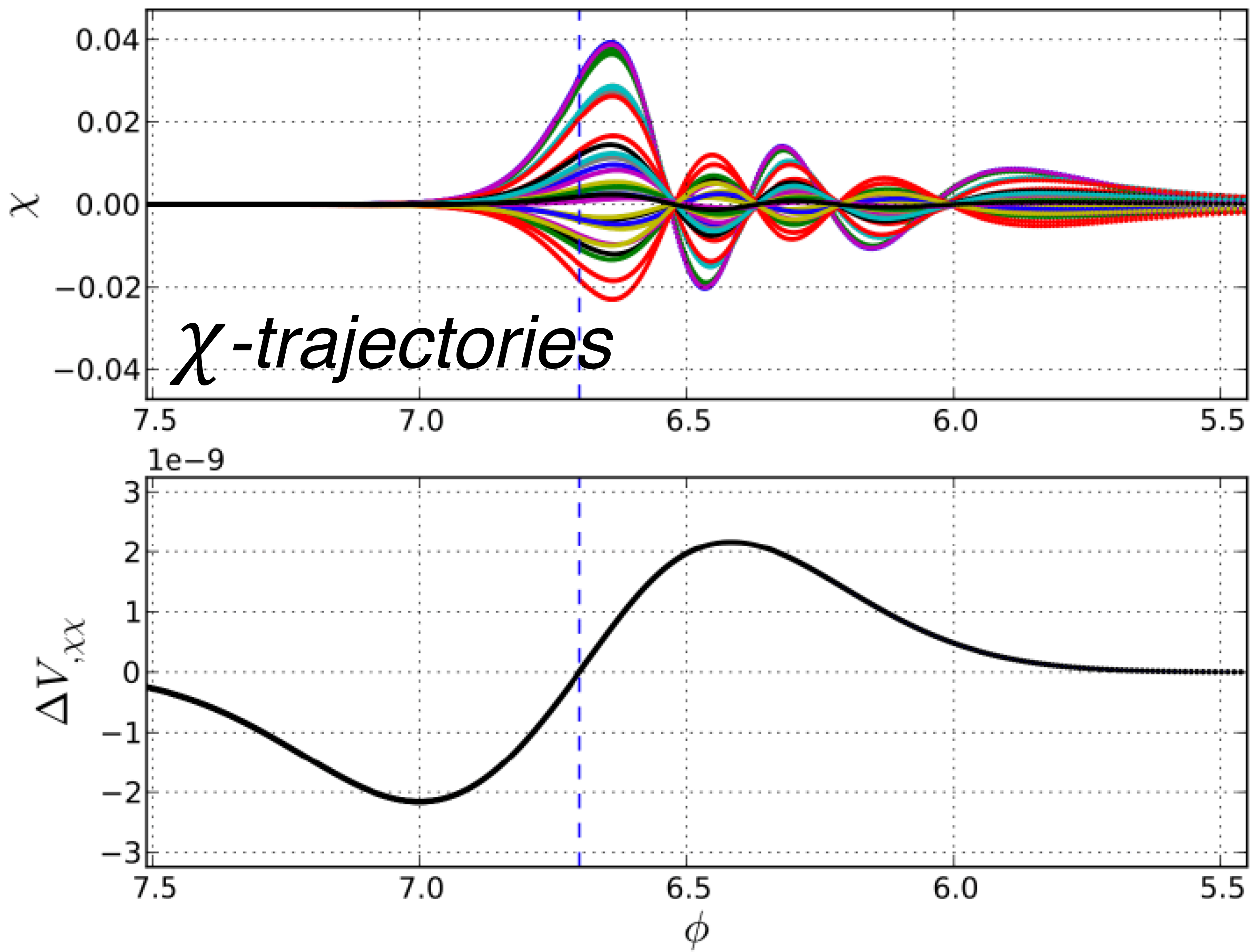
$$\Delta V = -\frac{A^2\sqrt{e}}{b}(\phi-\phi_p)\exp\left[-\frac{(\phi-\phi_p)^2}{2b^2}\right]\chi^2$$

$$= m_{\chi\chi}{}^2(\phi)\chi^2\ /2$$

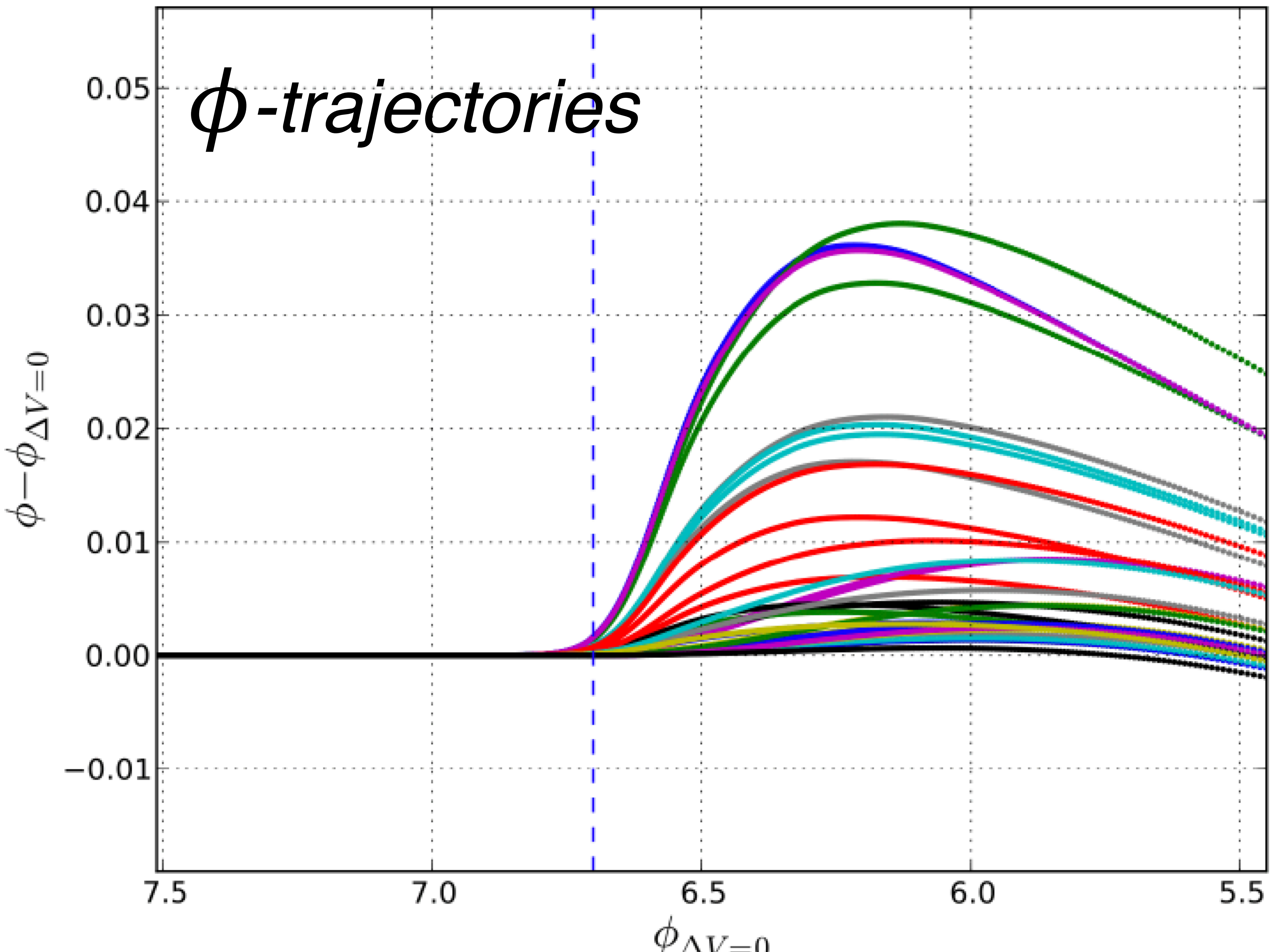
$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \sum_i \frac{\nabla\dot{\phi}_i\cdot\nabla\phi_i + \dot{\phi}_i\nabla^2\phi_i}{3a^2(\rho+P)}$$



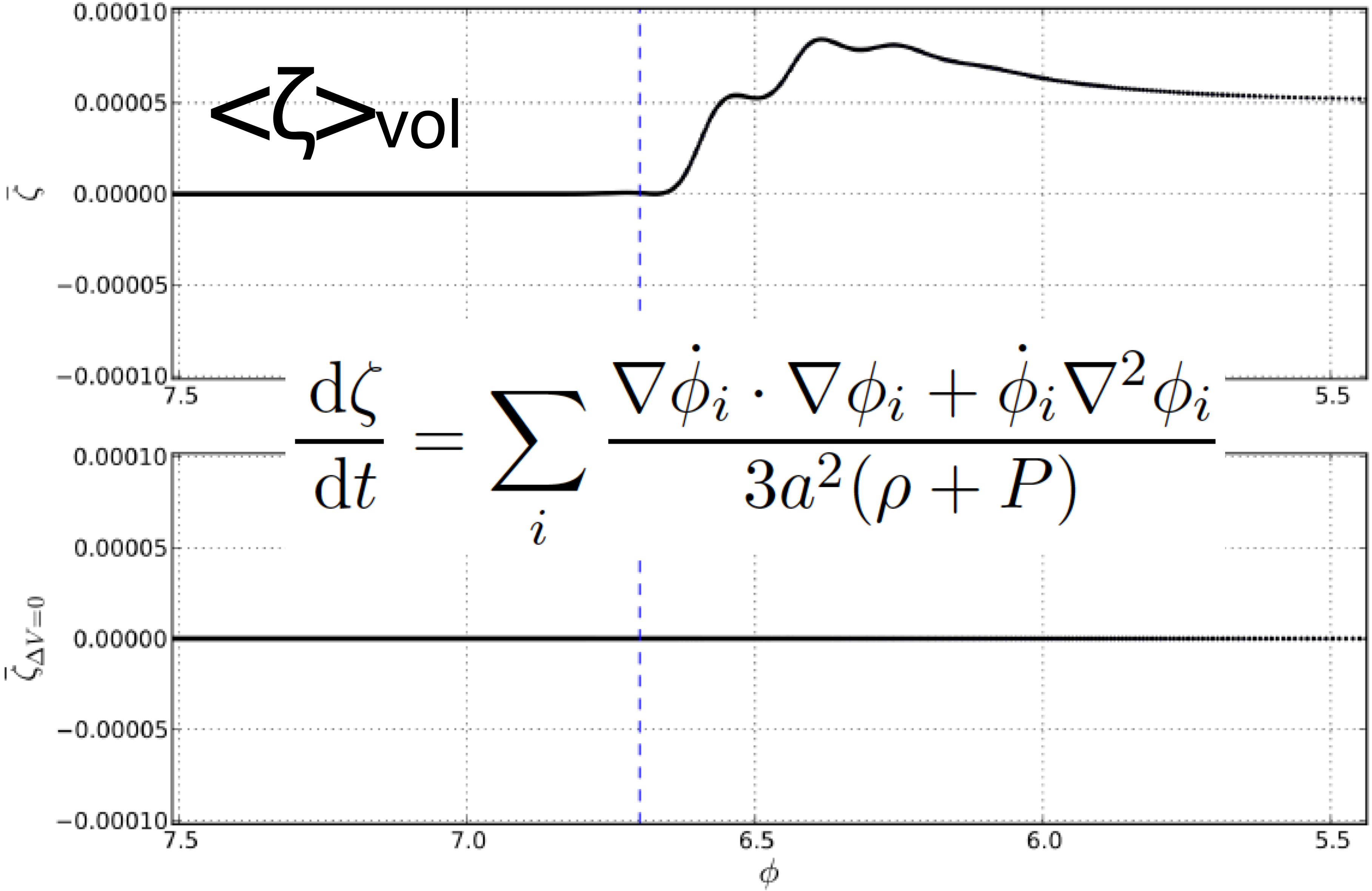
in states
instability
out states
1e−11
$V + \Delta V$
7
6
5
4
3
0.04
0.00
−0.04
χ
5.5
6.0
6.5
7.0
ϕ



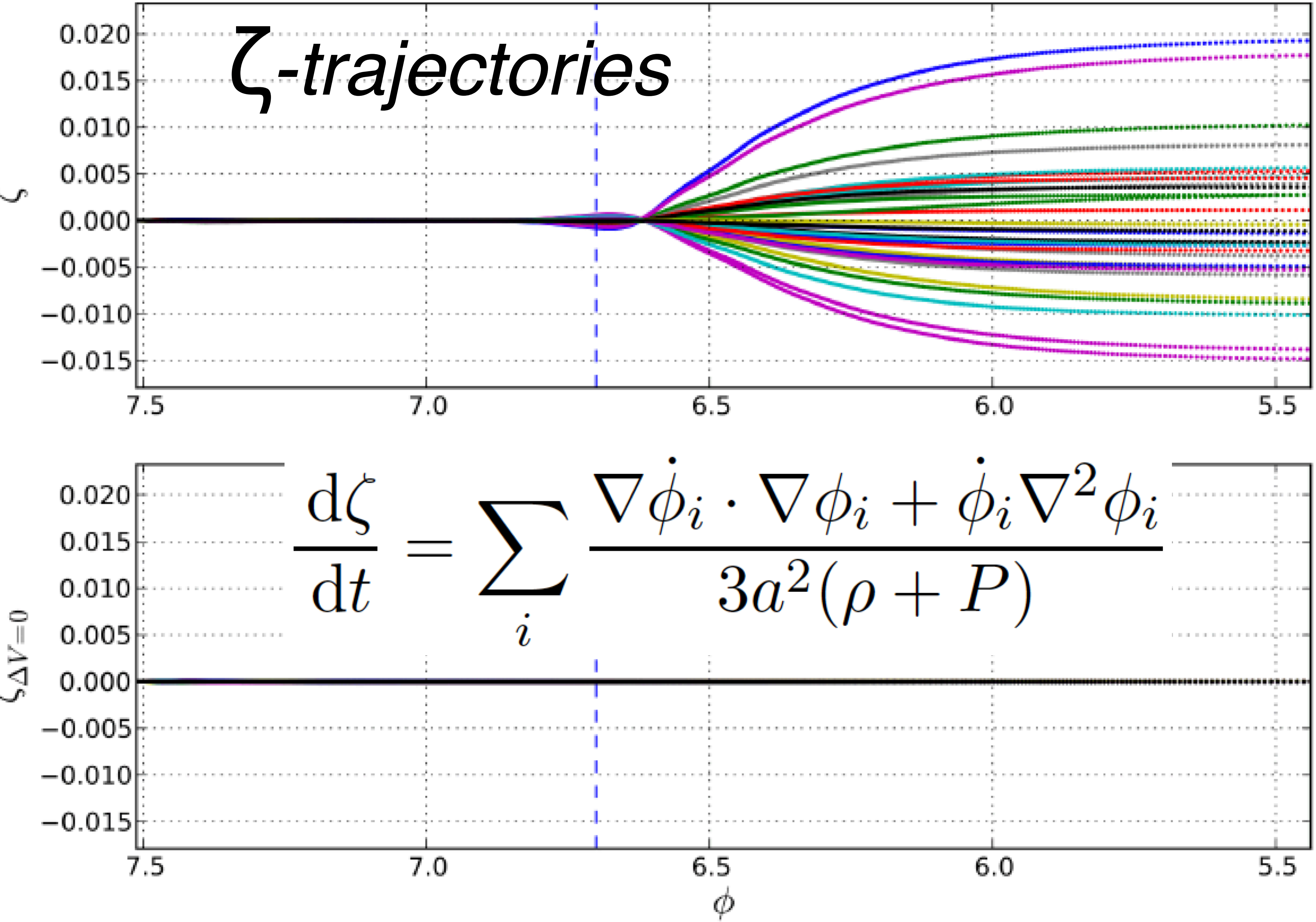
χ-trajectories
χ
0.04
0.02
0.00
−0.02
−0.04
7.5
7.0
6.5
6.0
5.5
1e−9
$\Delta V_{,\chi\chi}$
3
2
1
0
−1
−2
−3
ϕ



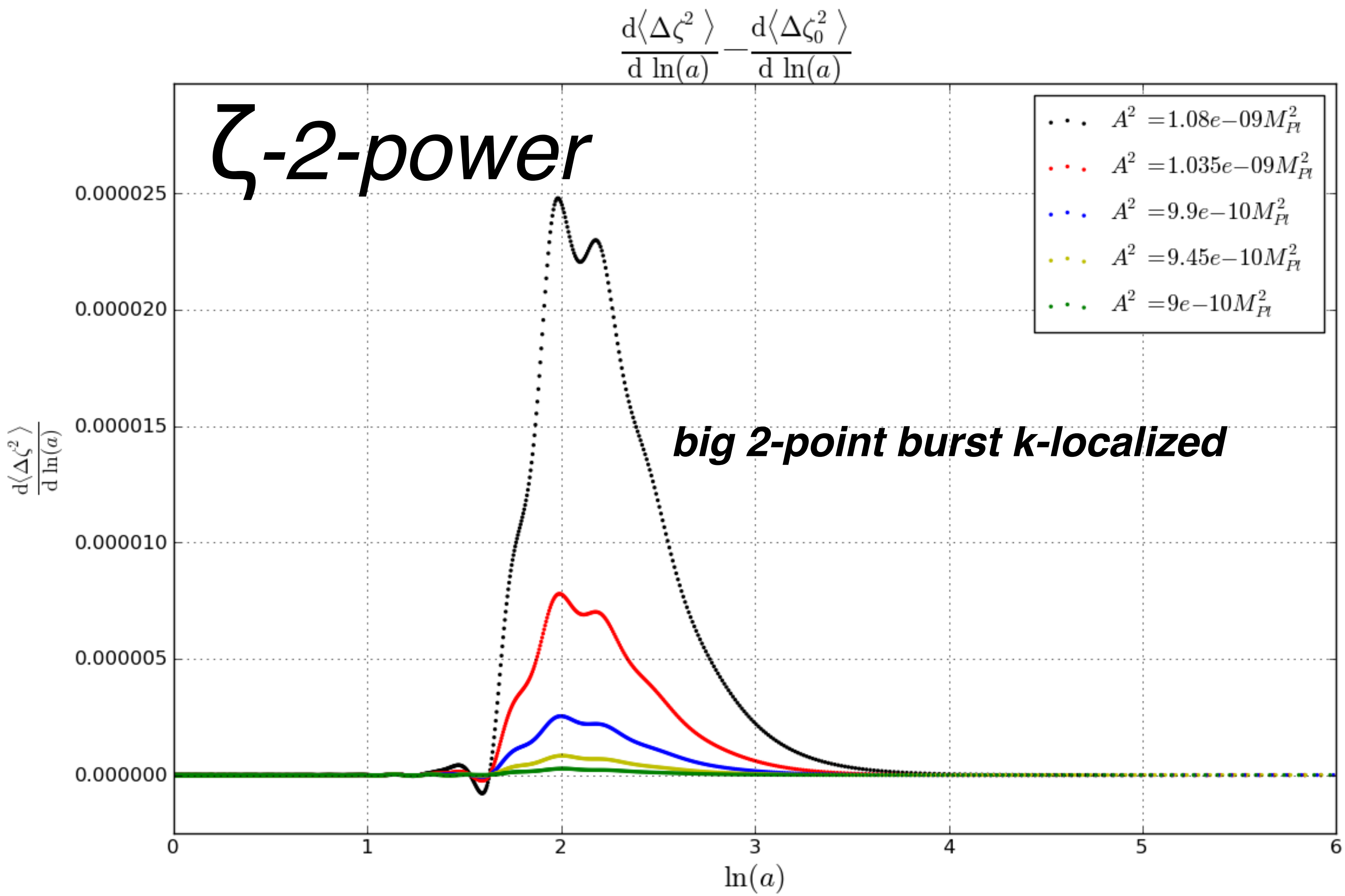
ϕ-trajectories
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
$\phi - \phi_{\Delta V=0}$
7.5
7.0
6.5
6.0
5.5
$\phi_{\Delta V=0}$

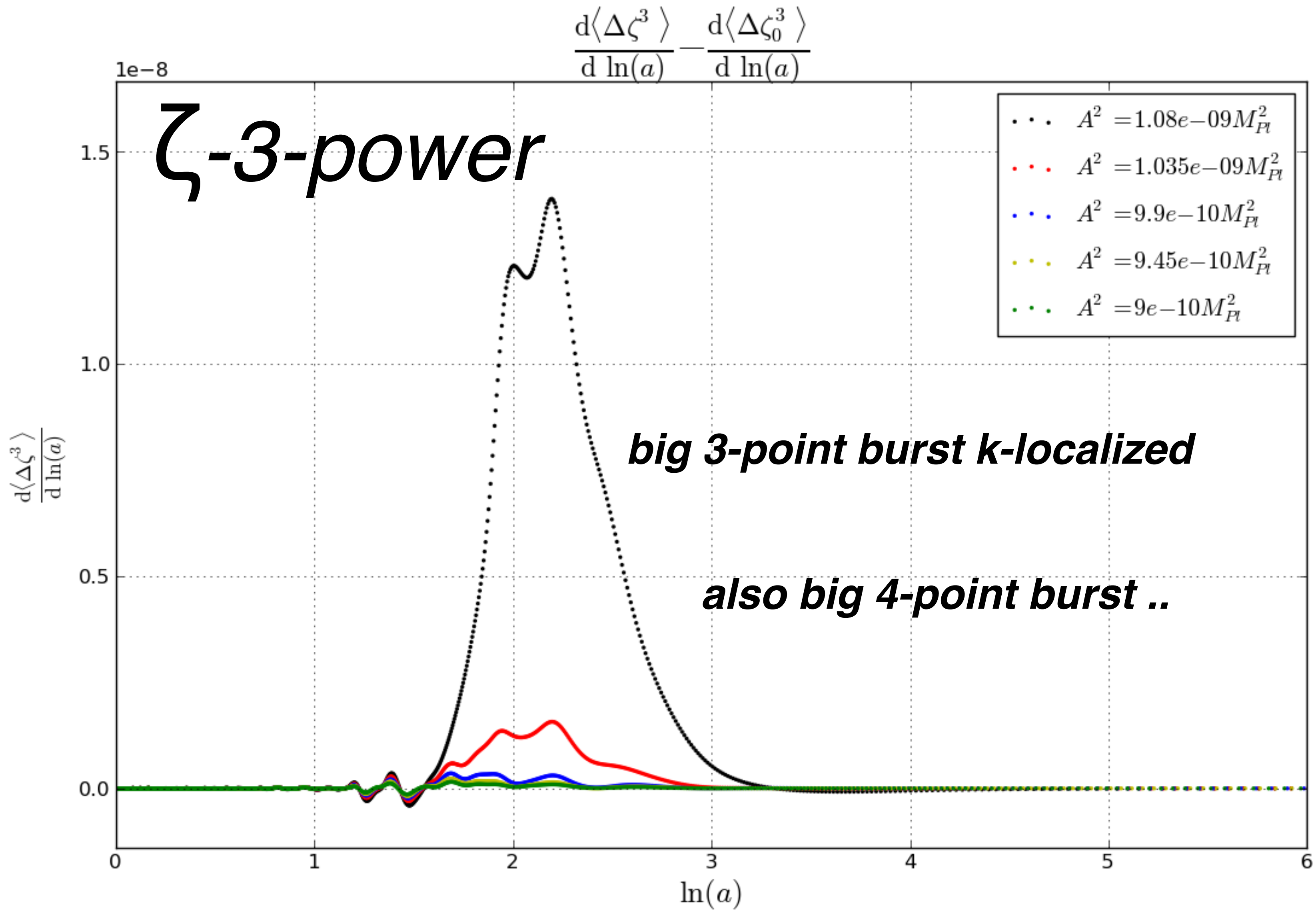


$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \sum_i \frac{\nabla\dot{\phi}_i \cdot \nabla\phi_i + \dot{\phi}_i \nabla^2 \phi_i}{3a^2(\rho + P)}$$

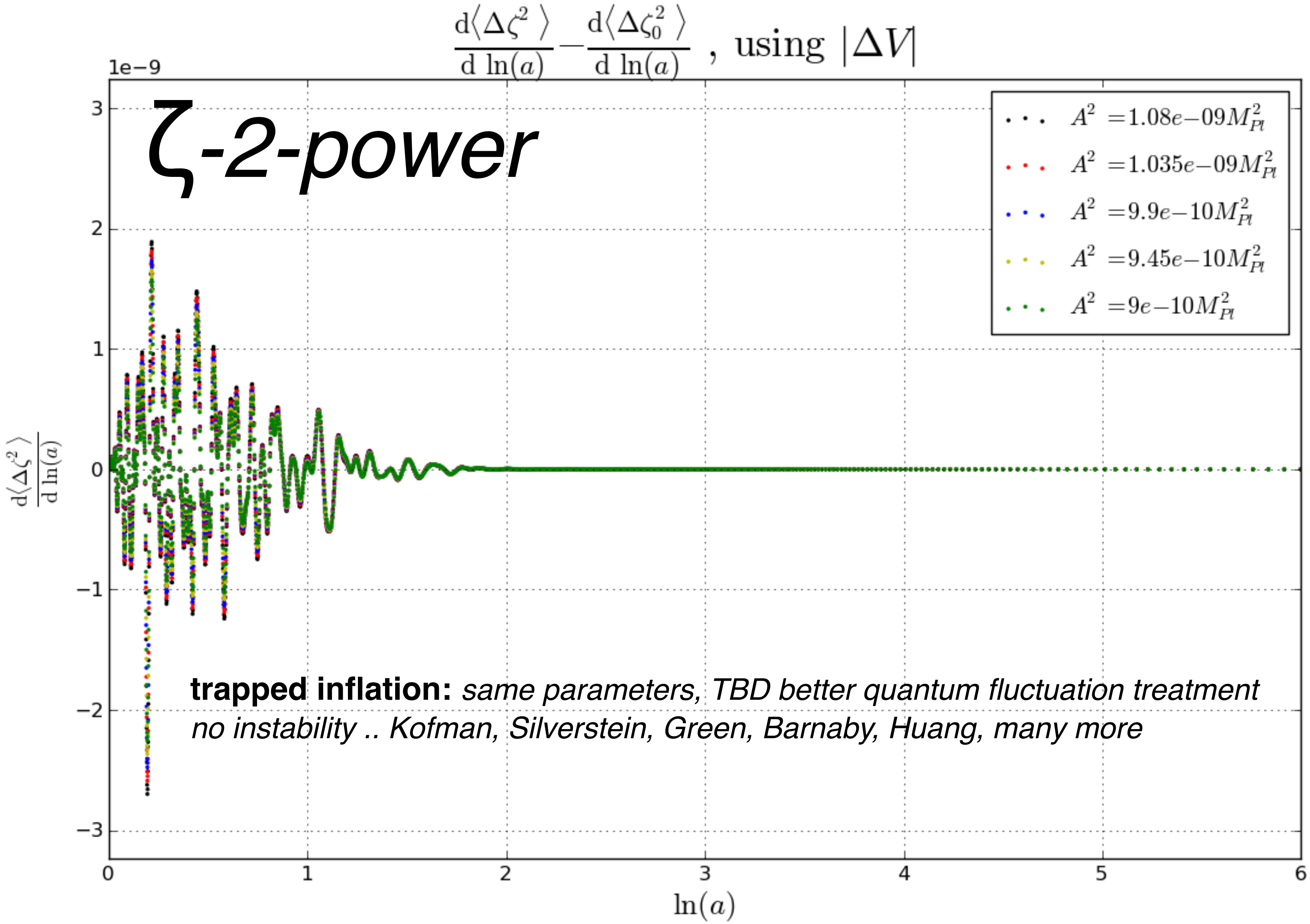


$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \sum_i \frac{\nabla\dot{\phi}_i \cdot \nabla\phi_i + \dot{\phi}_i \nabla^2 \phi_i}{3a^2(\rho + P)}$$

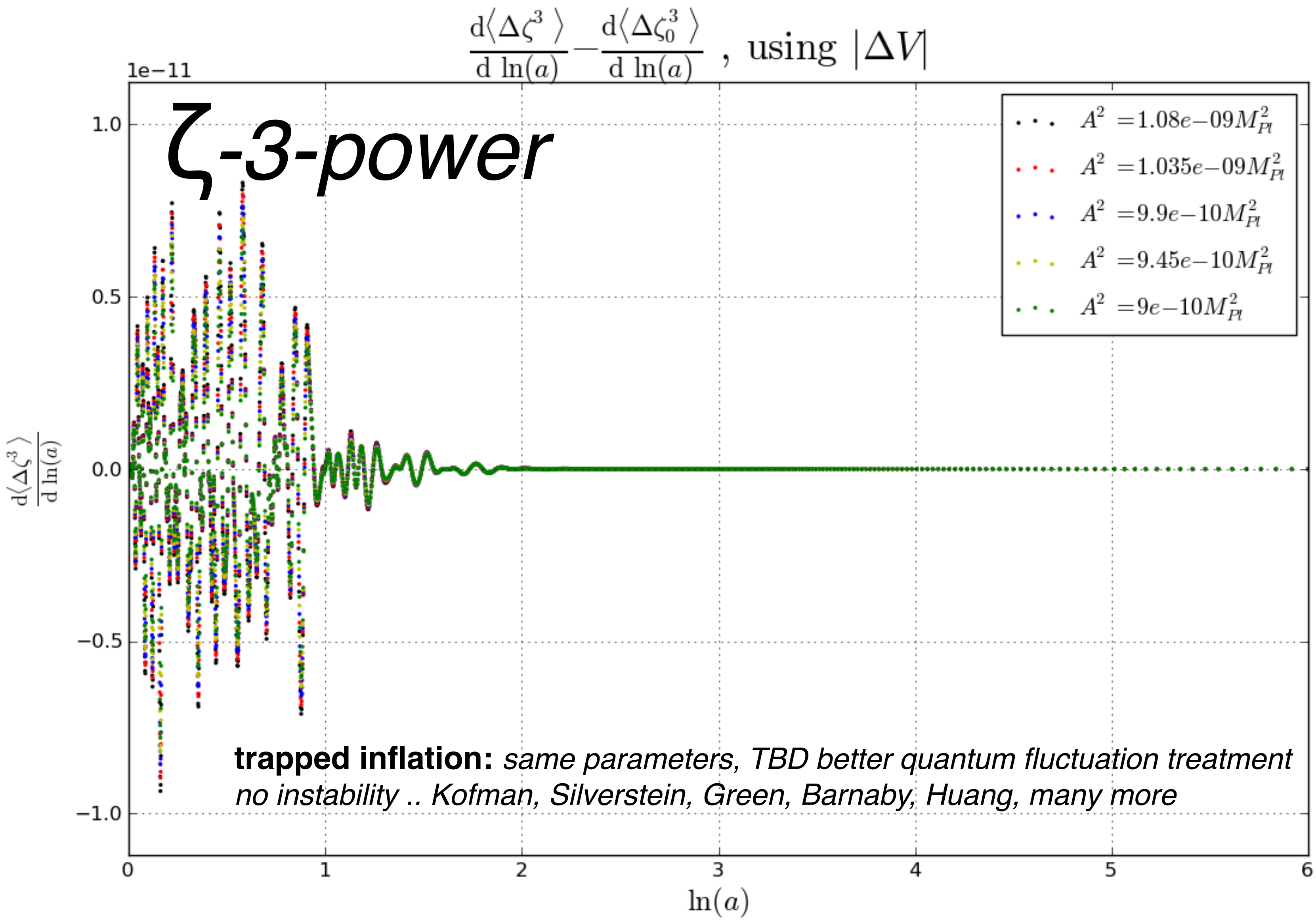

ζ-2-power
$\frac{d\langle\Delta\zeta^2\rangle}{d\ln(a)} - \frac{d\langle\Delta\zeta_0^2\rangle}{d\ln(a)}$
$A^2 = 1.08e{-}09 M_{Pl}^2$
$A^2 = 1.035e{-}09 M_{Pl}^2$
$A^2 = 9.9e{-}10 M_{Pl}^2$
$A^2 = 9.45e{-}10 M_{Pl}^2$
$A^2 = 9e{-}10 M_{Pl}^2$
big 2-point burst k-localized
$\frac{d\langle\Delta\zeta^2\rangle}{d\ln(a)}$
0.000025
0.000020
0.000015
0.000010
0.000005
0.000000
0
1
2
3
4
5
6
$\ln(a)$



$\frac{d\langle\Delta\zeta^3\rangle}{d\ln(a)} - \frac{d\langle\Delta\zeta_0^3\rangle}{d\ln(a)}$
1e−8
ζ-3-power
big 3-point burst k-localized
also big 4-point burst ..
$A^2 = 1.08e{-}09 M_{Pl}^2$
$A^2 = 1.035e{-}09 M_{Pl}^2$
$A^2 = 9.9e{-}10 M_{Pl}^2$
$A^2 = 9.45e{-}10 M_{Pl}^2$
$A^2 = 9e{-}10 M_{Pl}^2$
$\frac{d\langle\Delta\zeta^3\rangle}{d\ln(a)}$
$\ln(a)$



$\frac{d\langle\Delta\zeta^2\rangle}{d\ln(a)} - \frac{d\langle\Delta\zeta_0^2\rangle}{d\ln(a)}$, using $|\Delta V|$
1e−9
ζ-2-power
$A^2 = 1.08e{-}09 M_{Pl}^2$
$A^2 = 1.035e{-}09 M_{Pl}^2$
$A^2 = 9.9e{-}10 M_{Pl}^2$
$A^2 = 9.45e{-}10 M_{Pl}^2$
$A^2 = 9e{-}10 M_{Pl}^2$
$\frac{d\langle\Delta\zeta^2\rangle}{d\ln(a)}$
$\ln(a)$
trapped inflation: *same parameters, TBD better quantum fluctuation treatment*
no instability .. Kofman, Silverstein, Green, Barnaby, Huang, many more



$\frac{d\langle \Delta\zeta^3 \rangle}{d\ \ln(a)} - \frac{d\langle \Delta\zeta_0^3 \rangle}{d\ \ln(a)}$, using $|\Delta V|$
ζ-3-power
1e−11
$A^2 = 1.08e{-}09 M_{Pl}^2$
$A^2 = 1.035e{-}09 M_{Pl}^2$
$A^2 = 9.9e{-}10 M_{Pl}^2$
$A^2 = 9.45e{-}10 M_{Pl}^2$
$A^2 = 9e{-}10 M_{Pl}^2$
$\frac{d\langle \Delta\zeta^3 \rangle}{d\ \ln(a)}$
$\ln(a)$
trapped inflation: *same parameters, TBD better quantum fluctuation treatment*
no instability .. Kofman, Silverstein, Green, Barnaby, Huang, many more

occupation numbers & particle creation ~ "Gaussian entropy" in the single A-field

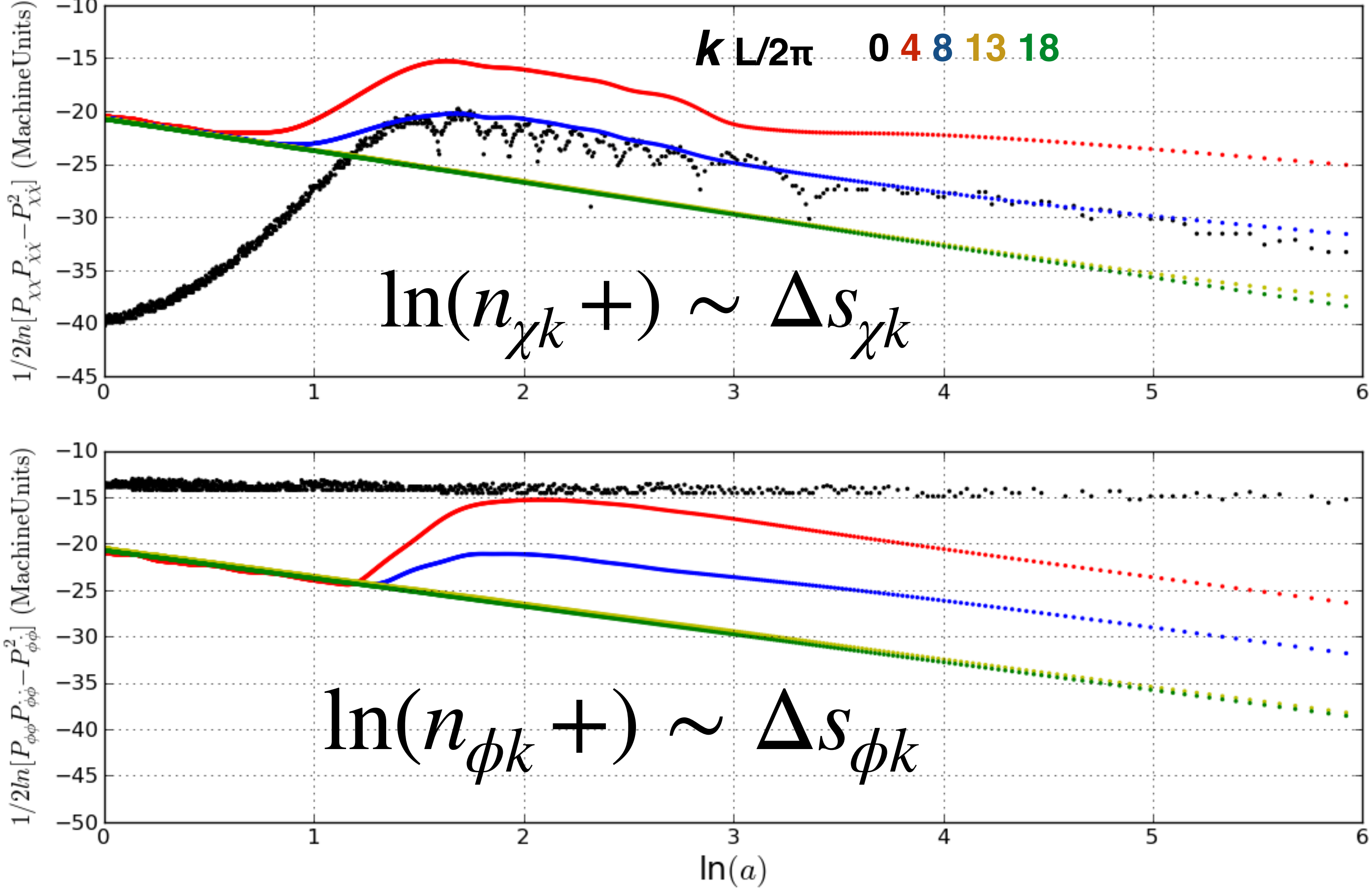
$$\ln(n_{\phi^A k} +) \sim \Delta s_{\phi^A k} \sim \qquad \frac{1}{2}\mathrm{Trace}\,\ln[C_{\phi^A\phi^A}C_{\Pi_A\Pi_A} - C^2_{\phi^A\Pi_A}]$$

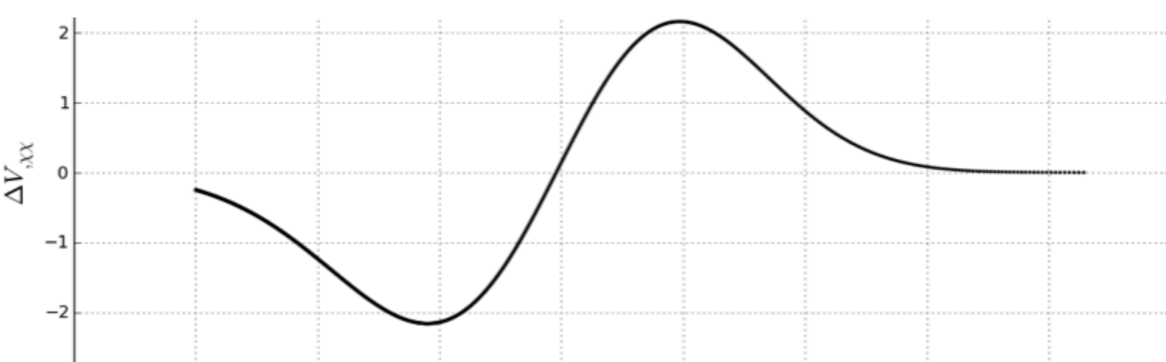
occupation ~ $n_{Ak} \sim e^{\Delta s_{Ak}}$

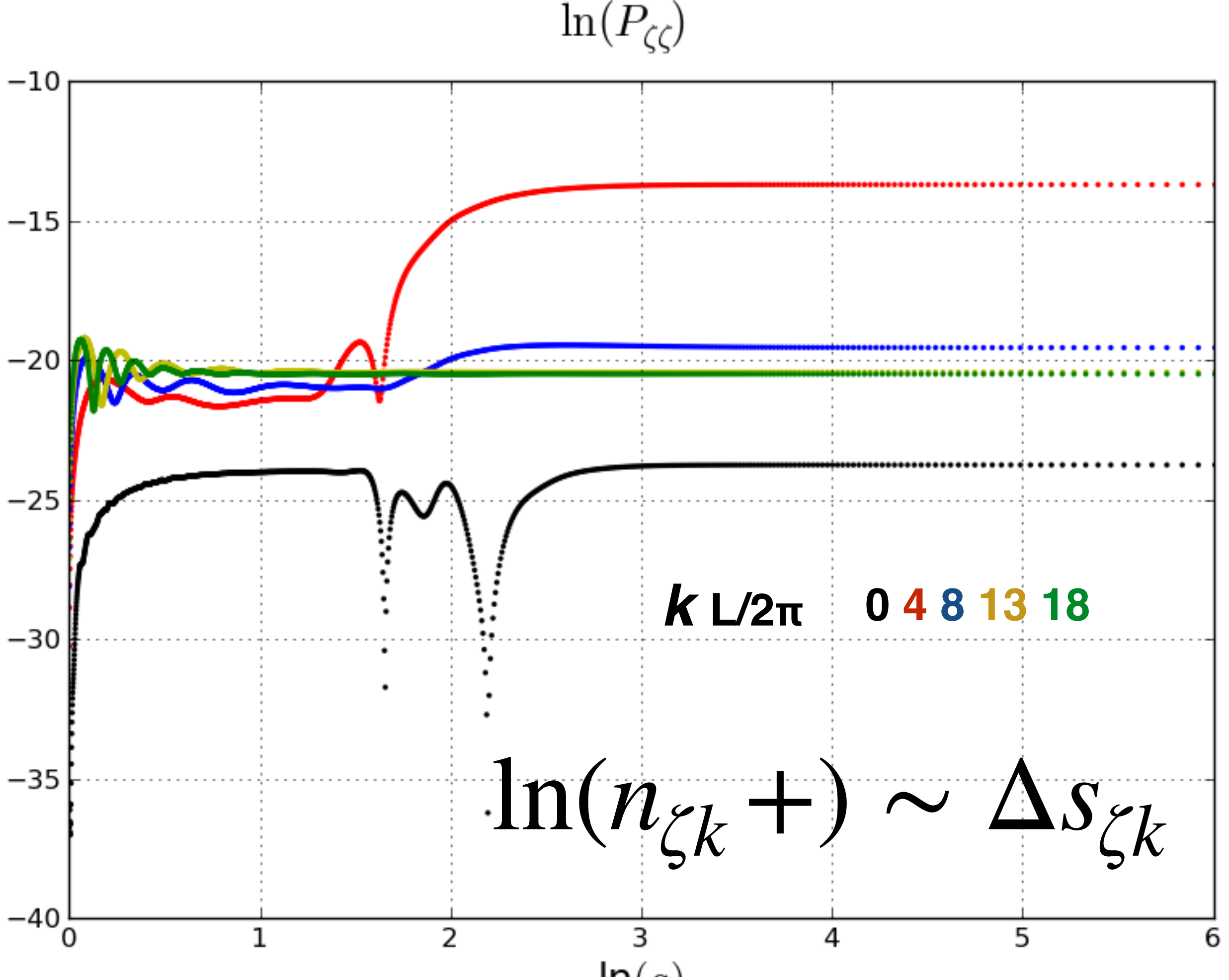
old way if well defined mode energies $\omega_{Ak}(t)$ $\quad \ln(n_{Ak} + 1/2) \sim \ln[\rho_A/\hbar\omega_{Ak}]$

full "Gaussian entropy" in the 2 fields, C are k-mode correlations = power spectra - generalized Sackur-Tetrode

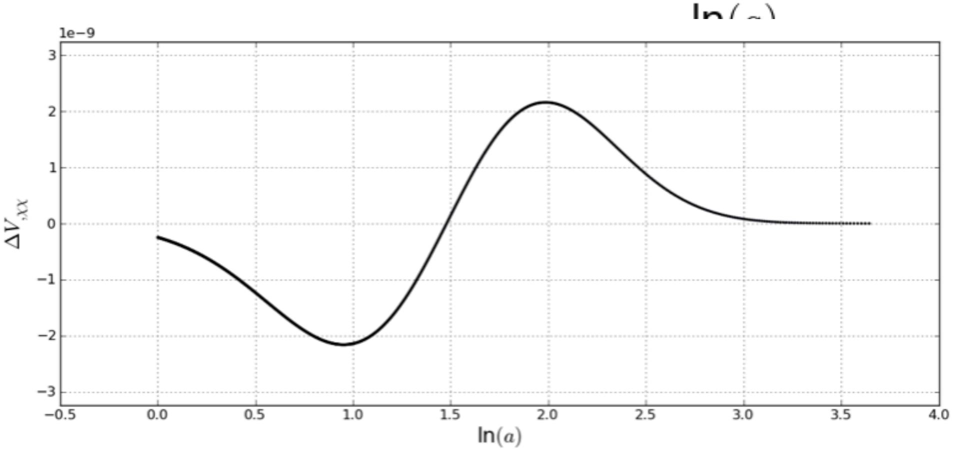
$$\Delta s_{A+B,k} = \frac{1}{2}\mathrm{Trace}\,\ln[C_{\phi^A\phi^B}C_{\Pi^A\Pi^B} - C_{\phi^A\Pi^B}C_{\Pi^A\phi^B}]$$

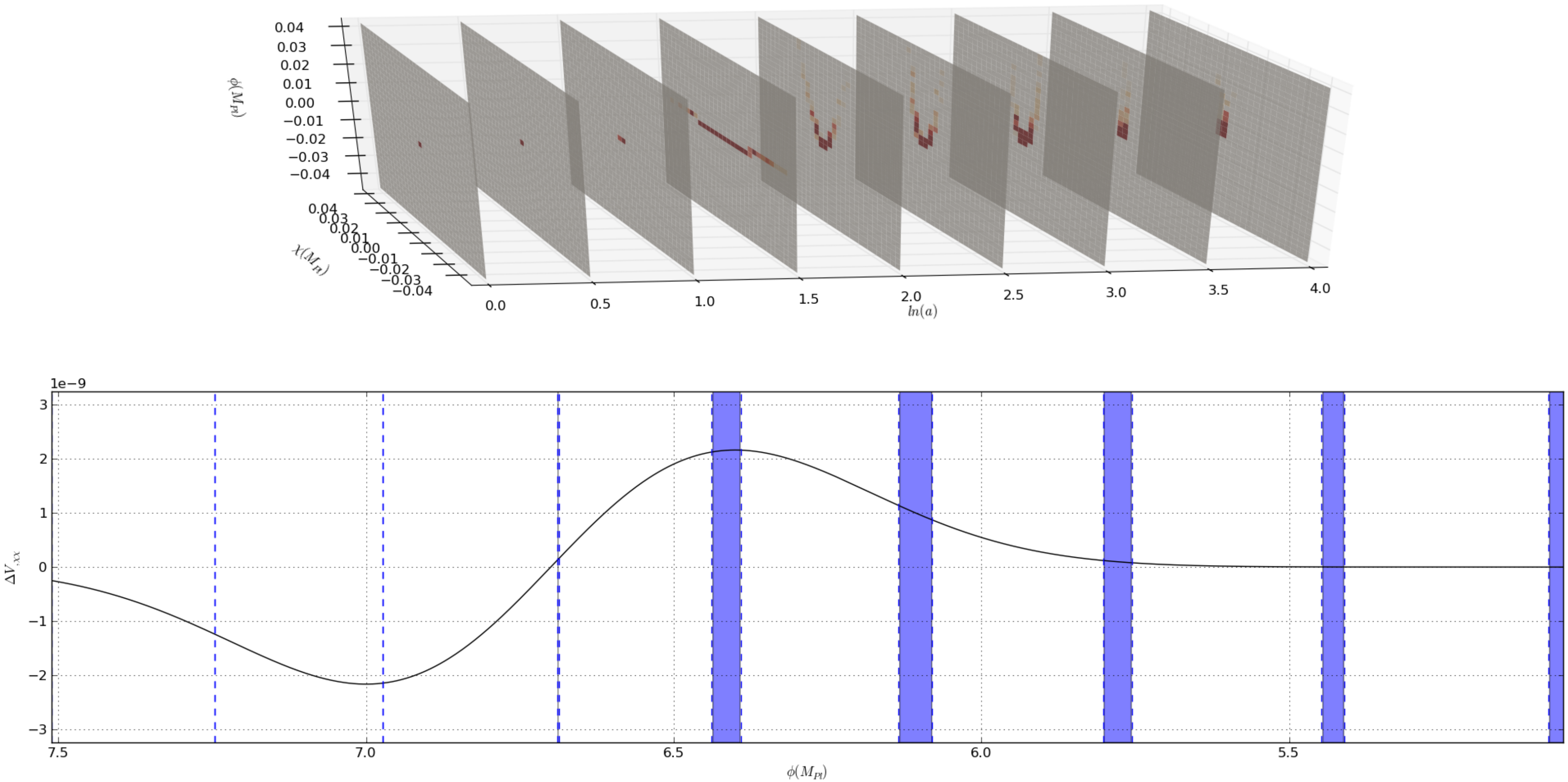

k L/2π 0 4 8 13 18
$\ln(n_{\chi k}+) \sim \Delta s_{\chi k}$
$1/2 ln[P_{\chi\chi} P_{\dot\chi\dot\chi} - P^2_{\chi\dot\chi}]$ (MachineUnits)
$\ln(n_{\phi k}+) \sim \Delta s_{\phi k}$
$1/2 ln[P_{\phi\phi} P_{\dot\phi\dot\phi} - P^2_{\phi\dot\phi}]$ (MachineUnits)
$\ln(a)$


$\Delta V_{,\chi\chi}$

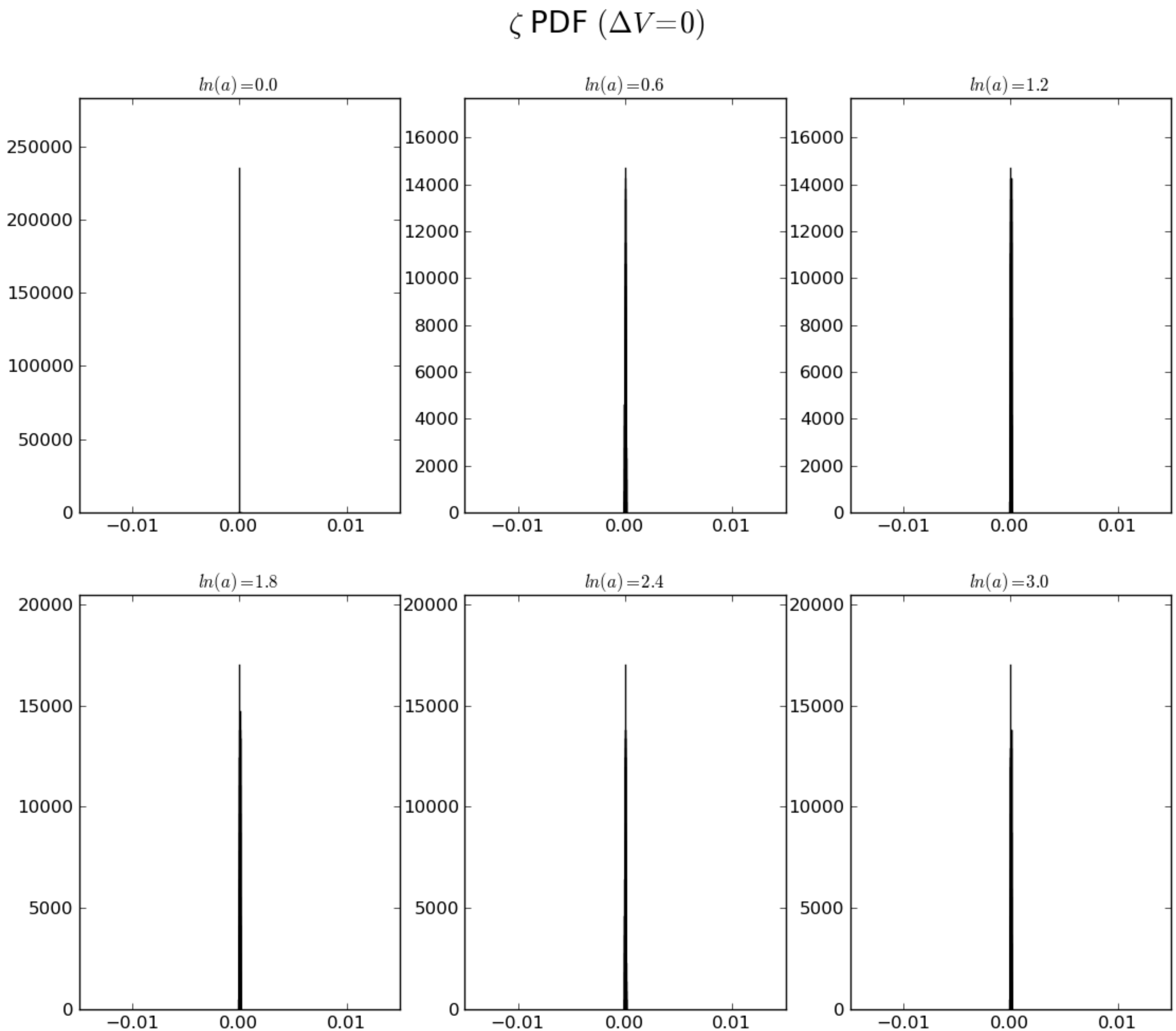


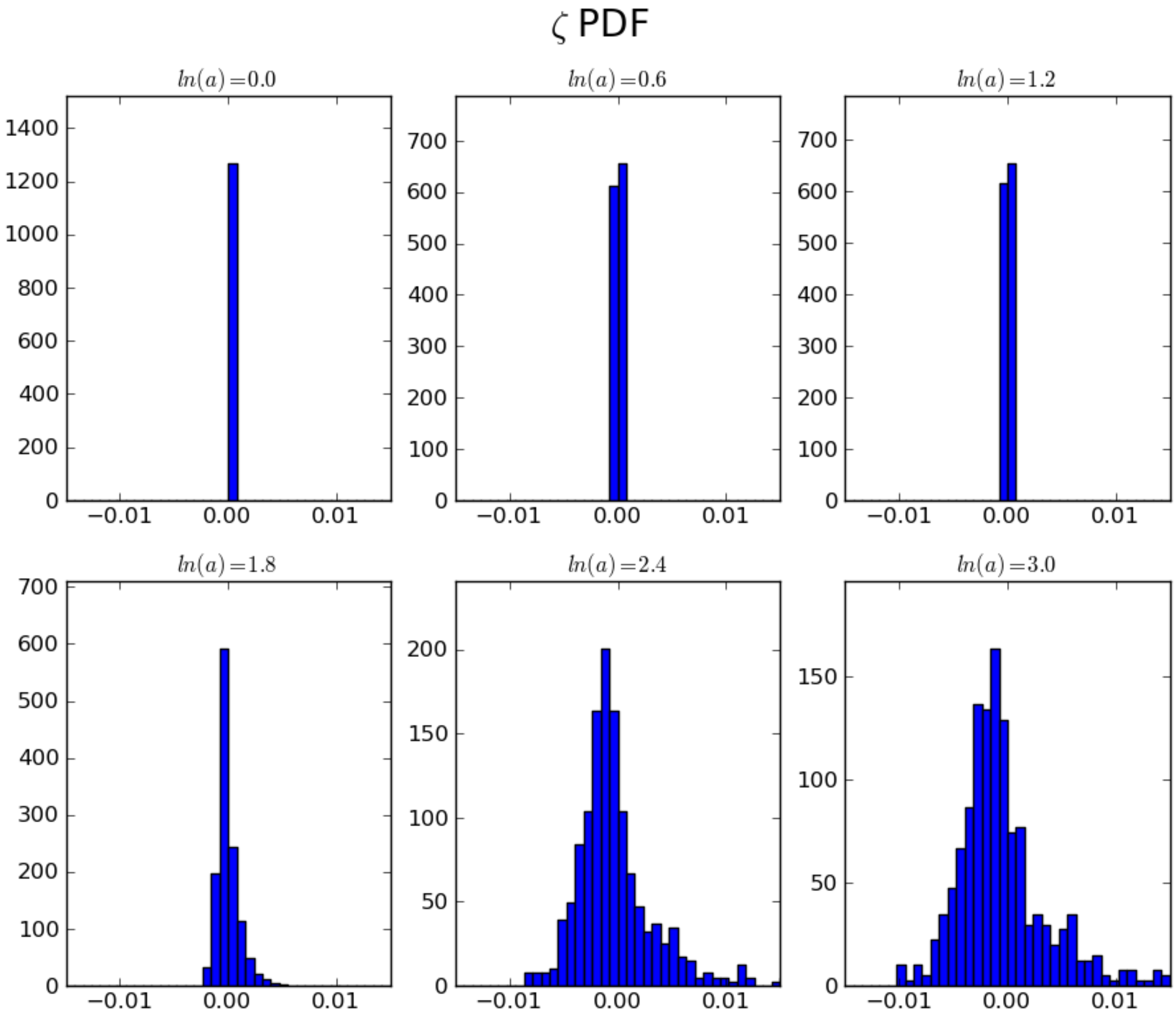
phonon occupation





$\phi(M_{Pl})$
$\chi(M_{Pl})$
$ln(a)$
1e−9
$\Delta V_{,\chi\chi}$
$\phi(M_{Pl})$


ζ PDF $(\Delta V = 0)$
$ln(a) = 0.0$
$ln(a) = 0.6$
$ln(a) = 1.2$
$ln(a) = 1.8$
$ln(a) = 2.4$
$ln(a) = 3.0$
0
50000
100000
150000
200000
250000
2000
4000
6000
8000
10000
12000
14000
16000
5000
10000
15000
20000
−0.01
0.00
0.01



ζ PDF
ln(a) = 0.0
ln(a) = 0.6
ln(a) = 1.2
ln(a) = 1.8
ln(a) = 2.4
ln(a) = 3.0
−0.01
0.00
0.01

condensate/fluctuation framework, classical-like approach with hbar + Bogoliubov transformations for fluctuations as condensate evolves => particle creation & fluctuation freeze-out into new condensate

stochastic inflation $| q_c ; q_f>$
Langevin network evolution step: $q_c(X,T+dT) = q_c(X,T) + V_c dT + \delta q_f$

cf. **condensate evolution** step $|q_c(T+dT)> = \exp(V_c dT) \exp(\delta q_f) |q_c(T)>$
schematic $\delta q_f(x,T) = \sum_{k\text{-band}} (Q_k^*(x,T) a_k^T - Q_k(x,T) a_k)$
$V_c dT = V_c(x,T) dT (a_x^T - a_x)$
annihilation/creation operators in position and momentum a_x, a_x^T a_k, a_k^T

fluctuating part $|q_f> \sim \exp(\sum \delta q_f) | 0>$ *a coherent state description?*
what is the relation to the usual $q_{f,op} = \sum_k (Q_k^*(x,T) a_k^T + Q_k(x,T) a_k)$ operator linear in Bunch Davies vacuum operators a_k, a_k^T (sign difference)

it is an **overcomplete basis representation**, **but it conforms to a classical lattice simulation of inflation (no bipartite split).**
still use the gradient expansion for $|q_c(T)>$
& mixed operators $V_c dT$ and $\delta q_f(x,T)$ - promising approach?

trajectory bundles & particle creation during inflation. **fast instabilities eg transverse + nonlinearity** or else **ζ-conservation** with no generation.
eg in state enters “chaotic unstable V-region” leaves as out state

END