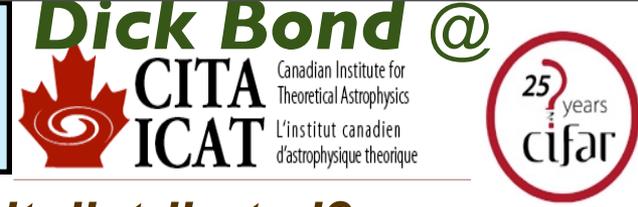


# Gaussian & non-G Random Fields in Early Universe Cosmology: Then and Now



## What was the Universe made of & how was it distributed?

**NOW:** baryons/leptons + (cold-ish) dark matter + dark energy/inflaton + tiny curvature energy (+photons+light neutrinos + gravity waves). ??a bit of strings/textures/PBHs??

Are there primordial non-Gaussian components - subdominant inflation-induced, preheating - induced or cosmic-string induced? *nonlinear & non-G web of galaxies/clusters*

**THEN:** coherent inflaton / "vacuum" energy + zero-point fluctuations in all fields (*Gaussian RF*) & then preheat via mode coupling to incoherent cascade to thermal equilibrium soup

**very early U**      early to middle to now U      **very late U**

*string theory/landscape/higher dimensions*

**inflation** cyclic  
 $V_{\text{eff}}(\psi_{\text{inf}}) ?$   
 $K_{\text{eff}}(\psi_{\text{inf}}) ?$   
**trajectory probability**

baryogenesis dark matter BBN  $\gamma$ dec

**dark energy**  
 $V_{\text{eff}}(\psi_{\text{inf}}) ?$   
 $K_{\text{eff}}(\psi_{\text{inf}}) ?$   
**trajectory probability**

$-\frac{d \ln \rho_{\text{tot}}}{d \ln a} / 2$   
 $= \mathcal{E}(k) = 1 + q, k \sim H a$

## cosmic mysteries

$n_b/n_\gamma$   $\rho_{\text{dm}}/\rho_b$   $z_{\text{eq}}/z_{\text{rec}}$   $\rho_{\text{curv}}$   $\rho_{\text{del}}/\rho_{\text{dm}}$   $\rho_{\text{de}} \sim H^2 M_{\text{Planck}}^2$   $\rho_{\text{mv}}/\rho_{\text{stars}}$

## brief history of bond's non-Gaussian exploration in inflation: THEN

early 80s: hot, warm & **cold** collisionless **dark matter** + inflation

⇒ **xCDM**: *86 extra power dilemma* ⇒ vary x:  $k_{\text{Heq}}$   $k_{\text{mn}}$   $k_{\text{features}}$

**87: X =**  $s/H_0$  /  $\Lambda$  / **Open** /  $i_s$  /  $i_s + a_d$  /  $h-c$  /  $h+$  /  $b/b$  /  **$\Lambda + b$**  /  $Op + b$  /  $\tau$  **BSI / BSI2**

90s-00s: data settled on **X =  $\Lambda$  + tilt** ⇒ **dark-energy + tilt**

Linde & Kofman 87, clipped pNGB  $(1 - \cos(\chi/f))$  string issue single field Grinstein, Allen & Wise 87

**SBB89:  $\delta H, \delta m^2_{ij}$** , **moguls, waterfalls** ⇒ plateau/mountain/valley

$V = \lambda_\phi \phi^4/4 + m_\phi^2 \phi^2/2 + \lambda_\chi \chi^4/4 + m_\chi^2 \chi^2/2 - \nu \phi^2 \chi^2/2 + 3\text{-leg}$  **HYBRID**; **inflaton + isocons**

trajectory bifurcation at  $m^2_{is, is} \leq 0$  **TACHYONIC** ⇒ **non-G** with & without 3-leg,  
avoid late domain walls (⇒ modify  $V_{\text{late time}}$ )

SB90/91: nonG technology:  $\mathbf{y}(\mathbf{r}, t) = \mathbf{y}_f(\mathbf{r}, t) + \mathbf{y}_b(\mathbf{r}, t) (+ \mathbf{y}_{>h}(\mathbf{r}, t))$  full  $EE + \phi/\chi$  eqs,  
restricted to the nonlinear longitudinal gauge (NL-LG); fluctuation/background split

⇒ **Langevin network**:  $\mathbf{y}_f$  linearized (fn of  $\mathbf{y}_b$ );  $\mathbf{y}_b(\mathbf{r}, t)$  super-"horizon"  $k < u H a$ ,

**drift+stochastic kicks**; reduced **Hamilton-Jacobi** eqn; identify  **$\ln a|_{H^*}$**  as the  
nonlinear generalization of  $\varphi_{\text{com}}$  or  $\zeta$

**setup for eternal alps (semi-eternal inflation, nonG at UUULSS, but  
nearly Gaussian over current horizon scales or baroque-ish V)**

**B91 full NL-LG nonG Langevin network:  $Y_{bi}(\mathbf{r}, t) \in \{\phi, \Pi_\phi, \chi, \Pi_\chi, \ln a, H \mid T = \ln H a\}$**

80-90s: arena for BSI & non-G near EOI, new fields coupling in  
 expected  $k \sim H a$  rule would apply. **pre-heating surprise!**

**$\ln a[\chi_i(x,t)]$**  from “subgrid”  $\propto H_e^{-1}$  lattice simulations of  $\phi_{\text{UHF}} \chi_{\text{UHF}}$

like stochastic f-b split, with no dropping of gradient or nonlinear terms

Why  **$\ln a[\chi_i]$** ? **ingredient 1** **chaotic zero modes** fill V arms, Lyapunov  $\log-\chi_i$   
 spacing, overtones as well; **ingredient 2** arm flow shuts off when  $m_{\chi\chi}^2$  rises  
 sharply at **vigorous preheating nonlinearity onset**  $\Rightarrow$  EOS change

$P(\chi | \chi_{\text{LF}}) \sim \exp[-(\chi - \chi_{\text{LF}})^2 / 2\sigma_{\text{HF}}^2]$  builds a usable **low-pass effective mean field**

does it work? linear  $\langle \chi | \chi_{\text{LF}} \rangle \sim \chi_{\text{LF}}$  is sharp-k filter f-b split BBKS86, BCEK90, BM96

fourier transform ( $F_{\text{NL}} - \langle F_{\text{NL}} | \chi_{\text{LF}} \rangle$ ) is small for  $k < k_{\text{LF}}$  for quadratic,  
 exponential & even **Gaussian spikes** (variance  $\sim 1\%$  at  $k_{\text{LF}} 0.15$  at  $k_{\text{LF}}/10$ )

**$\langle F_{\text{NL}} | \chi_b(x,t) + \chi_{>h} \rangle$  regimes** contrast with  $\phi_{>h}$ : may be way out there in eternal inflation  
 land, but not in a preheated  $\epsilon > 1$  patch

**LOW**  $\chi_{>h}$   $\beta_{\chi} \chi_b + f_{\chi} \chi_b^2$  subdominant linear, (much) less constrained  $f_{\chi}$  cf.  $f_{\text{NL}}$

**MEDIUM**  $\chi_{>h}$  encompass smoothed spikes, to be rare (for  **$\Delta T$  cold spot**, potential well  
 anomalies or not to be rare (and suffer constraints) 3

**LARGE**  $\chi_{>h}$  encompass part of a smoothed spike, upside, downside, topside

# Approaching the Planck Era in $\leq 1$ month

*status; impact of Planck on Planck era physics, early*

inflation  $n_s(k)$ , **GW: Tensor(k)**, subdominant isocurvature,

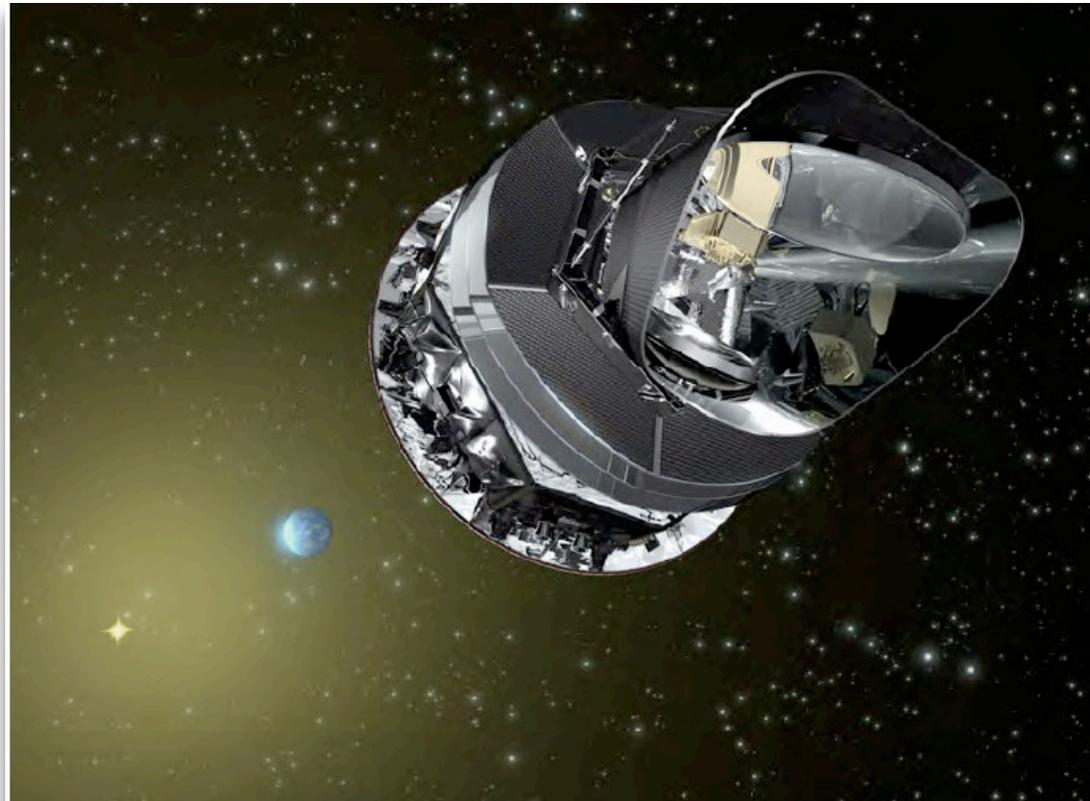
cosmic strings, textures, **nonGaussian  $F_{NL}$**  + late inflation  $w(z)$

**Launch planned for  
April 29, 2009 maybe  
(~10:30am Eastern)  
from Kourou, French  
Guiana 4°N**

Herschel in Kourou Feb 11

Planck in Kourou Feb 18

Launch window: 1 hour per day; 2  
days on, 2 weeks off to refill Herschel  
dewar



ESA /NASA /CSA Toronto HFI QLA/KST, TA, ... Barth & Dick, Marc-Antoine Miville-Deschenes, Carrie MacTavish, Brendan Crill, Olivier Dore, Mike Nolta, Peter Martin UBC LFI Douglas Scott et al.

**CBI** pol to Apr'05 @Chile  
*nonG*

**Boom03**@LDB +B98  
*nonG*

**WMAP** @L2 to 2010-20??  
*nonG*

DASI @SP

CAPMAP

**CBI2**  
**QUaD** @SP

**Bicep** @SP

**Planck09.3**  
*nonG*

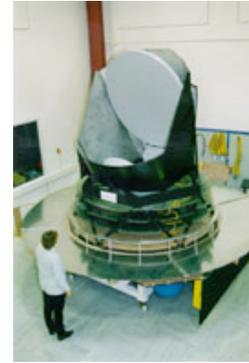
(52 bolometers)  
+ HEMTs @L2  
9 frequencies

**Herschel**

**BLAST**

**Quiet1**  
@Chile

**Bicep2**

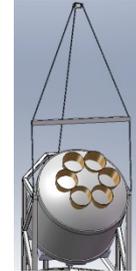


**Quiet2**  
1000 HEMTs

**EBEX**  
@LDB

**Spider**

2312 bolos  
@LDB



**CHIP**

2004

2006

2008

**LHC**

2011

**Bpol**  
@L2

2005

**Acbar** to Jan'06, 08f @SP

**SZA**  
@Cal

**AMI**



**GBT**

2007



**APEX**

~400 bolos  
@Chile

**SPT**

1000 bolos  
@SPole

**ACT**

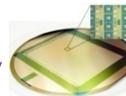
3000 bolos  
3 freqs @Chile

2009

**BLASTpol**

**SCUBA2**

12000 bolos  
JCMT @Hawaii



**Clover**  
@Chile

**Polarbear**

300 bolos  
@Cal/Chile

**SPTpol**

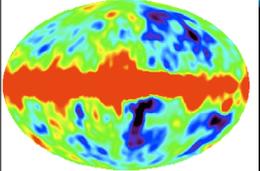
**ALMA**

@Chile

LMT@Mexico

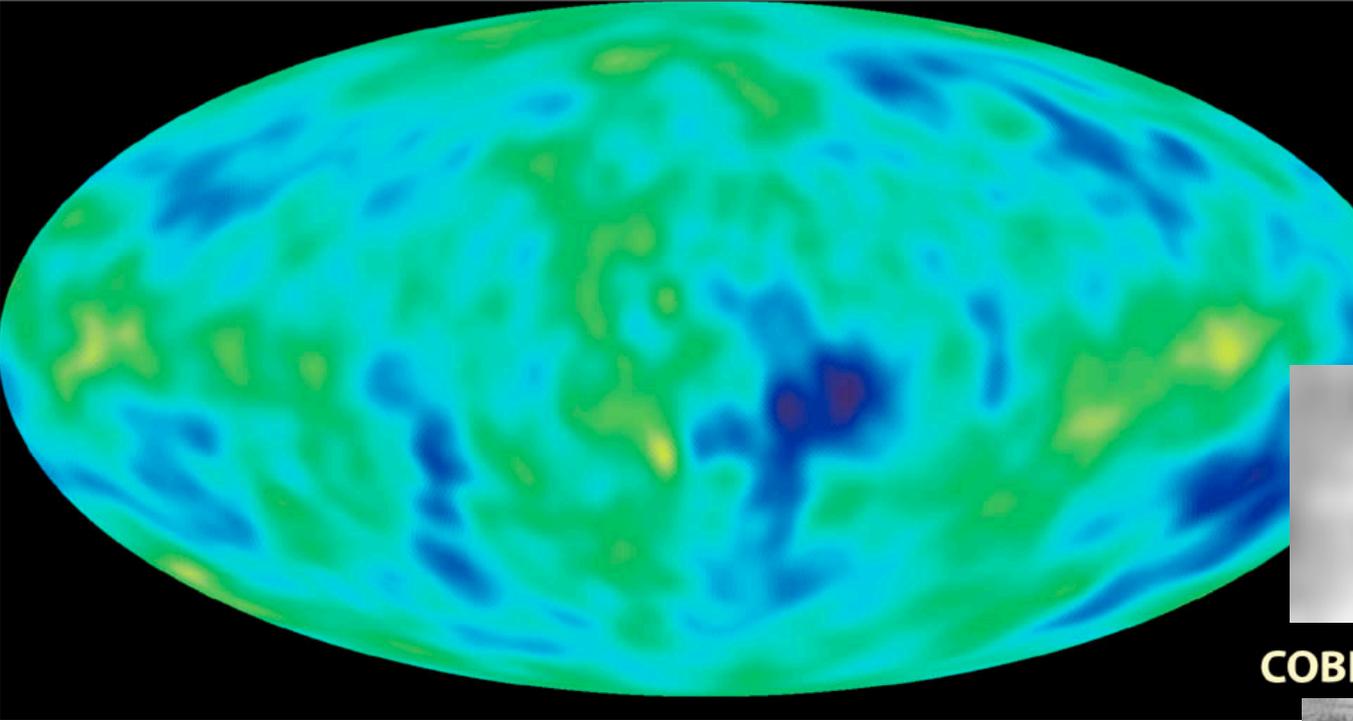
**CMB**  
Nearly Perfect Blackbody  
 $T=2.725 \pm .001$  K COBE/FIRAS

Dipole: flow of the earth in the CMB

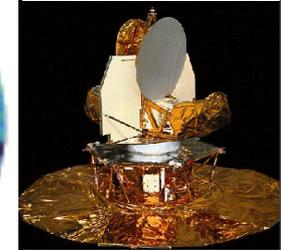
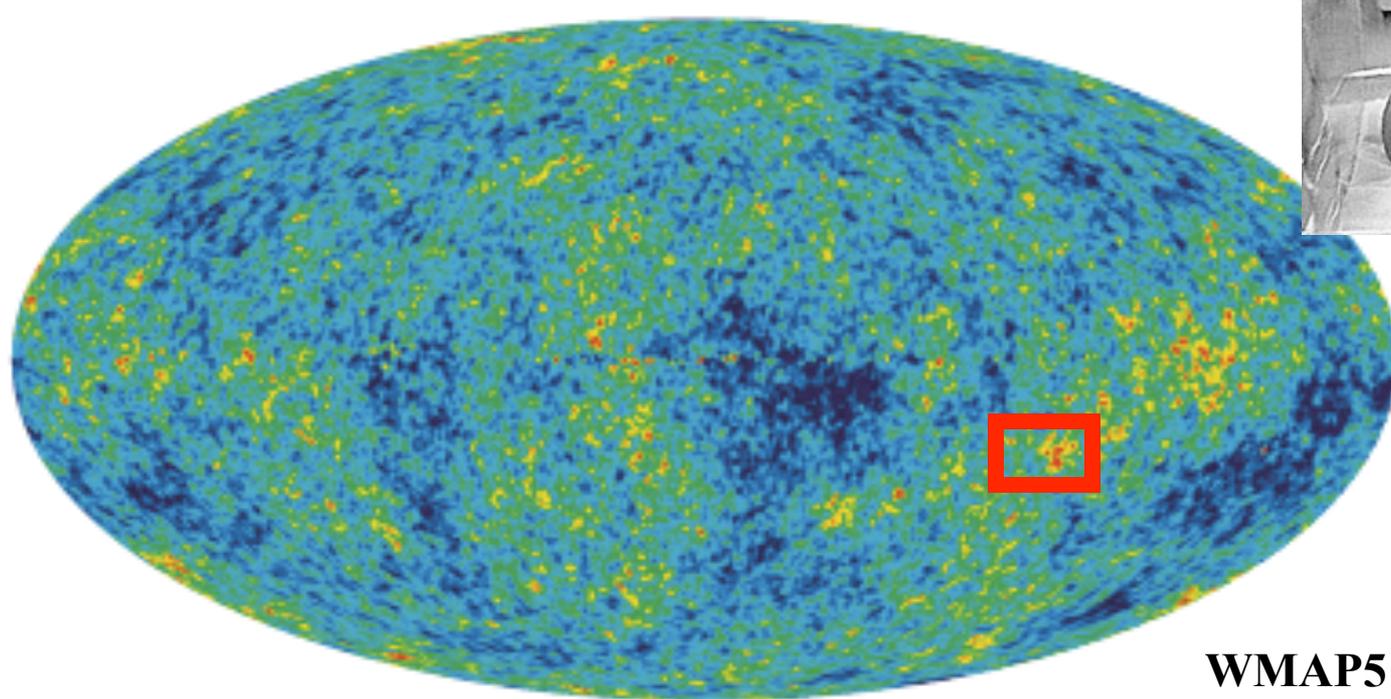


COBE/DMR:  
CMB + Galactic @  $7^\circ$

is this a statistically isotropic Gaussian random field, when account is taken of the Milky Way emissions & extra-galactic sources?  
**yes! maybe?**



**COBE 1992/96**



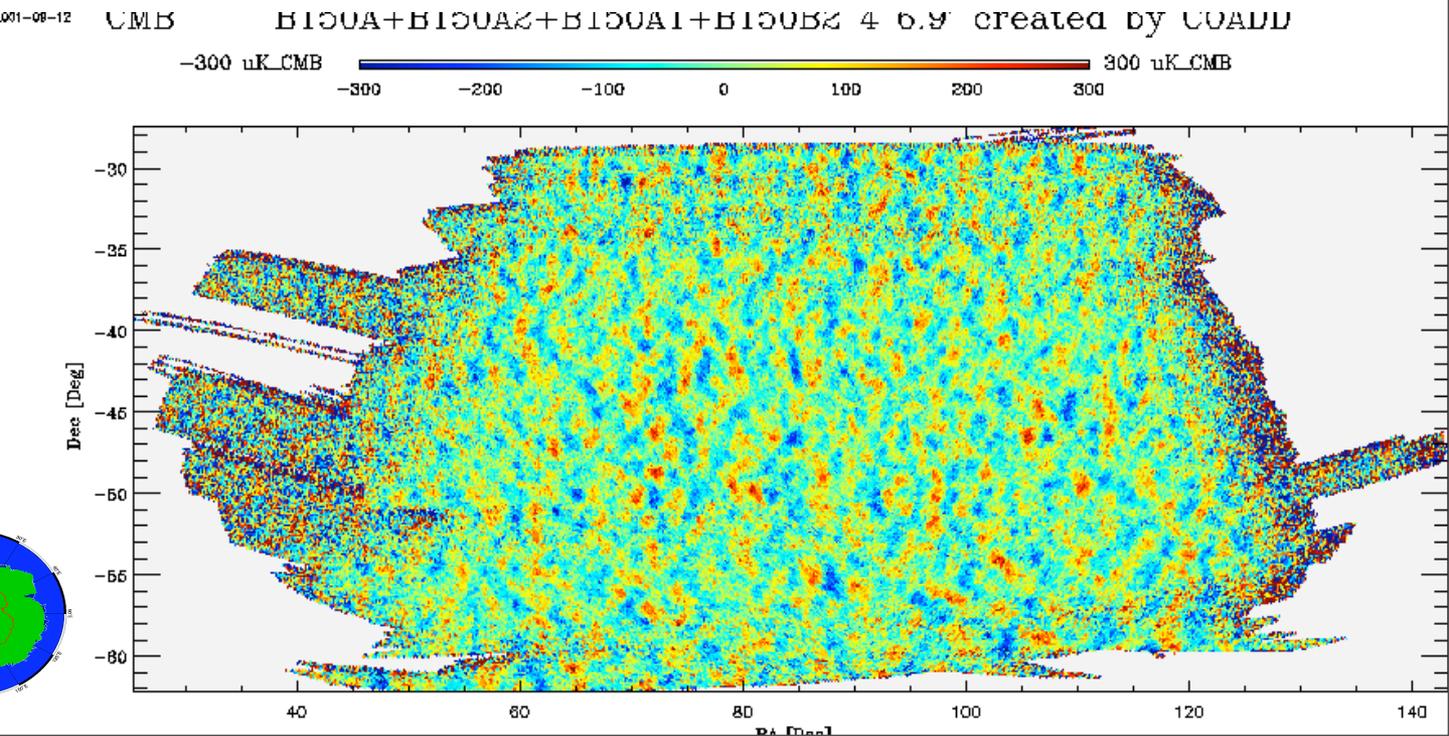
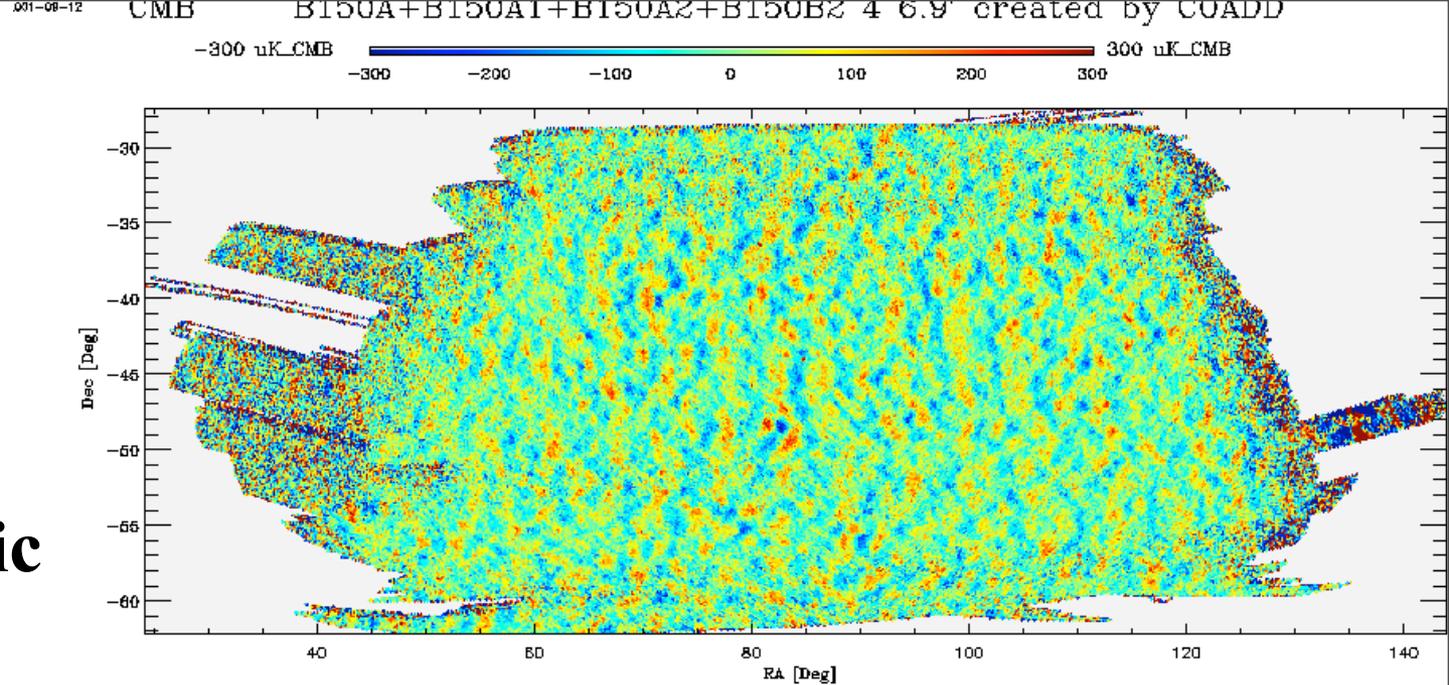
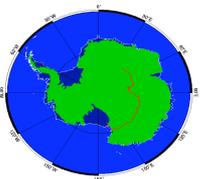
**Feb03  
Mar06  
Mar08**

**WMAP5**

Boomerang  
@150GHz is  
(nearly)  
Gaussian:  
Simulated vs  
Real

thermodynamic  
CMB

temperature  
fluctuations  
2.9% of sky  
 $\Delta T \sim 30$  ppm

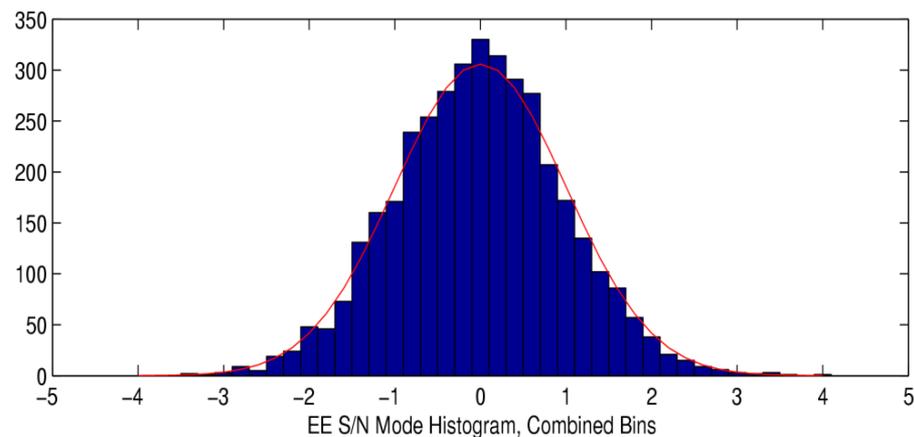
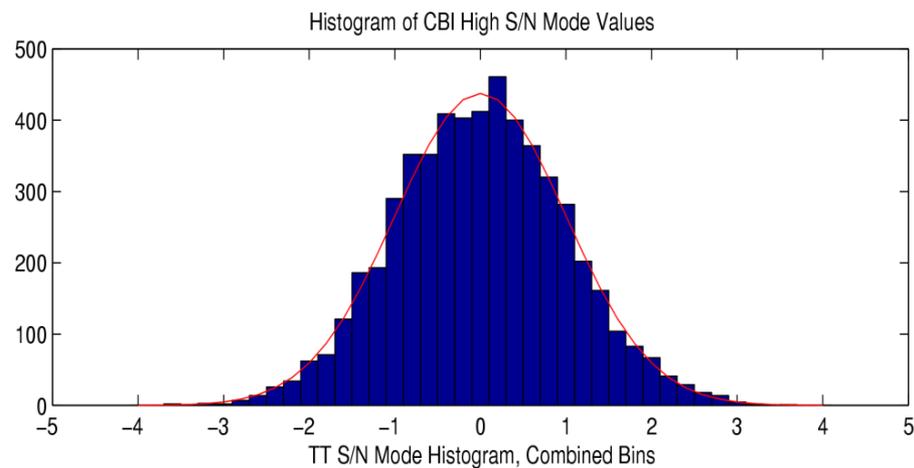


*All non-primary CMB components are non-Gaussian: extragalactic radio and submm sources; Galactic synchrotron bremsstrahlung & dust emission, CMB-upscattering from hot gas in clusters, gravitational lensing of the CMB, ...*

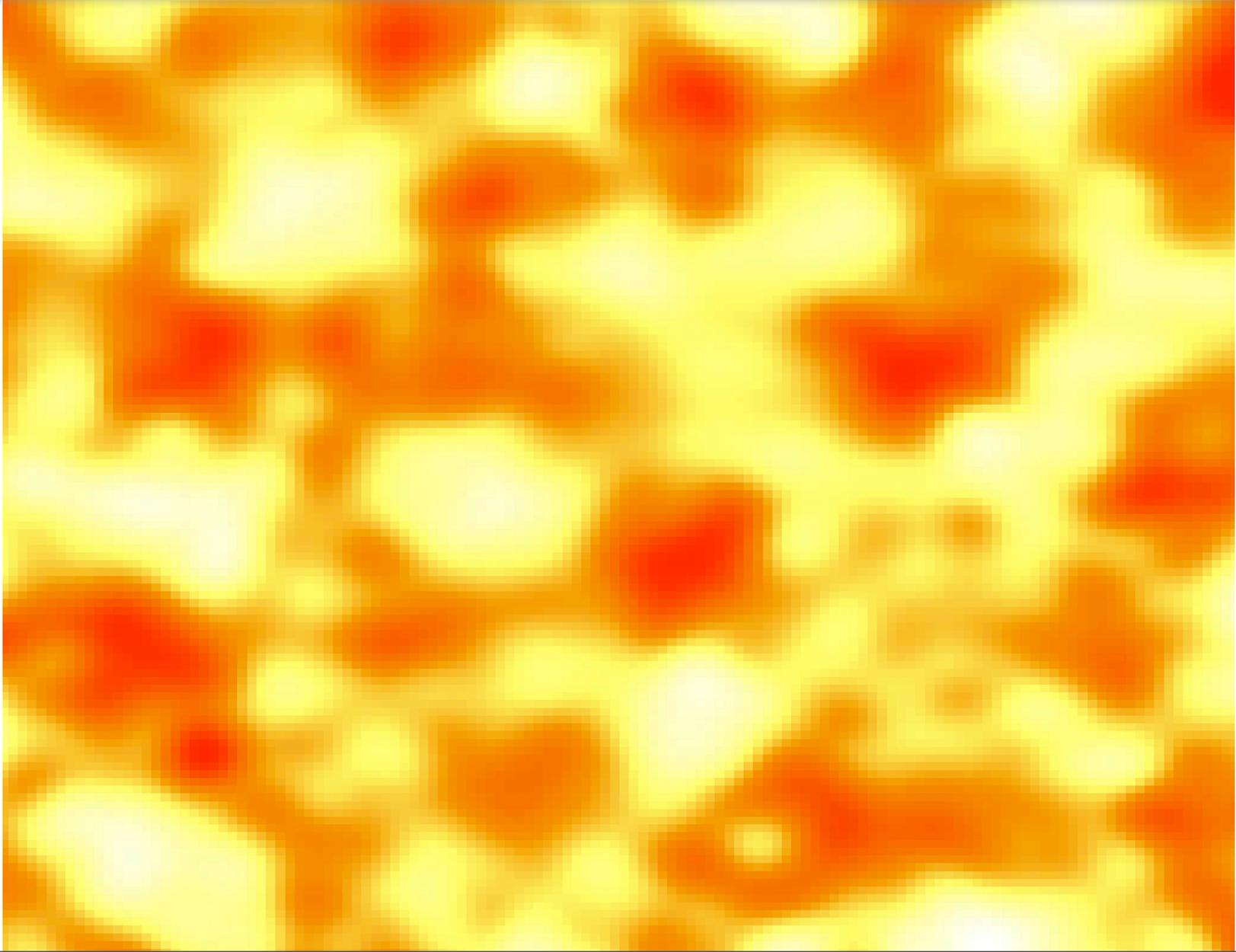


even the high resolution Cosmic Background Imager  $\Delta T$  is  $\sim$  Gaussian, & so is its CMB polarization signal

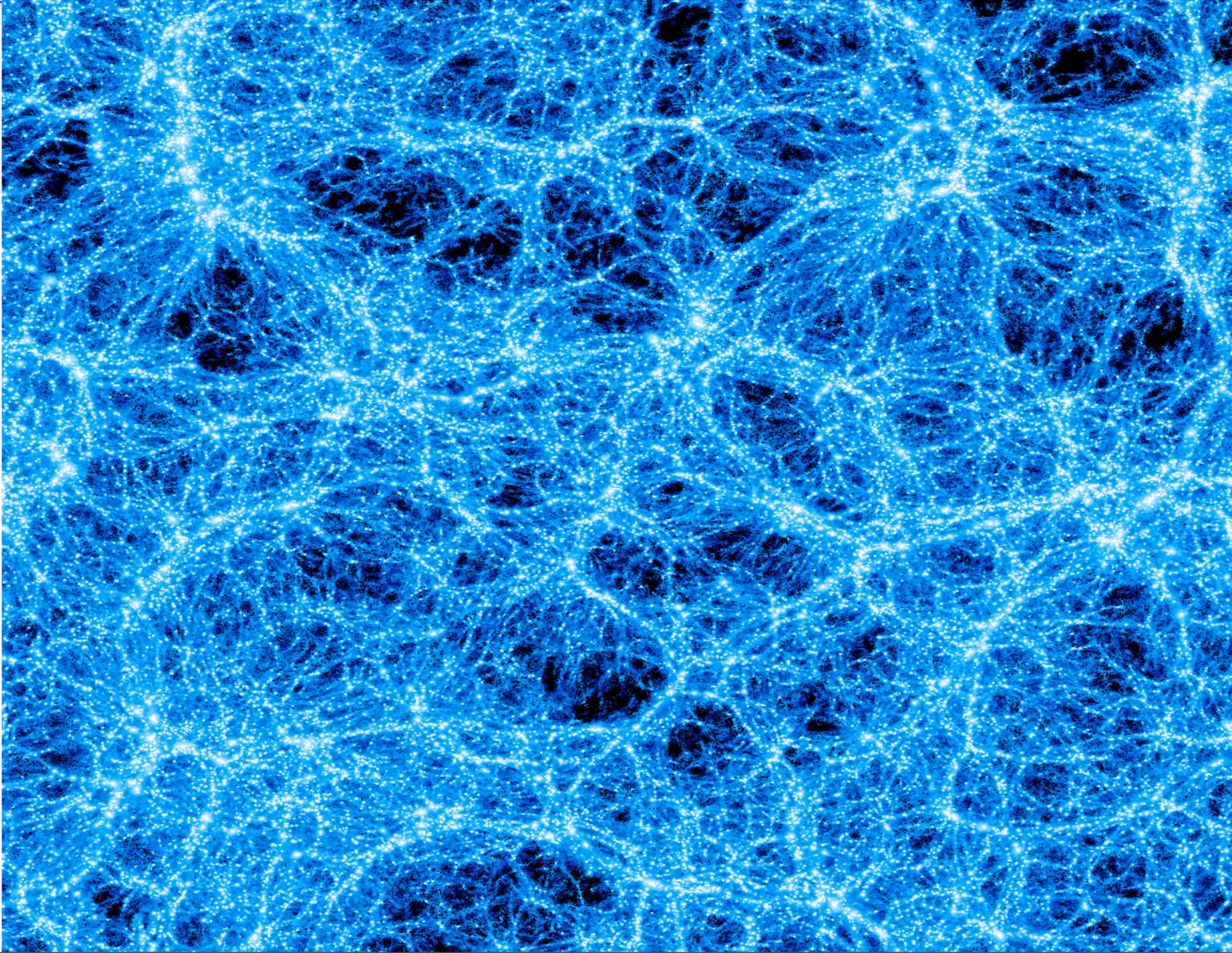
- Method: Decompose data (with extragalactic radio sources removed) into uncorrelated S/N eigenmodes for each bin; Pick out modes expected to have signal; Check distribution for non-Gaussianity
- We kept 5500 modes for TT  $\Delta T$ , 3800 for EE polarization
- all are consistent with Gaussian
- first check of EE polarization



# **nonlinear Gas & Dark Matter Structure in the Cosmic Web the cluster/gp web “now”, the galaxy/dwarf system “then”**

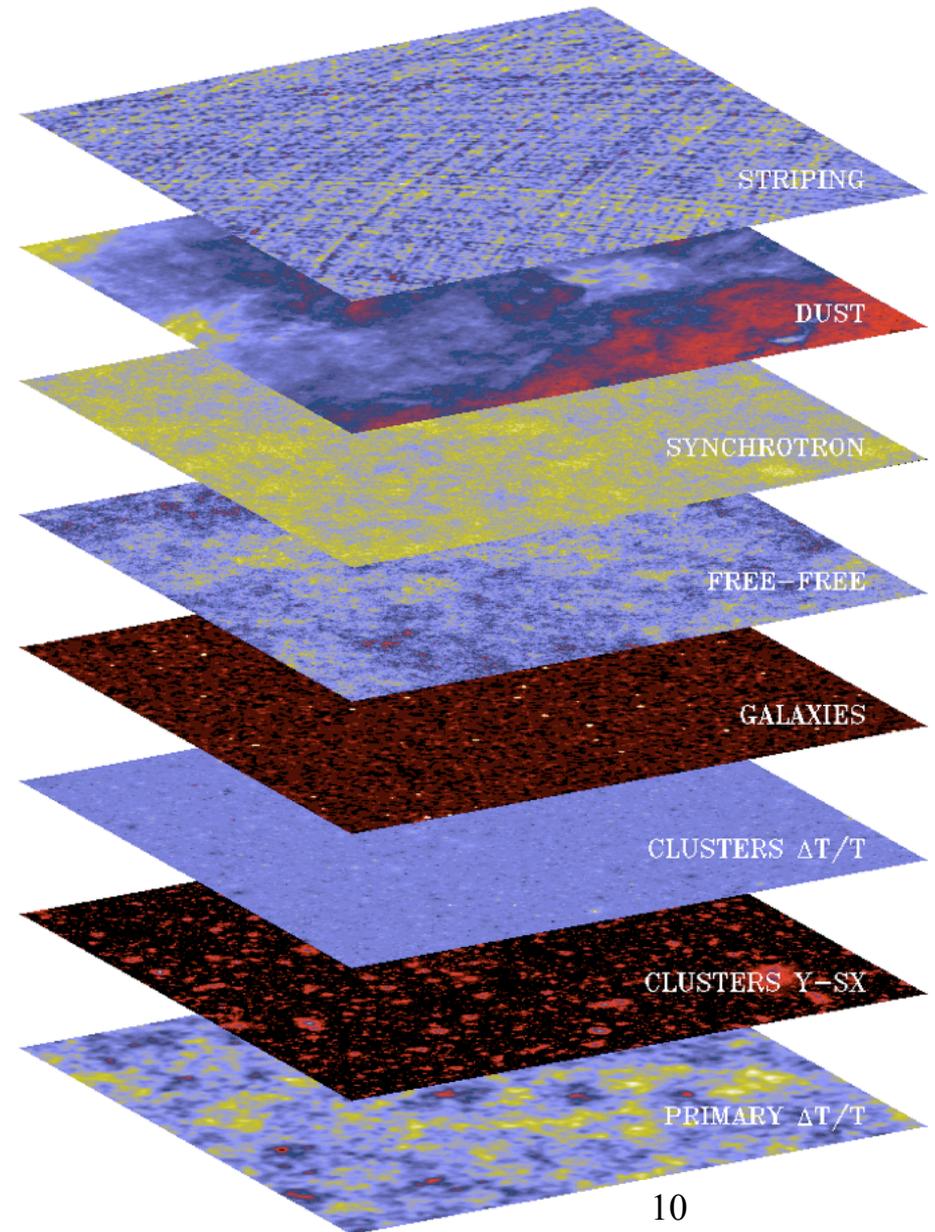


# **nonlinear Gas & Dark Matter Structure in the Cosmic Web the cluster/gp web “now”, the galaxy/dwarf system “then”**



*the quest for primordial non-Gaussianity within the primary CMB requires exquisite foreground removal, whether inflation-induced or cosmic-string-induced, ...*

striping  
dust  
synchrotron  
bremsstrahlung  
dusty galaxies  
kinetic SZ  
thermal SZ  
PRIMARY







AOCO

PLANCK  
TRANSPORT & STORAGE  
CONTAINER

ESA



Several workers in winter clothing are standing near the base of the container, observing the operation.

A worker in a dark jacket is standing in the foreground, looking towards the container.

Two workers in dark jackets are standing near the white truck, observing the operation.

A group of people is gathered in the background, watching the process.



АН-124-100

Volga-Dnepr

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<http://www.volga-dnepr.com>



ADCO

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esa

DNEB

MONDI  
TEL: 90 34 91 91

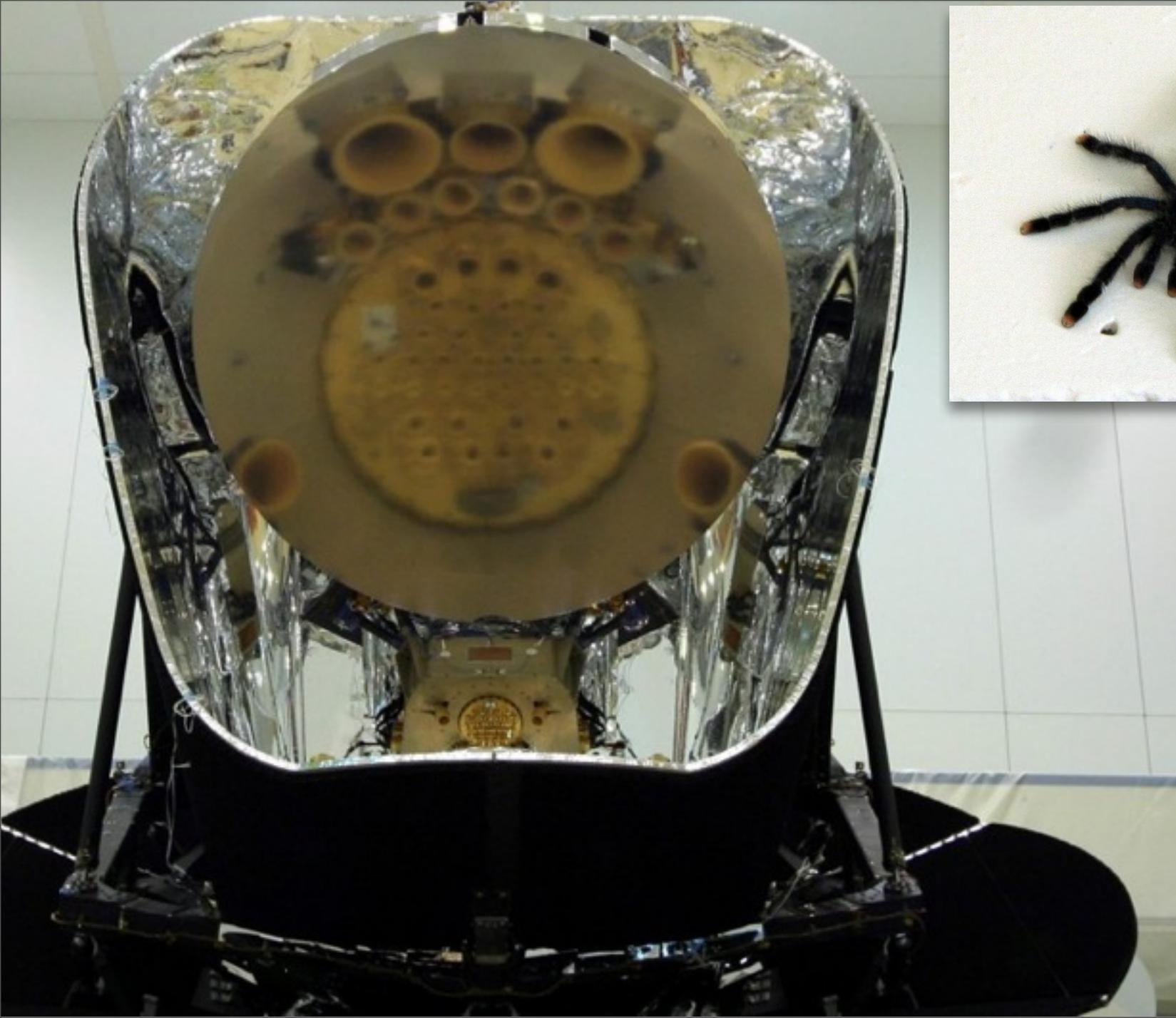


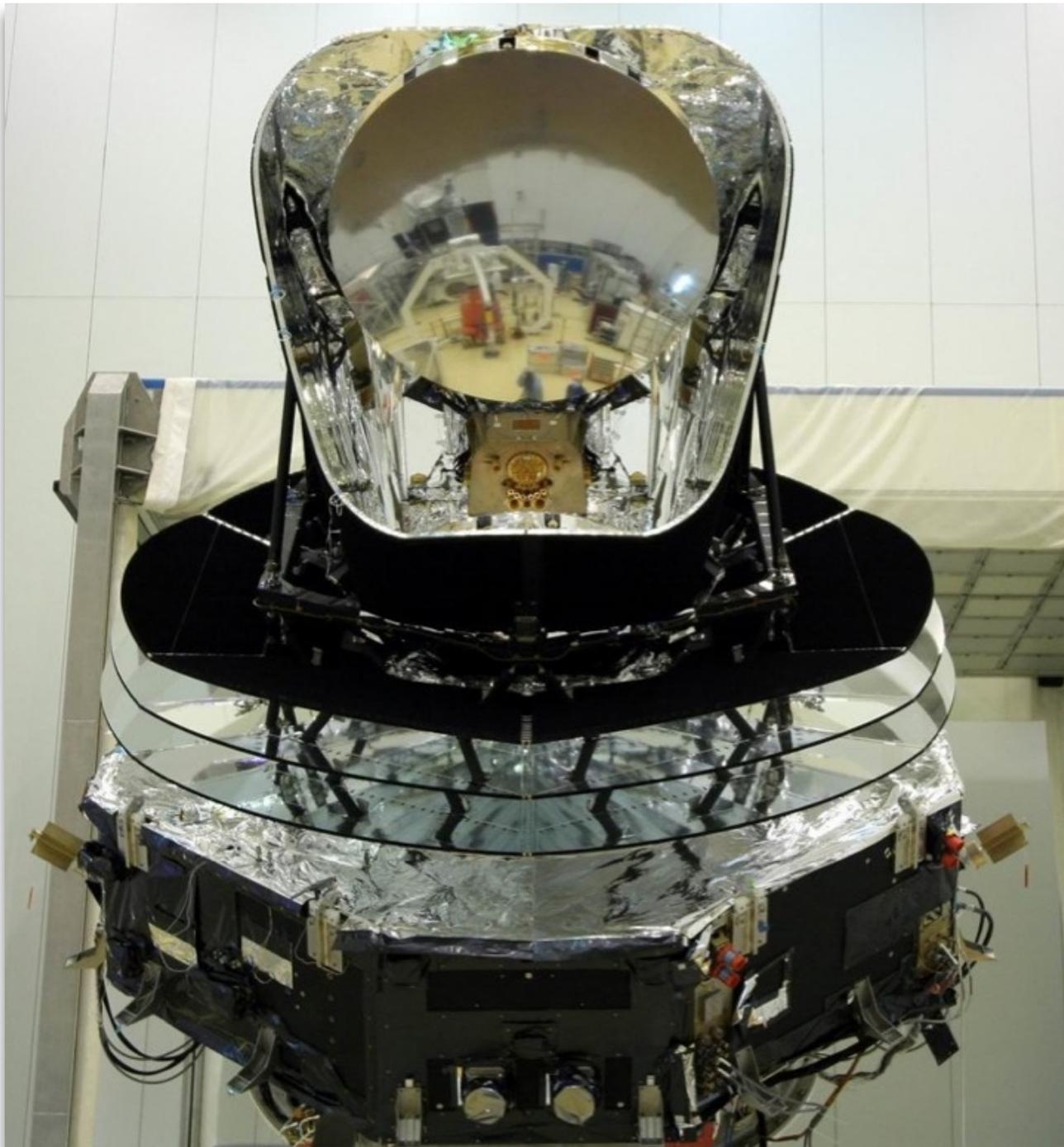
Volga-Dnepr

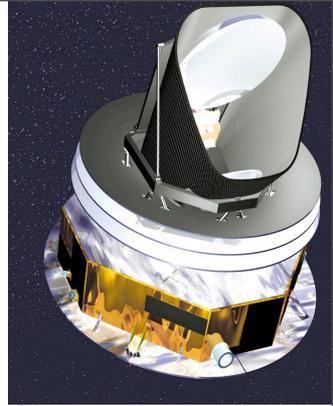
Волга-Днепр

АН-124-100









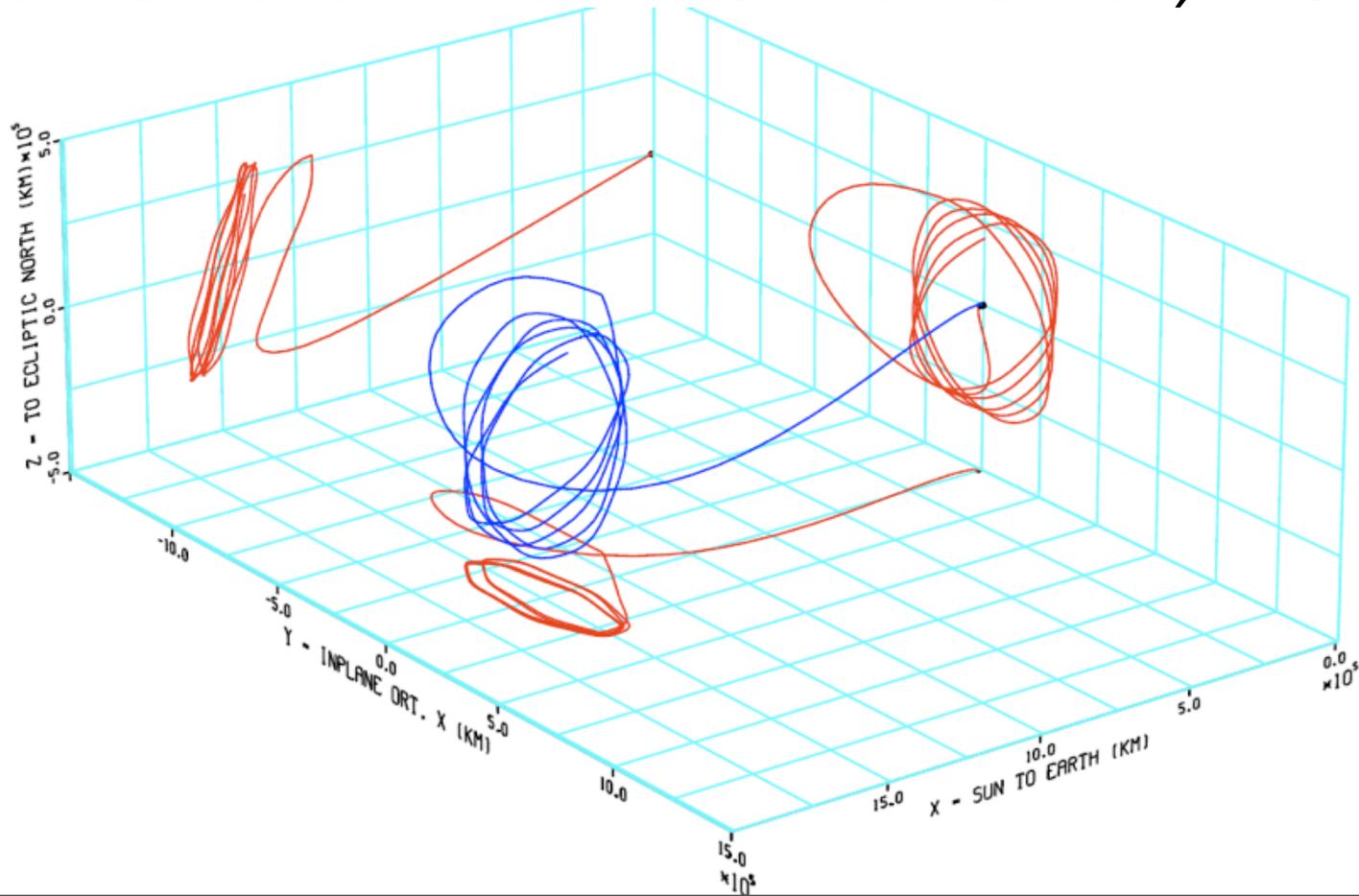
# Planck's Journey

*Trip to L2: ~ 30 days*

*. Decontamination & Cooldown ~ 45 days*

*. Detectors at 100mK at L2 around June 18*

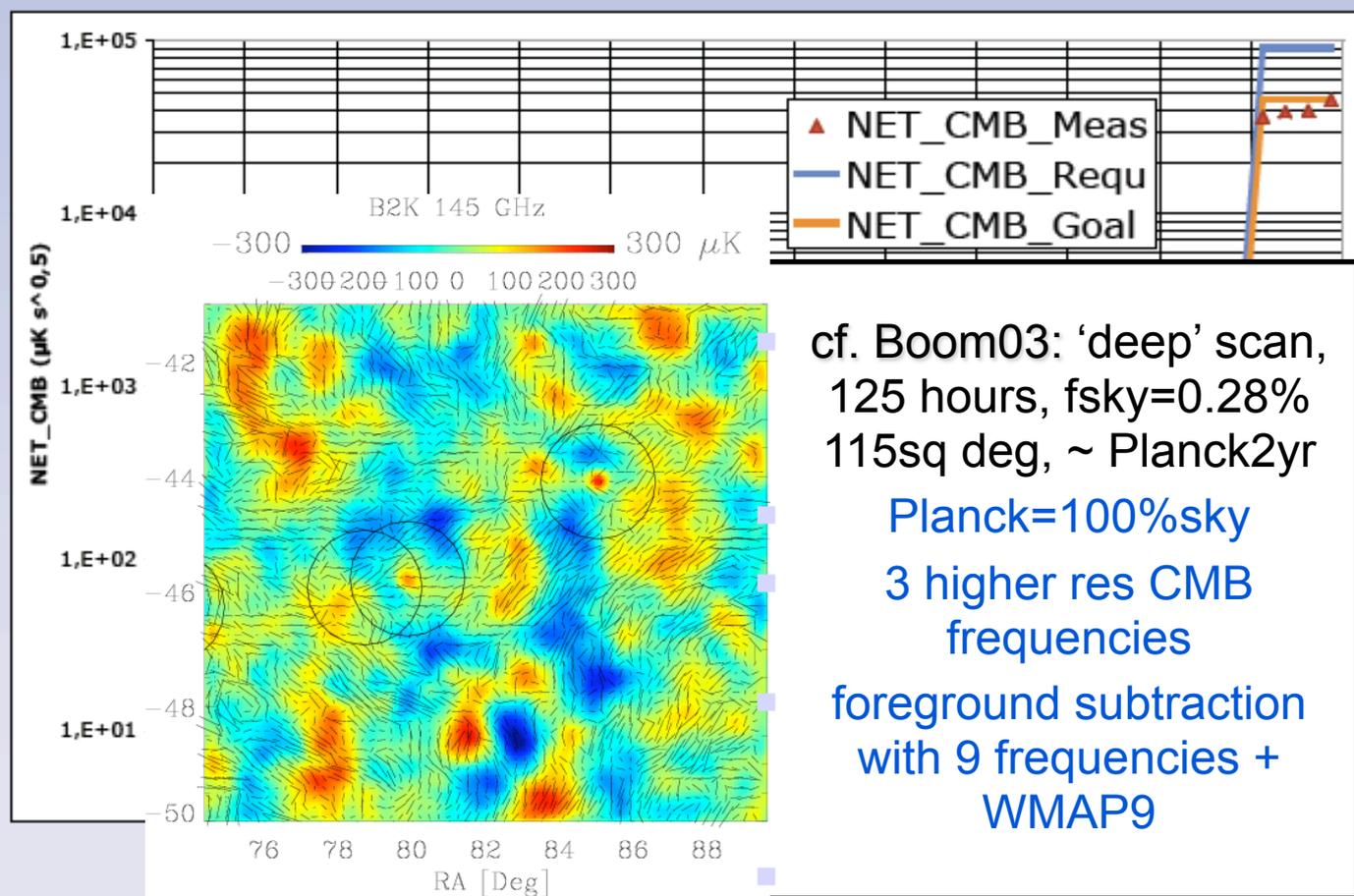
*. CPV (Checkout & Performance Verification) thru end-Jul*



NET requirement vs. NET measurement vs NET goal: very close to goal,  
 better than requirement in all channels: 100 GHz\_P 143 GHz\_P 143 GHz  
 217 GHz\_P 217 GHz 353 GHz\_P 545 GHz 857 GHz



## Expected performances (1) Noise Equivalent Temperature



# Schedule

*Launch: April 2009*

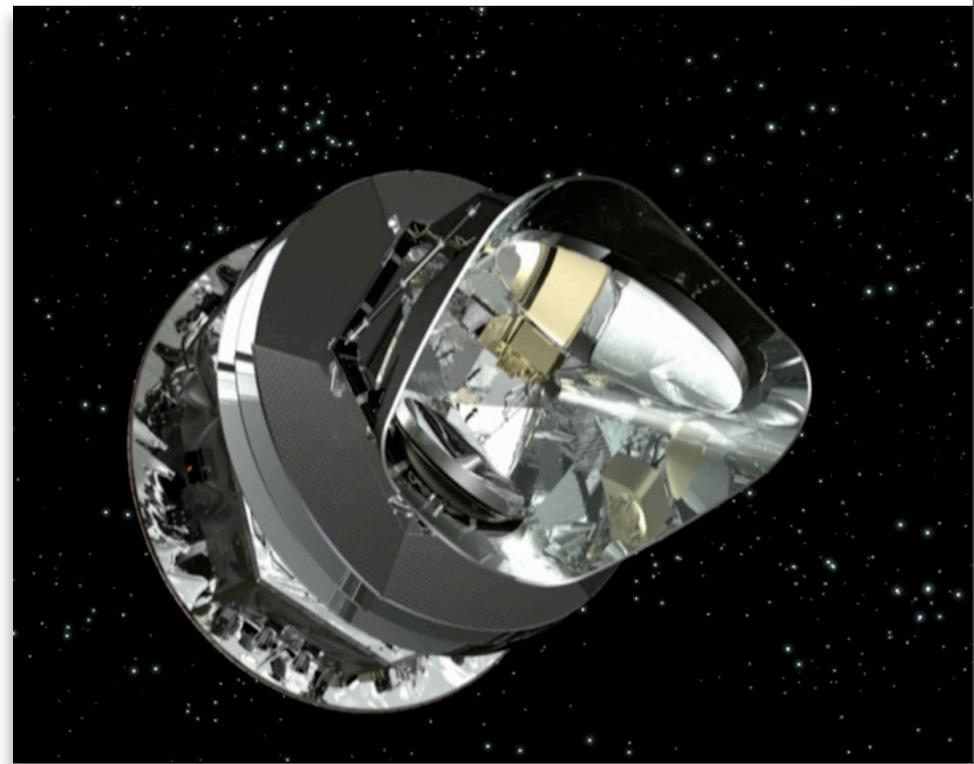
*. Cruise & CPV ends July 2009*

*. Two sky surveys finished July 2010*

*. Early Release Compact Source Catalog ~Dec 2010*

*. Four sky surveys finished: July 2011*

*. Public release of 1yr data, papers: July 2012*



brief history of bond's non-Gaussian exploration in inflation: THEN

early 80s: hot, warm & **cold** collisionless **dark matter** + inflation

⇒ **xCDM**: *86 extra power dilemma* ⇒ vary x:  $k_{\text{Heq}}$   $k_{\text{mn}}$   $k_{\text{features}}$

87: **X** =  $s/H_0$  /  $\Lambda$  / Open / is / is+ad / h-c / h+ / b / b /  **$\Lambda$ +b** / Op+b /  $\tau$  / **BSI / BSI2**

90s-00s: data settled on **X** =  **$\Lambda$  +tilt** ⇒ **dark-energy +tilt**

# COSMIC PARAMETERS THEN



e.g., **BBE1987 vary x in xCDM**

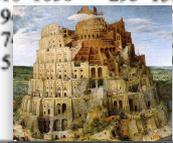
for xCDM, predict CMB (6deg, 5min); LSS cluster-cluster, cluster-galaxy, bulk flows,  $\sigma_8$ : redshift of “galaxy formation”

14 Gyr,  $\Omega_\Lambda=0.8$ ,  $H_0=75$ ,  $b\sim c$ ,  $50\mu\text{K}$  cf  $30\mu\text{K}$  coBE,  $\sigma_8\sim 0.72$

**X =** s /  $H_0$  /  $\Lambda$  / Open / is / is+ad / h-c / h+ / b / b /  **$\Lambda+b$**  / Op+b /  $\tau$  / **BSI** / **BSI2**

PREDICTIONS FOR MODELS

Parameter	OBS	CDM	C40	VAC/C	OP/C	ISO/C	ISO/AD	HOT	HC	C + B	B + C	BCV	BCO	CDM + dec	(CDM + X) <sub>2</sub> ( $k_s^{-1} = 300$ )	(CDM + X) <sub>2</sub> ( $k_s^{-1} = 200$ )
$\Omega, \Omega_b, H_0$ .....	...	1, 0.1, 50	1, 0.1, 40	1, 0.03, 50	0.2, 0.03, 50	1, 0.1, 50	1, 0.1, 50	1, 0.1, 50	1, 0.1, 50	1, 0.2, 40	1, 0.5, 50	1, 0.1, 75	0.2, 0.1, 75	1, 1, 50	1, 0.1, 40	1, 0.1, 50
$\Omega_x(\Omega_v), \Omega_{vac}$ .....	...	0.9, 0	0.9, 0	0.17, 0.8	0.17, 0	0.9, 0	0.9, 0	(0.9), 0	0.5(0.4), 0	0.8, 0	0.5, 0	0.1, 0.8	0.1, 0	1, 0	0.9, 0	0.9, 0
b .....	...	1.7	1.8	1	1	1.7	1.7	0.53	1.7	1.8	1.7	1	1	1.7	1.8	1.7
$t_0$ (by) .....	GC: 14-22 NC: 13-26	13	17	22	17	13	13	13	13	17	13	14	11	13	17	13
$\sigma_8(R_g = 0.35)$ ...	...	2.9	2.4	2.7	2.7	1.6	2.5	2.0	1.3	2.2	1.9	2.4	2.4	6.8	2.2	2.7
$z_g$ .....	...	3.7	2.9	2.3	4.0	1.3	3.1	1	1.1	2.5	2.0	1.3	2.0	13	2.6	3.4
$\sigma_8(R_{cl} = 5)$ .....	...	0.42	0.39	0.75	0.75	0.43	0.42	1.4	0.44	0.40	0.44	0.72	0.72	0.47	0.41	0.43
$\langle v \rangle_c$ .....	...	3.2	3.1	3.1	3.1	3.0	3.2	3.1	2.9	3.1	3.0	2.8	2.8	2.7	3.1	3.1
$\xi_{cc}(20)$ .....	1.5	0.15	0.26	1.7	1.7	0.70	0.35	1.1	1.0	0.49	1.3	2.2	2.2	1.8	1.0	0.85
$\xi_{cc}(25)$ .....	1.0	0.08	0.15	1.2	1.2	0.42	0.21	0.45	0.51	0.31	0.93	1.7	1.7	0.92	0.83	0.68
$\xi_{cc}(30)$ .....	0.72	0.03	0.07	0.85	0.85	0.25	0.11	0.20	0.24	0.20	0.61	1.4	1.4	0.49	0.64	0.51
$\xi_{cc}(50)$ .....	0.29	-0.01*	-0.006*	0.24	0.24	0.02	-0.01*	-0.009*	-0.02*	0.04	0.23	0.59	0.59	0.16	0.28	0.21
$\xi_{cc}(100)$ .....	0.08	-0.002*	-0.003*	0.02	0.02	-0.003*	-0.003*	-0.003*	-0.009*	-0.007*	-0.01*	0.36	0.36	0.02	0.08	0.06
$\xi_{cg}(20)$ .....	0.49	0.13	0.17	0.57	0.57	0.32	0.19	0.96	0.44	0.23	0.50	0.76	0.76	0.70	0.39	0.32
$\xi_{cg}(25)$ .....	0.33	0.04	0.06	0.37	0.37	0.16	0.08	0.35	0.23	0.11	0.32	0.54	0.54	0.42	0.26	0.20
$\xi_{cg}(30)$ .....	0.24	0.01	0.02	0.25	0.25	0.09	0.03	0.12	0.11	0.06	0.22	0.41	0.41	0.24	0.19	0.15
$\xi_{cg}(40)$ .....	0.14	-0.003	0.002	0.13	0.13	0.03	0.006	-0.001	0.02	0.03	0.13	0.26	0.26	0.09	0.12	0.10
$\tau(R_f = 3.2)$ .....	$610 \pm 50$	136-654	134-650	166-797	157-752	172-824	148-709	594-2850	185-889	149-714	208-1000	232-1120	218-1050	293-1399	280-1331	241-1151
$\tau(R_f = 15)$ .....	$599 \pm 104$	71-340	76-365	134-639	126-601	114-544	86-409	387-1850	124-587	95-450	154-735	206-987	19	70	250-1190	202-970
$\tau(R_f = 25)$ .....	...	53-250	56-269	115-550	108-516	89-421	64-309	419-1350	91-435	71-342	119-573	186-894	17	28	233-1106	185-882
$\tau(R_f = 40)$ .....	$970 \pm 300$	35-180	40-192	95-456	90-430	66-315	47-221	200-958	65-311	52-251	87-419	160-771	15	9	214-1016	165-787
$\Delta T/T$ (4:5) .....	<25	5	6	20	70	...	...	20	...	6	8	10	...	...	...	...
$\times 10^6$ (6°) .....	<48	7	8	20	40	60	30	20	8	8	15	25	...	72 (98)	...	40 (64)



# Delta T over Tea Toronto May 1987: first dedicated CMB conference, exptalists+theorists, primary+secondary $\Delta T/T$

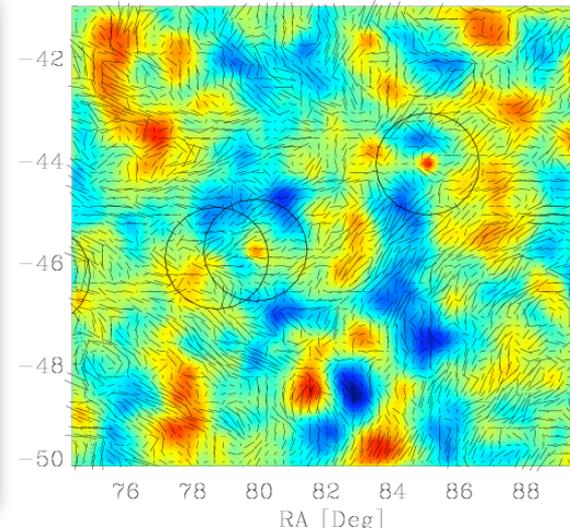
Primary Cosmic Microwave Background Radiation ~ a statistically isotropic all-sky GRF on the 2-sphere  $C_L = \langle |\Delta T(LM)|^2 \rangle$  with target  $C_L$  shapes

A tentative list of topics organized according to angular scale, with theory and observation intertwined, is:

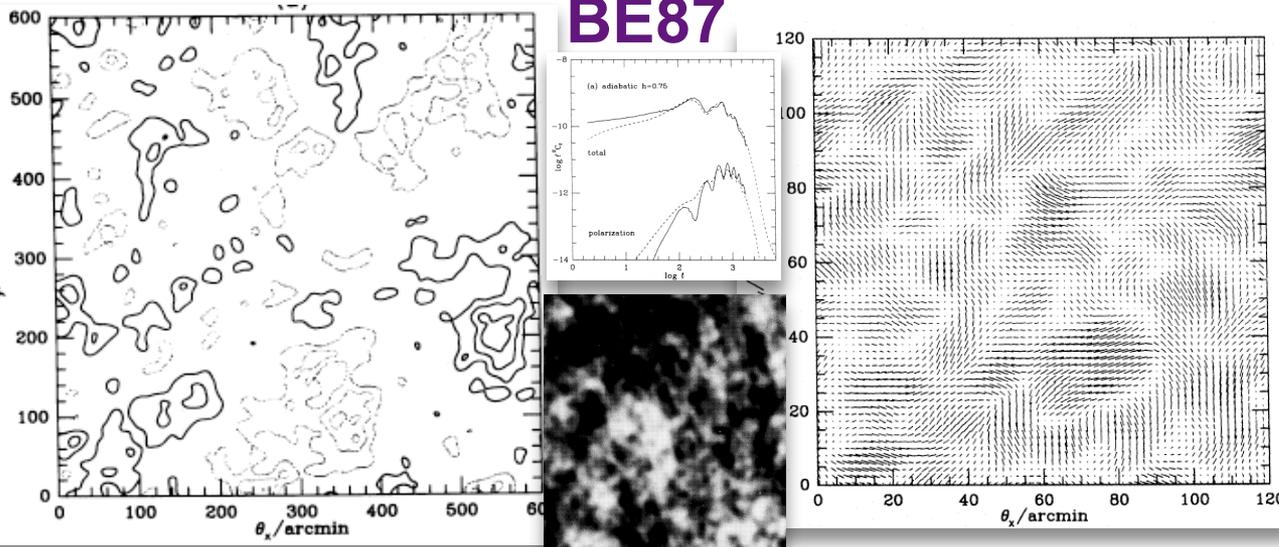
- very small angle anisotropies - VLA results, secondary fluctuations via the Sunyaev-Zeldovich effect, primeval dust emission, and radio sources
- small angle anisotropies - current results, optimal measuring strategies, statistical methods for small signals in larger noise, which universes can we rule out, the reheating issue, future detectors and techniques, **CMB map statistics, polarization**
- intermediate and large angle anisotropies -  $5^\circ - 10^\circ$  results, future experiments at  $\sim 1^\circ$ , COBE and other large angle analyses, theoretical  $C(\theta)$ 's and their angular power spectra, Sachs-Wolfe effect in open Universes, the isocurvature CDM and baryon stories,  $\Delta T/T$  from gravitational waves, the cosmic string story.

**Boom05 deep**

-300 200 100 0 100 200 300  $\mu K$



**BE87**



## brief history of bond's non-Gaussian exploration in inflation: THEN

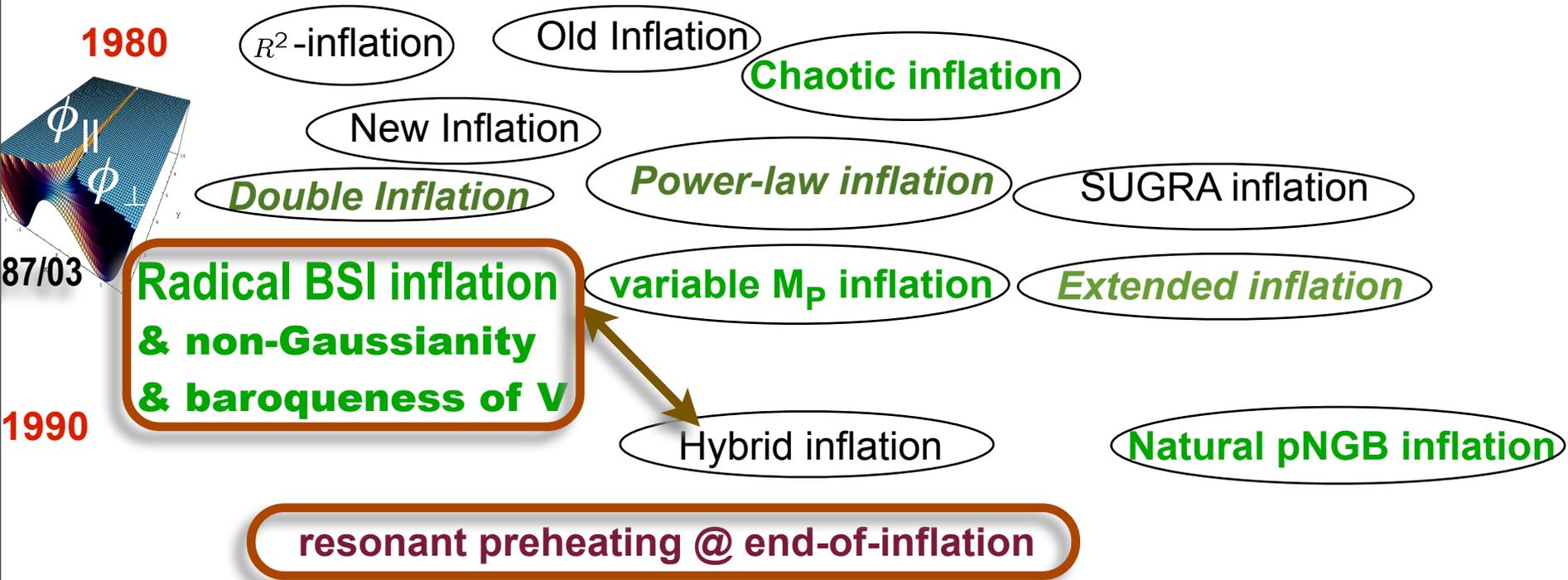
Linde & Kofman 87, clipped pNGB (1-cos( $\chi/f$ )) string issue single field Grinstein, Allen & Wise 87

SBB89:  $\delta H, \delta m^2_{ij}$ , moguls, waterfalls  $\Rightarrow$  plateau/mountain/valley

$V = \lambda_\phi \phi^4/4 + m_\phi^2 \phi^2/2 + \lambda_\chi \chi^4/4 + m_\chi^2 \chi^2/2 - \nu \phi^2 \chi^2/2 + 3\text{-leg}$  HYBRID; inflaton + isocons

trajectory bifurcation at  $m^2_{is, is} \leq 0$  TACHYONIC  $\Rightarrow$  non-G with & without 3-leg,  
avoid late domain walls ( $\Rightarrow$  modify  $V_{\text{late time}}$ )

**Old view:** Theory prior = delta function of THE correct one and only theory



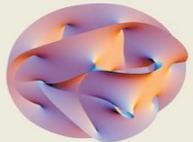
**2000**



**New:** Theory prior = probability distribution of late-flows on an energy **LANDSCAPE**

**moduli fields**

**Roulette inflation Kahler moduli/axion**



## Radical BSI inflation

**SBB89:** multi-field, the hybrid inflation prototype, with curvature & isocurvature &  $P_s(k)$  with any

**shape possible** &  $P_t(k)$  almost any shape

(mountains & valleys of power), gorges, moguls, waterfalls,  $m_{\text{eff}}^2 < 0$ , i.e., tachyonic, **non-Gaussian, baroque**ness, radically broken by variable braking  $\varepsilon(k)$ ; **SB90,91** Hamilton-Jacobi formalism to do **non-G (& Bardeen pix of non-G)  $\varepsilon(k) - H(\phi)$**

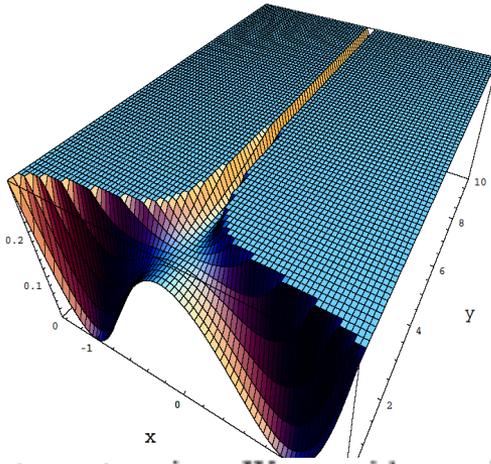
cf. gentle break by smooth brake in the slow roll limit.

than flat spectra give. We consider a wide variety of models based on the chaotic inflation paradigm and sketch the effects that varying the expansion rate, structure of the potential surface, and the curvature coupling constants have on the quantum fluctuation spectra. We calculate in detail the quantum generation of fluctuation spectra by numerically solving the linearized perturbation equations for multiple scalar fields, the metric, and the radiation into which the scalars dissipate, following the evolution from inside the horizon through reheating. We conclude that (1) useful extended nonflat power laws are very difficult to realize in inflation, (2) double inflation leading to a mountain leveling off at a high-amplitude plateau at long wavelengths is generic, but to tune the cliff rising up to the plateau to lie in an interesting wavelength range, a special choice of initial conditions and/or scalar field potentials is required, and (3) small mountains (moguls) on the potential surface lead to mountains of extra power in the fluctuations added on top of an underlying flat spectrum. For quadratic and quartic couplings, the mountain fluctuations may obey Gaussian statistics but the spectral form will be very sensitive to initial conditions as well as potential parameters; non-Gaussian mountain fluctuations which depend upon potential parameters but not on initial field conditions will be the more likely outcome. However, adding cubic couplings can give mountains obeying Gaussian statistics independently of initial conditions. Since observations only probe a narrow patch of the potential surface, it is possible that it is littered with moguls, leading to arbitrarily complex "mountain range" spectra that can only be determined phenomenologically. We also construct an inflation model which houses the chaotic inflation picture within the grand unified theory

**Blind power spectrum analysis cf. data, then & now measures matter "theory prior" informed priors?**

1987

2003



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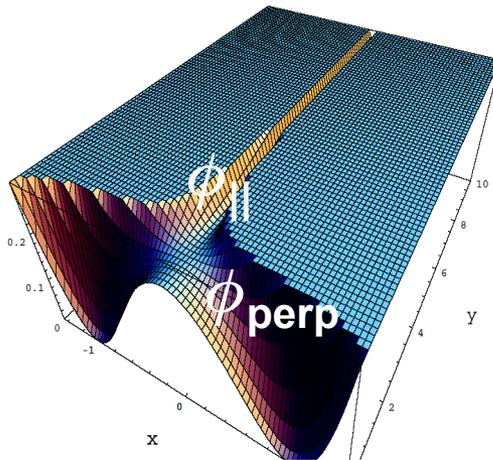
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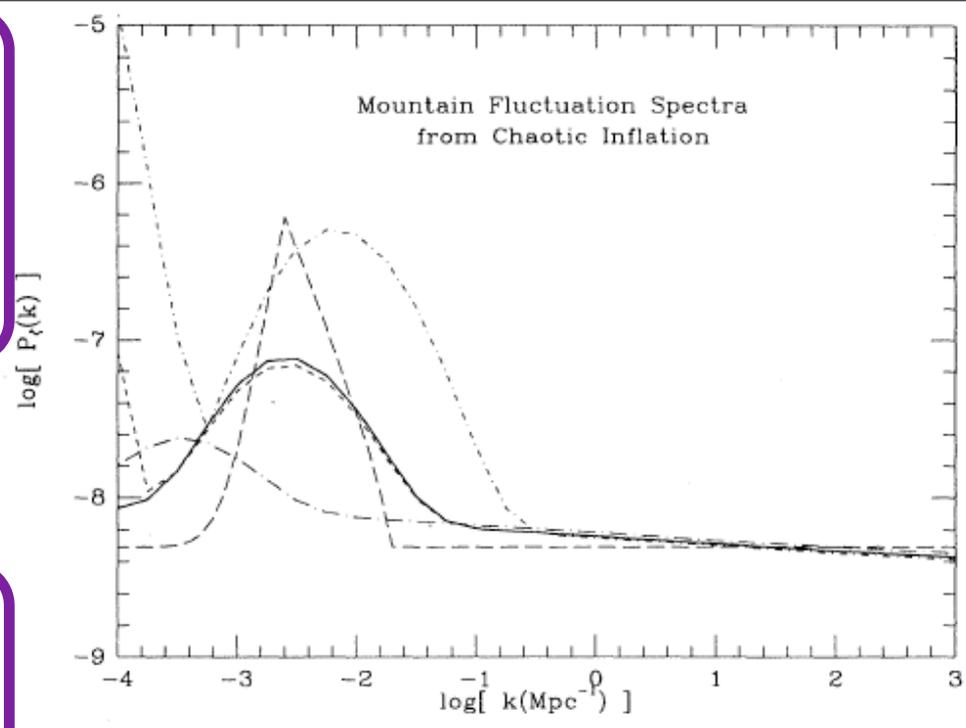
The net result would be non-Gaussian fluctuations in  $\phi_2$ . If the field  $\phi_1$  does not enter into a prolonged  $m_{11}^2 > 0$  phase after the  $m_{11}^2 < 0$  phase, significant non-Gaussian isocon fluctuations would survive. Thus the generic case for moguls centered about a  $\phi_{1ri}(\phi_2)$  ridge line which is continuous with the incoming  $\phi_{1tr}(\phi_2)$  trough line, as in the  $\nu < 0, \mu_1 = 0$  case of Sec. VIC, is a non-Gaussian "mountain," provided

$$\langle (\delta\phi_1)^2(\mathbf{x}, t) \rangle^{1/2} \gtrsim |\bar{\phi}_1 - \phi_{1ri}|. \quad (6.13)$$

In this expression, the quantum fluctuations  $\langle (\delta\phi_1)^2(\mathbf{x}, t) \rangle$  are assumed to have the short-distance rapidly oscillating  $k^{-1} \lesssim (Ha)^{-1}$  waves filtered out. Note that starting from  $\phi_1 = 0$  with a symmetric mogul leads to bifurcation of the field, which can lead to domain walls. To avoid a domain-wall density problem, we must suppose that either the mogul is localized in  $\phi_2$  or another interaction is present at lower energies to destroy the walls. In either case, the large-scale non-Gaussian metric perturbations will survive intact.

addition of cubic interaction terms to the potential (6.3) can also give  $m_{11}^2 < 0$  over a short range. If they are symmetric about  $\phi_{1tr}$ , the induced  $\phi_2$  fluctuations would again be non-Gaussian.

The nonlinear interaction of  $\delta\phi_1$  with  $\delta\phi_2$  generates adiabatic perturbations whose primordial amplitude  $\zeta$  is quadratic in the Gaussian field  $\delta\phi_1$  as evaluated at the end of the  $m_{11}^2 < 0$  period. The spectrum has a mountain centered on the comoving wave number leaving the horizon at the beginning of the  $m_{11}^2 < 0$  period.

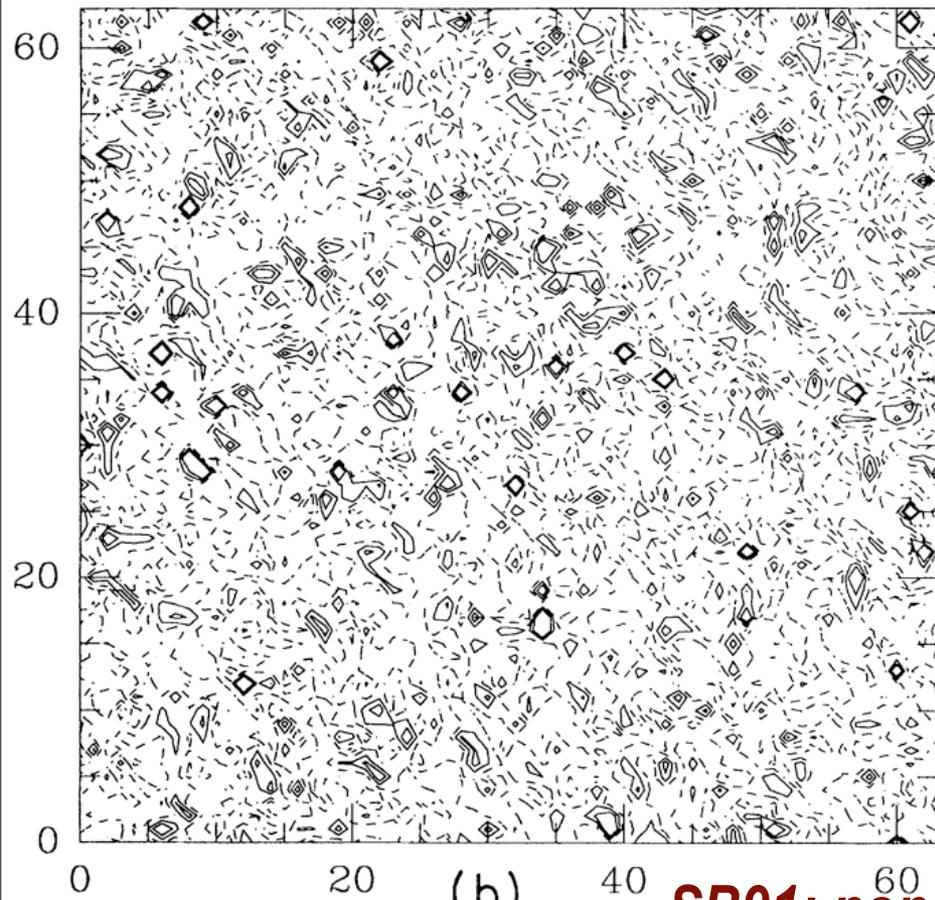


## brief history of bond's non-Gaussian exploration in inflation: THEN

SB90/91: nonG technology:  $\mathbf{y}(\mathbf{r},t) = \mathbf{y}_f(\mathbf{r},t) + \mathbf{y}_b(\mathbf{r},t) (+ \mathbf{y}_{>h}(\mathbf{r},t))$  full  $EE + \phi/\chi$  eqs, restricted to the nonlinear longitudinal gauge (NL-LG); fluctuation/background split  $\Rightarrow$  **Langevin network**:  $\mathbf{y}_f$  linearized (fn of  $\mathbf{y}_b$ );  $\mathbf{y}_b(\mathbf{r},t)$  super-"horizon"  $k < u H a$ , **drift+stochastic kicks**; reduced **Hamilton-Jacobi** eqn; identify  **$\ln a|_{H^*}$**  as the nonlinear generalization of  $\varphi_{\text{com}}$  or  $\zeta$   
**setup for eternal alps (semi-eternal inflation, nonG at UUULSS, but nearly Gaussian over current horizon scales or baroque-ish V)**

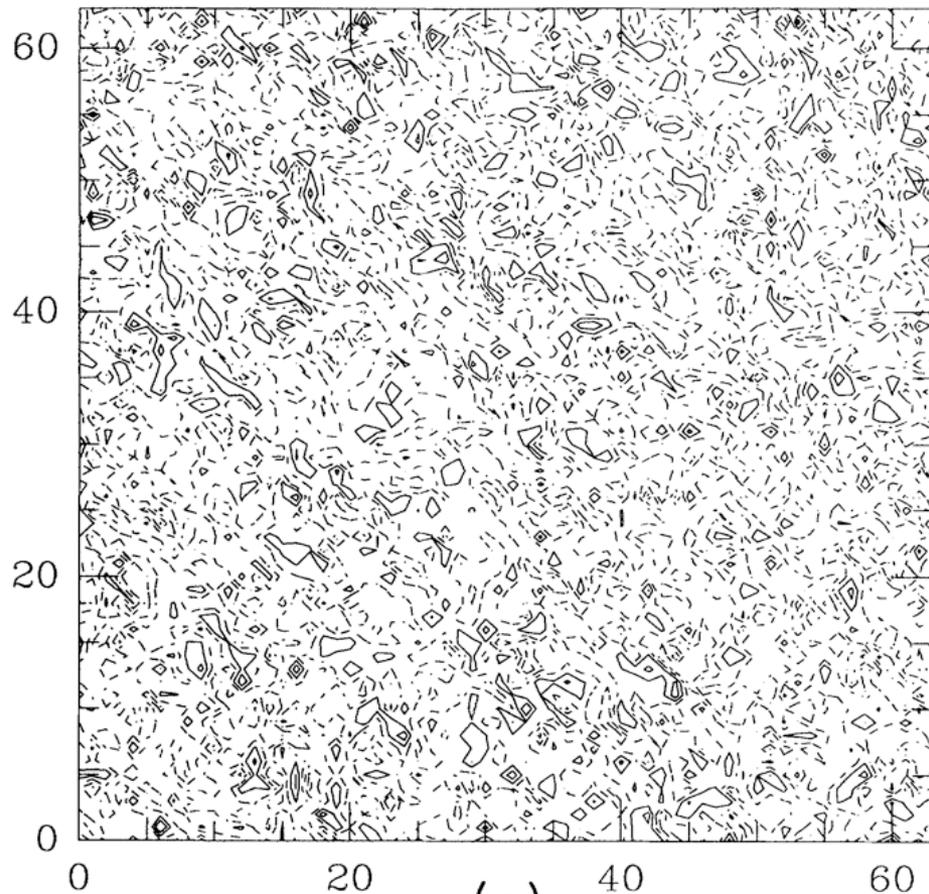
B91 **full NL-LG nonG Langevin network**:  $Y_{bi}(\mathbf{r},t) \in \{\phi, \Pi_\phi, \chi, \Pi_\chi, \ln a, H \mid T = \ln H a\}$

CONTOUR PLOTS FOR  $H(\phi_0) = 1.0 m_p$



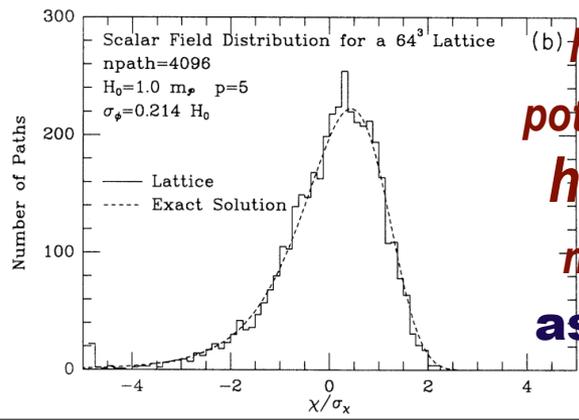
(b)

CONTOUR PLOTS FOR  $H(\phi_0) = 10^{-5} m_p$

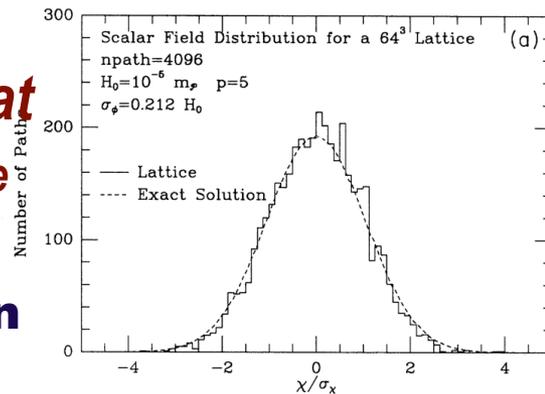


(a)

***SB91: non-G on uniform Ha-***  
***hypersurfaces from a simple exponential***  
***potential via quantum kicks > drift at***  
***high  $H_I \sim m_p$  uuUULSS cf. observable***  
***nearly-Gaussian at low  $H_I \sim 10^{-5} m_p$***   
**asymptotic flat eternal inflation**  
**V has similar behaviour**



(b)



(a)

# INFLATION

Inflaton Drifts & stochastic kicks @  $k=H a$   
 $\Rightarrow$  structure  $(x,t)$

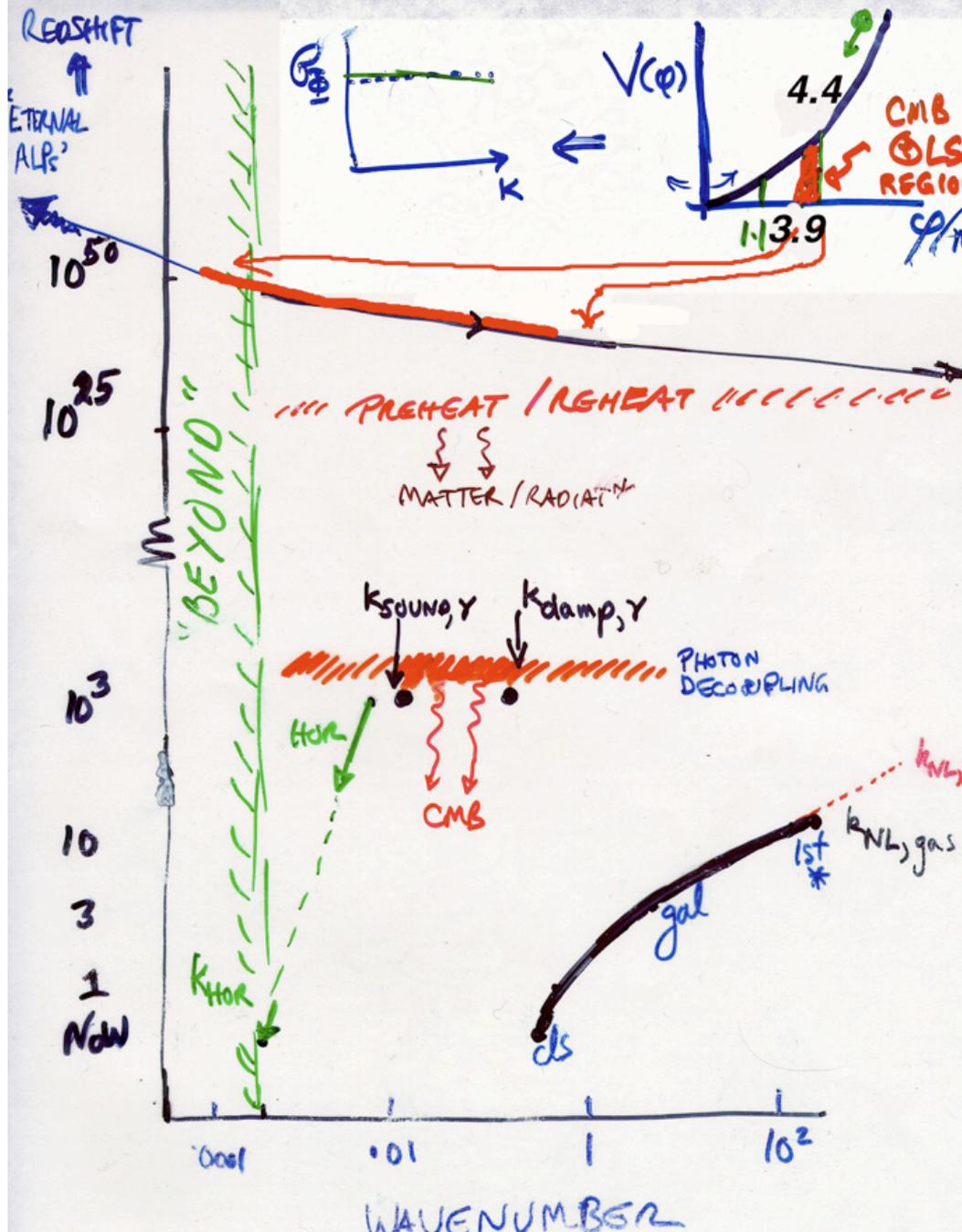
our Hubble patch: smooth + Gaussian fluctuations  
 observable scales are a narrow window on potential surface  $\Rightarrow$

featureless (?)  $P_{\phi com}$  &  $P_{GW}$

UUU...ULSS:

non-Gaussian  
 "eternal alps"

a very **VERYBIG U**



**B91 full NL-LG nonG Langevin network:  $Y_{bi}(r,t) \in \{\phi, \Pi_\phi, \chi, \Pi_\chi, \ln a, H \mid T = \ln H a\}$**

**NL-LG time is stochastic:  $N_b dt = f_t d \ln k_H + f_t q_t dW(\ln k_H)$**

**resolution dimension  $\sim \ln k_H$ ; Wiener increment  $dW(\ln k_H)$  obeys  
 $\langle dW(\ln k_{H1}) dW(\ln k_{H2}) \rangle = \delta(\ln k_{H1} - \ln k_{H2}) d \ln k_H$**

**mean drift:  $d \langle Y_{bi} \rangle = \langle f_t F_{bi} \rangle d \ln k_H$**

**bgnd-fluctuations about mean:  $d\delta Y_{bi} = \delta(f_t F_{bi}) d \ln k_H + [q_i + f_t F_{bi} q_t] d \ln k_H$**

**e.g.,  $q_\phi, q_\chi = u H_b / 2\pi$**

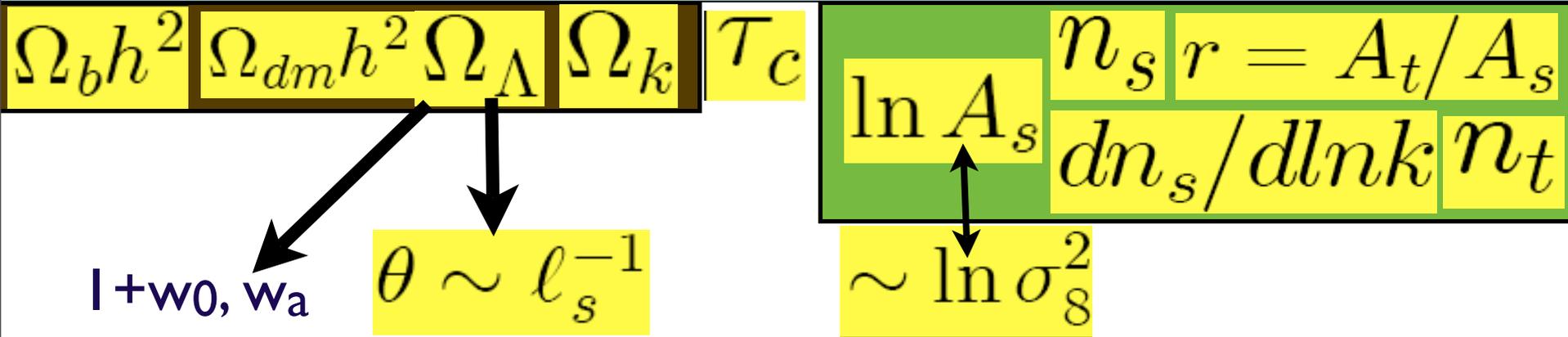
**generally:  $n$  scalar fields at  $m$  spatial points  $\Rightarrow nm$  independent Wiener increments**

80-90s: arena for BSI & non-G near EOI, new fields coupling in  
*expected  $k \sim H a$  rule would apply. **pre-heating surprise!***

**$\ln a[\chi_i(\mathbf{x}, t)]$**  from “subgrid”  $\propto H_e^{-1}$  lattice simulations of  $\phi_{\text{UHF}} \chi_{\text{UHF}}$

like stochastic f-b split, with no dropping of gradient or nonlinear terms

# Standard & Parameters of Cosmic Structure Formation



**+ subdominant isocurvature/ cosmic string & fgnds, tSZ, kSZ, ...**

$-9 < f_{NL} < 111 \Rightarrow -4 < f_{NL} < 80$

**WMAP5 ( $\pm 5-10$  Planck1yr)**

cosmic/fundamental strings/ defects from end-of-inflation & preheating

**+ primordial non-Gaussianity**

$\varphi(x) = \varphi_G(x) + \mathbf{f}_{NL} (\varphi_G^2(x) - \langle \varphi_G^2 \rangle)$   
local smooth

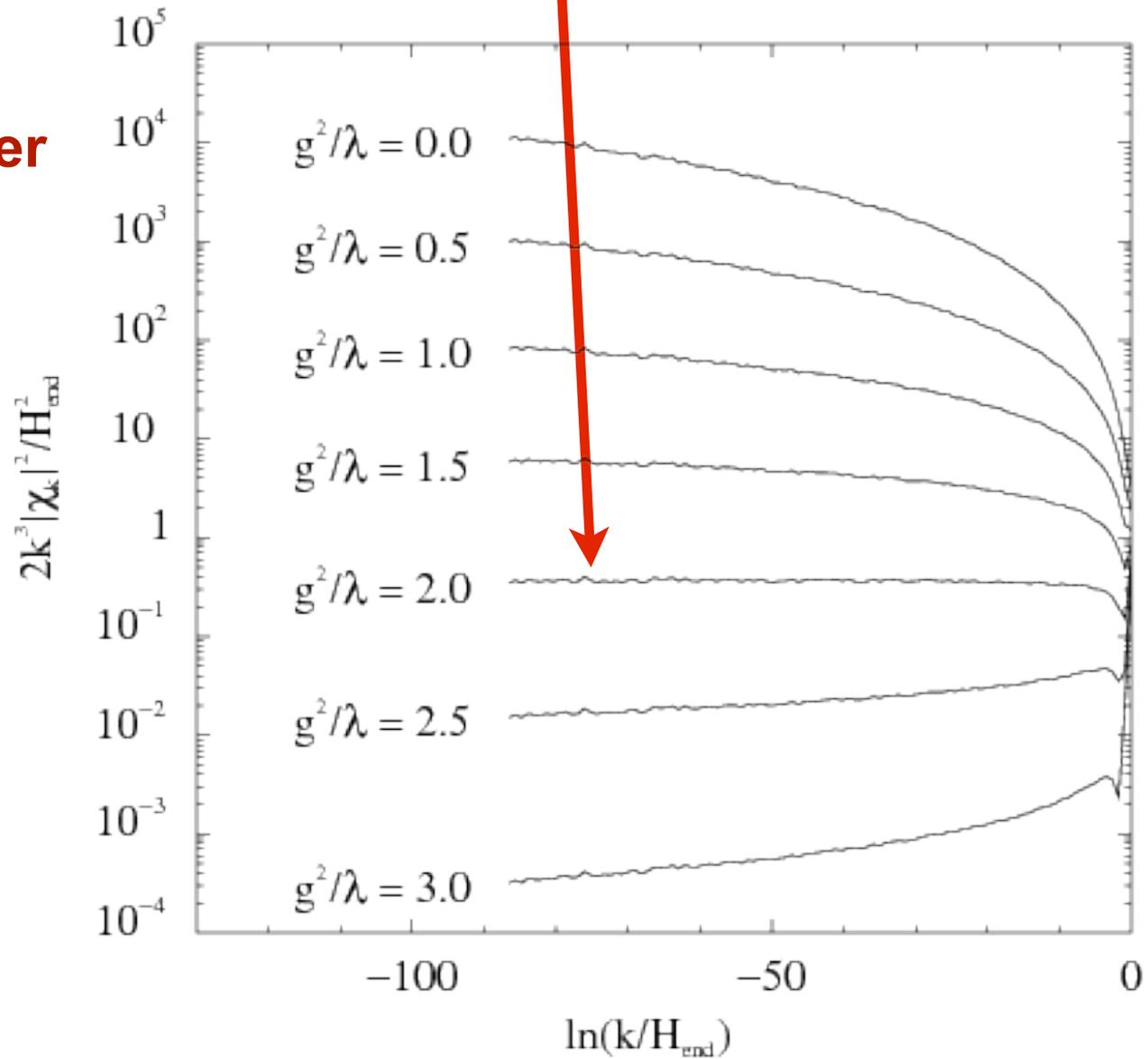
$\varphi(x) = \varphi_G(x) + \mathbf{F}_{NL} (\chi_b + \chi_{>h}) - \langle \mathbf{F}_{NL} \rangle$   
resonant preheating

**DBI inflation: non-quadratic kinetic energy**

**new parameters: trajectory probabilities for early-inflatons & late-inflatons (partially) blind cf. informed "theory" priors**

isocon power spectra are sensitive to  $g^2/\lambda$

$\chi_i(\mathbf{x},t)$  power



# characteristic smoothing scales

$$\chi_{\text{HF}} \quad \sigma_{\text{HF}} \sim 7 \times 10^{-7} \sim 50 \text{ e-folds}$$

field smoothing over  $\chi_{\text{HF}}$   $P(\chi | \chi_{\text{LF}}) \sim \exp[-(\chi - \chi_{\text{LF}})^2 / 2\sigma_{\text{HF}}^2]$

$$\chi_{\text{LF}} \quad \text{sqrt}(\sigma_{\text{b}}^2 + \sigma_{>\text{h}}^2) \quad \text{SSS} \sim 20 \text{ e-folds}$$

$$\text{LSS } \chi_{\text{b}} \quad \sigma_{\text{b}} \sim 3 \times 10^{-7} \sim 10 \text{ e-folds}$$

super-horizon  $\chi_{>\text{h}}$   $\sigma_{>\text{h}} \sim \text{sqrt}(\mathbf{N}_{>\text{h}}) \times 10^{-7}$   
 $\sim \mathbf{N}_{>\text{h}}$  e-folds  $\mathbf{N}_{>\text{h}} \sim 100$  to  $> 10^4$ ?

“observed”  $\chi_{>\text{h}}$  a random throw of the dice

dictates the nature of  $\langle F_{\text{NL}} | \chi_{\text{b}} + \chi_{>\text{h}} \rangle$

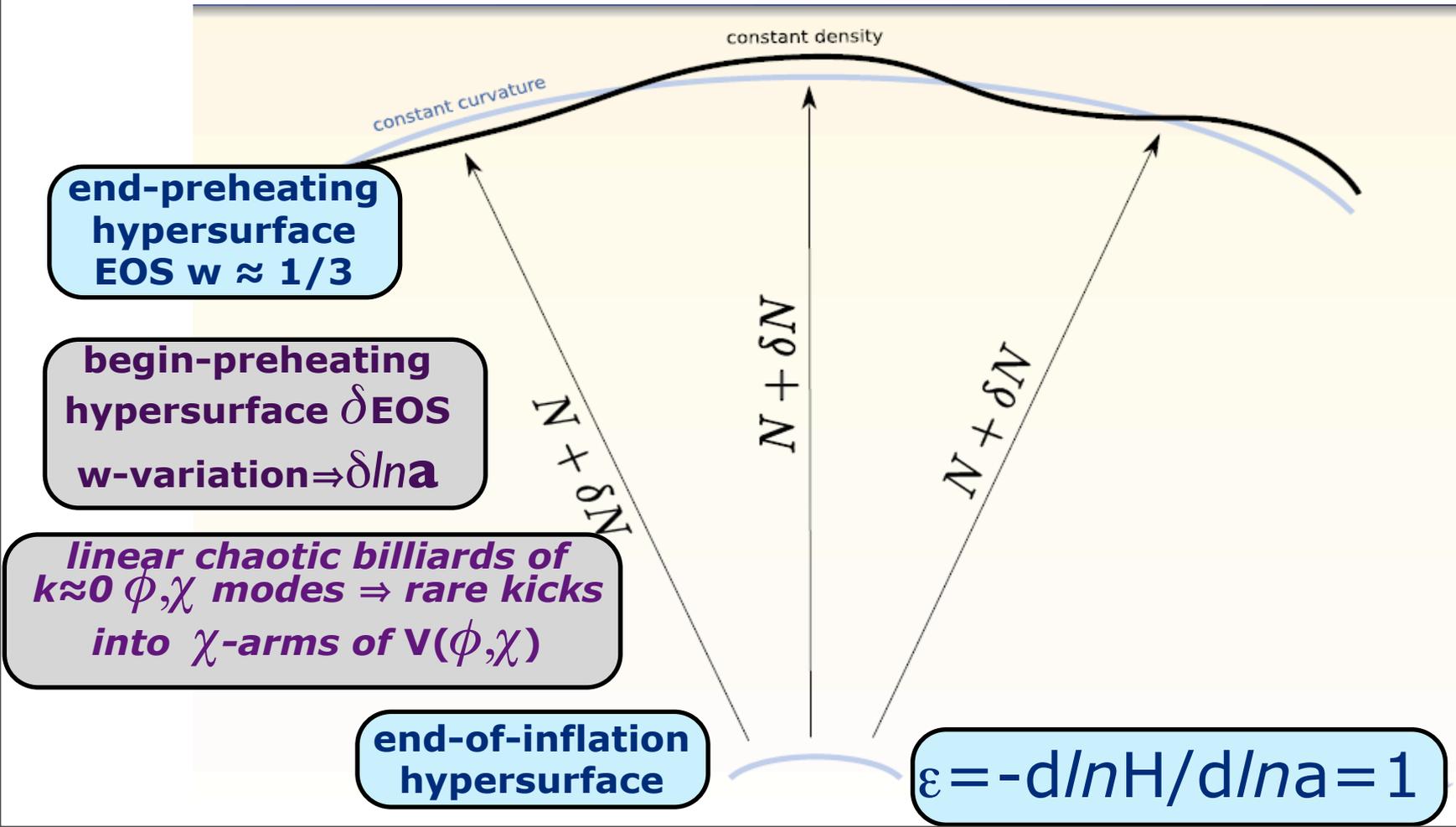
$$P(\chi_{>\text{h}}) \sim \exp[-\chi_{>\text{h}}^2 / 2\sigma_{>\text{h}}^2]$$

Why  $\ln a[\chi_i]$ ? ingredient 1 chaotic zero modes fill V arms, Lyapunov  $\log-\chi_i$  spacing, overtones as well; ingredient 2 arm flow shuts off when  $m_{\chi\chi}^2$  rises sharply at vigorous preheating nonlinearity onset  $\Rightarrow$  EOS change

*Bond, Andrei Frolov, Zhiqi Huang, Kofman 09:*

*calculate how the expansion factor from the end of accelerated expansion (end of inflation) through preheating (copious mode-mode-coupling aka particle creation) to the onset of thermal equilibrium depends on  $\chi_i(x,t)$*

$$\delta N = \delta \ln a|_H = \text{curvature fluctuation}$$



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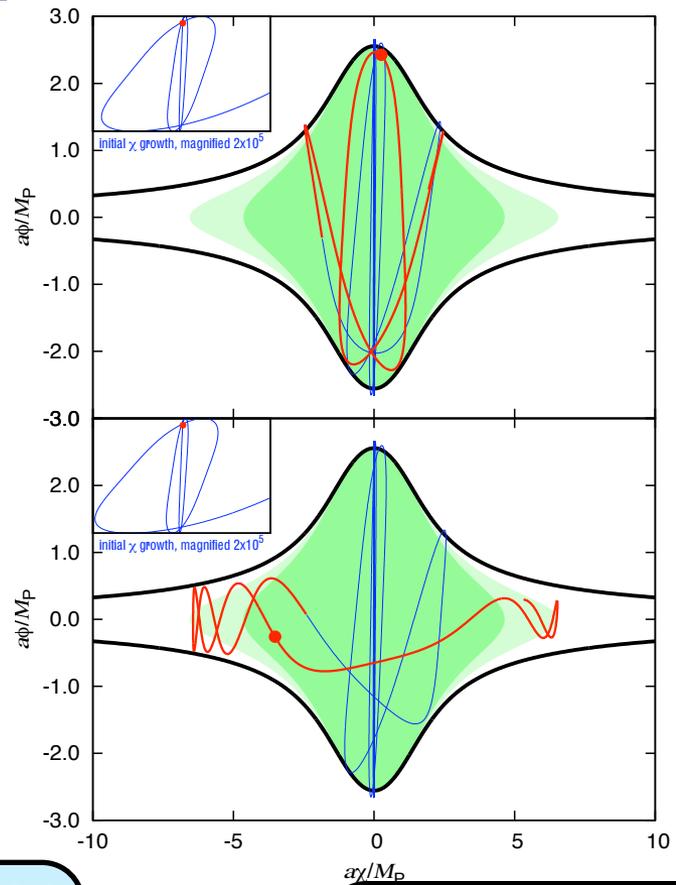
end-preheating hypersurface  
EOS  $w \approx 1/3$

begin-preheating hypersurface  $\delta$ EOS  
 $w$ -variation  $\Rightarrow \delta \ln a$

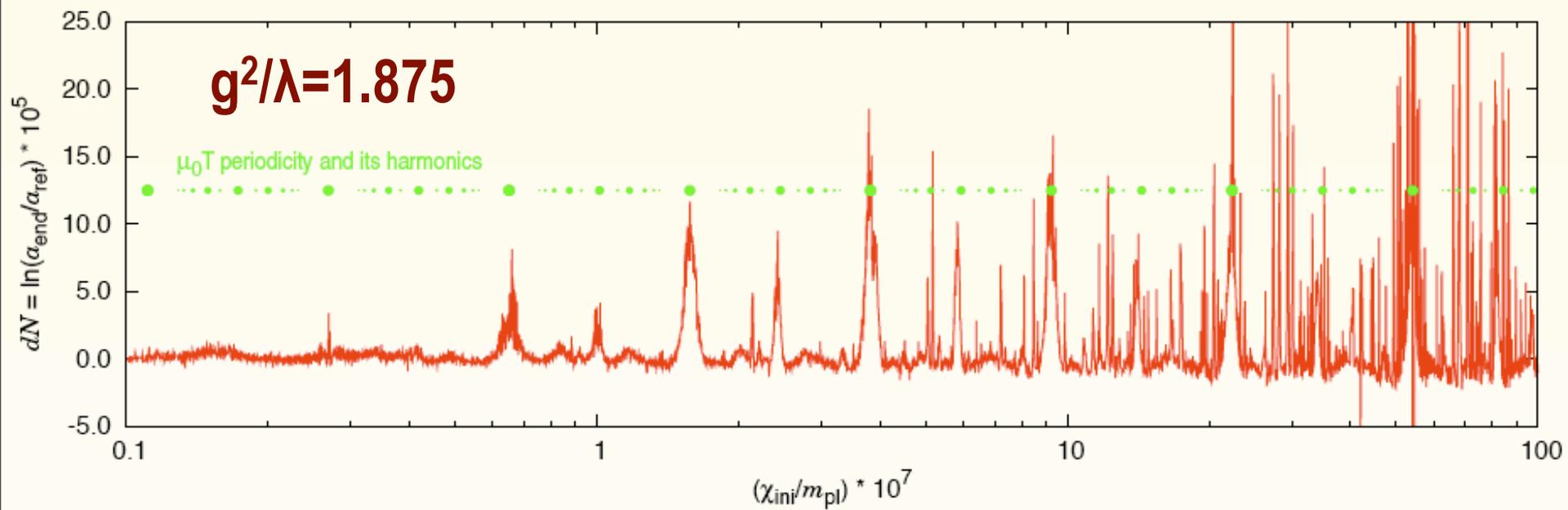
linear chaotic billiards of  $k \approx 0$   $\phi, \chi$  modes  $\Rightarrow$  rare kicks into  $\chi$ -arms of  $V(\phi, \chi)$

end-of-inflation hypersurface

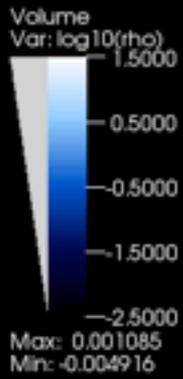
$$\epsilon = -d \ln H / d \ln a = 1$$



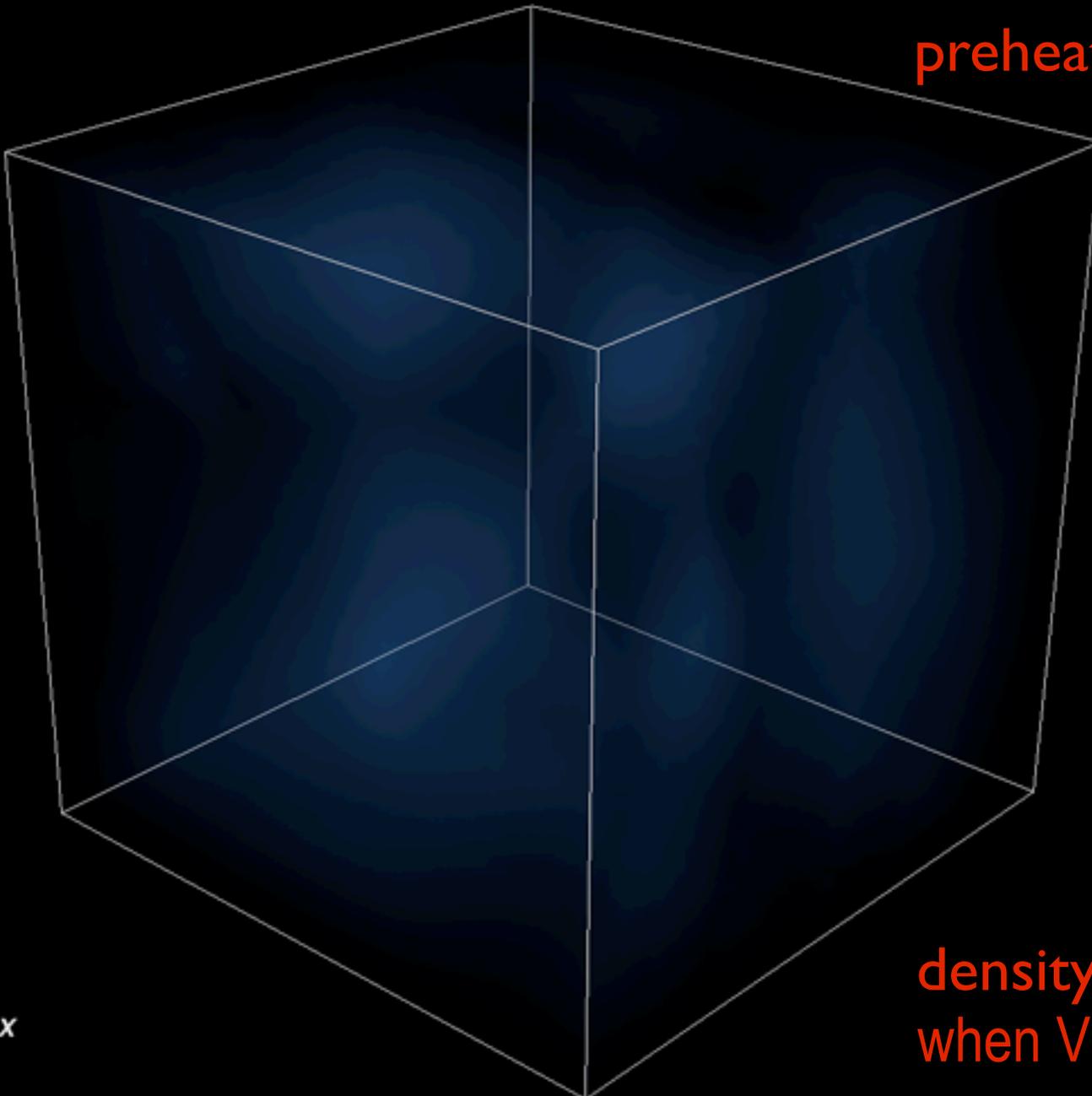
linear regime of zero-modes:  
 $\phi_0(t+T) = \phi_0(t)$   
 $\chi_0(t+T) = \chi_0(t) \exp[\mu_0 T]$   
 $\Rightarrow$  spikes are  $\log \chi_i$  spaced



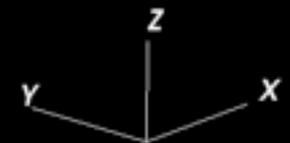
preheating in  $\lambda\phi^4$



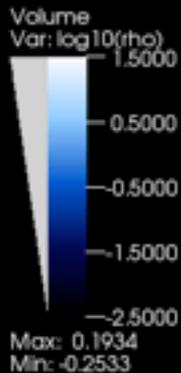
$256^3$



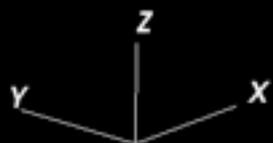
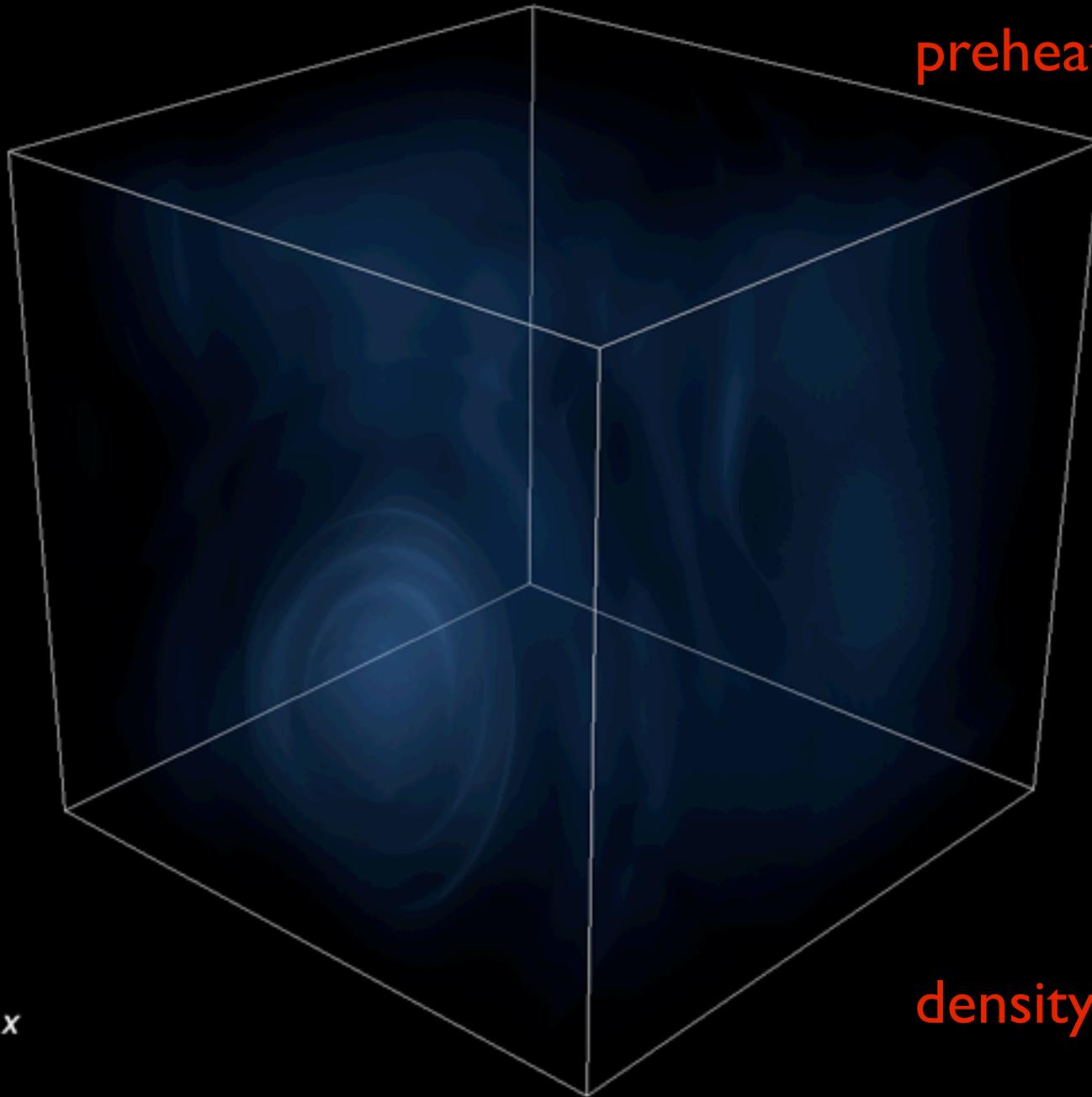
density: @begin NL,  
when V arms close



preheating in  $\lambda\phi^4$



$256^3$

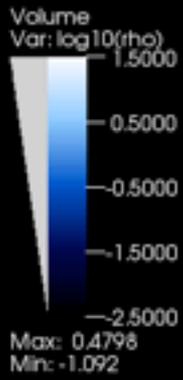


density: @NL waves

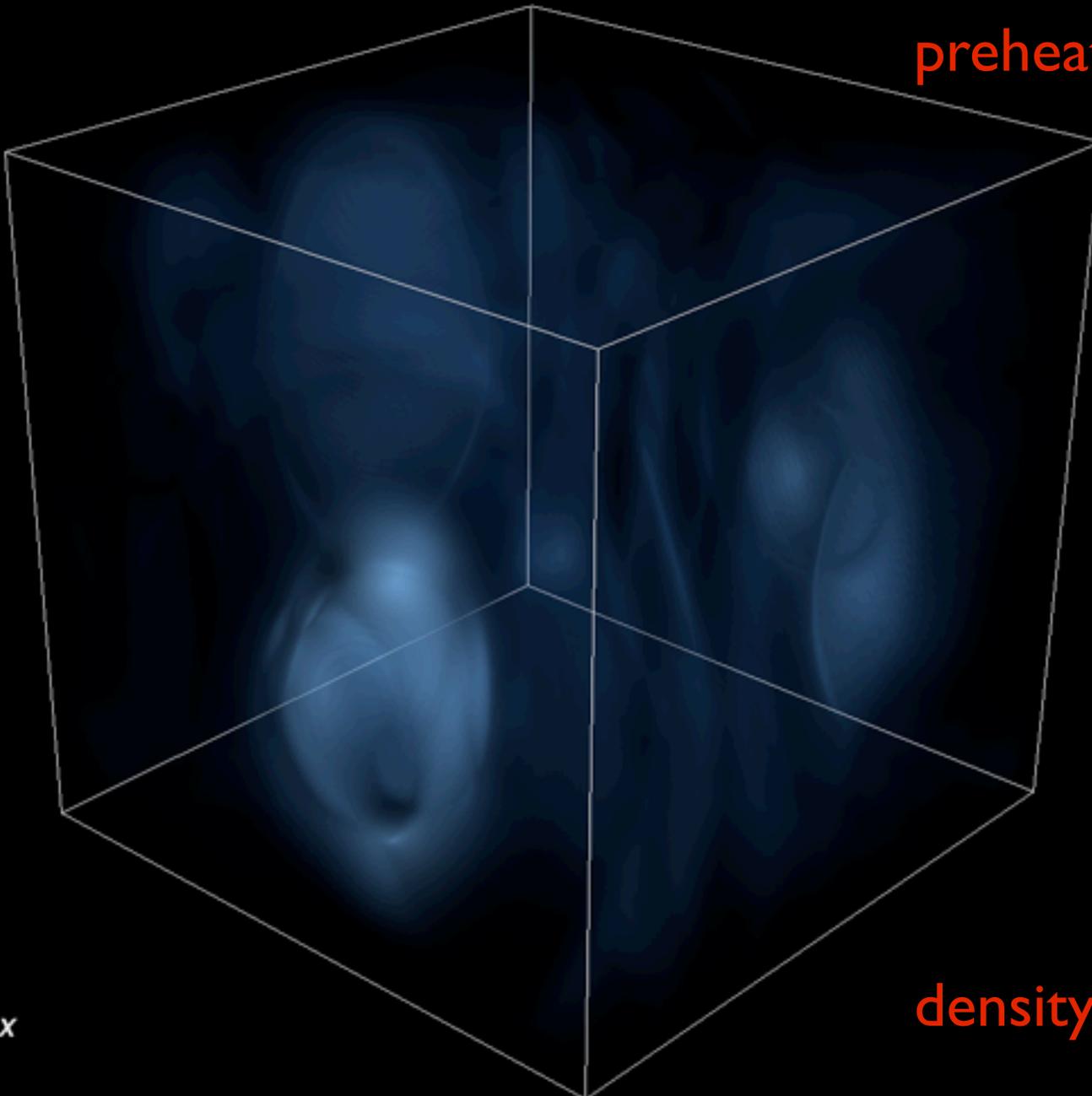
$t = 33.50$



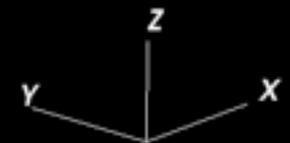
preheating in  $\lambda\phi^4$



$256^3$

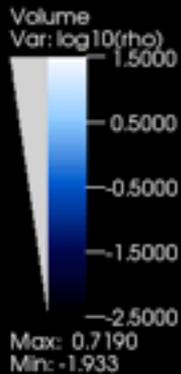


density: @NL waves

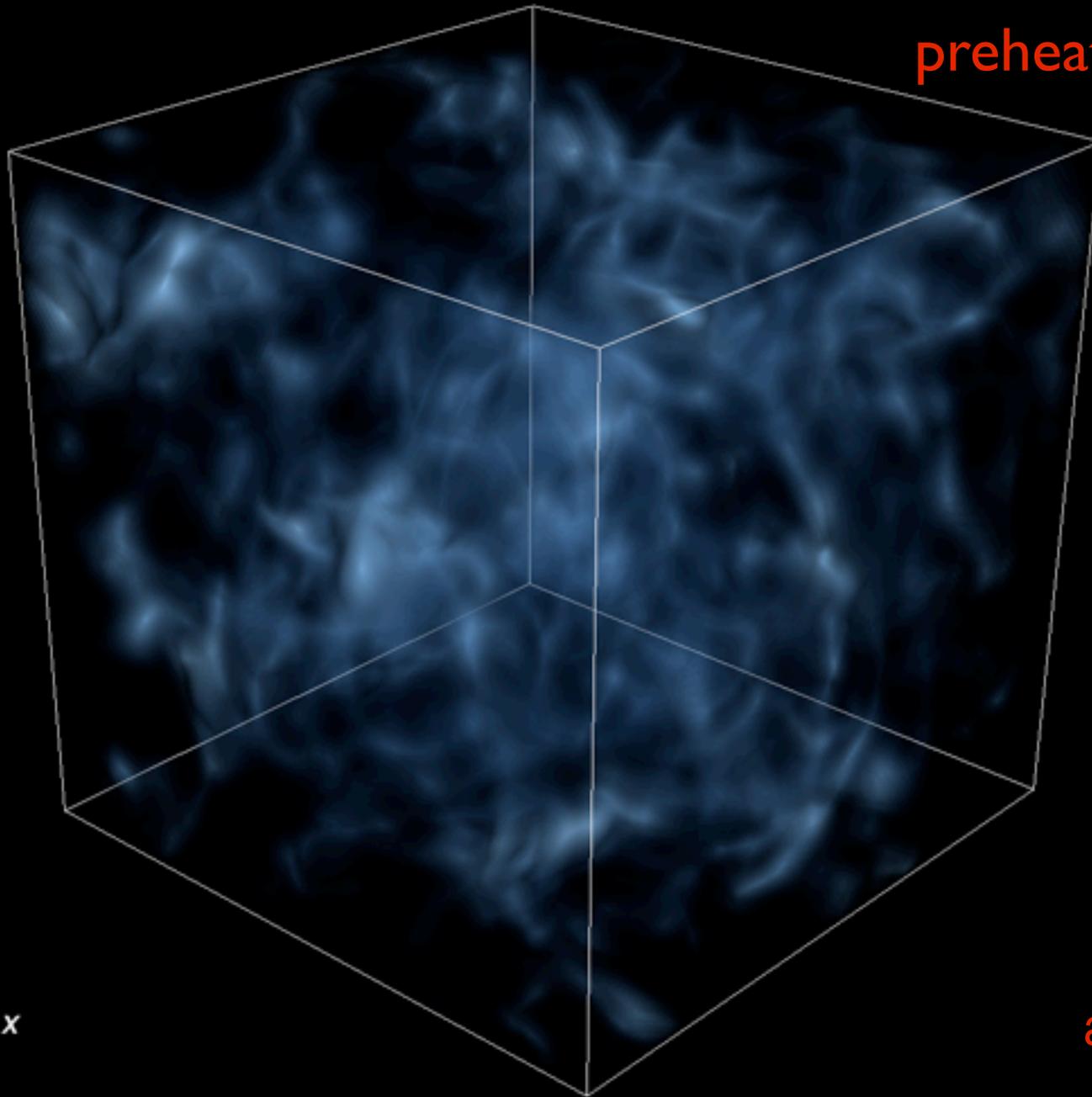


$t = 35.50$

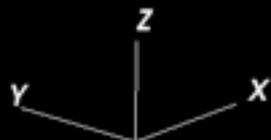
preheating in  $\lambda\phi^4$



$256^3$

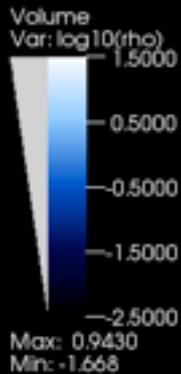


density:  
@mode  
cascade  
approach to  
equilibrium

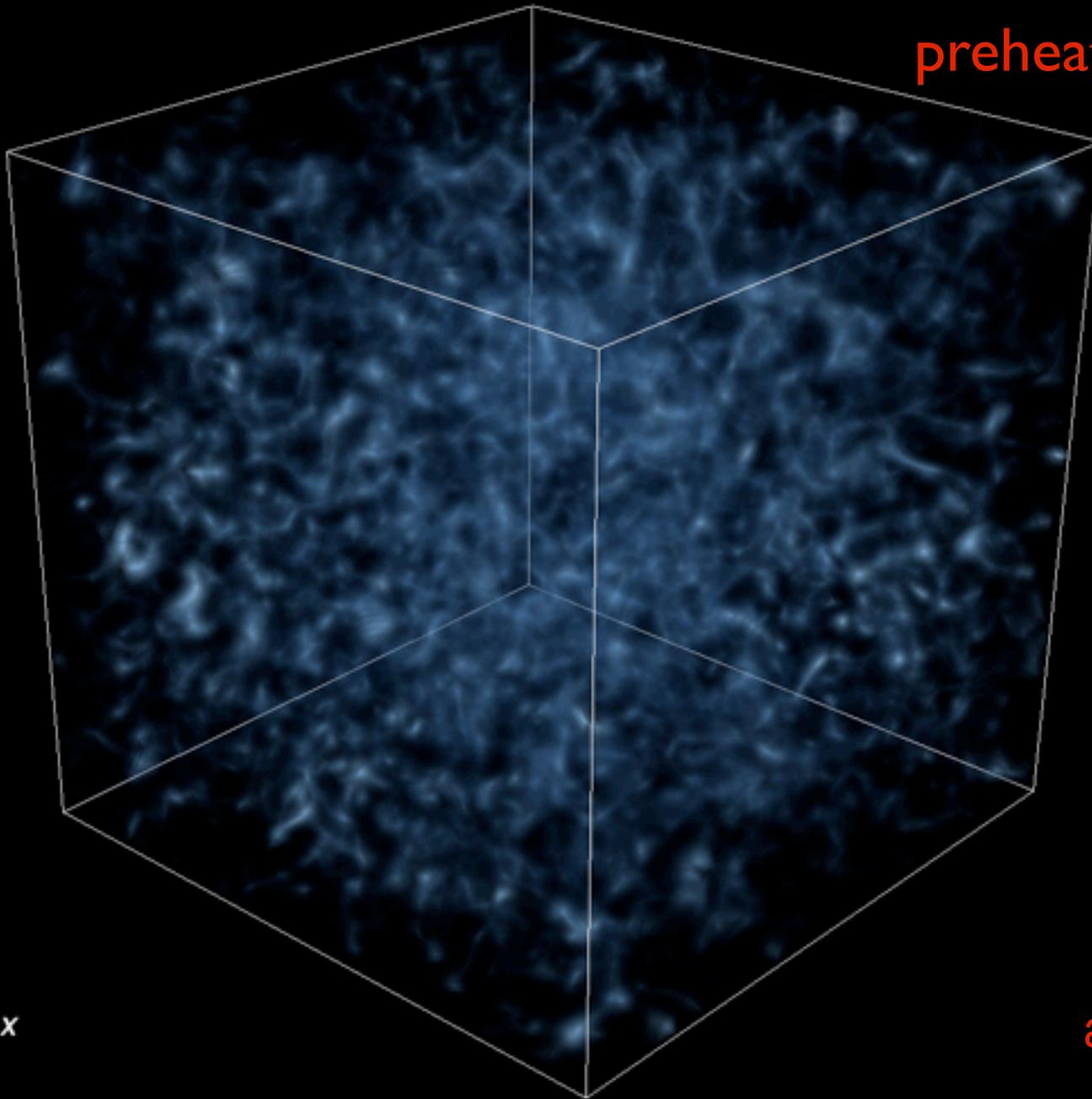


$t = 45.00$

preheating in  $\lambda\phi^4$



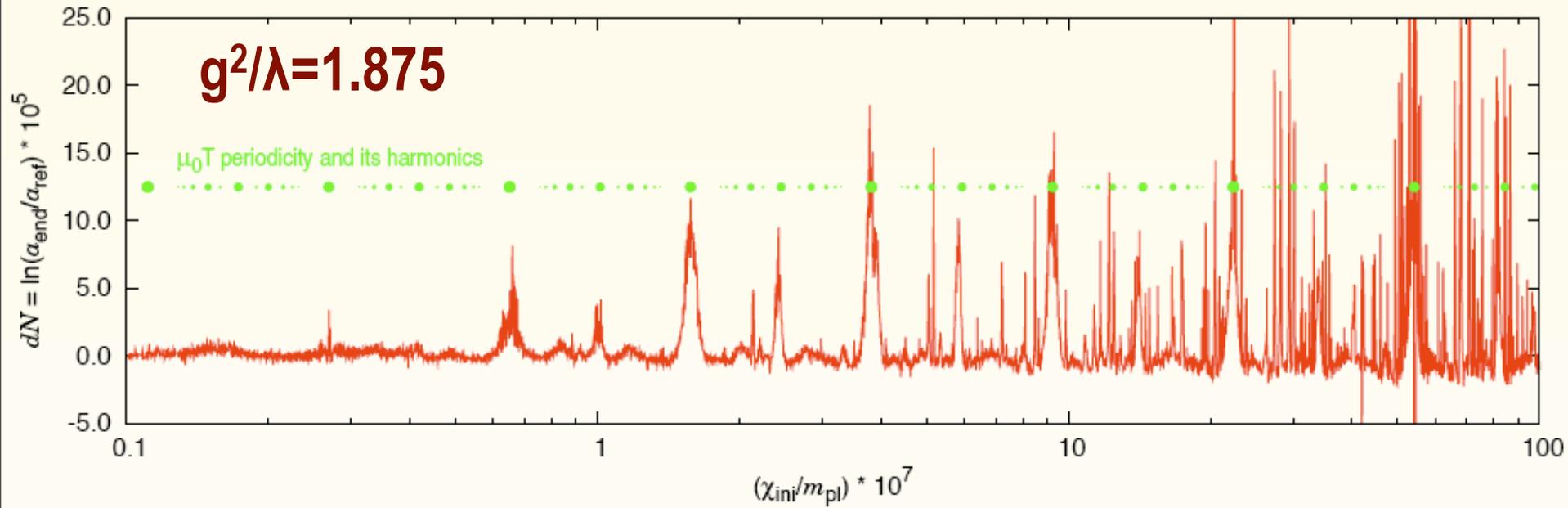
$256^3$

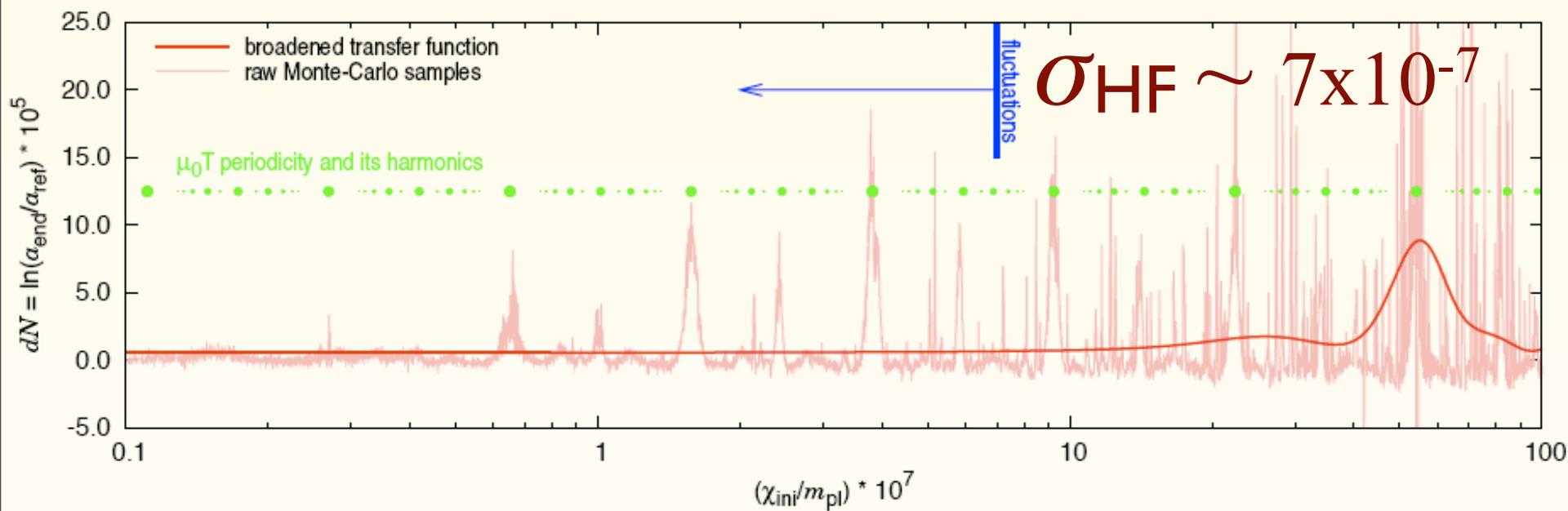
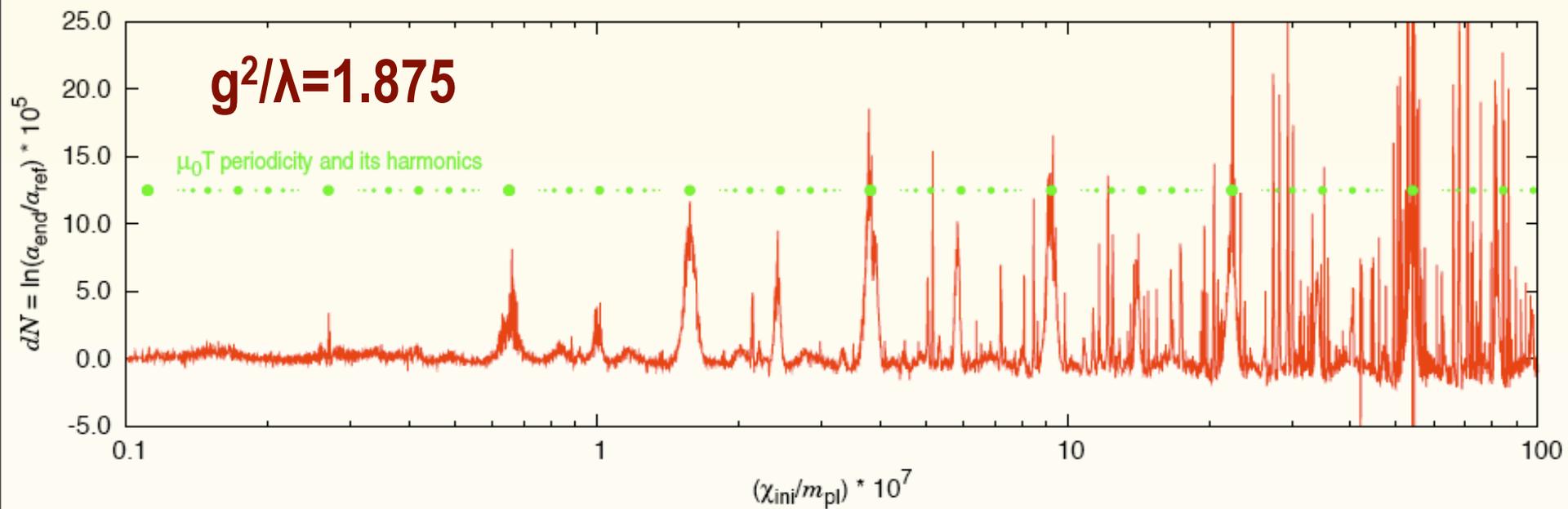


density:  
@mode  
cascade  
approach to  
equilibrium

$t = 128.00$

to develop the  $\ln a(\chi_i)$  response curve, we perform  $> 10^4$  lattice simulations for each  $g^2/\lambda$





$P(\chi | \chi_{LF}) \sim \exp[-(\chi - \chi_{LF})^2 / 2\sigma_{HF}^2]$  builds a usable **low-pass effective mean field**

does it work? linear  $\langle \chi | \chi_{LF} \rangle \sim \chi_{LF}$  is sharp-k filter f-b split BBKS86, BCEK90, BM96

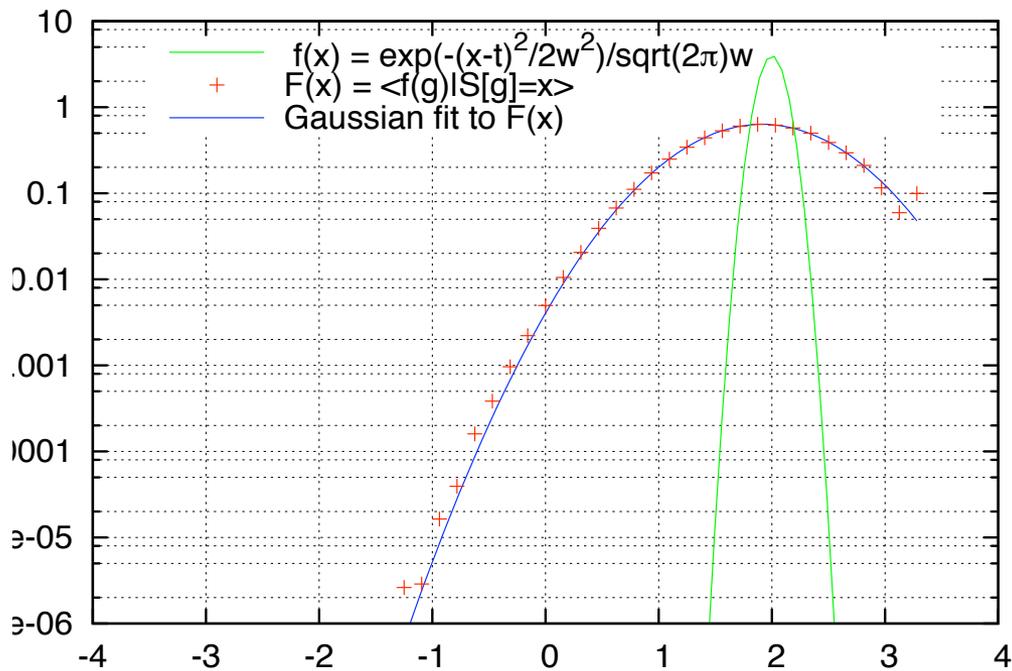
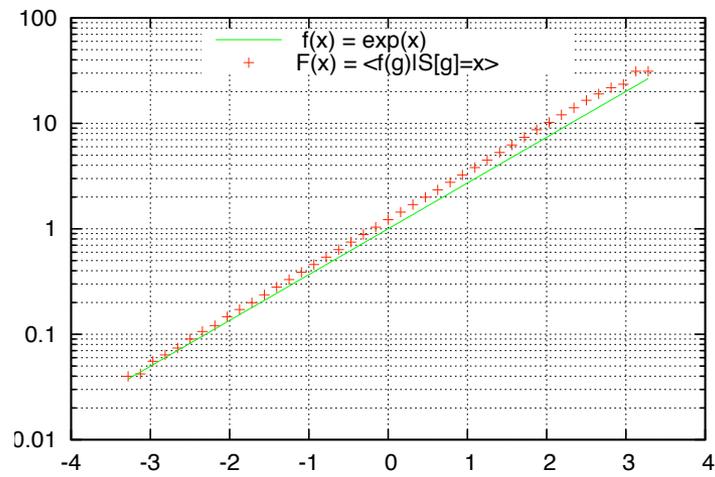
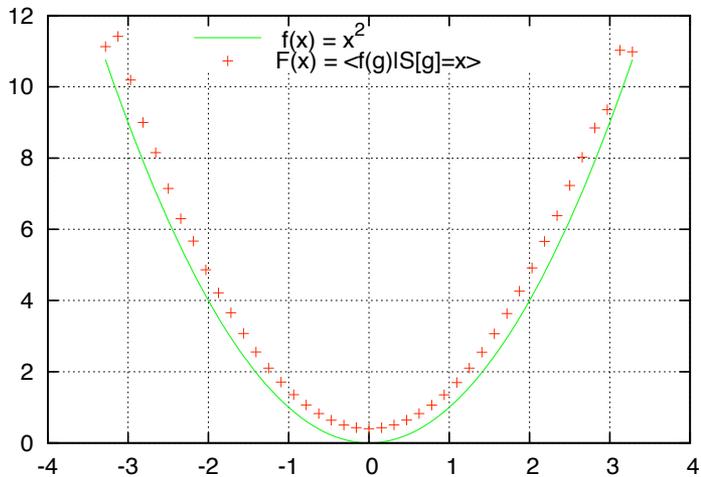
fourier transform ( $F_{NL} - \langle F_{NL} | \chi_{LF} \rangle$ ) is small for  $k < k_{LF}$  for quadratic, exponential & even **Gaussian spikes** (variance  $\sim 1\%$  at  $k_{LF} 0.15$  at  $k_{LF}/10$ )

$\langle F_{NL} | \chi_b(x,t) + \chi_{>h} \rangle$  regimes

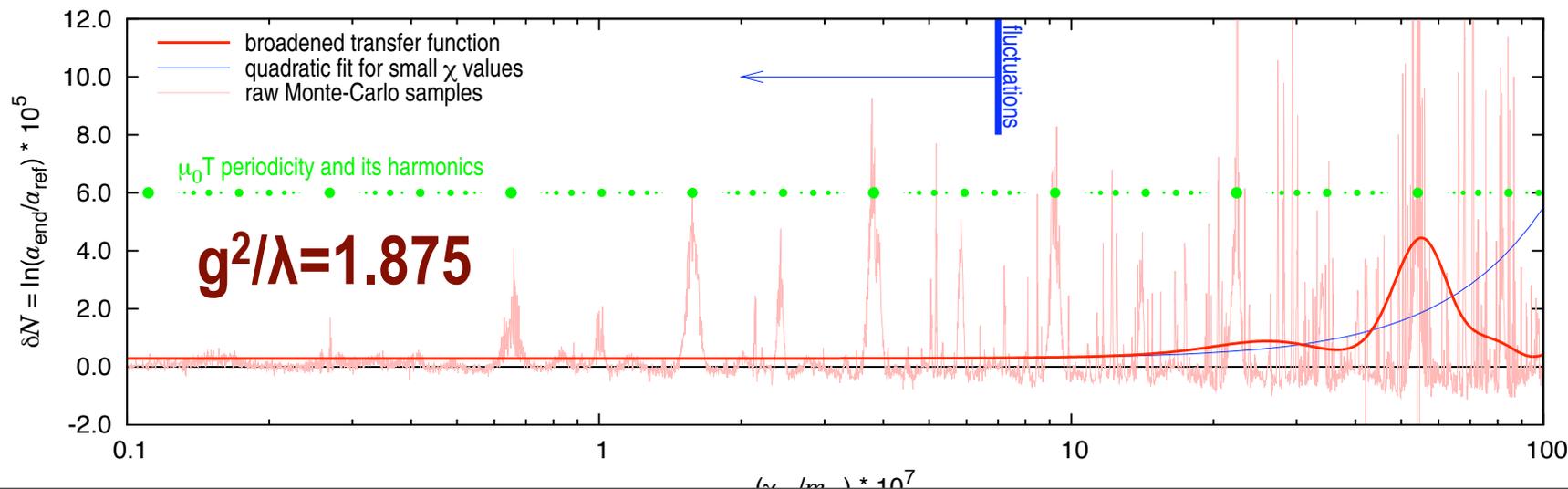
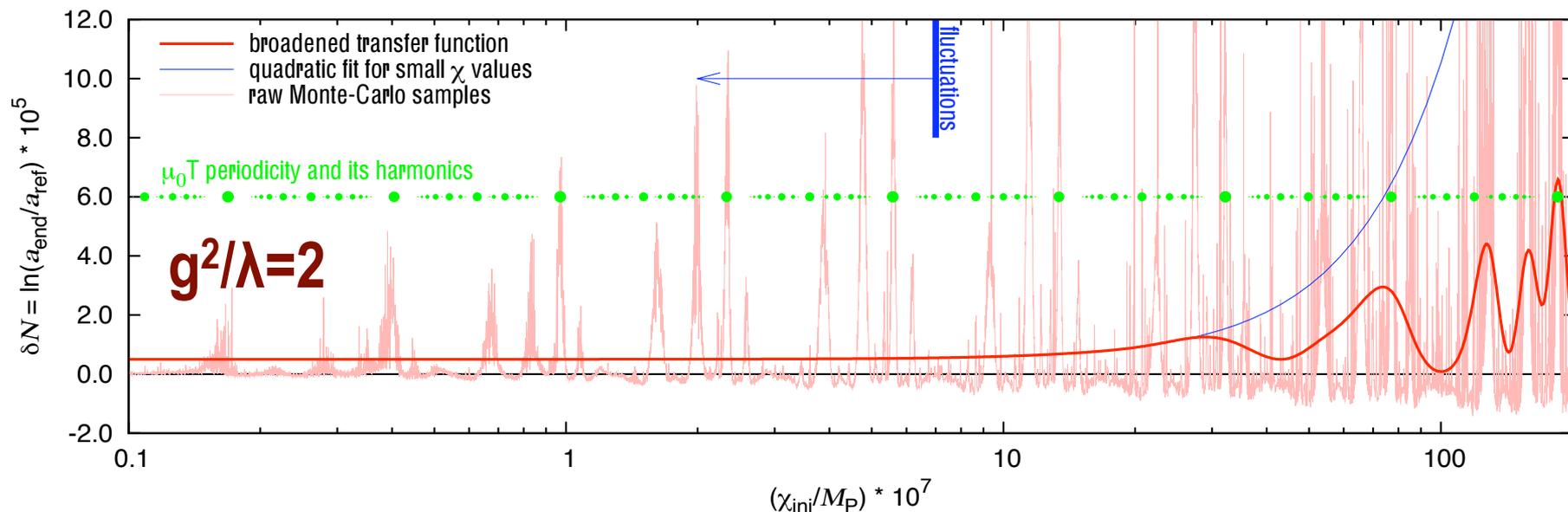
contrast with  $\phi > h$ : may be way out there in eternal inflation land, but not in a preheated  $\epsilon > 1$  patch

# $F_{NL}(\chi_{HF} + \chi_b + \chi_{>h}) - F_{NL}(\chi_b + \chi_{>h})$

small for low k-filter



if the  $k=0$  mode is in a parametric resonance band then  $\ln \mathbf{a}/\mathbf{a}_e$  is modulated by  $\chi_i(\mathbf{x},\mathbf{t}) = \chi_{\text{HF}} + \chi_{\text{b}} + \chi_{>\text{h}}$  with  $k < H_e a$ , treated as  $\sim$ uniform over “subgrid” lattice sim determining  $\chi_{\text{UHF}}$  &  $\phi_{\text{UHF}}$



$\langle F_{\text{NL}} | \chi_b(x,t) + \chi_{>h} \rangle$  regimes

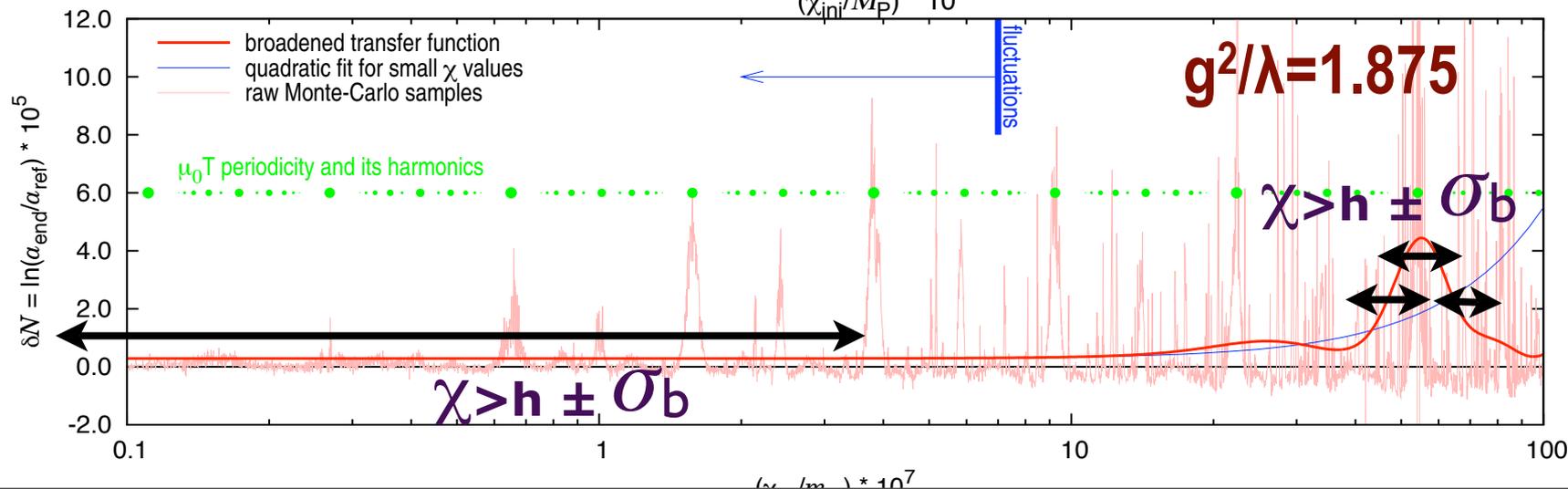
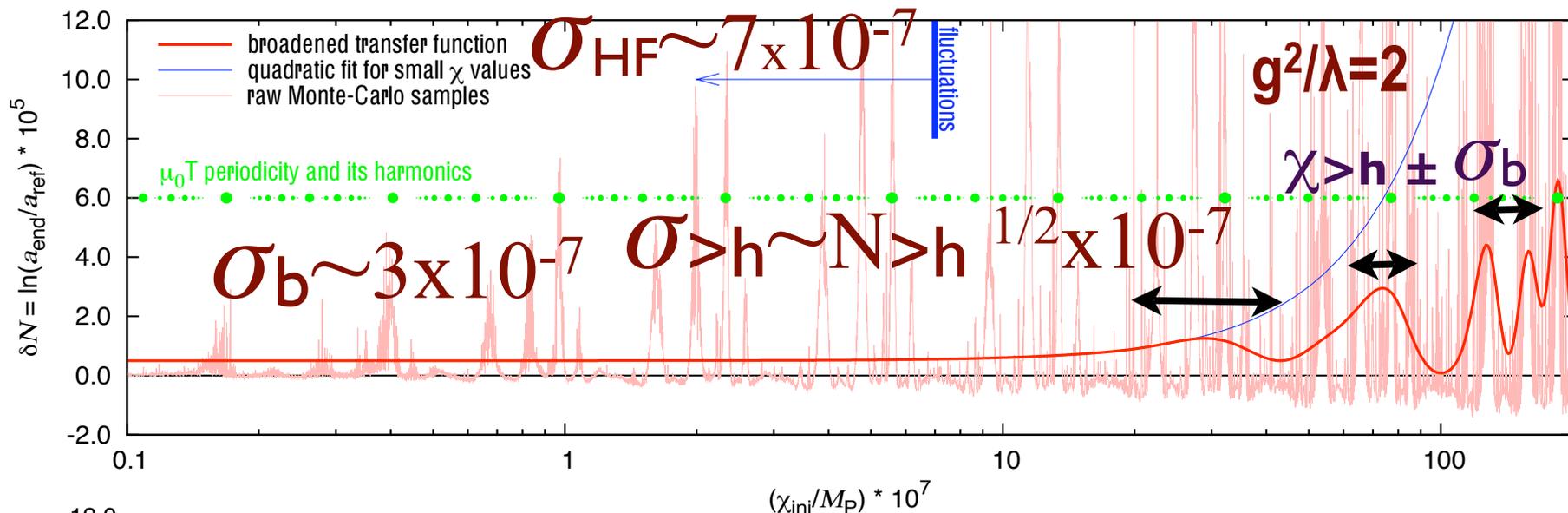
contrast with  $\phi_{>h}$ : may be way out there in eternal inflation land, but not in a preheated  $\epsilon > 1$  patch

LOW  $\chi_{>h}$   $\beta_\chi \chi_b + f_\chi \chi_b^2$  subdominant linear, (much) less constrained  $f_\chi$  cf.  $f_{\text{NL}}$

MEDIUM  $\chi_{>h}$  encompass smoothed spikes, *to be rare* (for  $\Delta T$  cold spot, potential well anomalies *or not to be rare* (and suffer constraints))

LARGE  $\chi_{>h}$  encompass part of a smoothed spike, upside, downside, topside

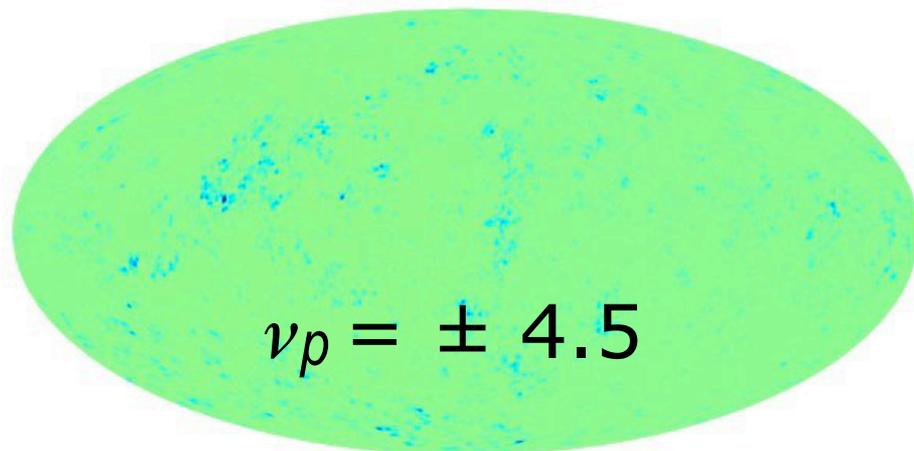
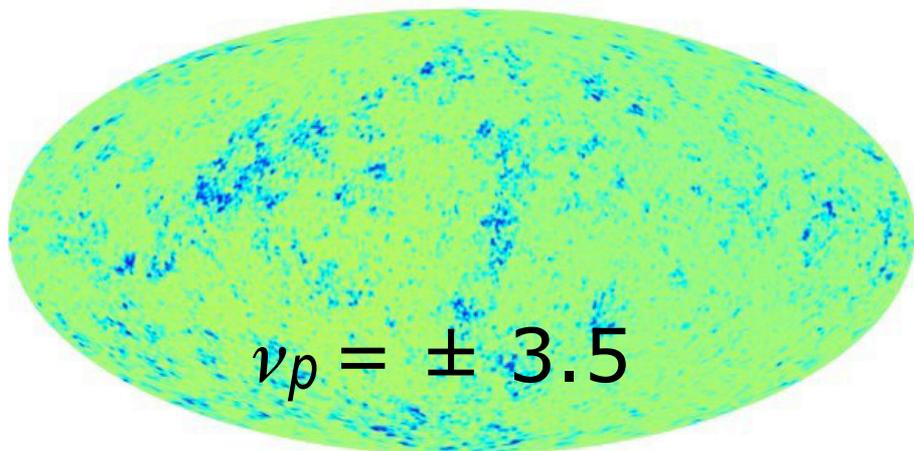
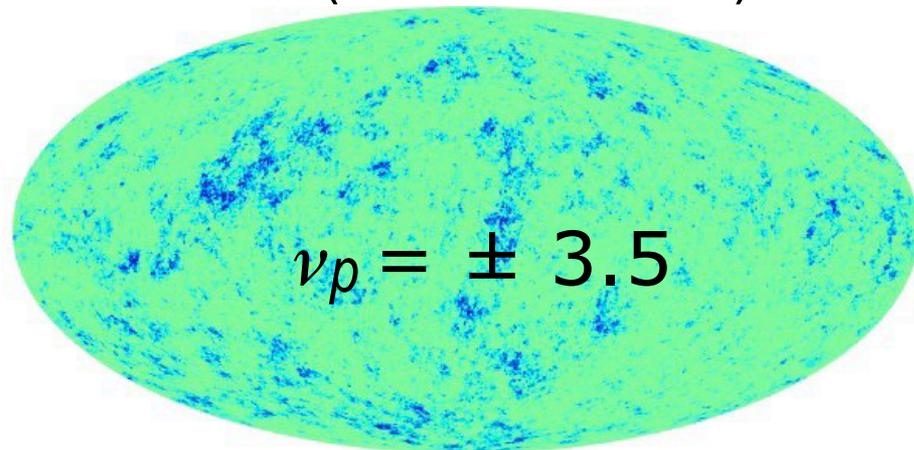
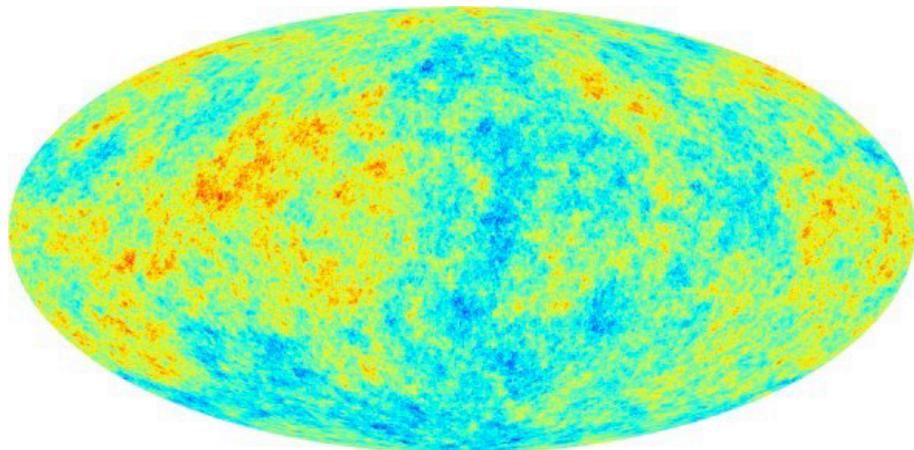
if the  $k=0$  mode is in a parametric resonance band then  $\ln \mathbf{a}/\mathbf{a}_e$  is modulated by  $\chi_i(\mathbf{x},t) = \chi_{\text{HF}} + \chi_{\text{b}} + \chi_{>h}$  with  $k < H_e a$ , treated as  $\sim$ uniform over “subgrid” lattice sim determining  $\chi_{\text{UHF}}$  &  $\phi_{\text{UHF}}$



$\langle F_{\text{NL}} | \chi_{\text{b}+\chi_{\text{h}}} \rangle \sim \pm \nu_{\text{p}} \sigma_{\text{b}}$  symmetrized Gaussian smoothed-spikes

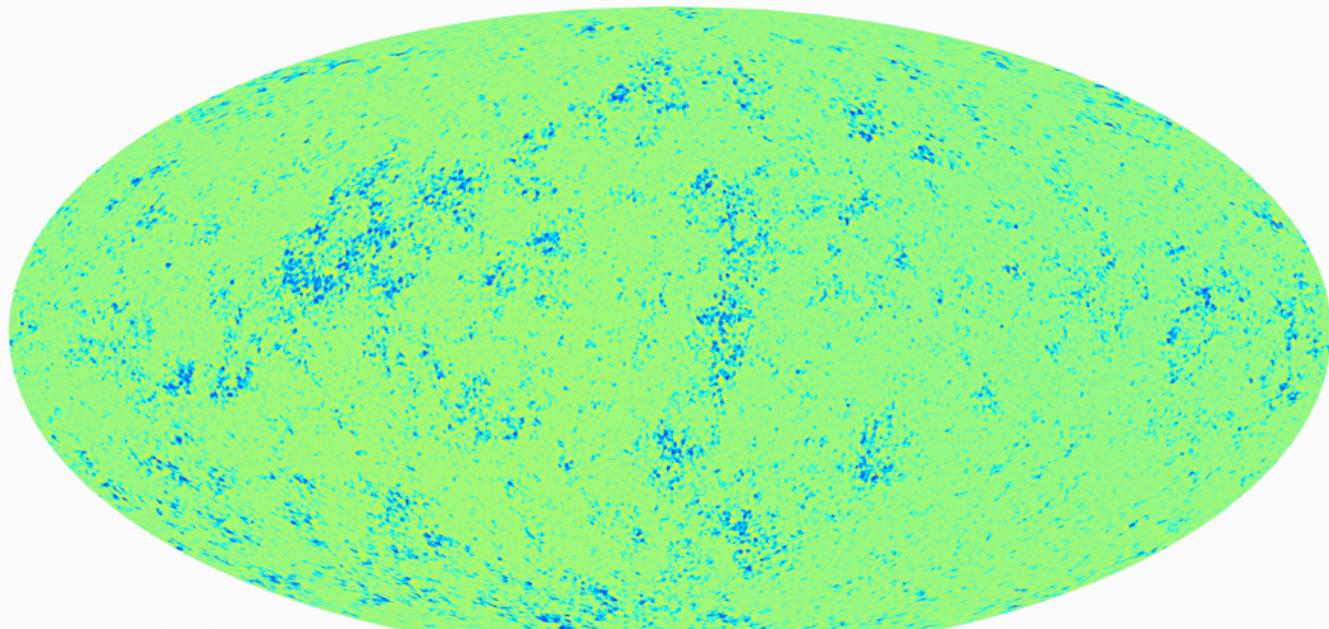
scale invariant

  $F_{\text{NL}}$  (scale invariant)

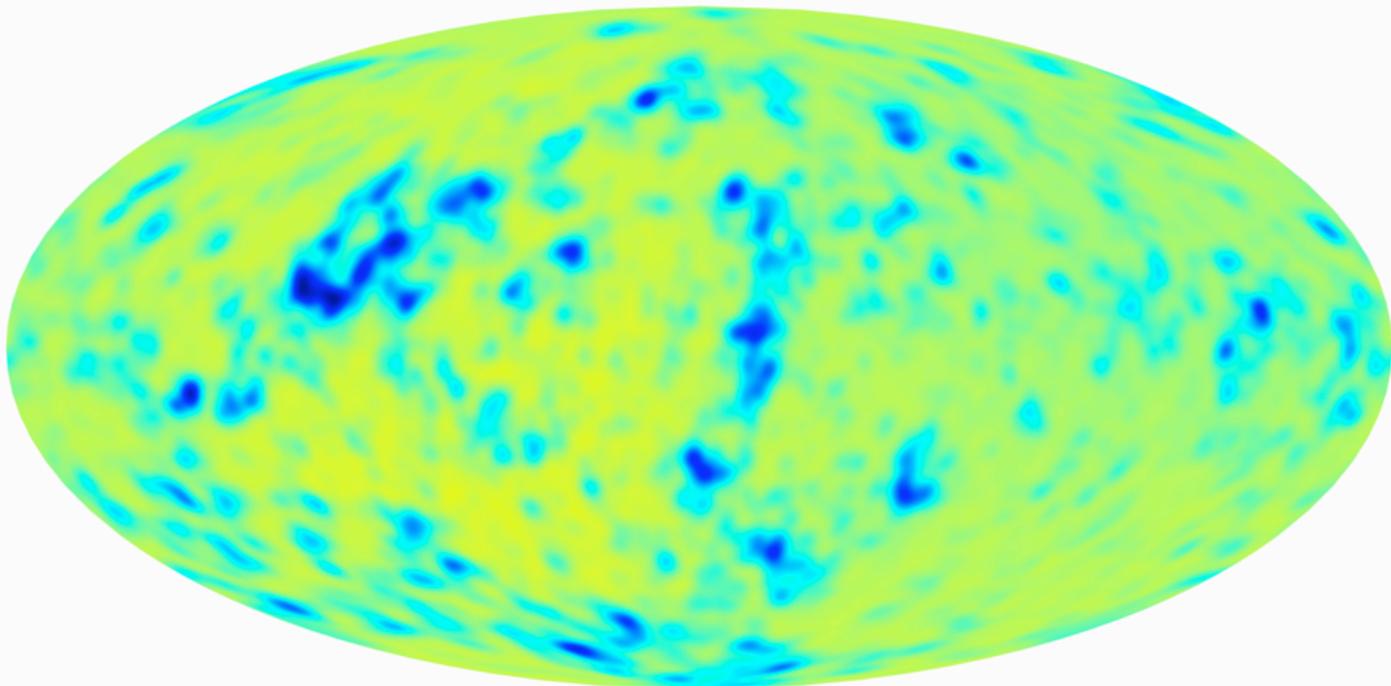


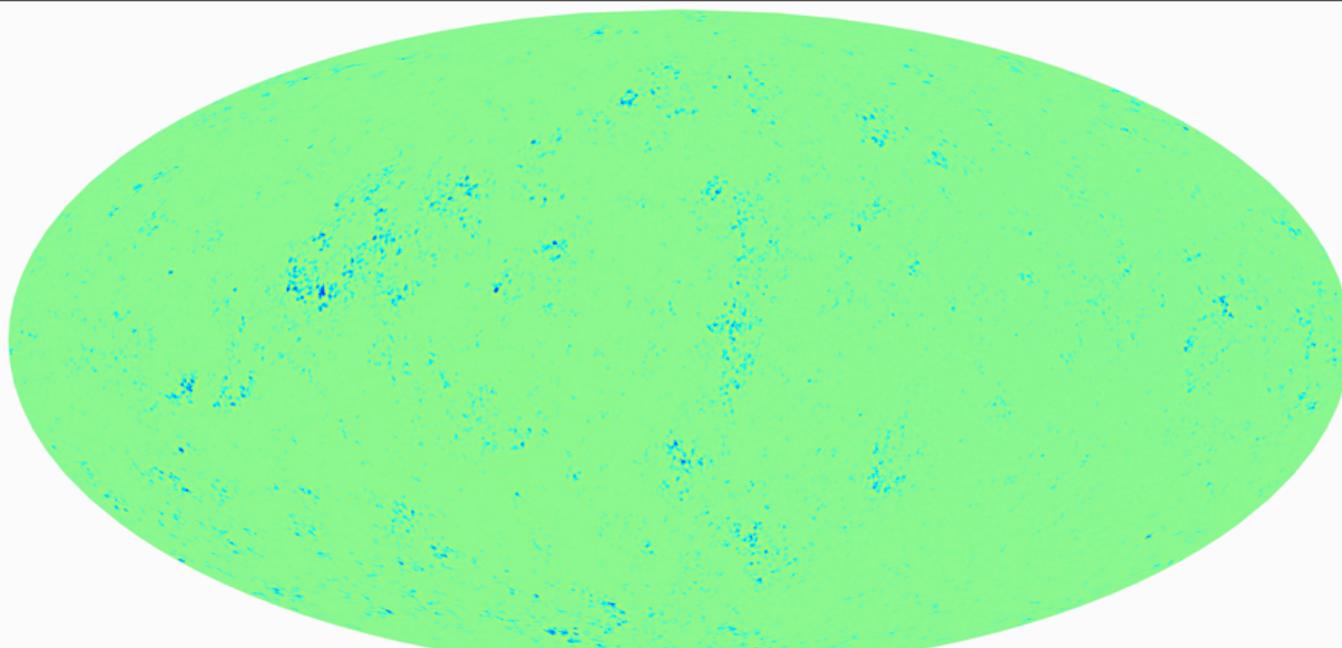
$\sim \Delta T[F_{\text{NL}} \text{ (scale invariant)}] * 1\text{deg}$

$\sim \Delta T[F_{\text{NL}} \text{ (scale invariant)}] * 1\text{deg}$

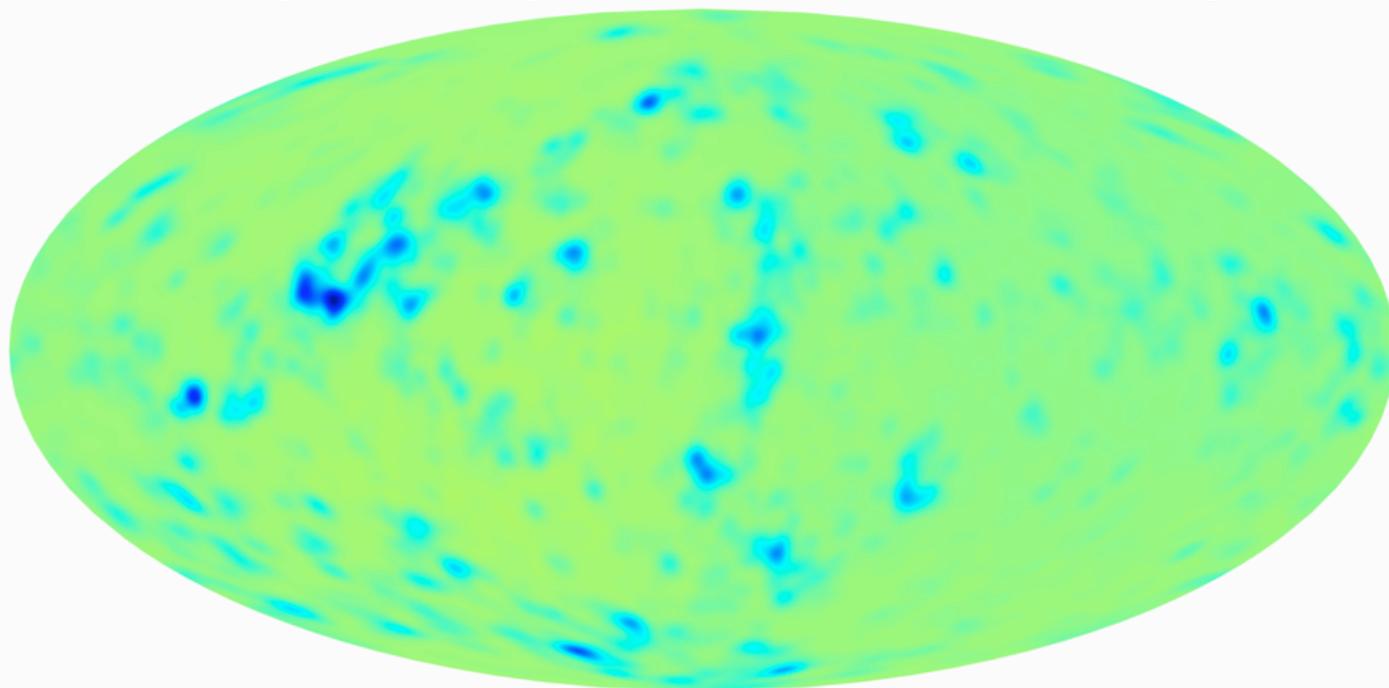


**smoothing: 0 deg cf. 4 deg fwhm  $\nu_p=3.5$**





**smoothing: 0 deg cf. 4 deg fwhm  $\nu_p=4.5$**



**Old view:** Theory prior = delta function of THE correct one and only theory

1980

$R^2$ -inflation

Old Inflation

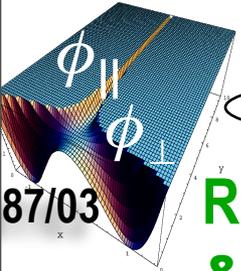
**Chaotic inflation**

New Inflation

Double Inflation

Power-law inflation

SUGRA inflation



**Radical BSI inflation  
& non-Gaussianity  
& baroqueness of V**

**variable  $M_p$  inflation**

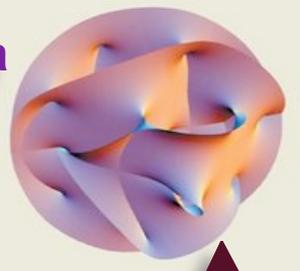
Extended inflation

1990

Hybrid inflation

**Natural pNGB inflation**

6/7 tiny extra dimensions



**New:** Theory prior = probability distribution of late-flows on an energy **LANDSCAPE**

**Old view:** Theory prior = delta function of THE correct one and only theory

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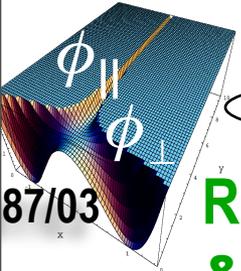
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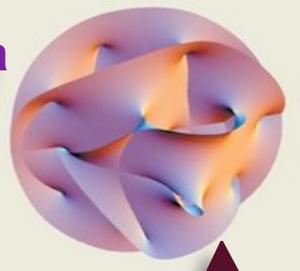
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**New:** Theory prior = probability distribution of late-flows on an energy **LANDSCAPE**

2000

SUSY F-term inflation

SUSY D-term inflation

Assisted inflation

SUSY P-term inflation

Super-natural Inflation

Brane inflation

2003 KKL

N-flation

K-flation

**D3,D7 brane inflation**

DBI inflation

*ekpyrotic/cyclic*

moving brane separations

Racetrack inflation

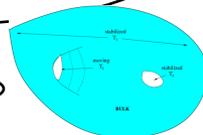
Tachyon inflation

Warped Brane inflation

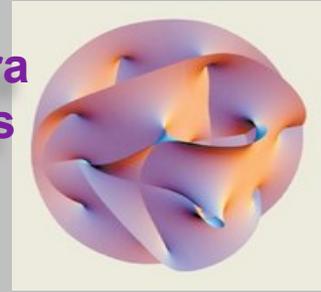
moduli fields

monodromy

**Roulette inflation Kahler moduli/axion**



6/7 tiny extra dimensions



New: Theory prior = probability distribution of late-flows on an energy LANDSCAPE

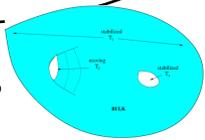
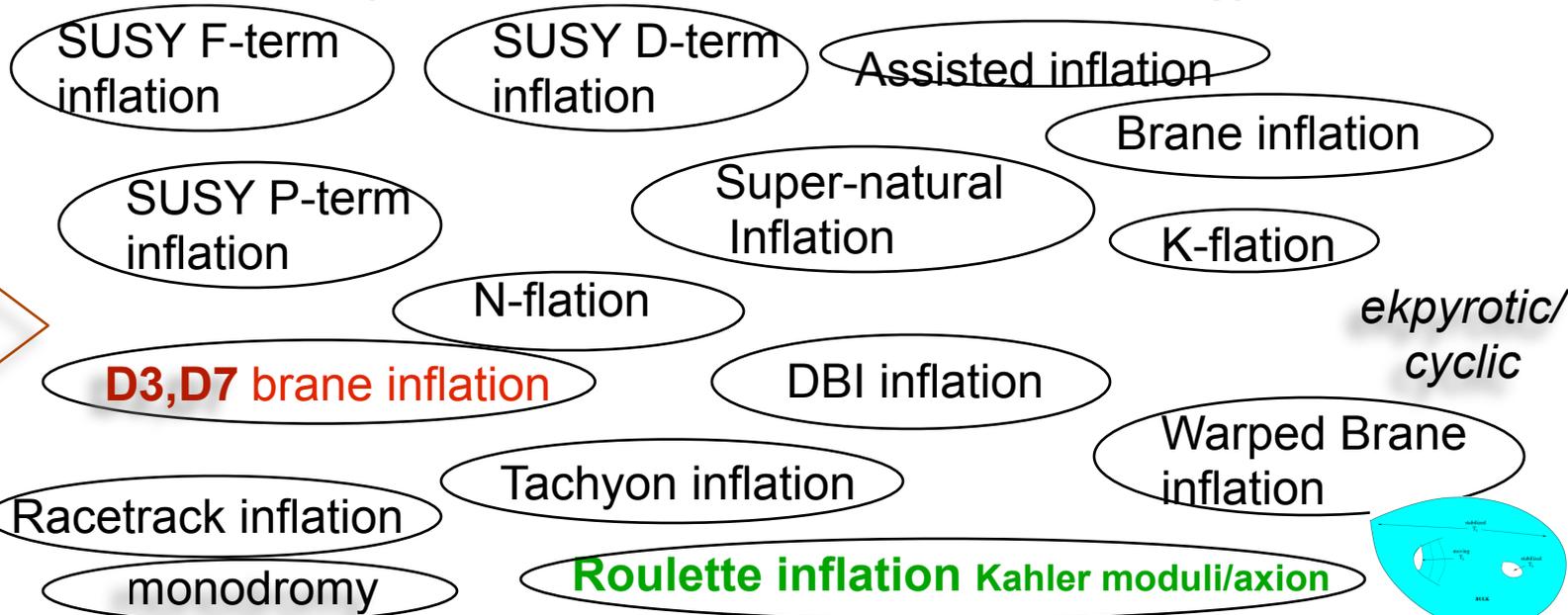
2000

2003 KKL



moving brane separations

moduli fields



# Roulette:

which minimum for the rolling ball depends upon the throw; but which roulette wheel we play is chance too.

The 'house' does not just play dice with the world.

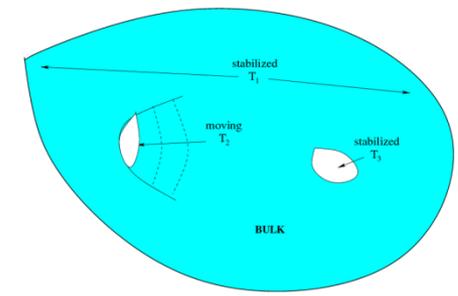
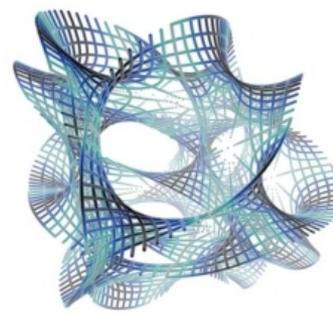
$$V \sim M_P^4 P_s r (1-\epsilon/3)^{3/2}$$

$$\sim (10^{16} \text{ GeV})^4 r/0.1 (1-\epsilon/3)$$

$$\sim (\text{few } \times 10^{13} \text{ GeV})^4$$

$$n_s \sim -d \ln \epsilon / d \ln k / (1-\epsilon)$$

i.e., a finely-tuned potential shape

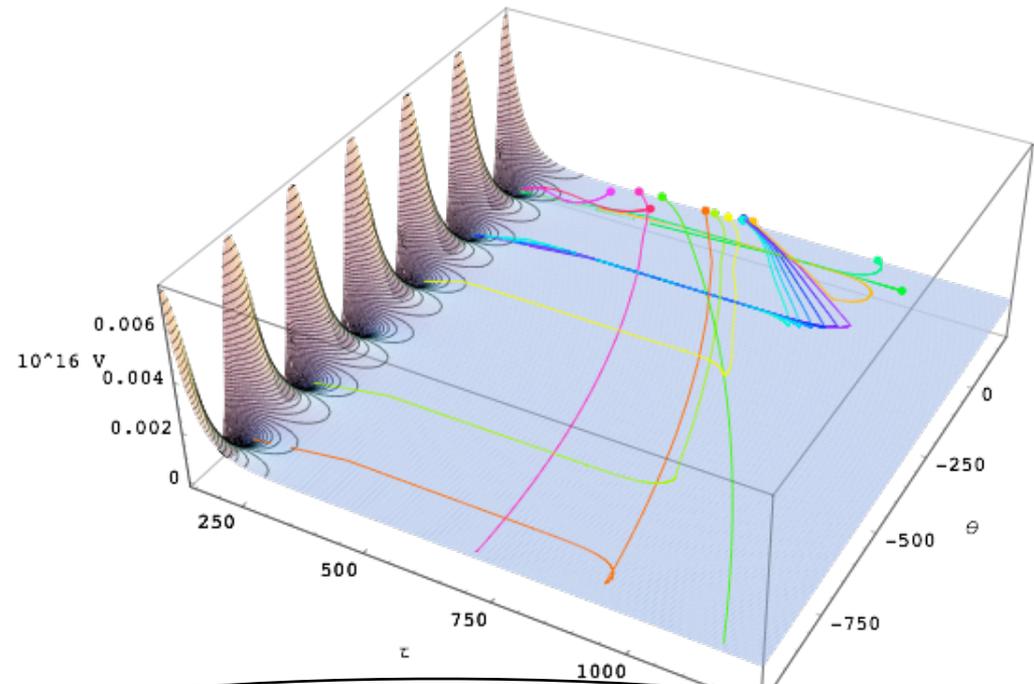


focus on “4-cycle Kahler moduli in large volume

limit of IIB flux compactifications” Balasubramanian, Berglund 2004, + Conlon, Quevedo 2005, + Suruliz 2005

Real & imaginary parts are both important BKPV06

{Number of Efolds:, 29, 211, 4, 12, 2, 285, 105, 8, 11, 18, 30, 53, 106, 0, 0, 0}



Roulette inflation Kahler moduli/axion

Trajectories in a Kahler modulus potential

$\tau_2$  VS  $\theta_2$

$T_2 = \tau_2 + i\theta_2$

Fixed  $\tau_i$   $\theta_i$

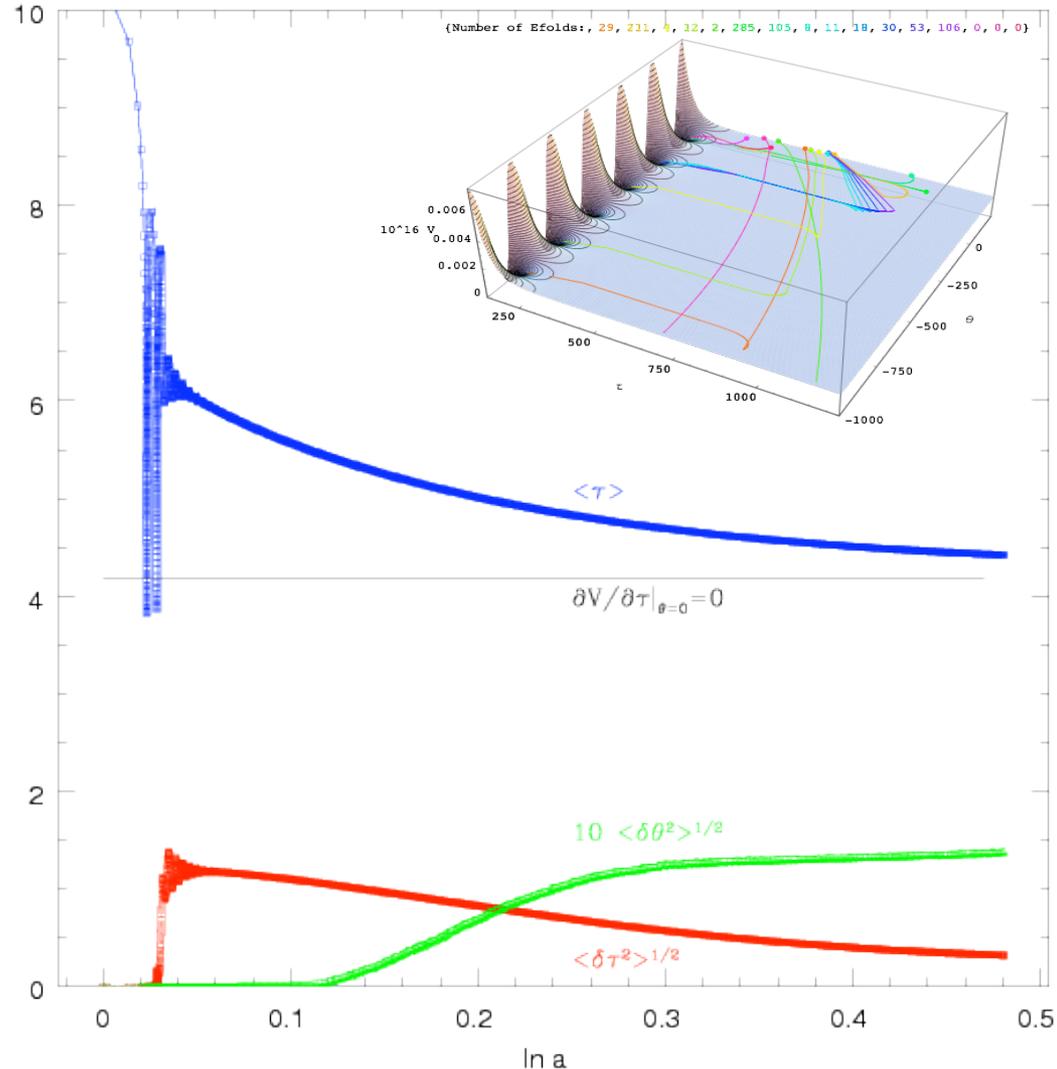
i.e.,  **$i \neq 2$**   
stabilized  
preheating

$\theta_2$  settled in so  
approach pre-h  
in  $\tau_2$  trough  
later nonlinear

$\theta_2$  is excited. **no**  
 **$\delta \ln a$**  but **other**  
**modes should**  
**be important**  
**in endgame**  
(stringy burst, SM  
coupling , ...)

$$V(\tau, \theta) = \frac{8(a_2 A_2)^2 \sqrt{\tau} e^{-2a_2 \tau}}{3\alpha\lambda_2 \mathcal{V}_m} - \frac{4W_0 a_2 A_2 \tau e^{-a_2 \tau} \cos(a_2 \theta)}{\mathcal{V}_m^2} + \Delta V$$

& modified kinetic energy



80-90s: arena for BSI & non-G near EOI, new fields coupling in  
 expected  $k \sim H a$  rule would apply. **pre-heating surprise!**

$\ln a[\chi_i(x,t)]$  from "subgrid"  $\propto H_e^{-1}$  lattice simulations of  $\phi_{\text{UHF}} \chi_{\text{UHF}}$

like stochastic f-b split, with no dropping of gradient or nonlinear terms

Why  $\ln a[\chi_i]$ ? **ingredient 1** chaotic zero modes fill V arms, Lyapunov  $\log-\chi_i$   
 spacing, overtones as well; **ingredient 2** arm flow shuts off when  $m_{\chi\chi}^2$  rises  
 sharply at **vigorous preheating nonlinearity onset**  $\Rightarrow$  EOS change

$P(\chi | \chi_{\text{LF}}) \sim \exp[-(\chi - \chi_{\text{LF}})^2 / 2\sigma_{\text{HF}}^2]$  builds a usable **low-pass effective mean field**

does it work? linear  $\langle \chi | \chi_{\text{LF}} \rangle \sim \chi_{\text{LF}}$  is sharp-k filter f-b split BBKS86, BCEK90, BM96  
 fourier transform ( $F_{\text{NL}} - \langle F_{\text{NL}} | \chi_{\text{LF}} \rangle$ ) is small for  $k < k_{\text{LF}}$  for quadratic,  
 exponential & even **Gaussian spikes** (variance  $\sim 1\%$  at  $k_{\text{LF}} 0.15$  at  $k_{\text{LF}}/10$ )

$\langle F_{\text{NL}} | \chi_b(x,t) + \chi_{>h} \rangle$  regimes contrast with  $\phi_{>h}$ : may be way out there in eternal inflation  
 land, but not in a preheated  $\epsilon > 1$  patch

LOW  $\chi_{>h}$   $\beta_{\chi} \chi_b + f_{\chi} \chi_b^2$  subdominant linear, (much) less constrained  $f_{\chi}$  cf.  $f_{\text{NL}}$

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 anomalies or not to be rare (and suffer constraints)

LARGE  $\chi_{>h}$  encompass part of a smoothed spike, upside, downside, topside

# Observables and conclusions

$$\varphi(\mathbf{x}) = \varphi_\phi(\mathbf{x}) + f_{\text{NL}} (\varphi_\phi^2(\mathbf{x}) - \langle \varphi_\phi^2 \rangle)$$

local quadratic non-G constraint:  $-9 < f_{\text{NL}} < 111 \Rightarrow -4 < f_{\text{NL}} < 80$  WM5 ( $\pm 5-10$  Planck1yr)

$$\Rightarrow \varphi(\mathbf{x}) = \varphi_\phi(\mathbf{x}) + F_{\text{NL}}(\chi_{\text{b}} + \chi_{>h}) - \langle F_{\text{NL}} \rangle$$

resonant preheating form

*modulated curvature fluctuations from preheating are superimposed on the usual curvature fluctuations from the inflaton. need  $\delta\varepsilon$*

*the peak values have  $\delta \ln a \sim 10^{-5} \Rightarrow$  comparable to standard Gaussian*

*temperature fluctuations, but spiky  $F_{\text{NL}} \Rightarrow$  non-Gaussian?*

*As long as  $g^2/\lambda \leq O(1)$  - SUSY-inspired is 2 - the  $\chi$  field has very long wavelength perturbations (similar to, but uncorrelated with, the inflaton field)*

*Large & Small Scale Structure statistics of spiky  $F_{\text{NL}}$  map: under investigation*

**Rich possibilities in theory space & on the sky**

e.g.,  $\langle \delta F_{\text{NL}} | \chi_{\text{LF}} \rangle \sim \text{linear} +$

e.g.,  $F_{\text{NL}}(\chi) \sim \sum_p F_p \exp(-(\chi \pm \chi_p)^2 / 2\gamma_p^2) \Rightarrow$  quadratic regime; **cold spot**

**regime: non-G ubiquitous**