

Scanning Inflation

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Introduction

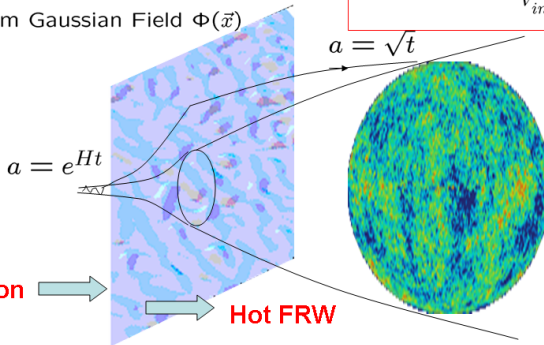
- Top-down approach to inflation
seeks to embed it in a fundamental theory
- Bottom-up approach to inflation
reconstruction of acceleration trajectories

Scalar metric Fluctuations from Inflation
 $ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\vec{x}^2$

Initial conditions from Inflation $\Rightarrow$



Random Gaussian Field $\Phi(\vec{x})$



$$\Omega_{tot} = 1$$

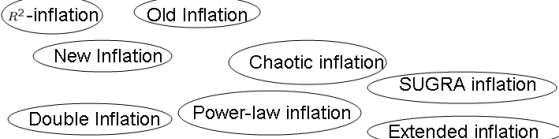
$$k^3 \Phi_k^2 \rightarrow P_s = A_s k^{n_s-1}$$

$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$

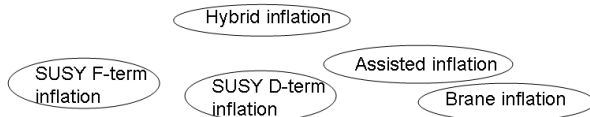
$$N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}}$$

Inflation in the context of ever changing fundamental theory

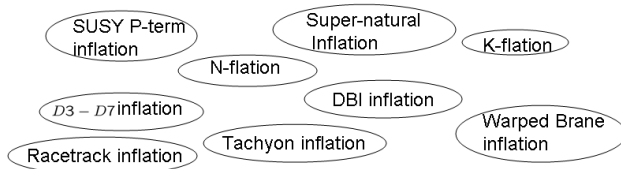
1980



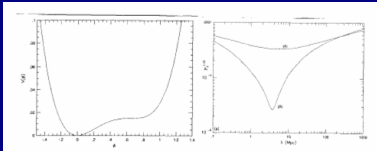
1990



2000



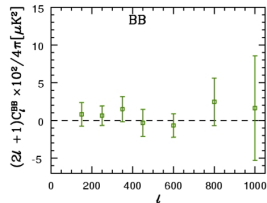
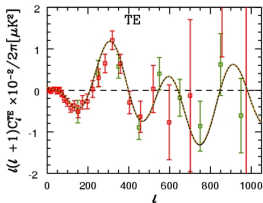
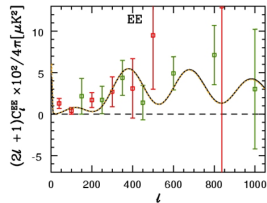
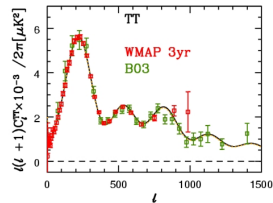
$$V = m^2\phi^2 + \lambda_1\phi^3 + \lambda_2\phi^4$$



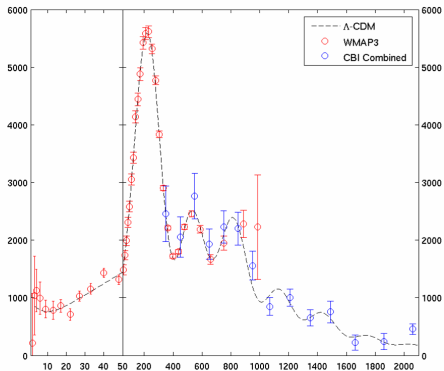
- renormalizable
- different choices of parameters give quite different powerspectra
- but also different shapes of the potential:
 - exponential
 - SUGRA
 - ...



BOOMERANG sees 4th peak



CBI sees 5th peak

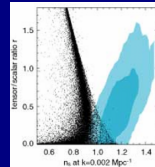


Slow-Roll Parameters

- Equations of Motion

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$\frac{8\pi}{3m_p^2} \left(\frac{1}{2}\dot{\phi}^2 + V \right) = H^2$$



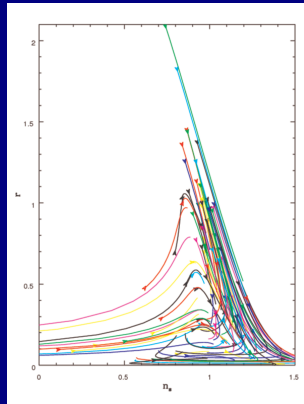
- Slow roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{m_p^2}{4\pi} \frac{H''}{H}, \quad \zeta^2 = \left(\frac{m_p^2}{4\pi} \right)^2 \frac{H''' H'}{H^2}, \quad \sigma = 2\eta - 4\epsilon$$

- Flow Equations

- Powerspectra

$$P_R \equiv A_S \left(\frac{k}{k_{\text{pivot}}} \right)^{n_s-1} \propto \frac{H^2}{\epsilon}, \quad P_G \equiv A_t \left(\frac{k}{k_{\text{pivot}}} \right)^{n_t} \propto H^2$$

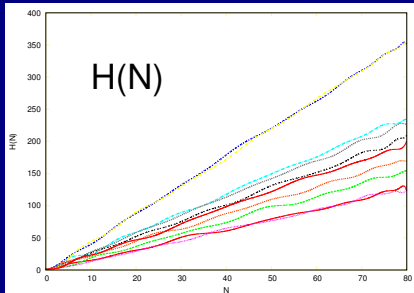


varying N moves the points in observable space around

Acceleration

reconstruction
←

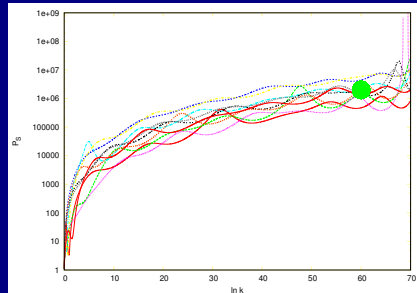
Powerspectrum



trajectory $H(N)$

(natural object from
Hamilton-Jacobi formalism)

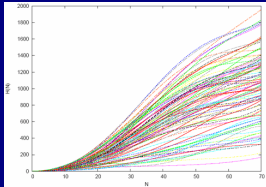
map
→



$$P_R \propto \frac{H^2}{\epsilon}$$

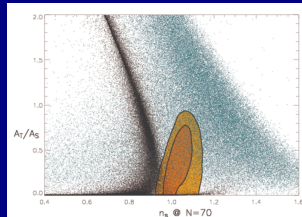
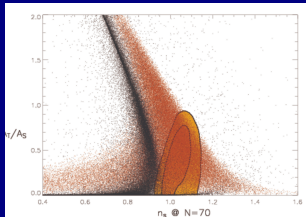
(mildly broken
scale invariance)

Ensemble of Inflationary Trajectories



various trajectories and time variables

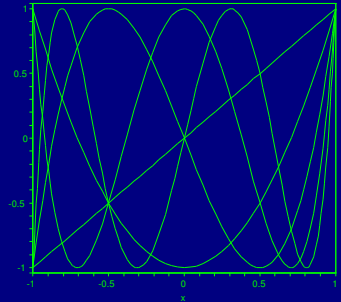
- $\epsilon, H, \ln(P_R), \ln(P_G)$
- $N, \ln(k)$
- ...



⇒ space opens up, fast

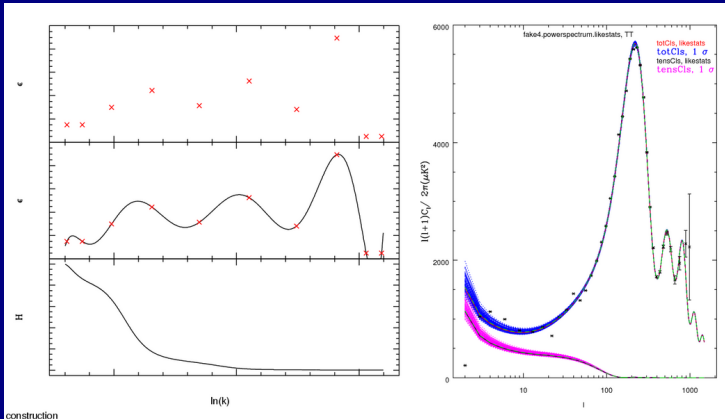
Trajectories

- N : # of efolds $dN = -Hdt$
- Constraints during inflation
 - $0 \leq \epsilon \leq 1$
 - $H > 0$
- at the end of inflation $\epsilon = 1$
- Expansion to arbitrary order
 - $H(N) = \sum_i c_i T_i(x)$
 - with $x = \frac{2N - N_{\max}}{N_{\max}}$, Chebyshev polynomials $T_i(x)$ (uniformly best approximation to “true” function)



$$T_n(\cos(x)) = \cos(nx)$$

Constructing Trajectories



- draw trajectories
- compute likelihood of model given data
- $\Rightarrow$ Markov-Chain Monte-Carlo

Nodal points

- choose $f(x_i)$ at nodal points x_i

- window function $w_j(x)$:

$$f(x_i) = \sum_j c_j T_j(x_i) \Rightarrow c_j = T_j(x_i)^{-1} f(x_i)$$

$$f(x) = \sum_i c_i T_i(x) = \sum_i f(x_i) \underbrace{\sum_j T_j(x_i)^{-1} T_j(x)}_{w_i}$$

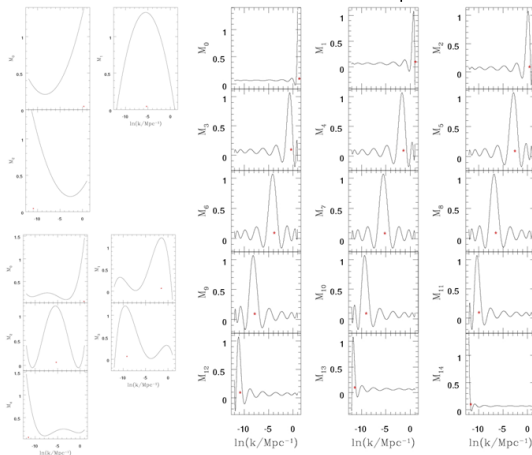
- $f(x) = H, \ln(H), \epsilon, \ln(\epsilon), P_R, \ln(P_R), P_G, \ln(P_G), \dots$
- $x = N, \ln(k)$

$$\Rightarrow \epsilon \in [0, 1]$$

Chebyshev nodal modes: order 3, 5, 15 (Fourier at high order)

Chebyshev modes are linear combination

How far does choice of values at nodal points “feel” out?



Displaying
Trajectory
constraints:

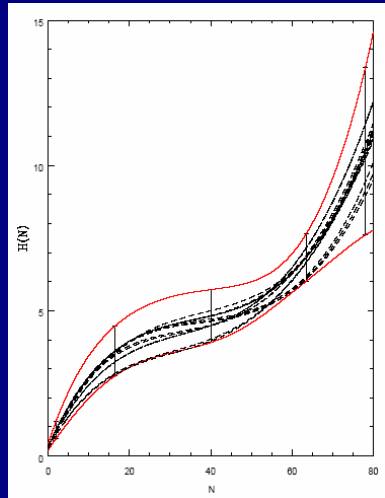
If Gaussian
likelihood,
compute χ^2
where 68%
probability, and
follow the ordered
trajectories to

$\ln L/L_m = -\chi^2/2$,
displaying a
uniformly sampled
subset.

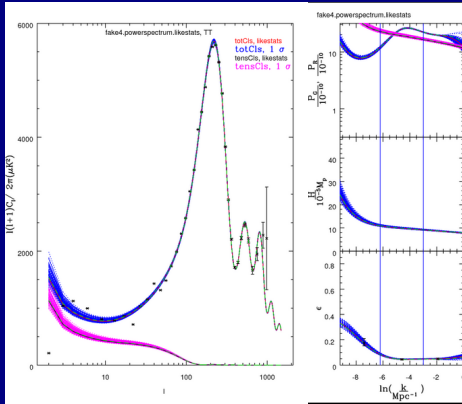
Errors at nodal
points in trajectory
coefficients can
also be displayed.

time variable $N \leftrightarrow \ln k$

- N : $N \in [0, 70]$
but: only interested in
observable interval
 $\Delta N \approx 5$
- $\ln k$: observable $k \in [10^{-4} \text{Mpc}^{-1}, 1 \text{Mpc}^{-1}]$



(Re-)Constructing Trajectories



- perfect experiment
- cosmic variance limited

(Re-)Constructing Trajectories

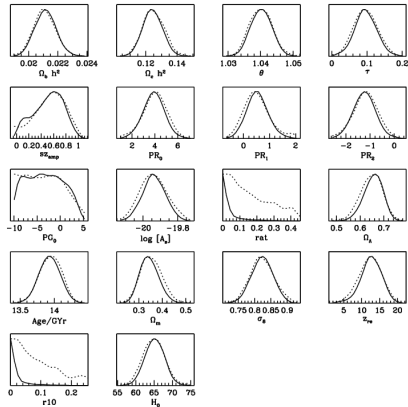
	Mock input value	reconstructed value
c_2	0.0640987	0.074 ± 0.011
c_3	0.0470099	0.049 ± 0.0023
c_4	0.0453124	0.046 ± 0.0016
c_5	0.1725266	0.17 ± 0.011
c_6	0.3512255	0.35 ± 0.015
c_7	0.1578866	0.13 ± 0.039
H_1	144	142 ± 19
Age of universe/GYr	14.1	14.1 ± 0.01
$\Omega_b h^2$	0.01977	0.01976 ± 0.000055
$\Omega_c h^2$	0.12535	0.12522 ± 0.00079
z_{re}	15.1	15.1 ± 0.3
σ_8	0.8548	0.8503 ± 0.0044

- Expansion of $\ln(P_R(\ln k))$ to order 3, $\ln(P_G(\ln k))$ to order 1
- Expansion of $\ln(P_R(\ln k))$ to order 3, $P_G(\ln k)$ to order 1
- Expansion of $P_R(\ln k)$ to order 5, $P_G(\ln k)$ to order 5
- Expansion of $\epsilon(\ln k)$ to order 10, monotonically increasing
- Expansion of $\epsilon(\ln k)$ to order 10, keeping outmost points fixed

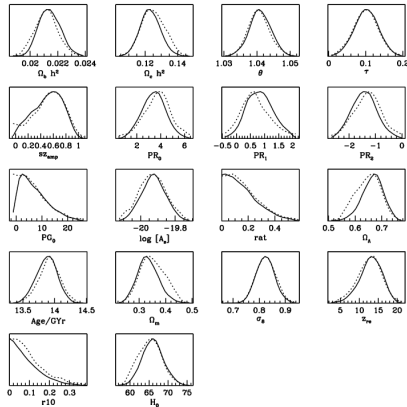
Datasets:

- ACBAR
- BOOMERANG
- CBI
- DASl
- MAXIMA
- VSA
- WMAP
- 2dF
- SDSS

lnPR3_1.all_paramsb



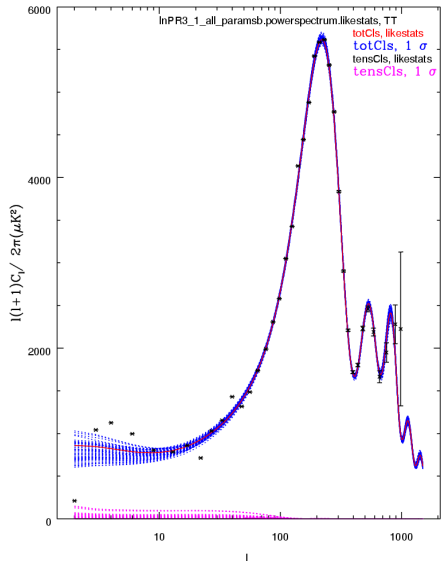
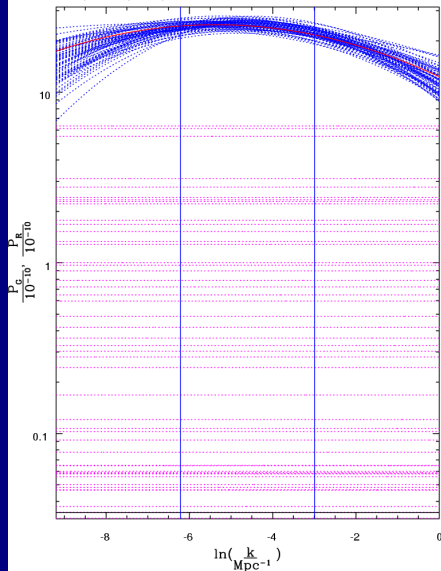
lnPR3_PG1.all_paramsb



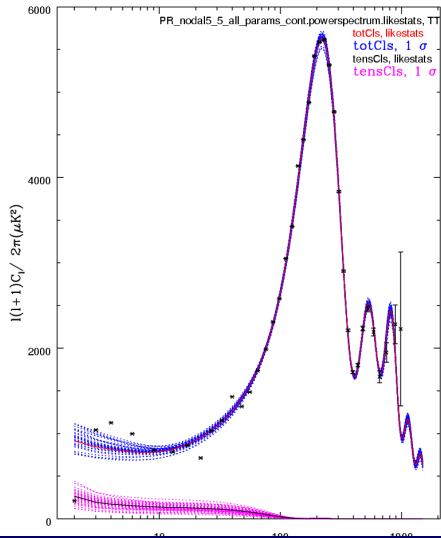
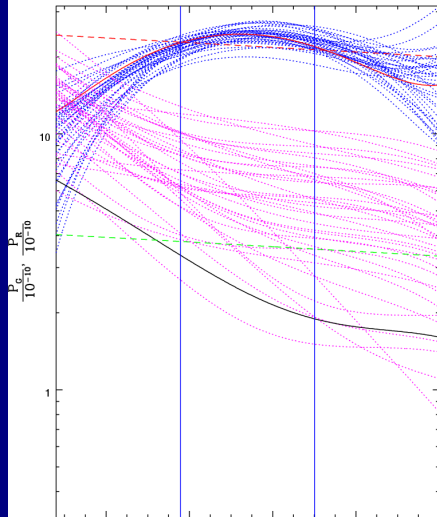
cosmological parameters:

parameter	WMAP3	lnPR 3 lnPG 1	lnPR3 PG1
$\frac{A}{10^{-10}}$	21^{+1}_{-2}	22^{+1}_{-2}	22^{+2}_{-1}
$n_s(0.002 Mpc^{-1})$	$1.21^{+0.1}_{-0.15}$	1.16	1.22
$n_s(0.05 Mpc^{-1})$	$0.845^{+0.036}_{-0.044}$	0.78	0.79
$\frac{dn_s}{d \ln k}$	$-0.112^{+0.1}_{-0.15}$	-0.117	-0.13
τ	$0.097^{+0.03}_{-0.036}$	$0.096^{+0.03}_{-0.03}$	$0.102^{+0.03}_{-0.03}$
...	...	...	...

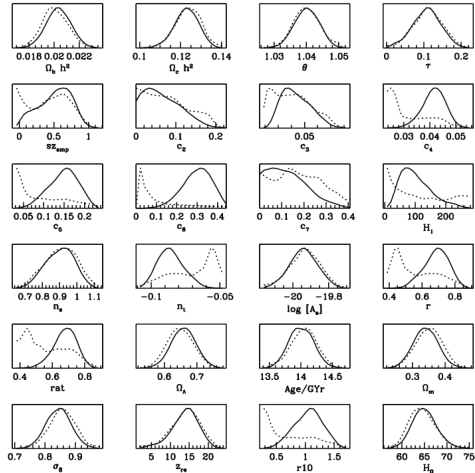
lnPR3_1_all_paramsb.powerspectrum.likestats



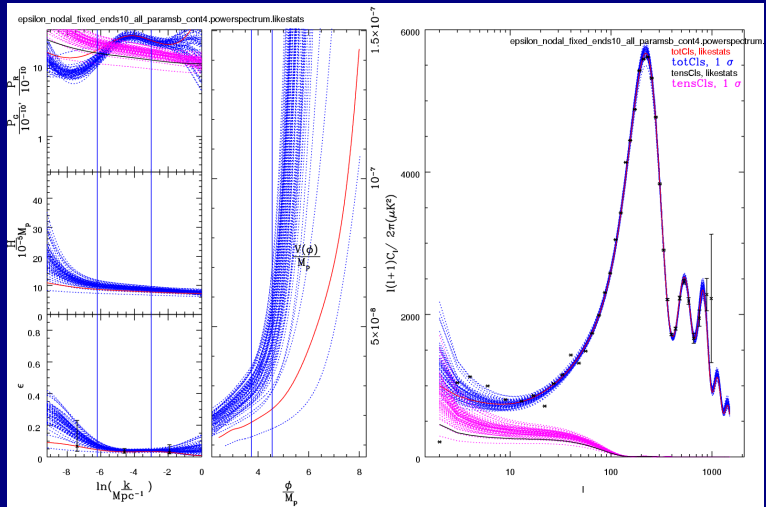
PR_nodal5_5_all_params_cont.powerspectrum.likestats

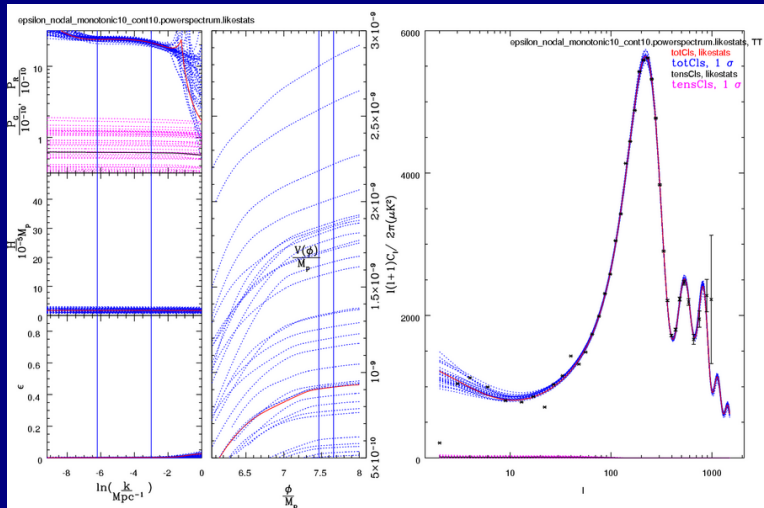


epsilon_nodal_fixed_ends10_all_paramsb_cont4

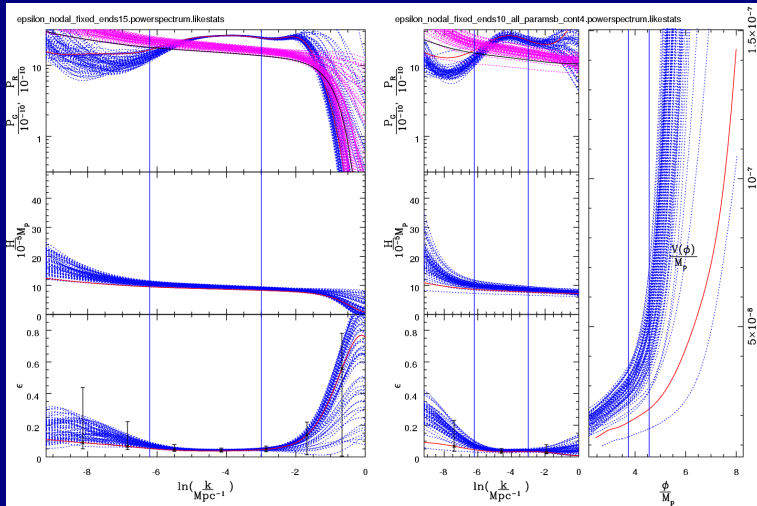


large tensors!
(compatible with
WMAP3 if run-
ning of n_s is al-
lowed)





WMAP1 $\leftrightarrow$ WMAP3

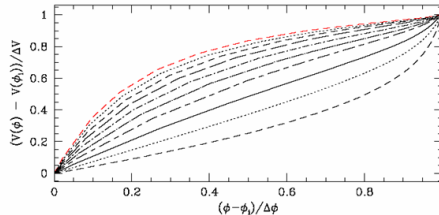


Degeneracy of the Potential Reconstruction

known $P_s(k) \rightarrow$ reconstruct $V(\phi)$

$$P_s(k) = \frac{8\pi H^4}{M_p^4 H'^2}$$

$$V(\phi) = \frac{M_p^4}{32\pi^2} \left(\frac{12\pi}{M_p^2} H^2 - H'^2 \right)$$



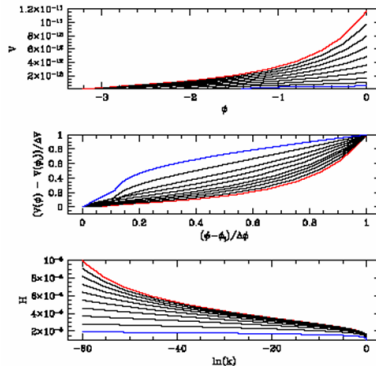
$$\phi - \phi_0 = \frac{M_p^2}{2} \int_{\ln k_0}^{\ln k} d \ln k' \frac{\sqrt{P_s(k')}}{H^2(k')} \frac{dH}{d \ln k'}$$

$$\frac{dH}{d \log k} = \frac{H^3}{H^2 - \pi M_p^2 P_s(k)}$$

Degeneracy is lifted by fixing $P_{GW} = \frac{H_*^2}{M_p^2}$

Example $P_s(k) = k^{n_s-1}$

$n_s = 0.98$



Conclusions

- Classical Observables $\leftrightarrow$ power spectra
- Increasing the order of Chebyshevization opens the space of classical observables
- Reconstruction of inflationary trajectories
- Priors
- Degeneracy