Non-Gaussian Spikes from Chaotic Billiards

Cosmological Fluctuations and Non-Gaussianity from Preheating

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Outline

Cosmological Fluctuations from Inflation

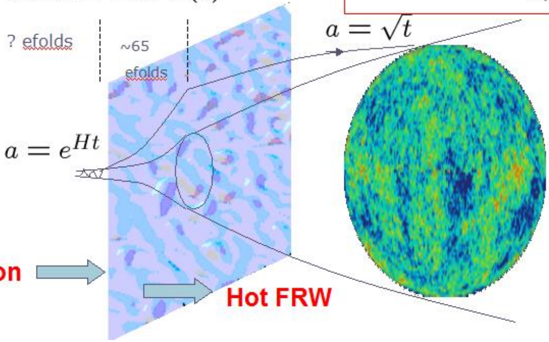
Scalar metric Fluctuations from Inflation

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Phi)a^2 d\vec{x}^2$$

Initial conditions from Inflation

$$\Omega_{tot} = 1$$
$$k^3\Phi_k^2 \to P_s = A_s k^{n_s - 1}$$
$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$
$$N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}}$$

Random Gaussian Field $\Phi(\vec{x})$

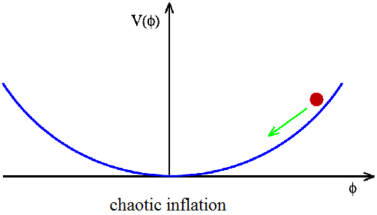


Preheating

- At the end of Inflation, Inflaton non-perturbatively decays into other fields.
- Most models are phenomenological (one exception is our recent work "Preheating After Modular Inflation" $\longrightarrow$ Neil Barnaby's talk).
- Usually happens at comoving scale $\lesssim 1$ m (causality $\Rightarrow$ no observable signatures on cosmological scales?)

Chaotic Inflation: $\lambda\phi^4$ Model

$$V = \frac{\lambda}{4}\phi^4$$

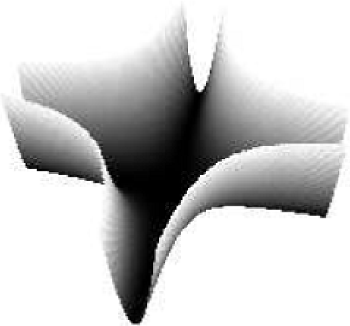


$$\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0$$

$$3M_p^2 H^2 = 1/2\dot{\phi}^2 + V(\phi)$$

Preheating: The Four-leg Interaction Model

$$V(\phi, \chi) = \frac{\lambda}{4}\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$



$$\frac{g^2}{\lambda} \gg 1 \Rightarrow m_\chi \gg H,$$

χ fluctuations suppressed (trivial case)

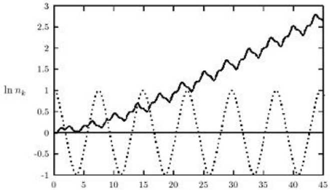
$$\frac{g^2}{\lambda} \sim 1 \Rightarrow m_\chi \lesssim H,$$

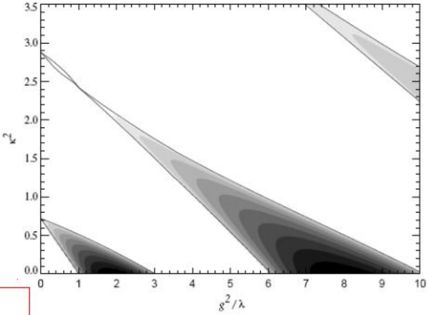
χ fluctuations $\surd$ (interesting case)

Parametric Resonance

Exponential Growth of χ_k

$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0$$





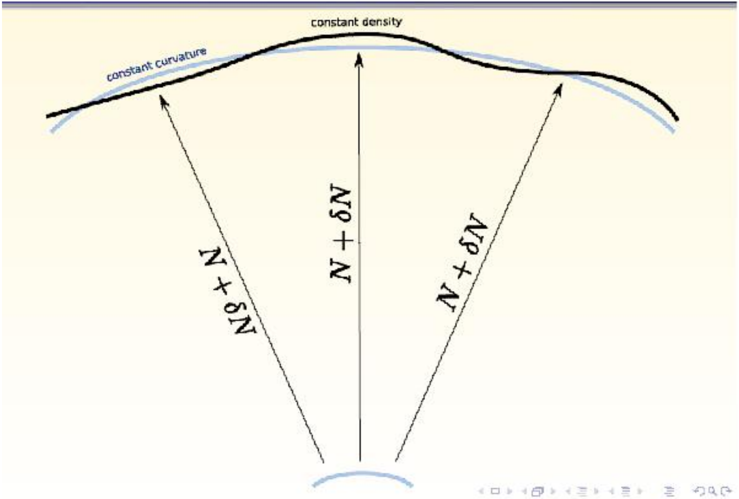
$$\boxed{n_k \sim e^{2\mu_k T}, \quad \chi_k \sim e^{\mu_k T}}$$

Green, Kofman, Linde, Starobinsky 1997

For $1 < g^2/\lambda < 3$, the k=0 mode (spatial average of χ) $\langle\chi\rangle$ grows.

δN formula

$$\mathcal{R}_c = \delta N$$



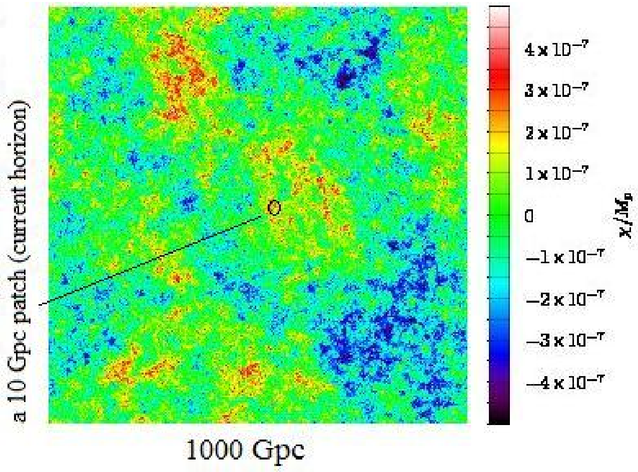
Choosing the Gauge

- Cosmological fluctuations from Inflaton
  spatial flat gauge $\Rightarrow$ comoving gauge($\phi = const$ hypersurface)

$$\mathcal{R}_c = \delta N = H \frac{\delta\phi}{\dot{\phi}}$$

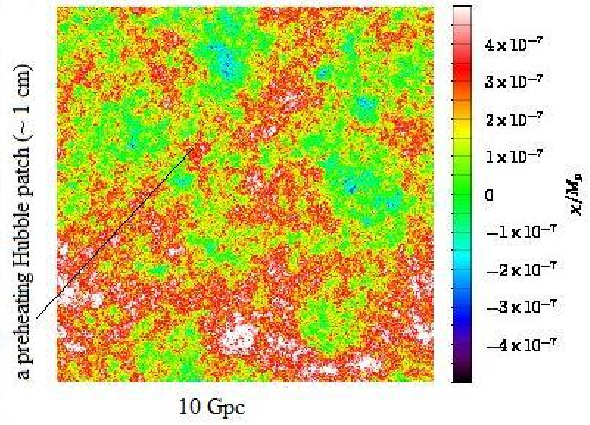
- Cosmological fluctuations from preheating
  Calculate δN from $\phi = const$ hypersurface to comoving slice at the end of preheating

Superhorizon "Landscape"



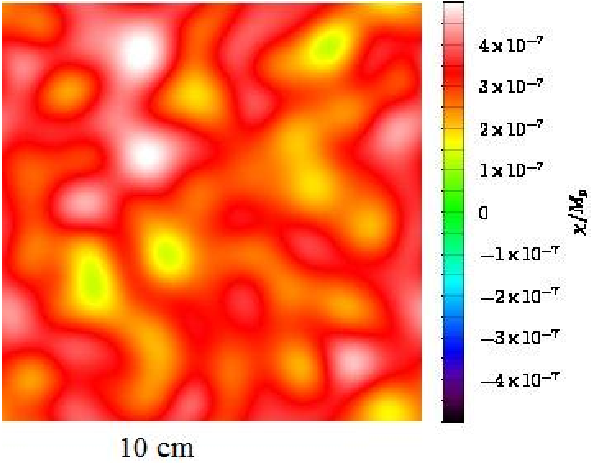
At Cosmological Scales ($\sim (aH)^{-1}$ Today)

mean value of χ on cosmo-scales $\langle\chi\rangle_{\rm cosm}$: random Gaussian with $\sigma \sim \sqrt{N_{>h}} \times 10^{-7} M_p$;
$N_{>h}$: # of efolds before cosmo-scales exit the horizon;
Measuring $\chi_{\rm cosm} \Rightarrow$ Prob($N_{>h}$); but limited by cosmic variance



Preheating Scales ($\sim (aH)^{-1}$ At the End of Inflation)

$\langle\chi\rangle = \langle\chi\rangle_{\rm cosm} + \langle\chi\rangle_{<h}$
$\langle\chi\rangle_{<h}$ random Gaussian, with $\sigma = \sqrt{60}\frac{H_{end}}{2\pi} = 7\times10^{-7}M_p$
vacuum fluctuations renormalized



Numerical Simulation

Goal: calculate δN to accuracy 10^{-8} (corresponds to $f_{\rm nl} \sim 100$)
A new lattice simulation code **HLattice**

- Higher (up to 8th) order PDE integrator: good accuracy of energy conservation
- Can simulate non-canonical scalar fields
- OMP-MPI hybrid parallel

For normal setups ($dt \sim 0.1dx$): energy conservation levels

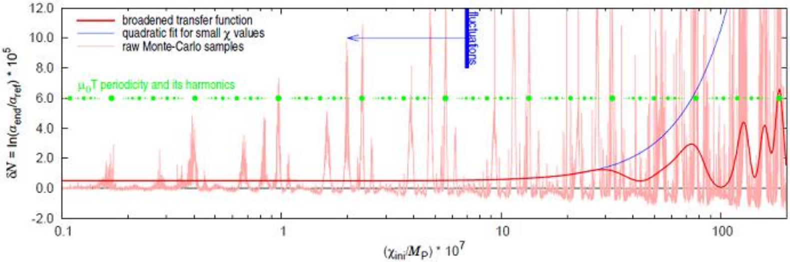
LatticeEasy (G. Felder)	$\sim 10^{-4}$ - 10^{-3}
DEFROST (A. Frolov)	$\sim 10^{-5}$ - 10^{-4}
HLattice (Z. Huang)	$\sim 10^{-12}$ - 10^{-8}

Methods:

- use DEFROST/LatticeEasy
  Brute-force calculation using very small timestep $dt \ll dx$.
- use HLattice

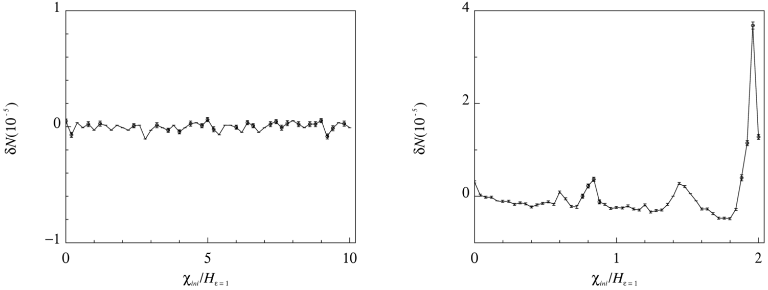
$\delta N(\langle\chi\rangle)$ Response $(g^2/\lambda = 2)$

Spikes: Raw data
Red line: smoothed to cosmo-scales (convolved with a Gaussian window with $\sigma = 7 \times 10^{-7} M_p$).
Blue line: qudratic fitting

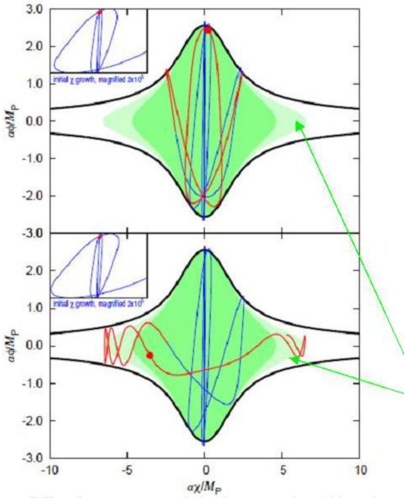


Numerical accuracy check

- HLattice agrees with brute-force DEFROST
- Null test
  $g^2/\lambda = 3$ (left panel) v.s. $g^2/\lambda = 2$ (right panel)



Billiards Entering the Arms



$\langle\chi\rangle = 3.6 \times 10^{-7} M_p$, "billiards" not entering the arms

$\langle\chi\rangle = 3.9 \times 10^{-7} M_p$, "billards" entering the arms

Potential walls close up in non-linear regime.

Numerical 3D simulation

A 4-panel movie

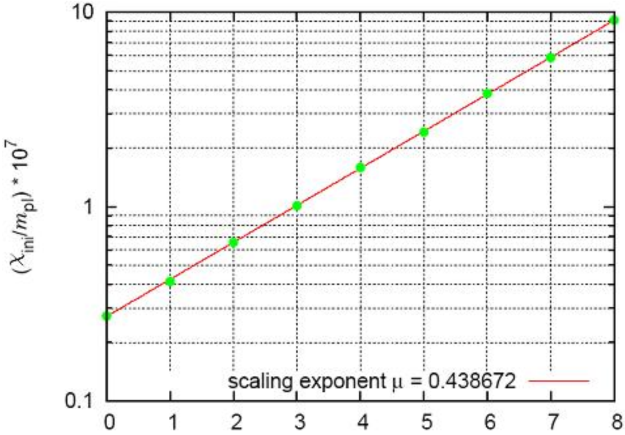
Visualized 2D slices of field values:

χ in simulation #1	ϕ in simulation #1
χ in simulation #2	ϕ in simulation #2

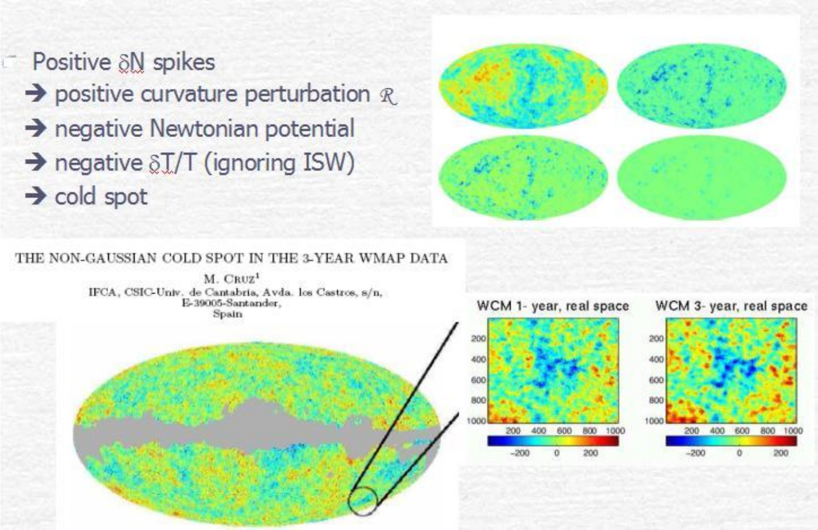
Guess which simulation box produces δN spike!!

Periodic Pattern of Spikes

During linear regime $\langle\chi\rangle = \chi_0 e^{n\mu T}$, $n = 0, 1, 2, \ldots$.
Replacing $\langle\chi\rangle = \chi_0$ with $\langle\chi\rangle = \chi_0 e^{\mu T}$ produce similar entering-the-arm dynamics.



CMB Cold Spot



New Form of Non-Gaussianity

The popular formalism of non-Gaussianity:

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{\rm nl}\big(\Phi_G^2(\mathbf{x}) - \langle\Phi_G^2(\mathbf{x})\rangle\big)$$

Our model

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + F\big(\chi(\mathbf{x})\big)$$

F a highly nonlinear spiky function

Need completely new analysis of CMB map (in progress).

Expecting new signature in CMB polarization (in progress).

End

Thanks