Primordial Power Spectra and Cosmological Observations

Zhiqi Huang

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Collaborators: J.R. Bond, C. Contaldi, L. Kofman, P. Vaudrevange

Outline

6-parameter Concordance Model

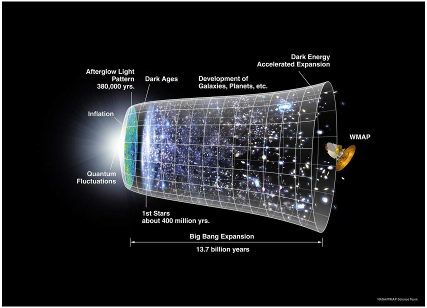


Figure from LAMBDA website

τ: reionization optical depth

$\Omega_b h^2$: today's physical density of baryon

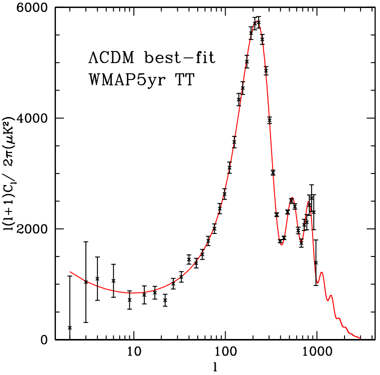
$\Omega_c h^2$: today's physical density of cold dark matter

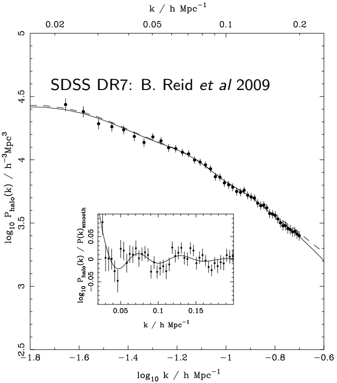
θ: the angle subtended by sound horizon on CMB sky

A_s: the amplitude of primordial scalar metric perturbation

n_s: the spectral index of primordial scalar metric perturbation

Observational Confirmation of the Concordance Model





Beyond 6 parameters:

- Late universe:
  dark energy (w): quintessence, $f(R)$, ...
  dark matter: annihilation, decay, ...
  reionization history
  neutrinos
  ...
- Early universe:

  $$\text{inflation \& (p)reheating} \begin{cases} \text{primordial power spectra} \\ \text{non-Gaussianity}(f_{nl}) \end{cases}$$

  phase transition
  topological defect
  CMB physics
  ...

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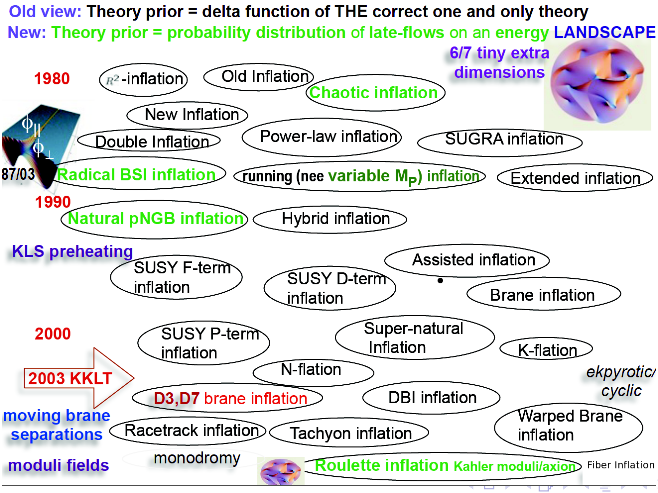
Inflation & (p)reheating: the zoology of theories



Primordial Power Spectra: Standard Extensions $n_{\rm run}$, r (and n_t).

$\mathbf{n}_{\rm run}$: the running of primordial scalar power spectrum

r: the tensor-to-scalar ratio of primordial metric perturbations

$\mathbf{n_t}$: the spectral index of primordial tensor power spectrum

$$\mathcal{P}_s(k) = A_s(\frac{k}{k_{\rm pivot}})^{n_s-1+\frac{1}{2}n_{\rm run}\ln(\frac{k}{k_{\rm pivot}})}$$
$$\mathcal{P}_t(k) = A_t(\frac{k}{k_{\rm pivot}})^{n_t},$$

where $A_t \equiv rA_s$, and often chosen $k_{\rm pivot}$: $0.002\mathrm{Mpc}^{-1}$ or $0.05\mathrm{Mpc}^{-1}$.

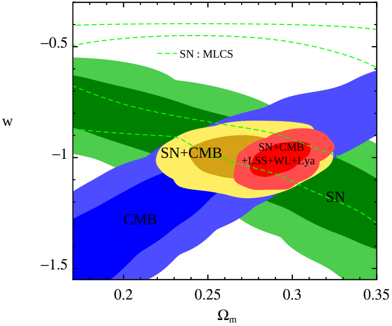
Cosmological data sets

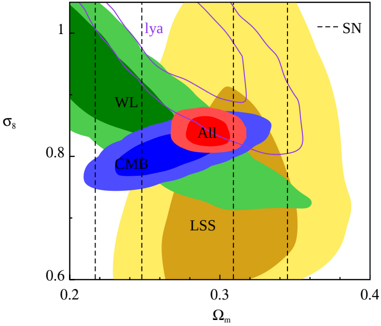
- Cosmic Microwave Background (**CMB**): WMAP5yr (09), Acbar (09), QUAD (09), BICEP (09), CBI (08), Boomerang (06), DASI (05), VSA (04), MAXIMA (00)
- Type Ia Supernova (**SN**): LOWZ + SDSS + ESSENCE + SNLS1yr + HST (Kessler et al 09) (soon will + SNLS3yr)
- Weak Lensing (**WL**): COSMOS + CFHTLS-wide + RCS + VIRMOS + GaBoDS (Massey et al 07, Lesgourgues et al 07, Benjamin et al 07)
- Large Scale Structure (**LSS**): SDSS-DR7 LRG (Reid et al 09)
- Lya Forest (**Lya**): SDSS Lya(McDonald et al 05, 06)
- Others: HST constraint on Hubble parameter (Riess et al 09); Cluster x-ray gas mass fraction (Allen et al 08)

Tool: Modified CosmoMC

- Arbitrary Primordial Power spectra functions $P_s(k)$ and $P_t(k)$.
- An integrator to calculate $P_s(k)$ and $P_t(k)$ from arbitrary single-field inflation model
- Automatic adjusting l, k interpolation for more oscillatory primordial power spectra.
- Dark energy equation of state arbitrary function $w(z)$, with an analytic quintessence/phantom motivated parametrization built-in.
- Decaying dark matter
- CMB, WL, SN, BAO mock data simulator, add an action $= -1$ for running simulations
- Self-written GetDist to do more statistics & visualizations

The Consistency between Cosmological Data Sets





default SN filter: SALT II

Forecasts

- **CMB**: Planck2.5yr, using 3 channels (70GHz, 100GHz, 143GHz), assuming 5% foreground residual (synchrotron + dust), $f_{\rm sky} = 3/4$, $l_{\rm max} = 2500$.

  other future polarization experiments: SPIDER, EBEX, QUIET, KECK, CMBPol ...

- **WL**: DUNE-like weak lensing tomography, 20000 degree^2, depth $z \sim 1$, 35 galaxies/arcmin^2, two redshift bins, $l_{\rm max} = 1500$.

  the other proposed deep and wide WL surveys: JDEM, LSST, ...

- **SN**: JDEM, 500 LOWZ ($z < 0.03$) + 2500 HIGHZ ($0.03 < z < 1.7$)

  other ongoing/future SN surveys: SNLS, SDSS, LSST ...

- **BAO**: JDEM, 10000 degree^2, $0.5 < z < 2$, 10 redshift bins

  other ongoing/future BAO surveys: CHIME, WIGGLEZ, BOSS, LSST, ...

Prediction from Single-Field Slow-Roll Inflation

Slow-Roll Approximation:

$$\begin{aligned} n_s - 1 &\approx 2\eta_v - 6\epsilon_v \\ r &\approx -8n_t \approx 16\epsilon_v \\ n_{\rm run} &\approx -16\epsilon_v\eta_v + 24\epsilon_v^2 + 2\zeta_v^2 \approx 0, \end{aligned}$$

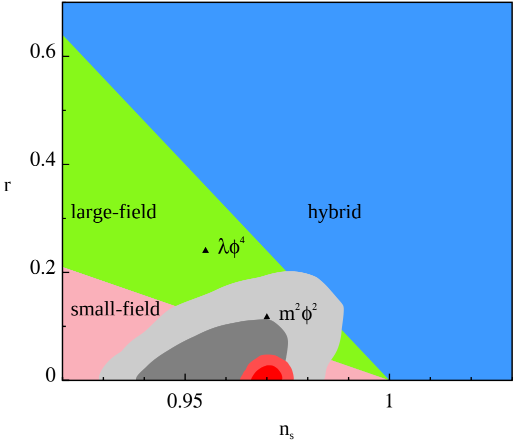
where

$$\begin{aligned} \epsilon_v &\equiv \frac{M_p^2}{2}(\frac{V'}{V})^2 \\ \eta_v &\equiv M_p^2(\frac{V''}{V}) \\ \zeta_v^2 &\equiv M_p^4(\frac{V'V'''}{V^2}) \end{aligned}$$

Classification in ϵ_v-η_v space (or r-n_s space):

- Small Field Inflation: $\eta_v < 0$
- Large Field Inflation: $0 < \eta_v < 2\epsilon_v$
- Hybrid Inflation: $\eta_v > 2\epsilon_v$

Constraining Single-Field Slow-Roll Inflation



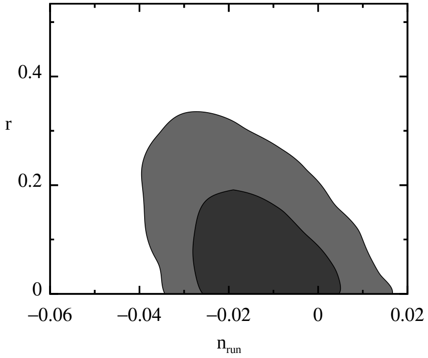
68.3% and 95.4% CL constraints, current data

Forecast (fiducial $n_s = 0.97$, $r = 0$)

current $r < 0.16$ (95% CL)

forecast $r < 0.037$ (95% CL)

The Degeneracy between Scalar and Tensor Spectrum



constraint on r (95%CL):

- $r < 0.16$ (no running, data sets: all)
- $r < 0.32$ (no running, data sets: CMB all)
- $r < 0.27$ (with running, data sets: all)

Releasing the Slow-Roll Assumptions:

If allow V''' to be big, ζ_V not necessarily small $\rightarrow$ nontrivial $n_{\rm run}$.
Single field models are not limited in the $r - n_s$ plane.
Two approaches to extend the parametrization of primordial power spectra:

- **Phenomenologically** take $P_s(k)$ and $P_t(k)$ as arbitrary functions.
- **Consistently** solve $P_s(k)$ and $P_t(k)$ for all possible expansion histories.

Previous works on this topic:

- Simple binning techniques: Briddle et al 03; Hannestad 04; Bridges et al 06, 07; Spergel et al 07;
- Direct inversion: Shafieloo et al 04, 08; Kogo et al 04; Tocchini-Valentini et al 05 06; Nagata et al 08; Nicholson et al 09a, 09b;
- Basis function expansion: Mukherjee 05; Leach 06;
- Cubic spline interpolation: Sealfon et al 05; Peris et al 08, 09;
- Slow-roll reconstruction (flow equations): Peris et al 03, 06a, 06b; Easther 06; Adshead et al 09;

Phenomenological Parametrization

conventional "$3+2$" $\Rightarrow$ "$n+2$".

- For the well constrained scalar power spectrum, take $\ln P_s(k)$ at n fixed knots $k_1, k_2, ..., k_n$ in the observable range, interpolate in between.

  n parameters

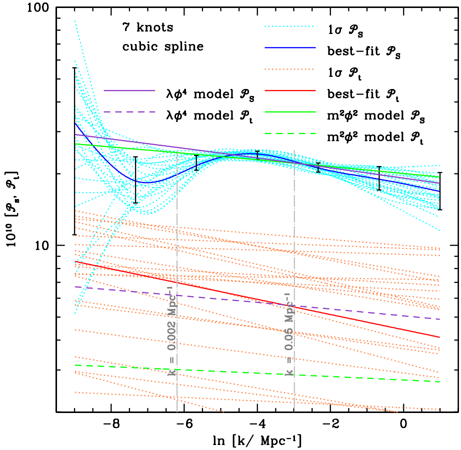
- For the poorly constrained tensor power spectrum, use simple

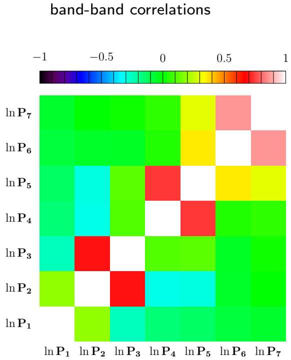
$$P_t(k) = A_t\left(\frac{k}{k_{\mathrm{pivot}}}\right)^{n_t}, \text{with prior } -0.1 < n_t < 0$$

  2 parameters

  (optional: do similar expansion as $P_s \Rightarrow$ "$n+m$" parametrization)

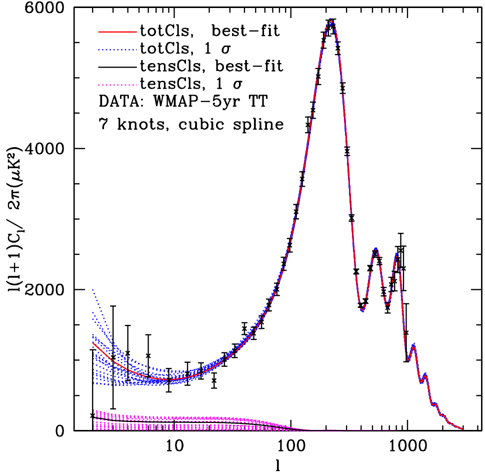
Reconstructed Primordial Power Spectra



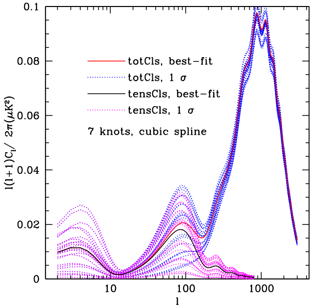


$r(0.002\mathrm{Mpc}^{-1}) < 0.89$ (95% CL)

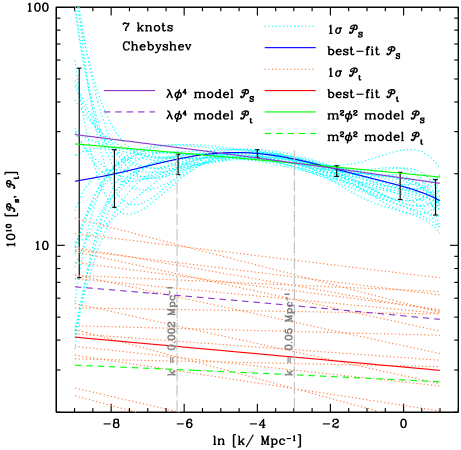
Reconstructed Trajectories in C_l Space

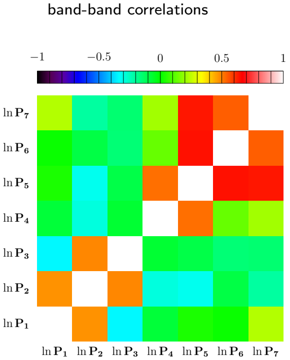


BB spectrum



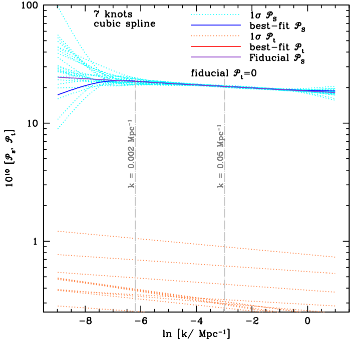
Vary the Interpolation Method: Chebyshev Interpolation

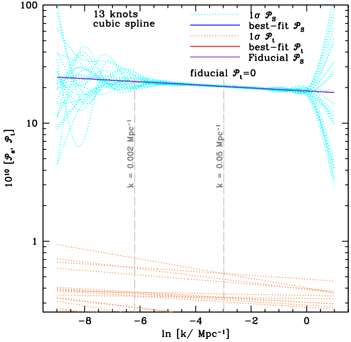




$r(0.002\mathrm{Mpc}^{-1}) < 0.68$ (95% CL)

Forecast: 7 knots and 13 knots

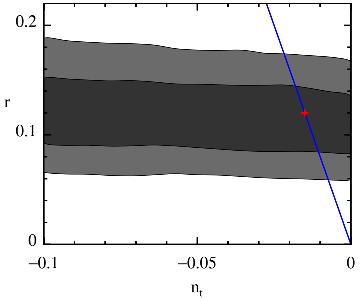




$r(0.002\mathrm{Mpc}^{-1}) < 0.041$ (95% CL) $r(0.002\mathrm{Mpc}^{-1}) < 0.044$ (95% CL)

What is Consistency and Why Consistency

- In the context of single-field inflation:
  uniform acceleration $\epsilon \approx const$: $n_t \approx -2\epsilon \approx -\frac{r}{8}$
  generic $\epsilon(\ln k)$: more complicated functional relationship.
- Even the next generation data can not measure the consistency
  $\Rightarrow$ need to be treated as a theoretical prior.



Consistency Treatment

Expansion history $H(\ln a) \Rightarrow P_s(k)$ and $P_t(k)$.

Method #1: slow-roll approximation

$$P_s(k) \approx \frac{1}{8\pi^2 M_p^2}\frac{H^2}{\epsilon}|_{k=aH},$$
$$P_t(k) \approx \frac{2}{\pi^2 M_p^2}H^2|_{k=aH},$$

where

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

Inflation $\Leftrightarrow 0 < \epsilon < 1$.

Method #2: numerical exact solution

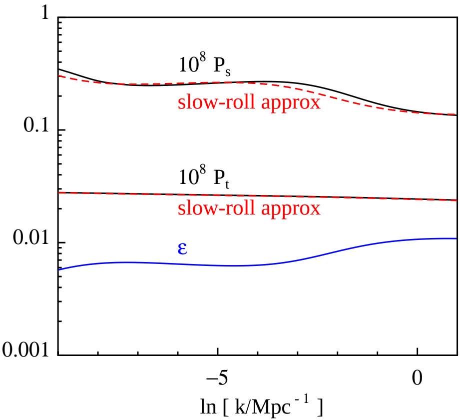
$$P_s(k) \propto |\mathcal{R}_k|^2,$$
$$P_t(k) \propto |h_k|^2,$$

where $\mathcal{R}_k$ and h_k can be solved from:

$$\mathcal{R}_k'' + 2\frac{(\sqrt{\epsilon}a)'}{\sqrt{\epsilon}a}\mathcal{R}_k + k^2\mathcal{R}_k = 0,$$

$$h_k'' + 2\frac{a'}{a}h_k + k^2 h_k = 0.$$

How Good Is the Slow-Roll Approximation?



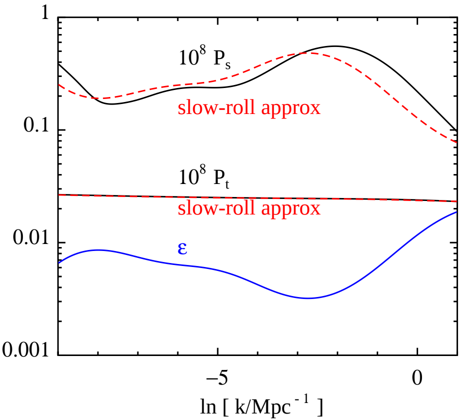
If $\left|\frac{d\ln\epsilon}{d\ln k}\right| \lesssim 0.1$,

- P_s: a few percents error
- P_t: sufficiently good

probably ok for current data

For future precision cosmology better use exact power spectra.

A wilder example.



When $\left|\frac{d\ln\epsilon}{d\ln k}\right| \sim 1$,

- P_s: errors of order 1.
- P_t: still not bad (this is generic given small ϵ).

Constraining Expansion Histories

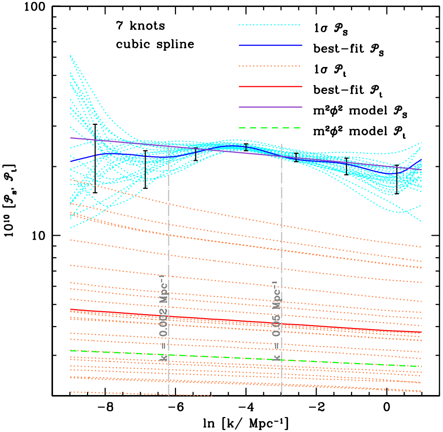
"$2+n$" parametrization:

- $\ln A_p \equiv \ln(\frac{H^2_{\rm pivot}}{\epsilon_{\rm pivot}})$
- $\epsilon_{\rm pivot}$
- Take $\frac{d\ln\epsilon}{d\ln k}$ at n fixed knots $k_1, k_2, ..., k_n$, interpolate in between (at knots prior $-1 < \frac{d\ln\epsilon}{d\ln k} < 1$).

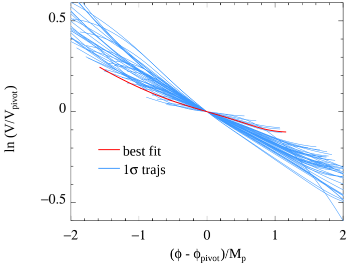
Each MCMC step numerically solve $P_s(k)$ and $P_t(k)$ (CPU time $\sim$ a few WMAP likelihood evaluations).

$$\begin{array}{ccccc} \boxed{\text{prior } V(\phi)} & & & & \boxed{\text{posterior } V(\phi)} \\ \Updownarrow & & & & \Updownarrow \\ \boxed{\text{expansion histories}} & \Longrightarrow & \boxed{P_s, P_t} & \xRightarrow{\text{DATA}} & \boxed{\text{posterior expansion histories}} \end{array}$$

Reconstructed Power Spectra and Potentials

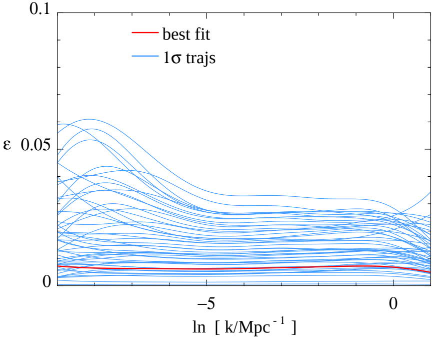


reconstructed potentials

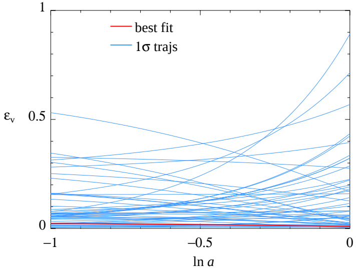


$r(0.002\mathrm{Mpc}^{-1}) < 0.66$ (95% CL)

Reconstructed ϵ trajectories

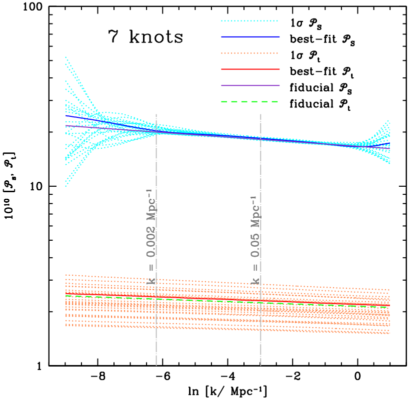


comparing to dark energy ϵ_v (just for fun)

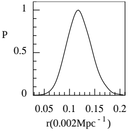


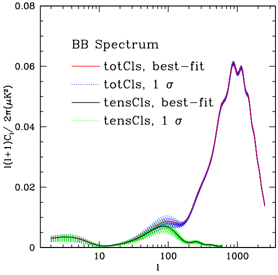
Reconstructing $m^2\phi^2$ model with mock data

Fiducial model: $V(\phi) = \frac{1}{2}m^2\phi^2$, $m = 5.5 \times 10^{-6} M_p$, $\phi|_{0.002Mpc^{-1}} = 16.5 M_p$



$r = 0.119^{+0.024+0.051}_{-0.024-0.047}$





Directly Reconstruct $V(\phi)$

Parametrization:

$$V(\phi) = V_0\alpha^2\left(1 + \alpha\Delta\phi + \frac{\beta}{2}\Delta\phi^2 + \frac{\gamma}{3!}\Delta\phi^3 + \frac{\omega}{4!}\Delta\phi^4\right),$$

where

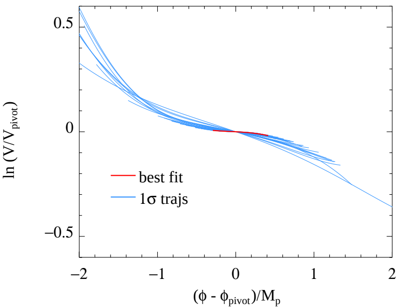
$$\Delta\phi \equiv (\phi - \phi_{\mathrm{pivot}})/M_p$$

Priors:

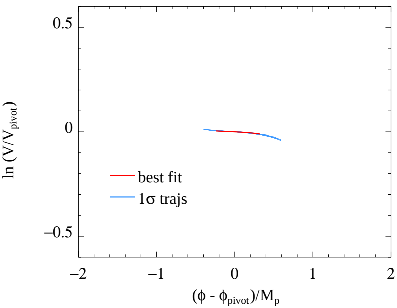
- $V(\phi)$ positive and monotonic (WLOG assume $dV/d\phi < 0$).
- $0 < \epsilon < 1$
- (optional) small-field assumption $\phi_{\mathrm{max}} - \phi_{\mathrm{min}} < M_p$

Reconstructed potentials

without small-field assumption



with small-field assumption



Summary and Conclusions

- Current constraint $r < 0.16$ strongly depends on the uniform acceleration assumption.
- Completely releasing slow-roll assumption $\Rightarrow$ degeneracy between P_s and $P_t \Rightarrow r \lesssim 0.7$
- The next generation cosmological observations can break the degeneracy between P_s and P_t at a much better level (constrain $r < 0.04$ without assuming slow-roll).