

Second-order Boltzmann Code and CMB Bispectrum from Recombination

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Outline

Introduction

Second-order Boltzmann Code

CMB bispectrum

Results

Planck Results on Primordial Non-Gaussianity

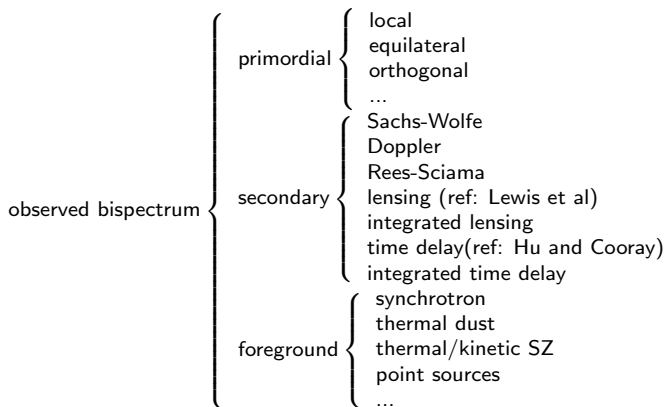
Foreground and ISW-lensing subtracted:

$$f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8$$

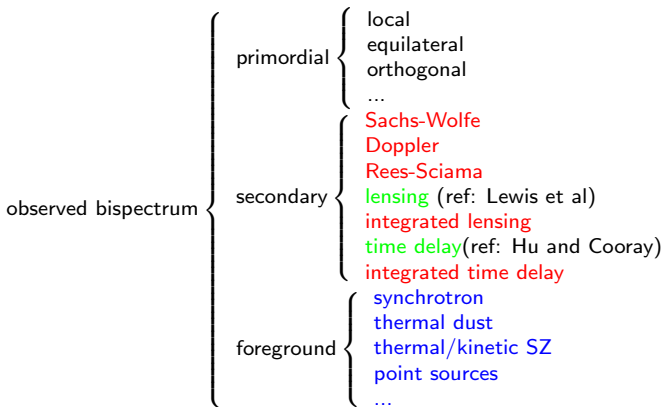
$$f_{\text{NL}}^{\text{equilateral}} = -42 \pm 75$$

$$f_{\text{NL}}^{\text{orthogonal}} = -25 \pm 39$$

In a Nutshell



In a Nutshell



The debate about $f_{\text{NL}}^{\text{local}}$ contamination

CMBquick era:

Khatri et al 2009 : -1

Nitta et al 2009 (only quadratic terms): 1

Pitrou et al 2010: ~ 5

Senatore et al 2010: -3.5 (*)

Creminelli et al 2011: 0.94

Bartolo et al. 2011: $O(1)$

Post-CMBquick era:

Huang and Vernizzi: 0.82 (for $\ell_{\text{max}} = 2000$)

Su et al. 2012: 0.88 (for $\ell_{\text{max}} = 2000$)

Pettinari et al. 2013: 0.5 (for $\ell_{\text{max}} = 2000$)

Second-order Boltzmann code

Linear-order perturbations

$$\frac{d\delta X_i}{d\eta} + A_{ij}(\eta)\delta X_j = 0$$

δX_i ($i = 1, 2, \dots$) are linear-order perturbations and $A_{ij}(\eta)$ are known background functions.

Perturbations include: baryon (density & velocity), CDM (density & velocity), neutrinos (phase-space distribution), radiation (phase-space distribution), and also DE if not a cosmological constant.

codes: CMBfast, CAMB, CMBEasy, CLASS, CosmoLib, CMBquick, ...

Second-order perturbations

$$\frac{d\delta X_i^{(2)}}{d\eta} + A_{ij}(\eta)\delta X_j^{(2)} = S_i$$

$\delta X_i^{(2)}$ ($i = 1, 2, \dots$) are second-order perturbations (in Fourier space), $A_{ij}(\eta)$ remain the same, and the sources S_i are convolutions of linear order perturbations.

Bruni *et al* 97; Pitrou *et al* 09, 10; Beneke & Fidler 10;
Christopherson *et al* 08, 09, 11; Bartolo 07, 11; Senatore 08; Nitta
et al 09; Khatri *et al* 09; Creminelli, Pitrou, and Vernizzi 11; Lewis
12 ...

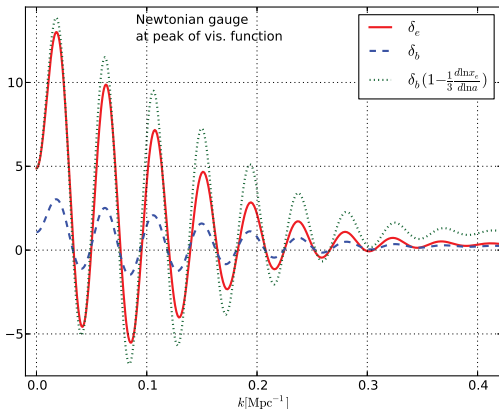
mathematica code CMBquick2 by Cyril Pitrou

Fortran code CosmoLib^{2nd}

Comparison with CMBQuick2 by Cyril Pitrou

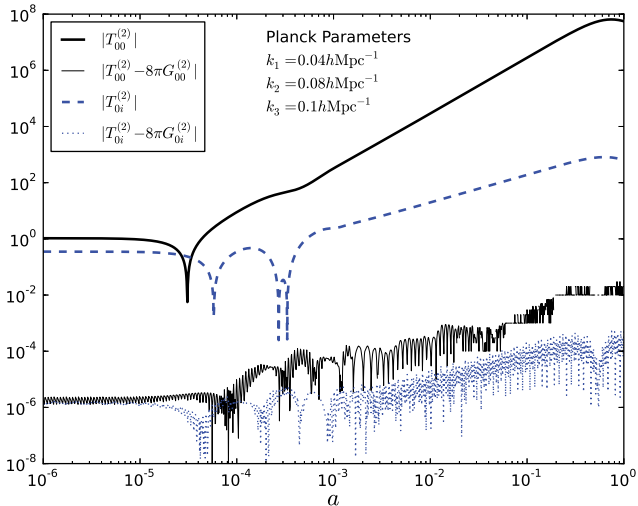
- ▶ Written in Fortran, no license constraint.
- ▶ Faster and parallelized.
- ▶ Much more accurate (energy and momentum constraint $\sim 10^{-6}$).
- ▶ Consistent treatment of perturbed RECFast (including Helium)
- ▶ Better scheme to integrate the CMB bispectrum (split out lensing and time delay boundary terms).
- ▶ Full-sky bispectrum.

Perturbed electron number density $\delta_e \equiv \delta n_e/n_e$

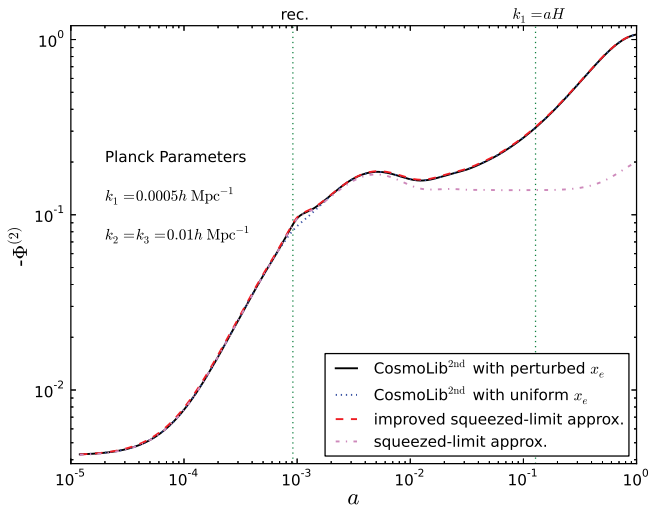


- ▶ Agree with the calculation in Senatore et al.
- ▶ $\delta_e \sim 5$ is large (expect large effective $f_{\text{NL}}?$).

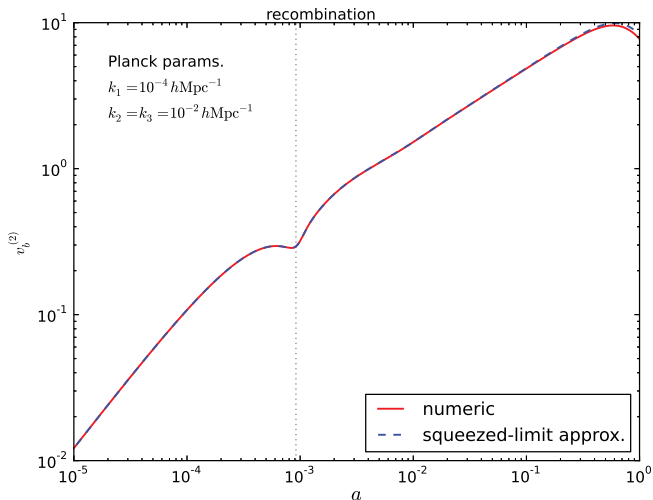
Einstein equations: energy/momentum constraints



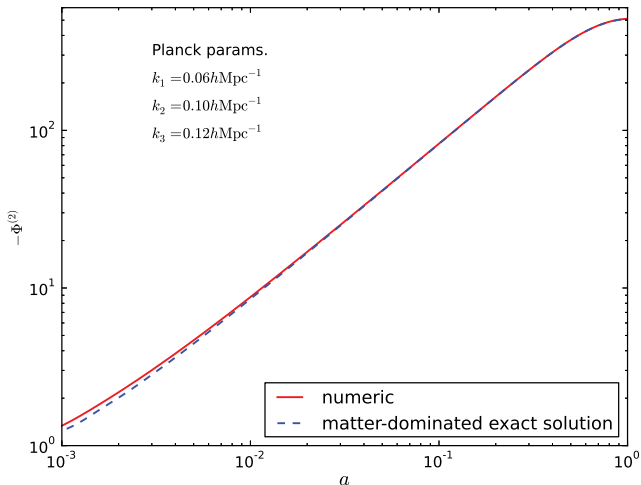
Squeezed limit: gravitational potential



Squeezed limit: baryon velocity



Late-time exact solution



CMB bispectrum

The bispectrum: definition

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{l,m} a_{lm} Y_{lm}(\mathbf{n}).$$

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = B_{l_1 l_2 l_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}.$$

$$B_{l_1 l_2 l_3} = b_{l_1 l_2 l_3} \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$

Primordial non-Gaussianity: a difficult integral

Primordial bispectrum \Rightarrow CMB angular bispectrum

$$b_{l_1 l_2 l_3} = \int dx dk_1 dk_2 dk_3 (x k_1 k_2 k_3)^2 B_{\text{prim}}(k_1, k_2, k_3) \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3) j_{l_1}(k_1 x) j_{l_2}(k_2 x) j_{l_3}(k_3 x)$$

Solution: Reduce the 4D integral into products of 1D integrals by factorizing the primordial bispectrum

$$B_{\text{prim}}(k_1, k_2, k_3) = \sum_i X_i(k_1) Y_i(k_2) Z_i(k_3).$$

See e.g. Fergusson *et al.* 09.

Second-order perturbs: a more difficult integral

$$b_{l_1 l_2 l_3} \propto \sum_{m_3} \int dk_1 dk_2 d\mu d\eta S_{l_3 m_3}(k_1, k_2, \mu, \eta) j_{l_3}^{(l_3 m_3)}[k(\eta_0 - \eta)] \sum_{m_1 m_2} Y_{l_1 m_1}^* Y_{l_2 m_2}^* \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \dots + \text{perms.}$$

Brute force almost impossible. However, the following tricks can be used to make it work:

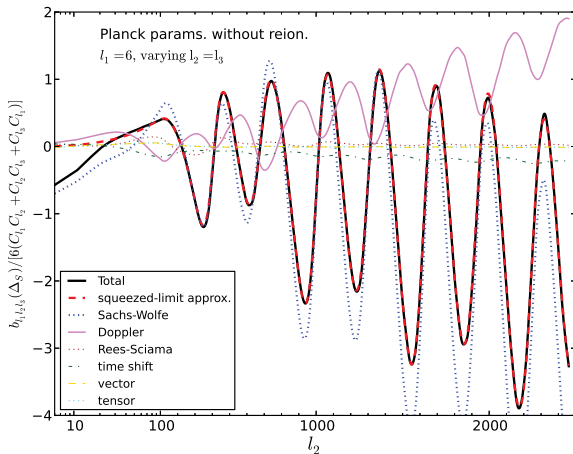
- ▶ The cosmology-independent geometrical factors (product of Y_{lm} and 3- j symbols) can be precomputed and saved for interpolation.
- ▶ $j_l^{(l' m)}$ can be replaced with j_l if we integrate by part (in a discrete way, using SVD decomposition).
- ▶ Use adaptive mesh.

The squeezed-limit consistency relation

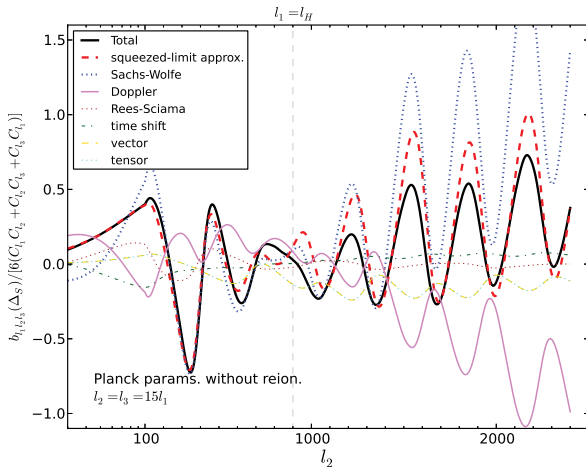
$$b_{l_S l_L l_L} = 2C_{l_L}^{TT} C_{l_S}^{TT} - C_{l_L}^{T\zeta} \tilde{C}_{l_S}^{TT} \frac{d \ln \left[\left(l_S + \frac{1}{2} \right)^2 \tilde{C}_{l_S}^{TT} \right]}{d \ln \left(l_S + \frac{1}{2} \right)}$$

,
if $l_L \ll l_S$ and $l_L \ll l_H \sim 60$.

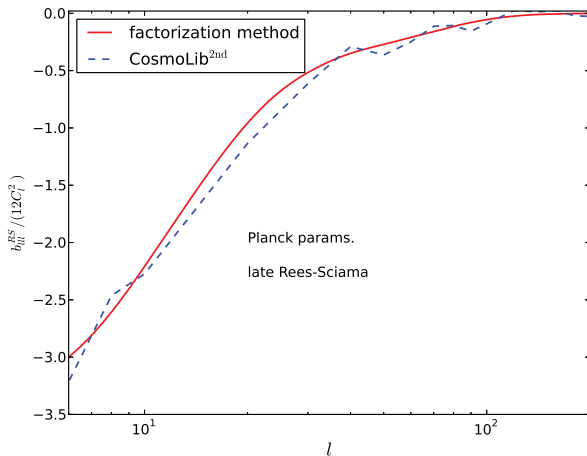
Testing the consistency relation



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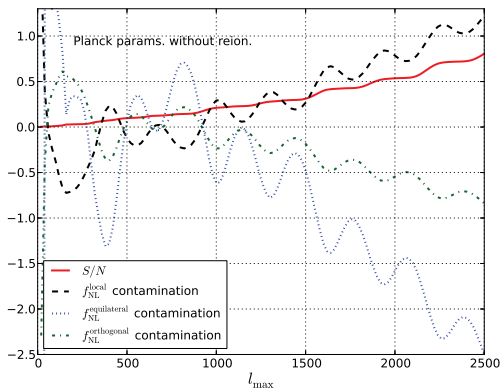
Testing the integrator in non-squeezed limit



Results and Conclusions

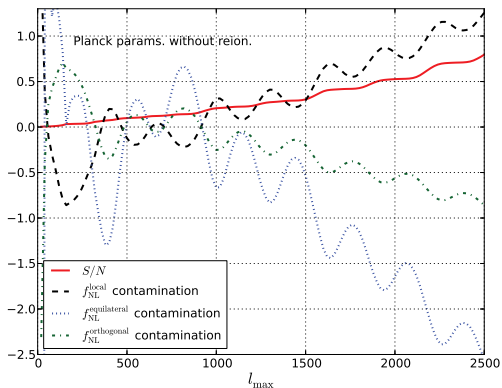
Results

All contributions $m = 0$ only:



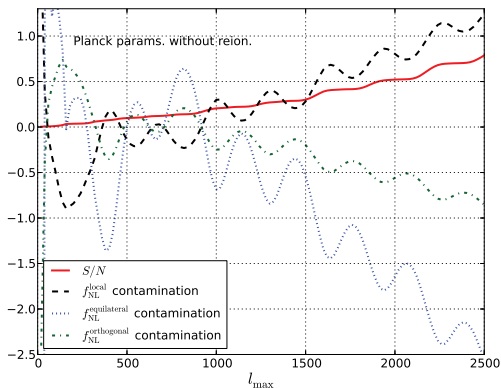
Results

All contributions $m = 0, 1$:



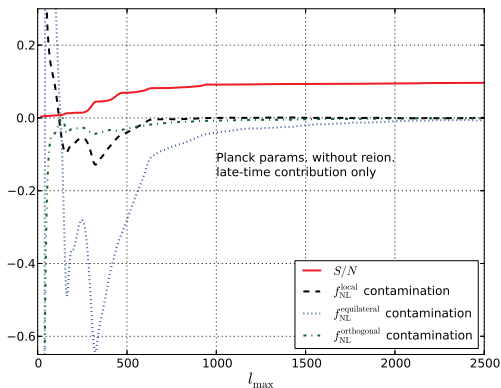
Results

All contributions $m = 0, 1, 2$:



Late-time integrated effects

Rees-Sciama + integrated lensing + integrated time delay



Conclusions

- ▶ We did a very complicated calculation and we found nothing important.

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- ▶ Secondary effect (not including lensing) biases f_{NL} by ≈ 1 , if we use temperature data only. This bias can be removed using our code.
- ▶ The measurement of secondary effect itself is limited by cosmic variance if we use temperature data only.
- ▶ What if we use polarization? (to be done)