

Cosmic ray transport in galaxy clusters – Implications for radio halos and gamma-rays

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in collaboration with

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Outline

- 1 **Cosmological simulations**
 - Introduction
 - Simulated physics
 - Cosmic rays in galaxy clusters
- 2 **Non-thermal emission**
 - Overview
 - Radio emission
 - Gamma-ray emission
- 3 **Cosmic ray transport**
 - Observations and models
 - CR pumping and streaming
 - Radio and gamma-ray bimodality



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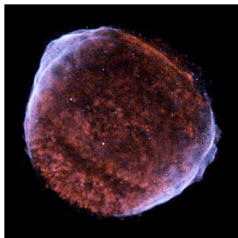
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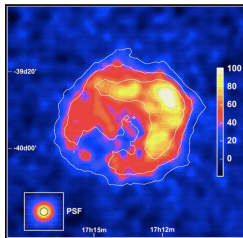
Collisionless shocks in supernova remnants

Astrophysical collisionless shocks can:

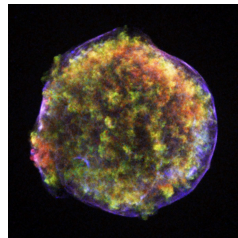
- accelerate particles (electrons and ions)
- amplify magnetic fields (or generate them from scratch)
- exchange energy between electrons and ions



SN 1006 X-rays (CXC/Hughes)



G347.3 HESS TeV
(Aharonian et al. 2006)



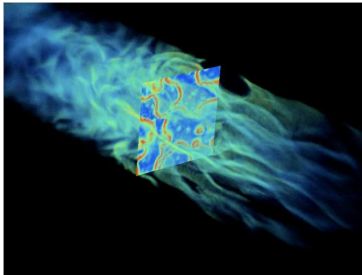
Tycho X-rays (CXC)

Collisionless shocks

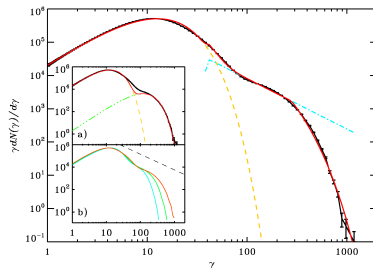
Astrophysical collisionless shocks can:

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Particle-in-cell simulations of unmagnetized, relativistic pair shocks that are mediated by the Weibel instability (Spitkovsky 2008)



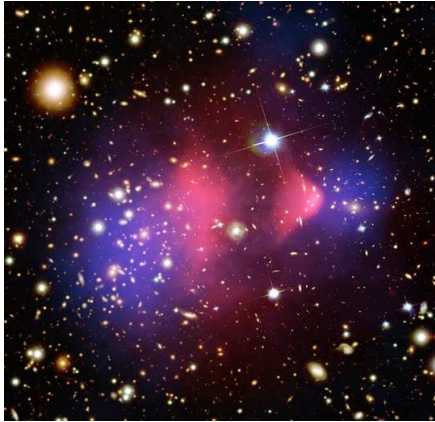
magnetic energy density (Spitkovsky 2008)



post-shock Maxwellian and accelerated CR power-law

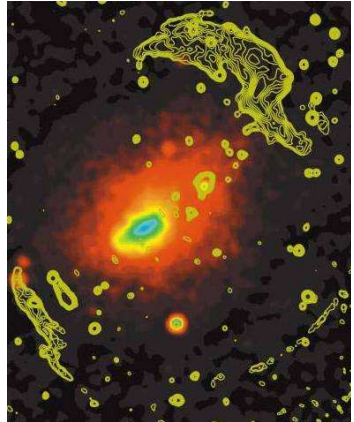


Shocks in galaxy clusters



1E 0657-56 ("Bullet cluster")

(X-ray: NASA/CXC/CfA/M.Markevitch et al.; Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.; Lensing: NASA/STScI; ESO WFI; Magellan/U.Arizona/D.Clowe et al.)

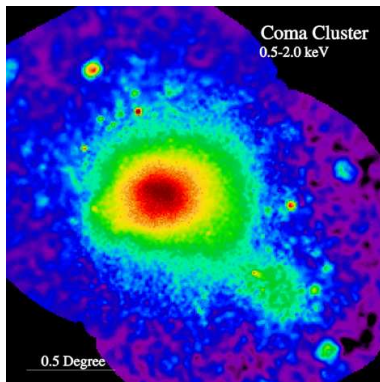


Abell 3667

(radio: Johnston-Hollitt. X-ray: ROSAT/PSPC.)

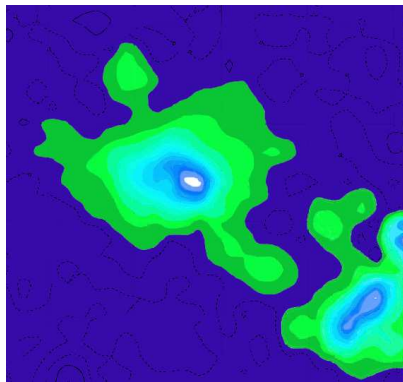


Giant radio halo in the Coma cluster



thermal X-ray emission

(Snowden/MPE/ROSAT)



radio synchrotron emission

(Deiss/Effelsberg)



How universal is diffusive shock acceleration?

What can galaxy clusters teach us about shock acceleration and beyond?

Cosmological structure formation shock physics complementary to interplanetary and SNR shocks:

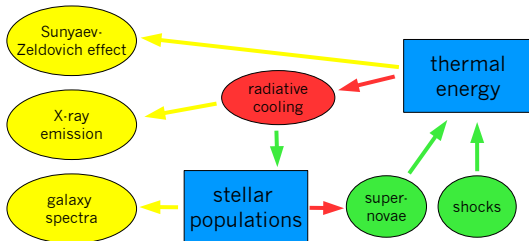
- **probing unique regions of shock acceleration parameter space:**
 - Mach numbers $\mathcal{M} \sim 2 \dots 10$ with ‘infinitely’ extended (Mpc) and lasting (Gyr) shocks (observationally accessible @ $z = 0$)
 - plasma- β factors of $\beta \sim 10^2 \dots 10^5$
- consistent picture of non-thermal processes in galaxy clusters (radio, soft/hard X-ray, γ -ray emission)
 - illuminating the **process of structure formation**
 - history of individual clusters: **cluster archeology**
 - calibrating thermal cluster observables: **cluster cosmology**



Radiative simulations – flowchart

Cluster observables:

Physical processes in clusters:



- loss processes
- gain processes
- observables
- populations

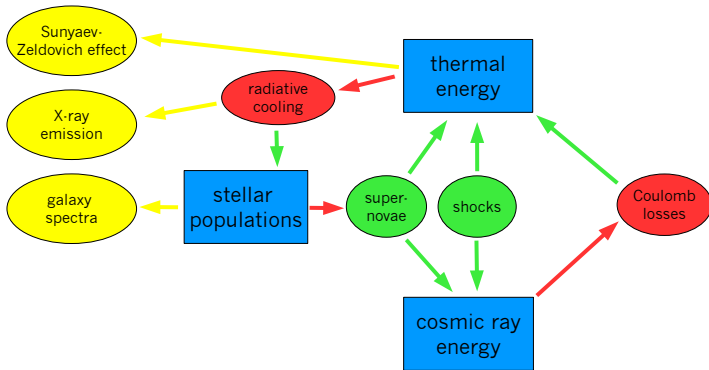
C.P., Enßlin, Springel (2008)



Radiative simulations with CR physics

Cluster observables:

Physical processes in clusters:



C.P., Enßlin, Springel (2008)

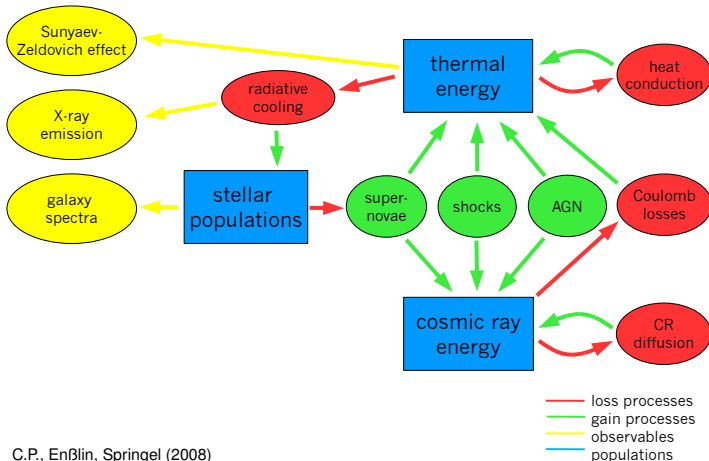
— loss processes
— gain processes
— observables
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Radiative simulations with extended CR physics

Cluster observables:

Physical processes in clusters:



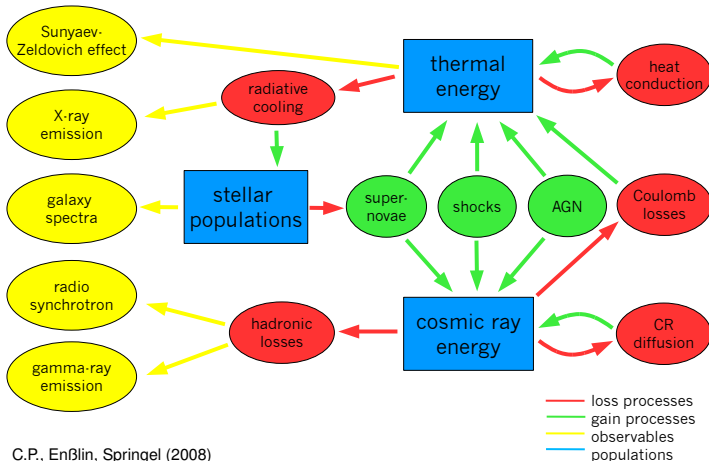
C.P., Enßlin, Springel (2008)



Radiative simulations with extended CR physics

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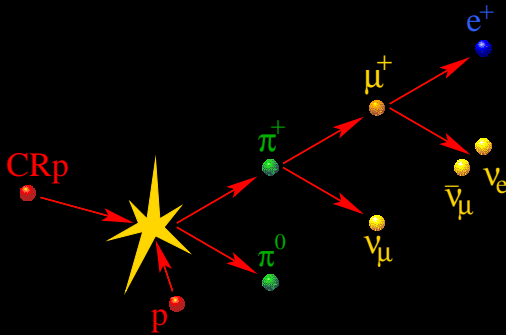
Physical processes in clusters:



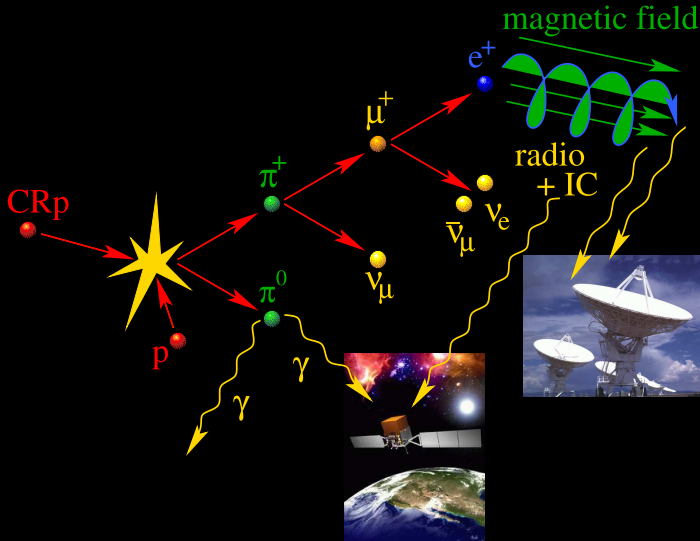
C.P., Enßlin, Springel (2008)



Hadronic cosmic ray proton interaction



Hadronic cosmic ray proton interaction



Our philosophy and description

An accurate description of CRs should follow the evolution of the spectral energy distribution of CRs as a function of time and space, and keep track of their dynamical, non-linear coupling with the hydrodynamics.

We seek a compromise between

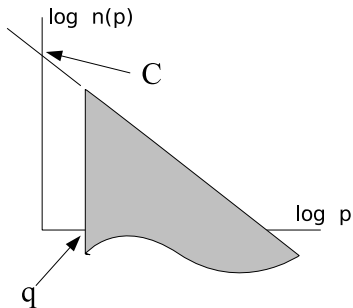
- capturing as many physical properties as possible
- requiring as little computational resources as necessary

Assumptions:

- protons dominate the CR population
- a momentum power-law is a typical spectrum
- CR energy & particle number conservation



CR spectral description



$$p = P_p / m_p c$$

Enßlin, C.P., Springel, Jubelgas (2007)

$$f(p) = \frac{dN}{dp dV} = C p^{-\alpha} \theta(p - q)$$

$$q(\rho) = \left(\frac{\rho}{\rho_0} \right)^{\frac{1}{3}} q_0$$

$$C(\rho) = \left(\frac{\rho}{\rho_0} \right)^{\frac{\alpha+2}{3}} C_0$$

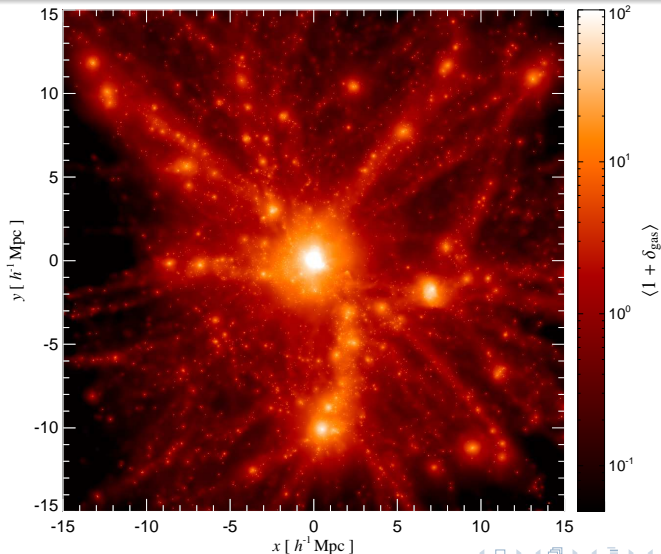
$$n_{\text{CR}} = \int_0^{\infty} dp f(p) = \frac{C q^{1-\alpha}}{\alpha-1}$$

$$P_{\text{CR}} = \frac{m_p c^2}{3} \int_0^{\infty} dp f(p) \beta(p) p$$

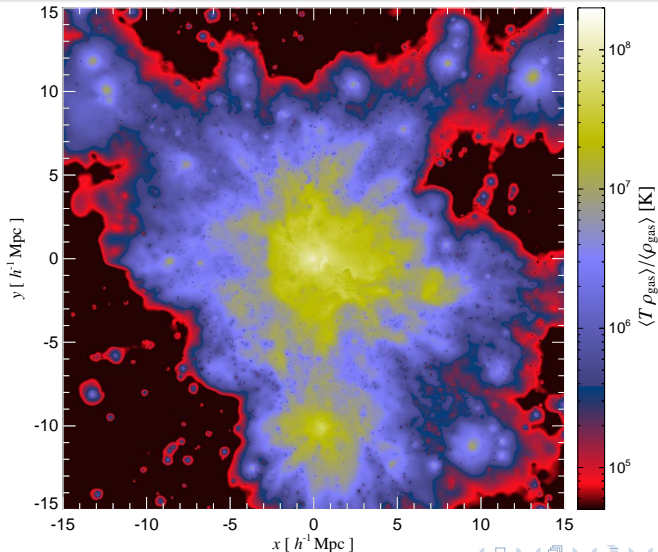
$$= \frac{C m_p c^2}{6} \mathcal{B}_{\frac{1}{1+q^2}} \left(\frac{\alpha-2}{2}, \frac{3-\alpha}{2} \right)$$



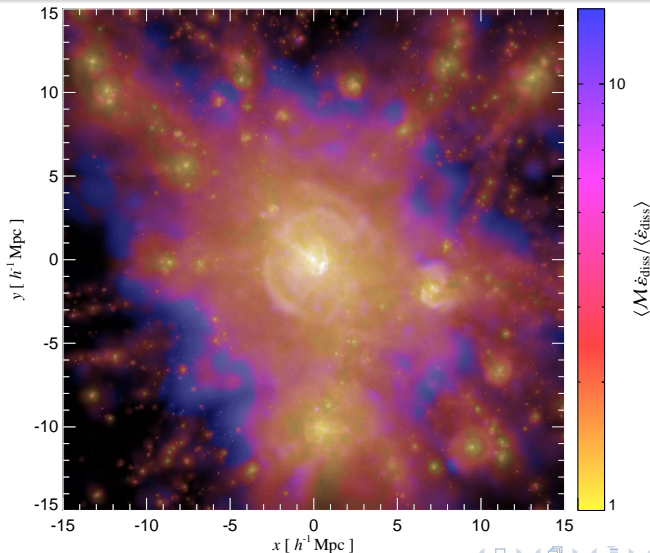
Cosmological cluster simulation: gas density



Mass weighted temperature

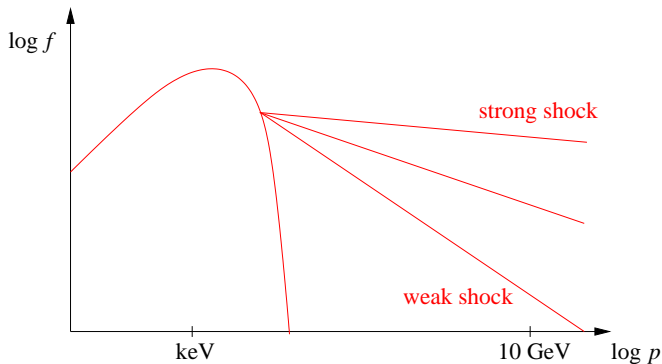


Mach number distribution weighted by ϵ_{diss}



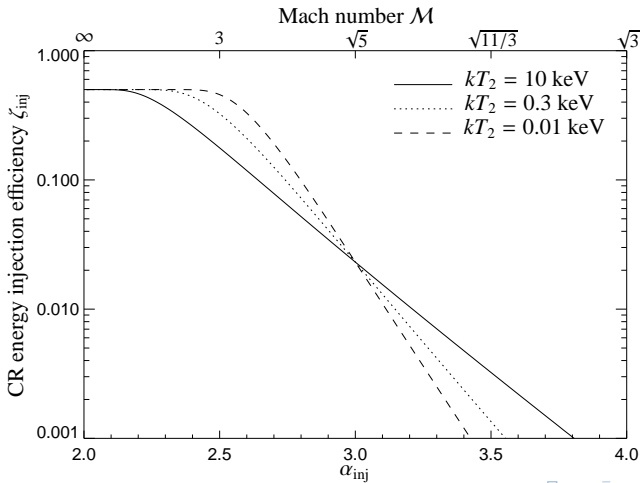
Diffusive shock acceleration – Fermi 1 mechanism (1)

Spectral index depends on the Mach number of the shock,
 $\mathcal{M} = v_{\text{shock}}/c_s$:

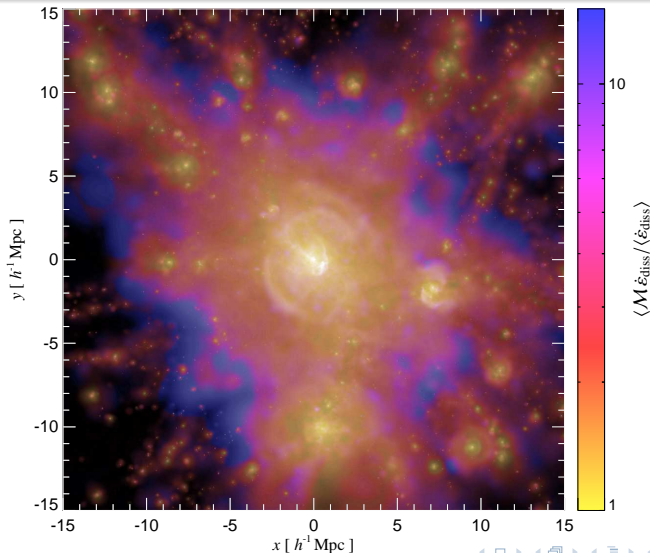


Diffusive shock acceleration – Efficiency (2)

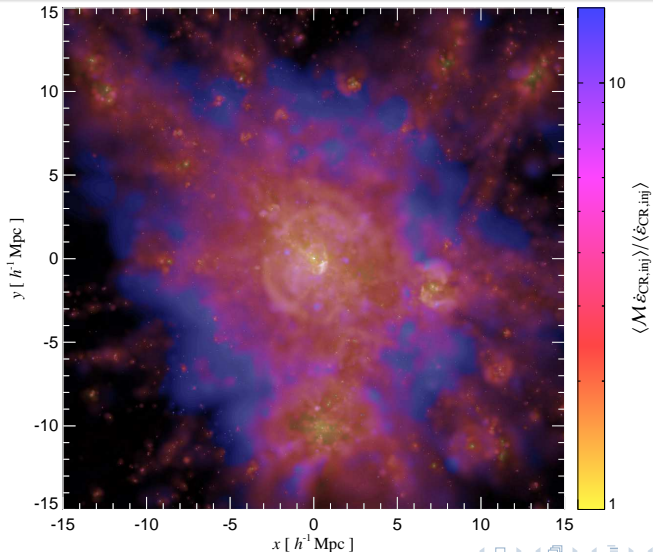
CR proton energy injection efficiency, $\zeta_{\text{inj}} = \varepsilon_{\text{CR}}/\varepsilon_{\text{diss}}$:



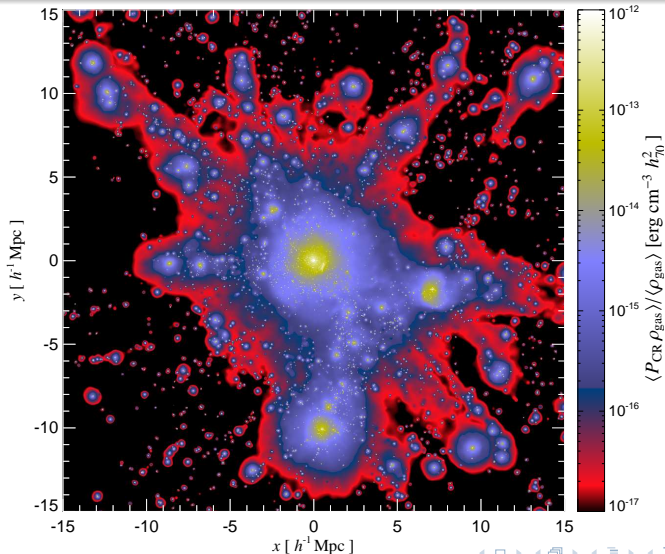
Mach number distribution weighted by ϵ_{diss}



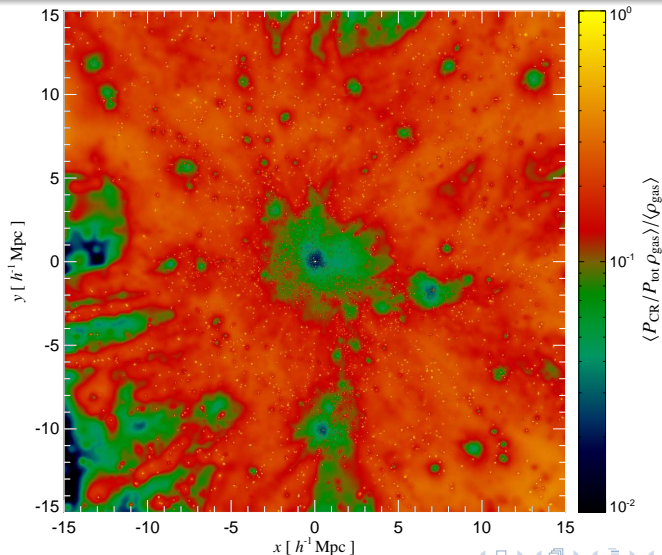
Mach number distribution weighted by $\varepsilon_{\text{CR},\text{inj}}$



CR pressure P_{CR}



Relative CR pressure $P_{\text{CR}}/P_{\text{total}}$



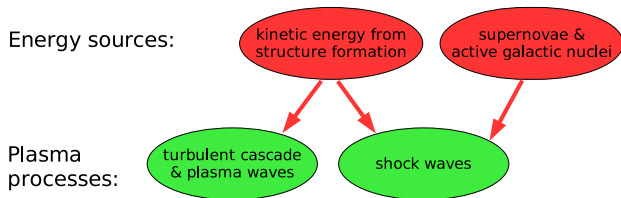
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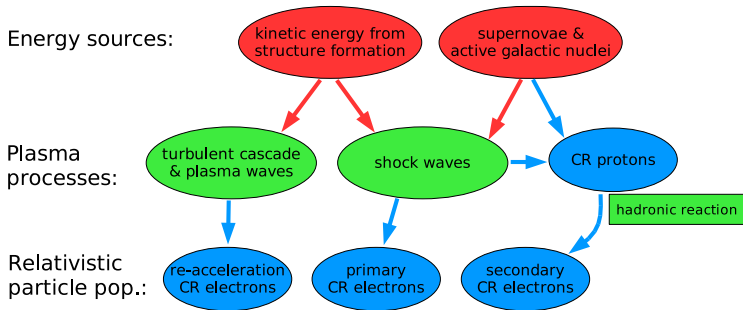
Multi messenger approach for non-thermal processes

Relativistic populations and radiative processes in clusters:



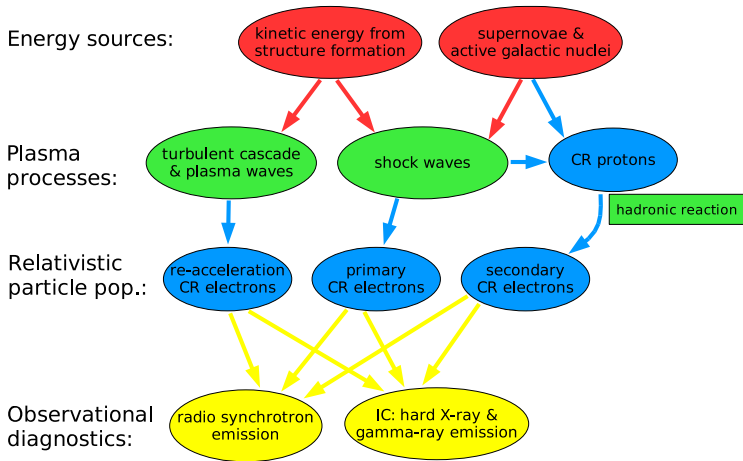
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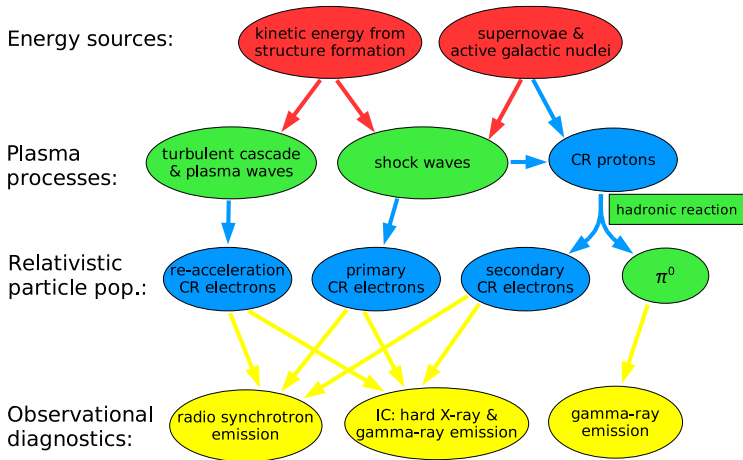
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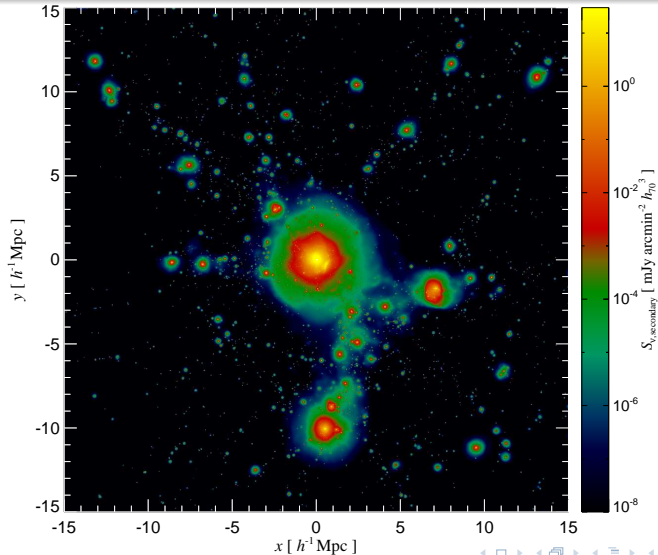


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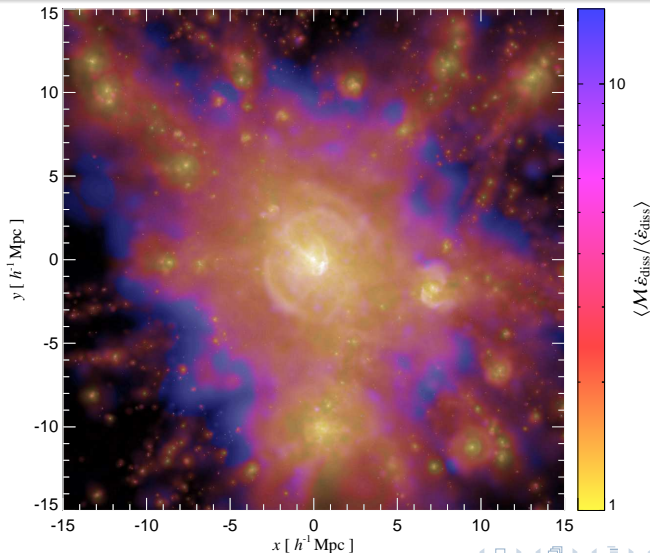
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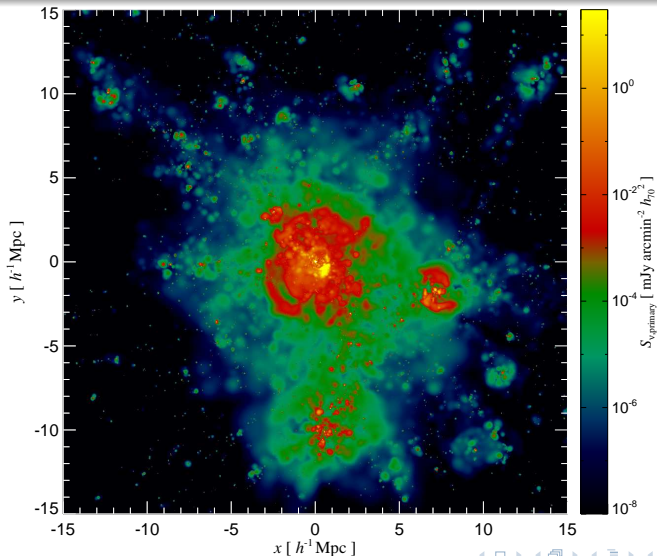
Cluster radio emission by hadronically produced CRe



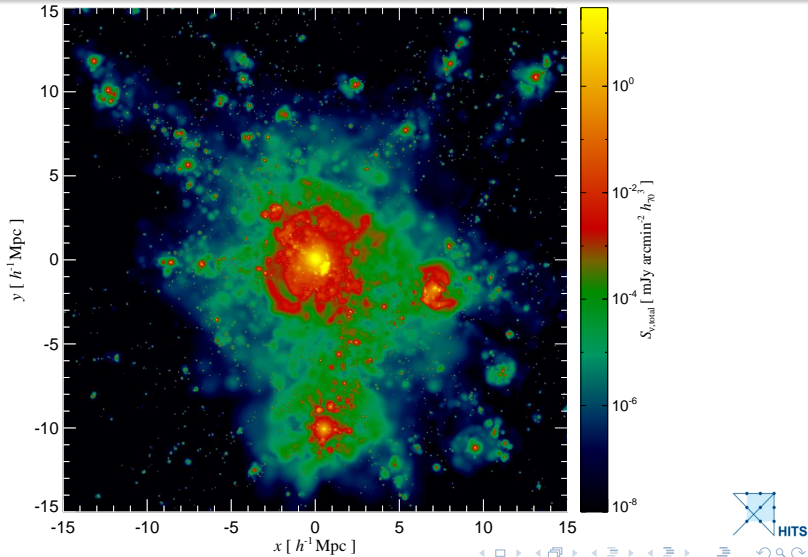
Cosmic web: Mach number



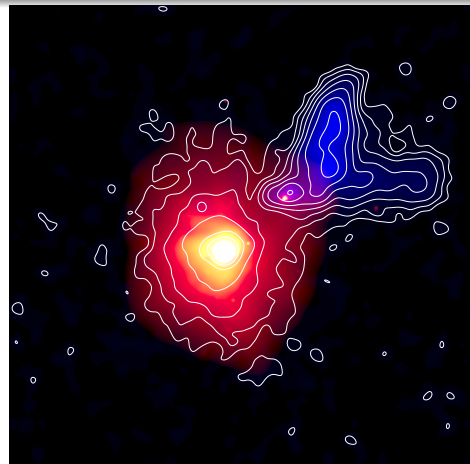
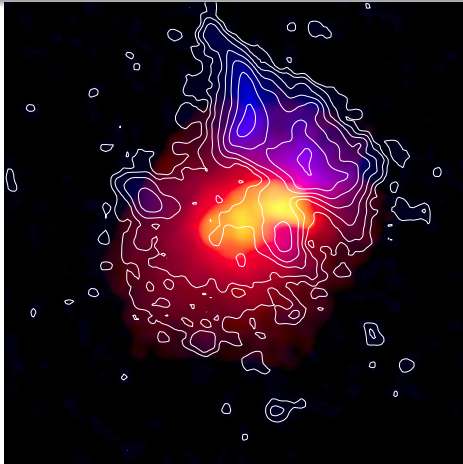
Radio gischt: primary CRe (150 MHz)



Radio gischt + central hadronic halo = giant radio halo



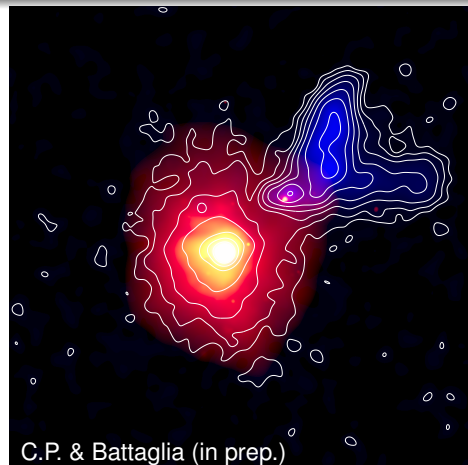
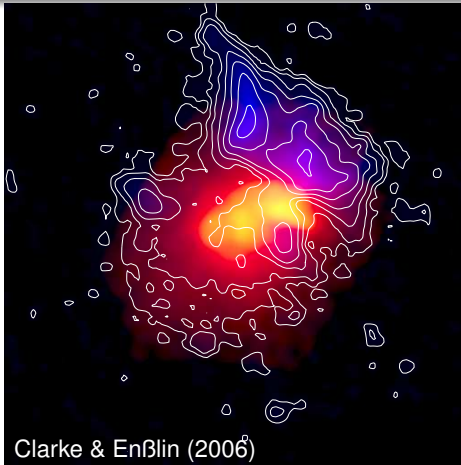
Which one is the simulation/observation of A2256?



red/yellow: thermal X-ray emission,
blue/contours: 1.4 GHz radio emission with giant radio halo and relic



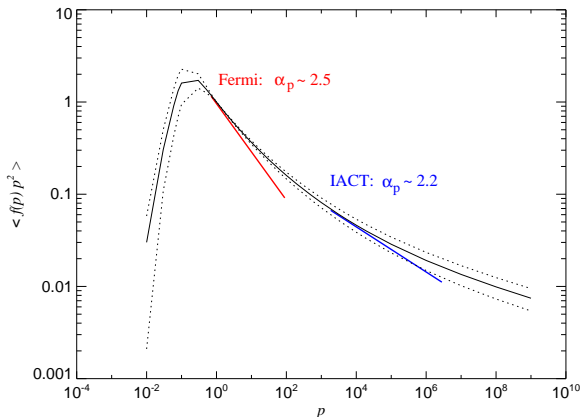
Observation – simulation of A2256



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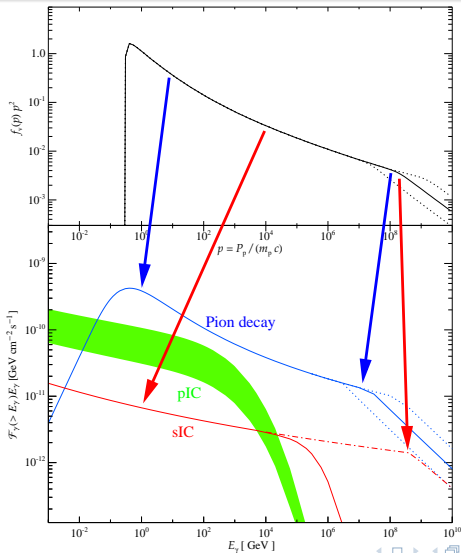
Universal CR spectrum in clusters (Pinzke & C.P. 2010)



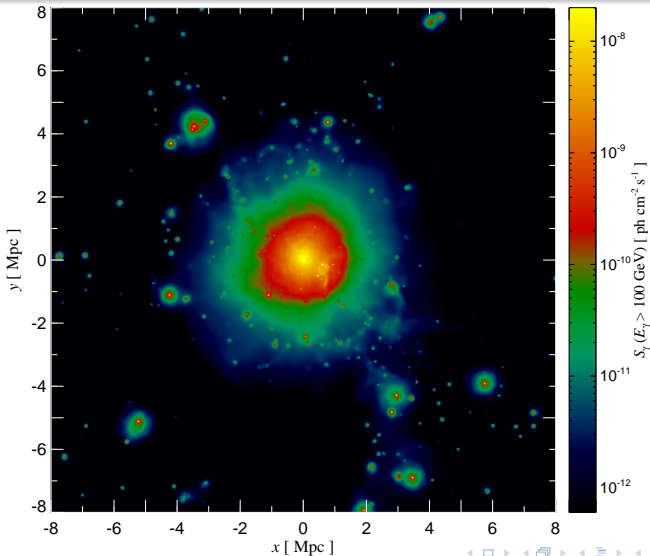
Normalized CR spectrum shows **universal concave shape** → governed by hierarchical structure formation and the implied distribution of Mach numbers that a fluid element had to pass through in cosmic history.



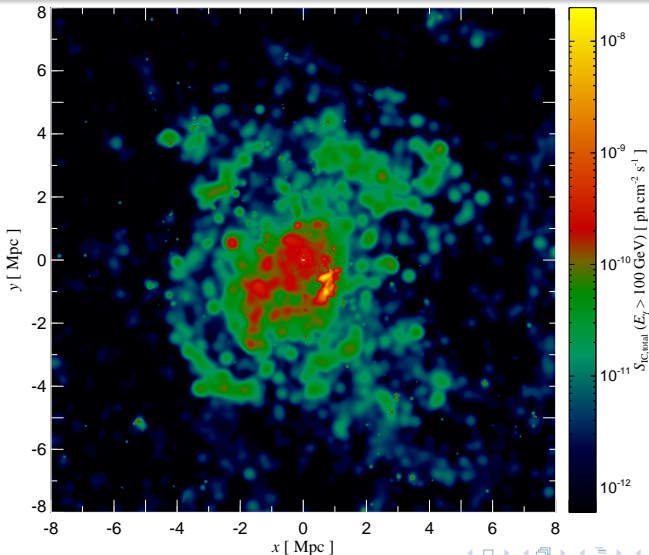
CR proton and γ -ray spectrum (Pinzke & C.P. 2010)



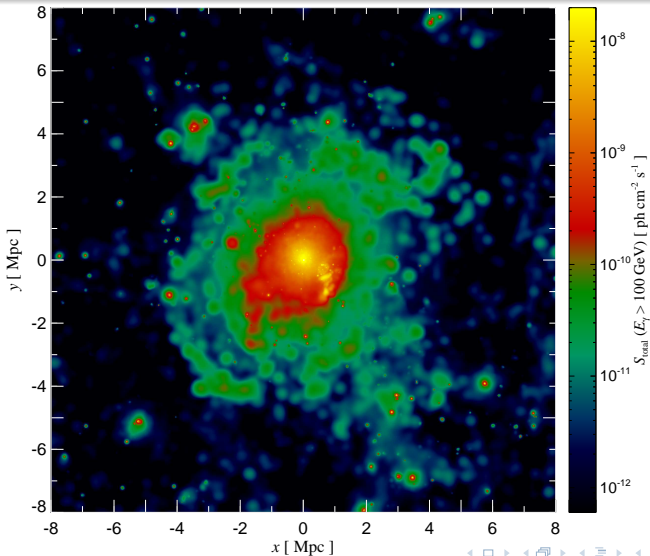
Hadronic γ -ray emission, $E_\gamma > 100$ GeV



Inverse Compton emission, $E_{IC} > 100$ GeV

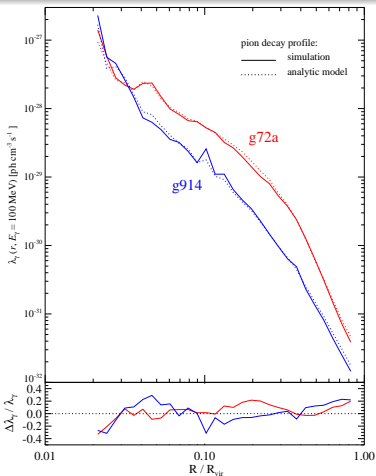


Total γ -ray emission, $E_\gamma > 100$ GeV

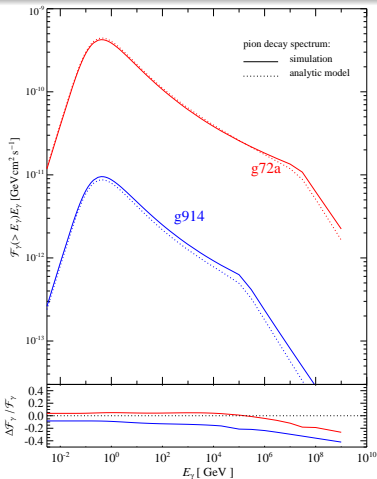


An analytic model for the cluster γ -ray emission

Comparison: simulation vs. analytic model, $M_{\text{vir}} \simeq (10^{14}, 10^{15}) M_{\odot}$



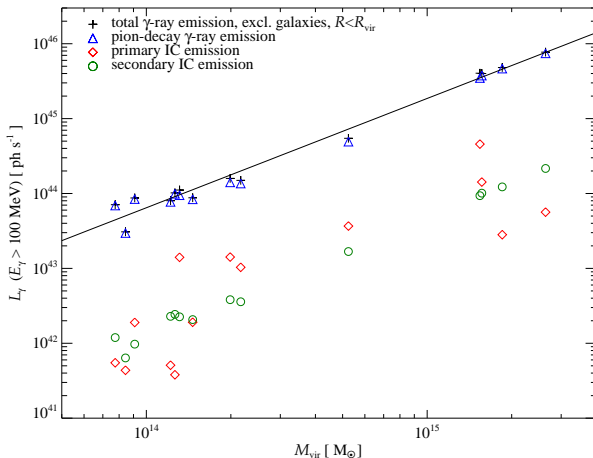
Spatial γ -ray emission profile



Pion decay spectrum



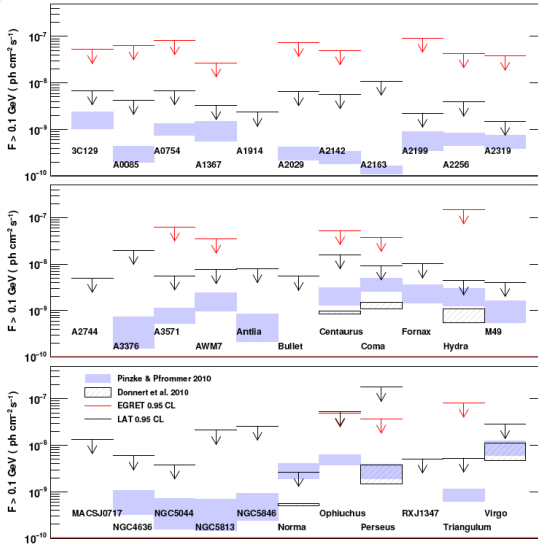
Gamma-ray scaling relations



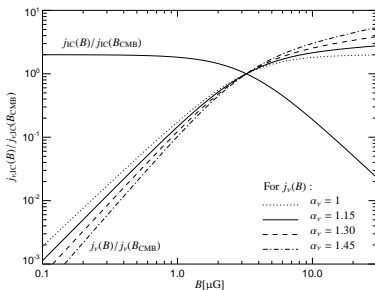
Scaling relation + complete sample of the brightest X-ray clusters (HIFLUGCS) \rightarrow predictions for *Fermi* and *IACT*'s



γ -ray limits and hadronic predictions (Ackermann et al. 2010)



Minimum γ -ray flux in the hadronic model



Synchrotron emissivity of **steady state CRes** is independent of the magnetic field for $B \gg B_{\text{CMB}}$!

Synchrotron luminosity:

$$L_{\nu} = A_{\nu} \int dV n_{\text{CR}} n_{\text{gas}} \frac{\epsilon_B^{(\alpha_{\nu}+1)/2}}{\epsilon_{\text{CMB}} + \epsilon_B}$$

$$\rightarrow A_{\nu} \int dV n_{\text{CR}} n_{\text{gas}} \quad (\epsilon_B \gg \epsilon_{\text{CMB}})$$

γ -ray luminosity:

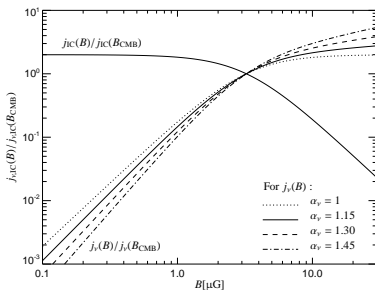
$$L_{\gamma} = A_{\gamma} \int dV n_{\text{CR}} n_{\text{gas}}$$

\rightarrow minimum γ -ray flux:

$$\mathcal{F}_{\gamma, \text{min}} = \frac{A_{\gamma}}{A_{\nu}} \frac{L_{\nu}}{4\pi D^2}$$



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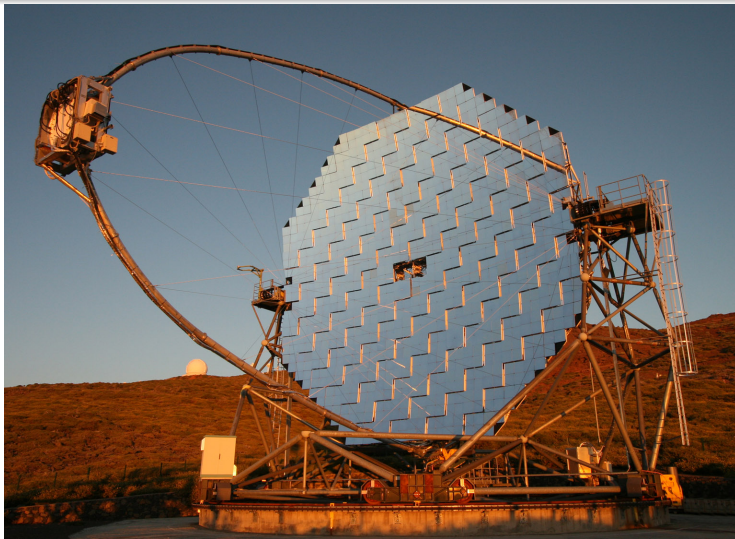
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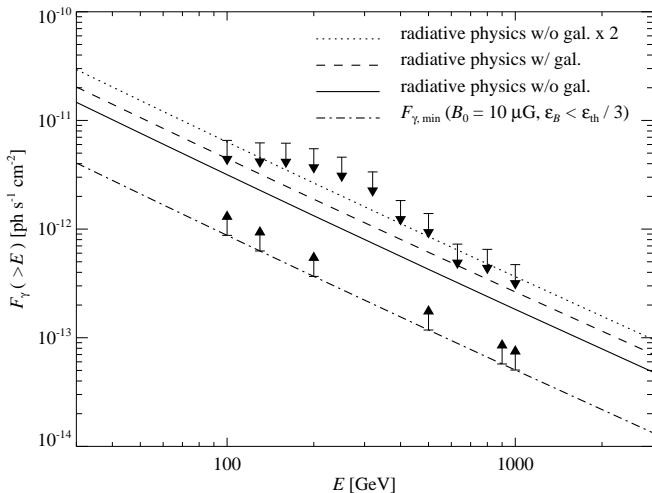
$$\mathcal{F}_{\gamma, \text{min}} = \frac{A_\gamma}{A_\nu} \frac{L_\nu}{4\pi D^2}$$



MAGIC observations of Perseus



Upper limit on the TeV γ -ray emission from Perseus



The MAGIC Collaboration: Aleksic et al. 2010



Results from the Perseus observation by *MAGIC*

- assuming $f \propto p^{-\alpha}$ with $\alpha = 2.1$, $P_{\text{CR}} \propto P_{\text{th}}$:
 $\langle P_{\text{CR}} \rangle < 0.02 \langle P_{\text{th}} \rangle \rightarrow$ **most stringent constraint on CR pressure!**
- **upper limits consistent with cosmological simulations:**
 $F_{\text{upper limits}}(100 \text{ GeV}) = 2 F_{\text{sim}}$ (optimistic model)
- simulation modeling of pressure constraint yields
 $\langle P_{\text{CR}} \rangle / \langle P_{\text{th}} \rangle < 0.04$ (0.08) for the **core** (entire cluster)
- resolving the apparent discrepancy:
 - concave curvature 'hides' CR pressure at GeV energies
 - relative CR pressure increases towards the outer parts (adiabatic compression and softer equation of state of CRs)



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Conclusions on non-thermal emission from clusters

Exploring the memory of structure formation

- **primary, shock-accelerated CR electrons** resemble current accretion and merging shock waves
- **CR protons/hadronically produced CR electrons** trace the time integrated non-equilibrium activities of clusters that is modulated by the recent dynamical activities

How can we read out this information about non-thermal populations?

→ **new era of multi-frequency experiments**, e.g.:

- **LOFAR, GMRT, MWA, LWA, SKA**: interferometric array of radio telescopes at low frequencies ($\nu \simeq (15 - 240)$ MHz)
- **NuSTAR**: future hard X-ray satellites ($E \simeq (1 - 100)$ keV)
- **Fermi** γ -ray space telescope ($E \simeq (0.1 - 300)$ GeV)
- Imaging air **Čerenkov telescopes** ($E \simeq (0.1 - 100)$ TeV)



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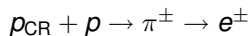


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Radio halo theory – (i) hadronic model



strength:

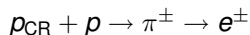
- all required ingredients available:
shocks to inject CRp, gas protons as targets, magnetic fields
- predicted luminosities and morphologies as observed without tuning
- power-law spectra as observed

weakness:

- all clusters should have radio halos
- does not explain all reported spectral features
- ...



Radio halo theory – (i) hadronic model



strength:

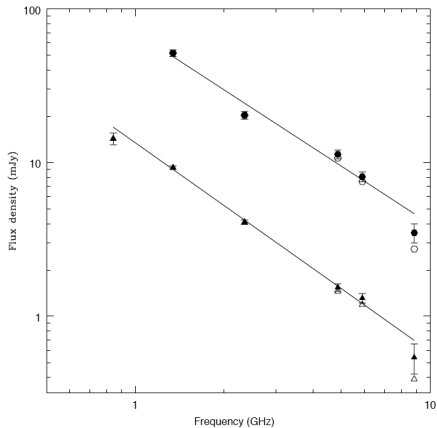
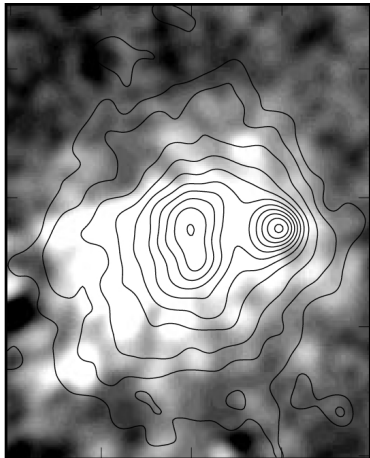
- all required ingredients available:
shocks to inject CRp, gas protons as targets, magnetic fields
- predicted luminosities and morphologies as observed without tuning
- power-law spectra as observed

weakness:

- ~~all clusters should have radio halos~~
- does not explain all reported spectral features
- ...



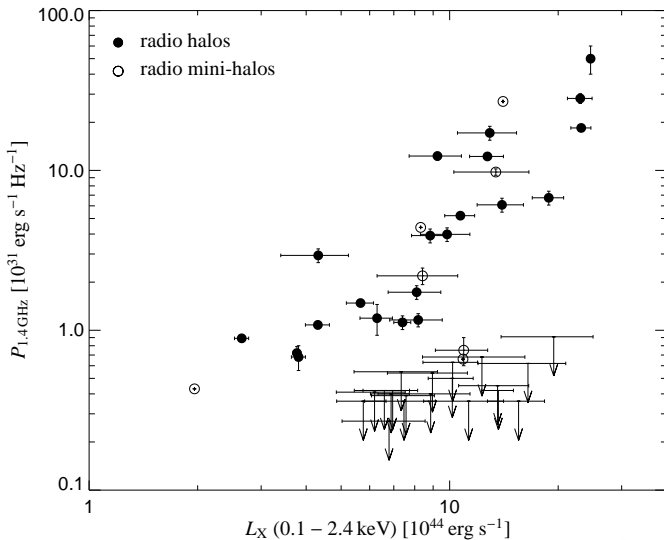
Radio halo and spectrum in the Bullet cluster



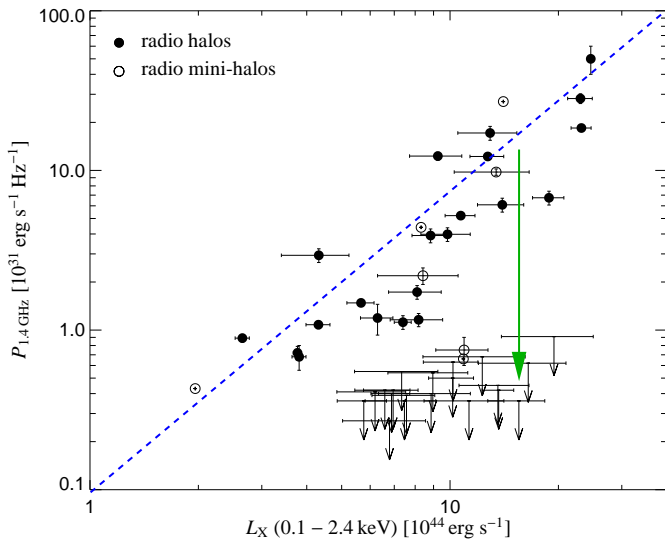
Liang et al. (2000): SZ-corrected



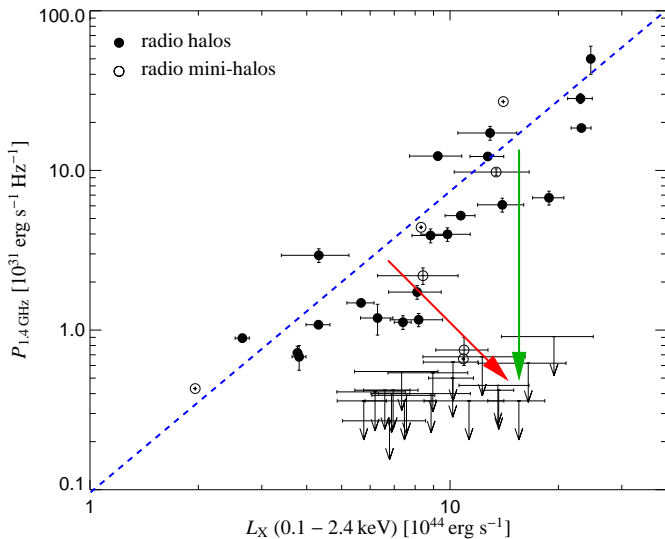
Radio luminosity - X-ray luminosity



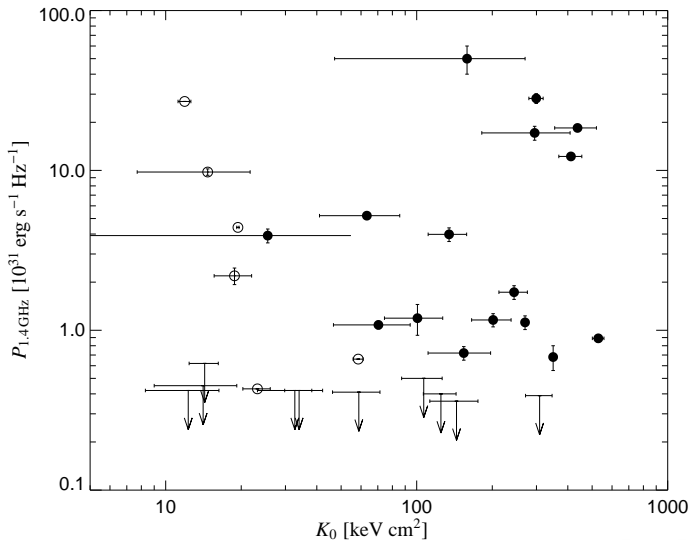
Radio luminosity - X-ray luminosity



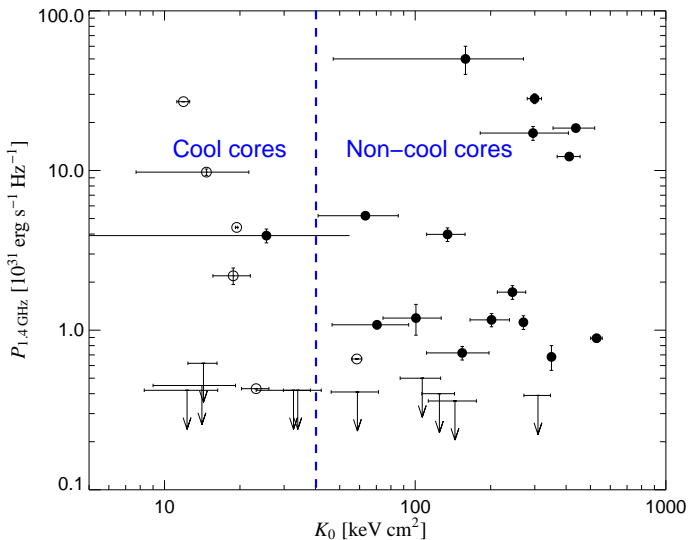
Radio luminosity - X-ray luminosity



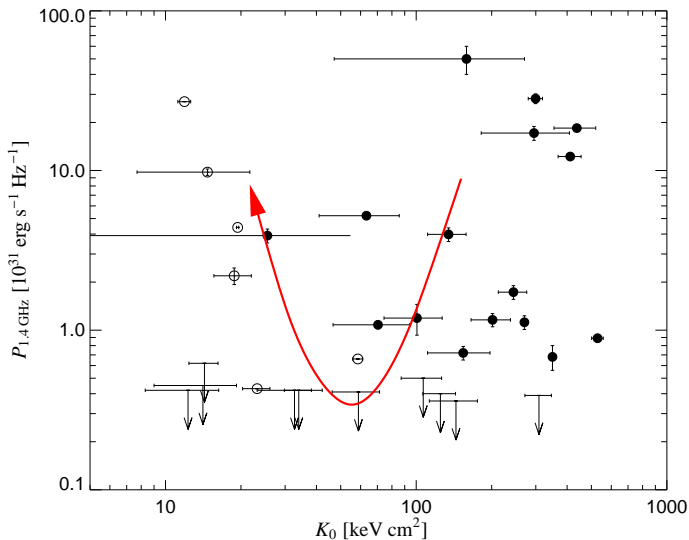
Radio luminosity - central entropy



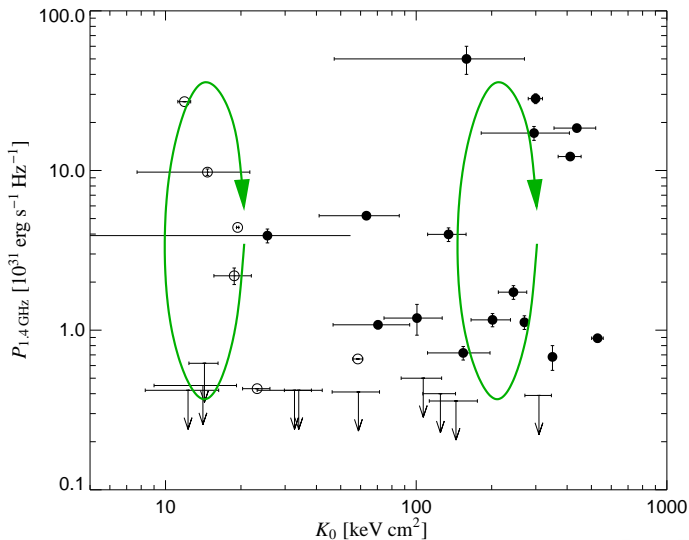
Radio luminosity - central entropy



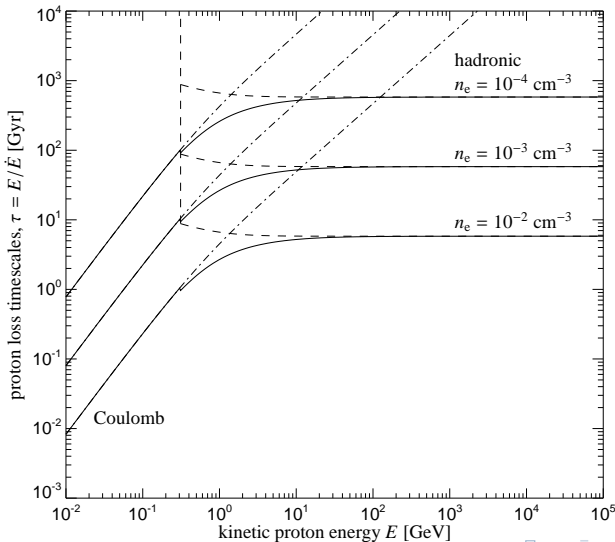
Radio luminosity - central entropy



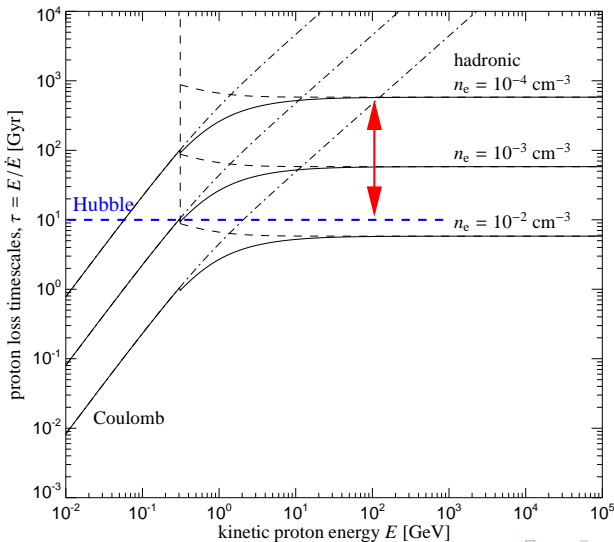
Radio luminosity - central entropy



Proton cooling times



Proton cooling times



Radio halo theory – (ii) re-acceleration model

strength:

- all required ingredients available:
radio galaxies & relics to inject CRe, plasma waves to re-accelerate, ...
- reported complex radio spectra emerge naturally
- clusters without halos ← less turbulent

weakness:

- Fermi II acceleration is inefficient – CRe cool rapidly
- observed power-law spectra require fine tuning
- ...



Radio halo theory – (ii) re-acceleration model

strength:

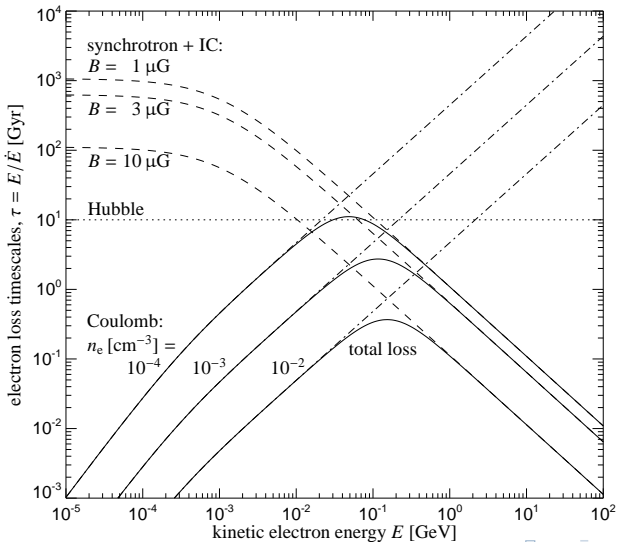
- all required ingredients available:
radio galaxies & relics to inject CRe, plasma waves to re-accelerate, ...
- reported complex radio spectra emerge naturally
- clusters without halos ← less turbulent

weakness:

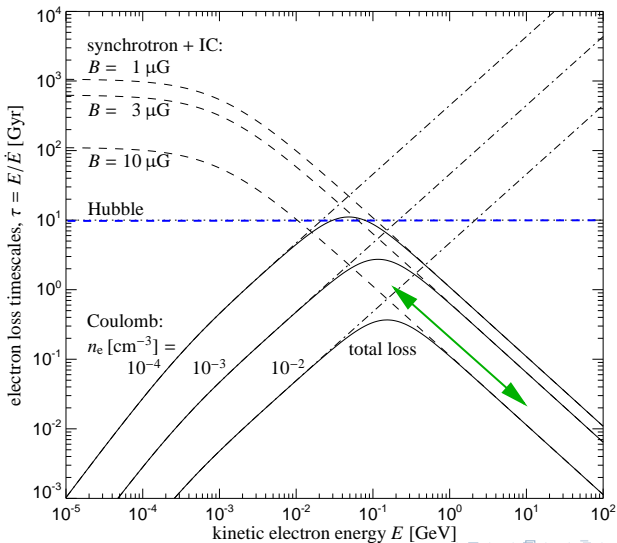
- Fermi II acceleration is inefficient – **CRe cool rapidly**
- observed power-law spectra require fine tuning
- ...



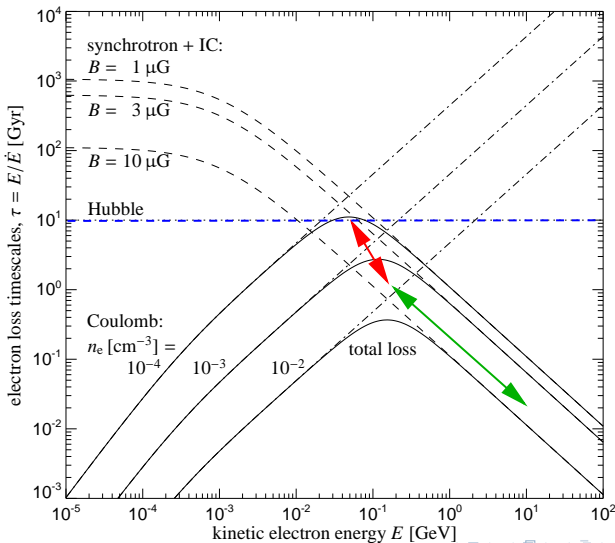
Electron cooling times



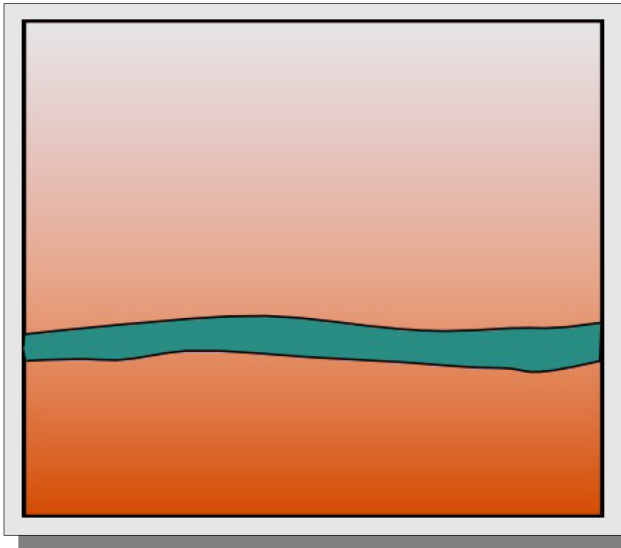
Electron cooling times



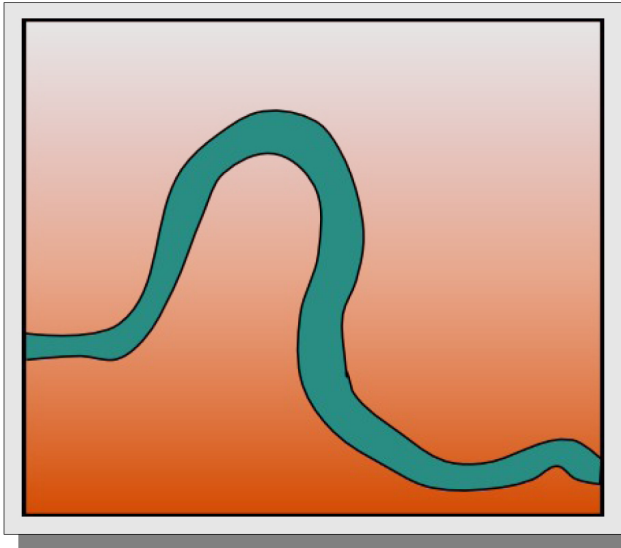
Electron cooling times



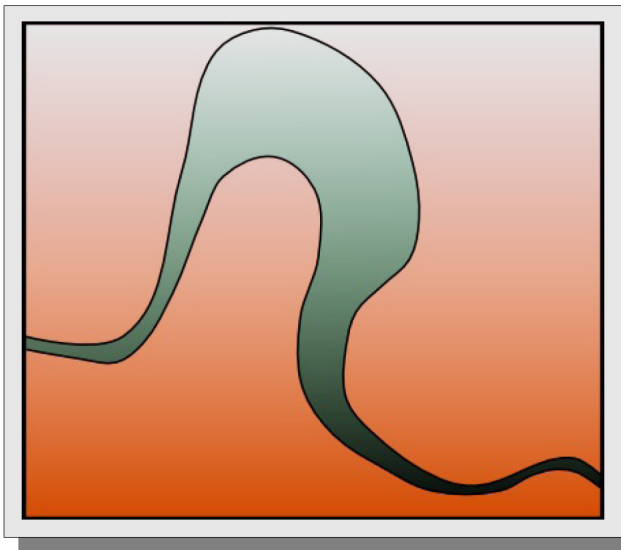
Cosmic ray transport – magnetic flux tube with CRs



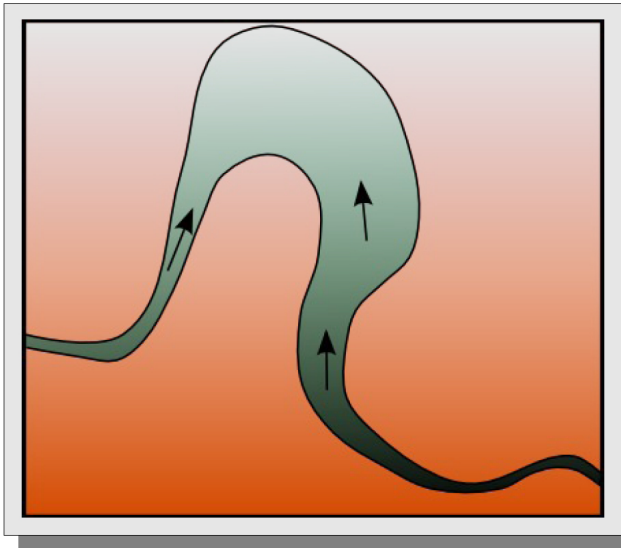
Cosmic ray advection



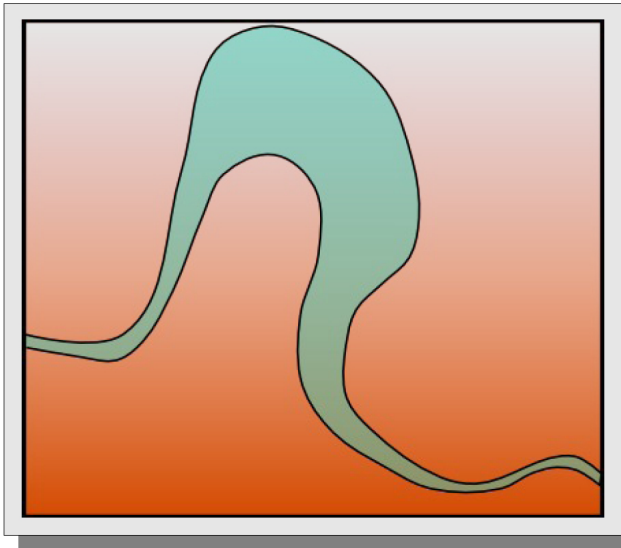
Adiabatic expansion and compression



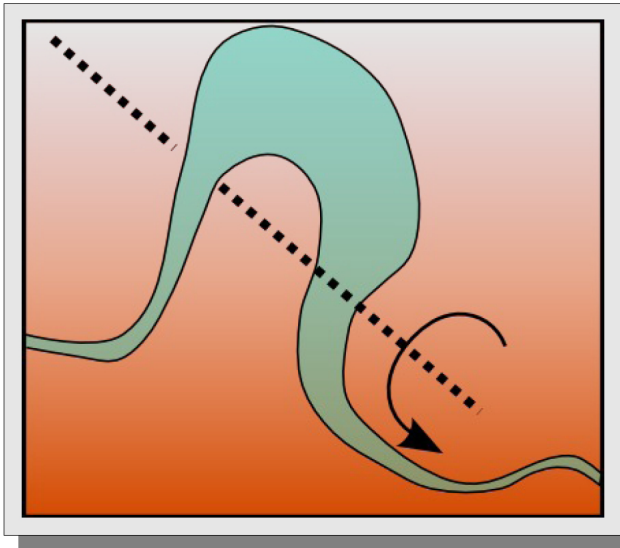
Cosmic ray streaming



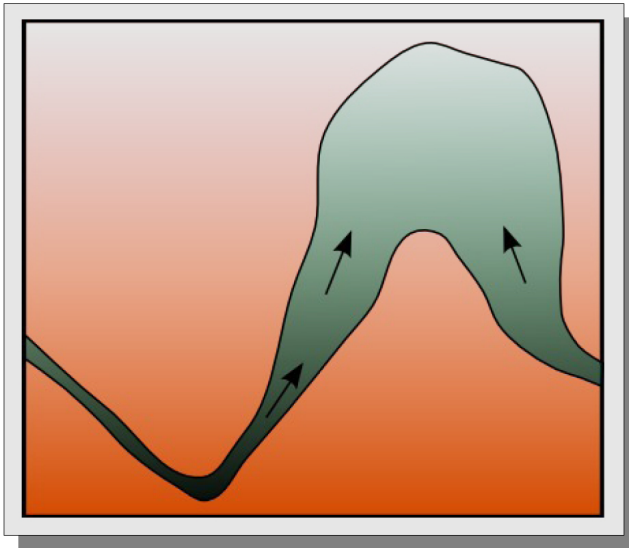
Expanded CRs



Turbulent pumping

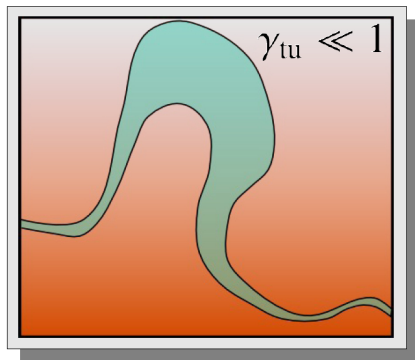
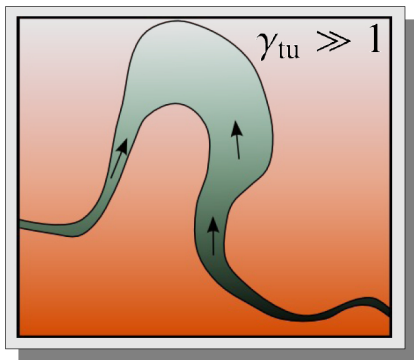


Turbulent pumping

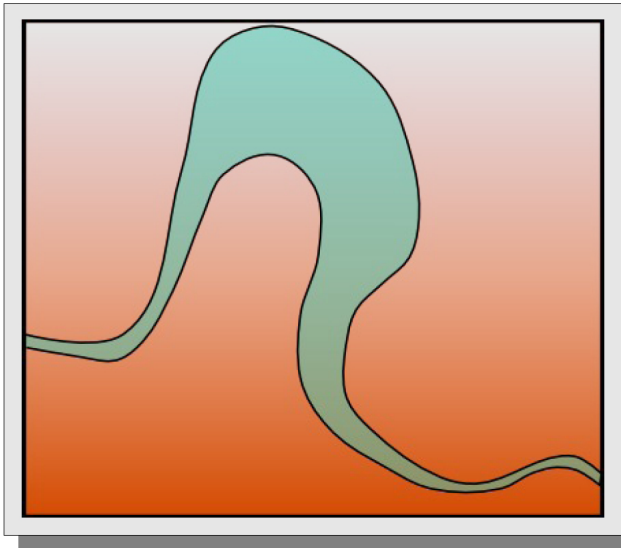


Turbulent-to-streaming ratio

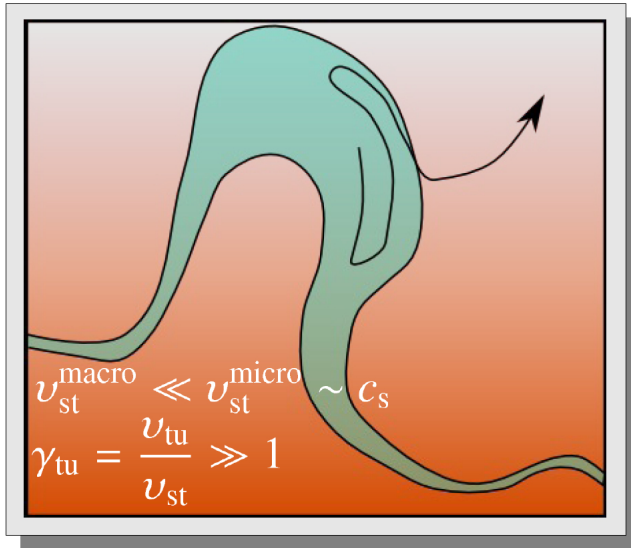
$$\gamma_{\text{tu}} = \frac{u_{\text{tu}}}{u_{\text{st}}}$$



Are CRs confined to magnetic flux tubes?



Escape via diffusion: energy dependence



CR transport theory

CR continuity equation in the absence of sources and sinks:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\mathbf{v} \rho) = 0$$

$$\mathbf{v} = \mathbf{v}_{\text{ad}} + \mathbf{v}_{\text{di}} + \mathbf{v}_{\text{st}}$$

$$\mathbf{v}_{\text{st}} = -v_{\text{st}} \frac{\vec{\nabla} \rho}{|\vec{\nabla} \rho|}$$

$$\mathbf{v}_{\text{di}} = -\kappa_{\text{di}} \frac{1}{\rho} \vec{\nabla} \rho$$

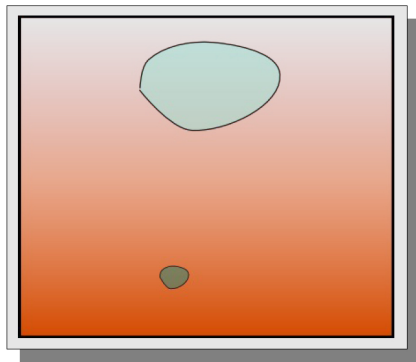
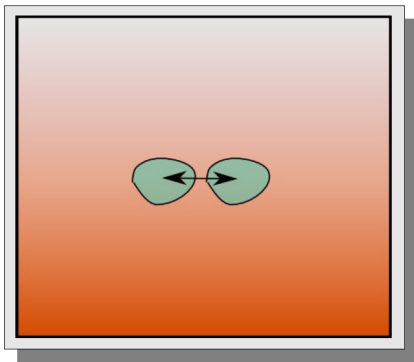
$$\mathbf{v}_{\text{ad}} = -\kappa_{\text{tu}} \frac{\eta}{\rho} \vec{\nabla} \frac{\rho}{\eta}$$

$$\kappa_{\text{tu}} = \frac{L_{\text{tu}} v_{\text{tu}}}{3}$$

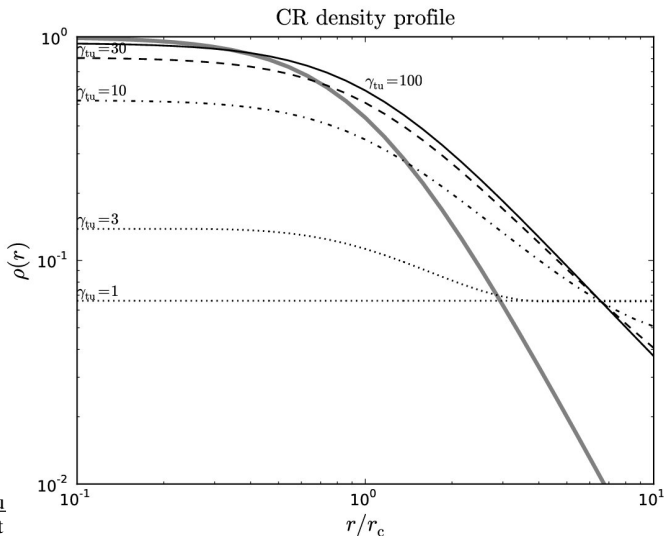


CR profile due to advection

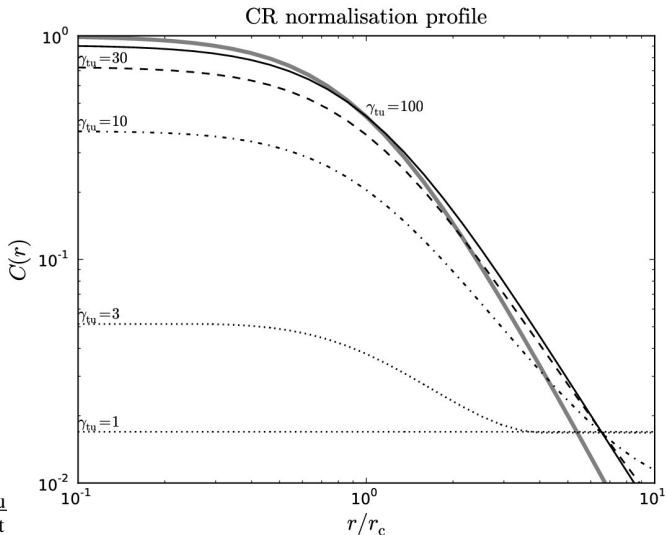
$$\eta(r) = \left(\frac{P(r)}{P_0} \right)^{\frac{3}{5}}$$



CR density profile

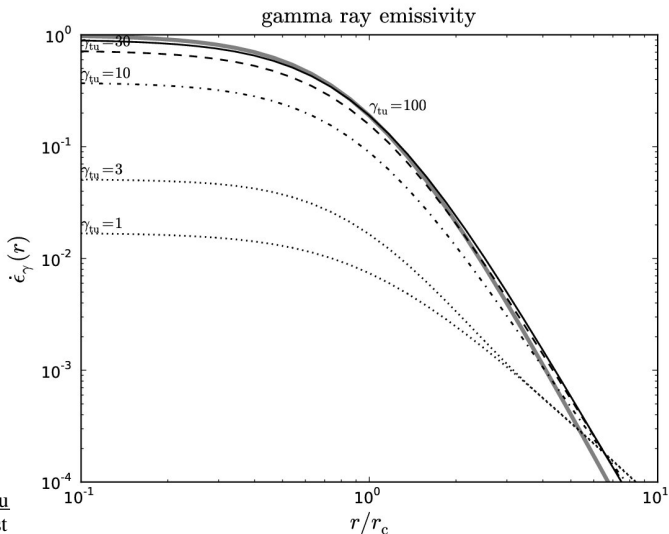


CR density at fixed particle energy



Gamma-ray emission profile

$$p_{\text{CR}} + p \rightarrow \pi^0 \rightarrow 2\gamma$$

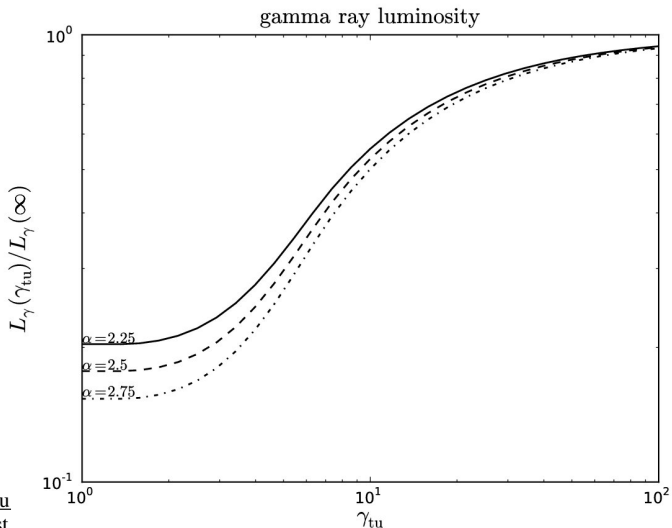


$$\gamma_{tu} = \frac{v_{tu}}{v_{st}}$$



Gamma-ray luminosity

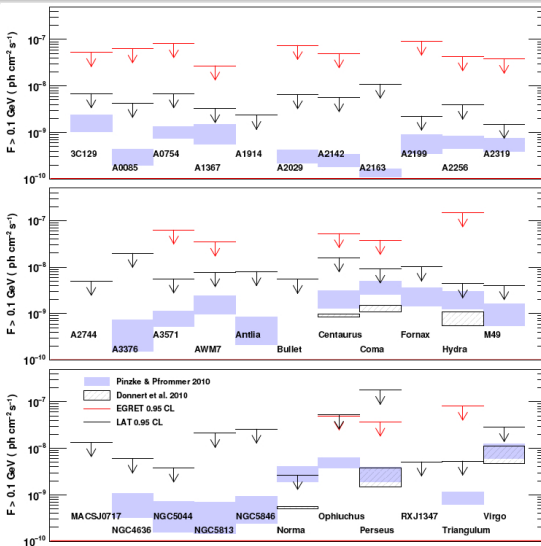
$$p_{\text{CR}} + p \rightarrow \pi^0 \rightarrow 2\gamma$$



$$\gamma_{\text{tu}} = \frac{v_{\text{tu}}}{v_{\text{st}}}$$

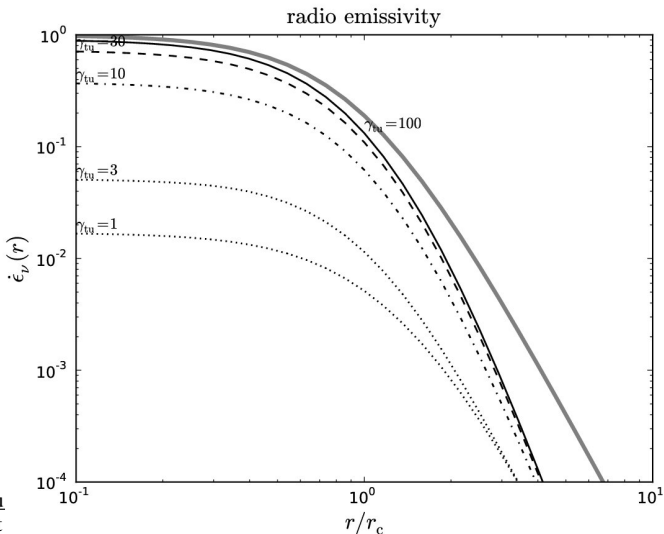


γ -ray limits and hadronic predictions (Ackermann et al. 2010)



Radio emission profile

$$p_{\text{CR}} + p \rightarrow \pi^{\pm} \rightarrow e^{\pm} \rightarrow \text{radio}$$

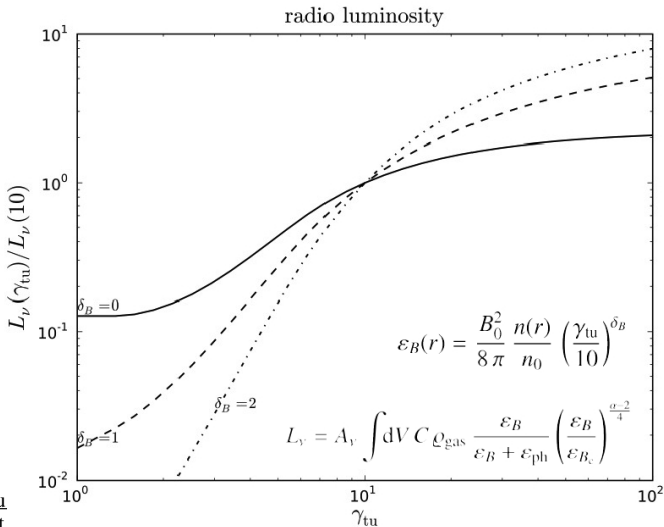


$$\gamma_{tu} = \frac{v_{tu}}{v_{st}}$$



Radio luminosity

$$p_{\text{CR}} + p \rightarrow \pi^{\pm} \rightarrow e^{\pm} \rightarrow \text{radio}$$



$$\gamma_{\text{tu}} = \frac{v_{\text{tu}}}{v_{\text{st}}}$$



Conclusions

- cosmological simulations predict universal CR spectrum and distribution (ignoring active CR transport)
 - Fermi limits consistent with simulations that use most optimistic assumptions of CR acceleration and transport
- streaming & diffusion produce spatially flat CR profiles
advection produces centrally enhanced CR profiles
 - profile depends on advection-to-streaming-velocity ratio
- turbulent velocity \sim sound speed ← cluster merger
CR streaming velocity \sim sound speed ← plasma physics
 - peaked/flat CR profiles in merging/relaxed clusters
- energy dependence of v_{st}^{macro} → CR & radio spectral variations
 - outstreaming CR: dying halo ← decaying turbulence

→ bimodality of cluster radio halos & gamma-ray emission!



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Literature for the talk

- Enßlin, Pfrommer, Miniati, Subramanian, 2011, A&A, 527, 99, *Cosmic ray transport in galaxy clusters: implications for radio halos, gamma-ray signatures, and cool core heating*
- Pinzke & Pfrommer, 2010, MNRAS, 409, 449, *Simulating the gamma-ray emission from galaxy clusters: a universal cosmic ray spectrum and spatial distribution*
- Pfrommer, 2008, MNRAS, 385, 1242, *Simulating cosmic rays in clusters of galaxies – III. Non-thermal scaling relations and comparison to observations*
- Pfrommer, Enßlin, Springel, 2008, MNRAS, 385, 1211, *Simulating cosmic rays in clusters of galaxies – II. A unified scheme for radio halos and relics with predictions of the γ -ray emission*
- Pfrommer, Enßlin, Springel, Jubelgas, Dolag, 2007, MNRAS, 378, 385, *Simulating cosmic rays in clusters of galaxies – I. Effects on the Sunyaev-Zel'dovich effect and the X-ray emission*

