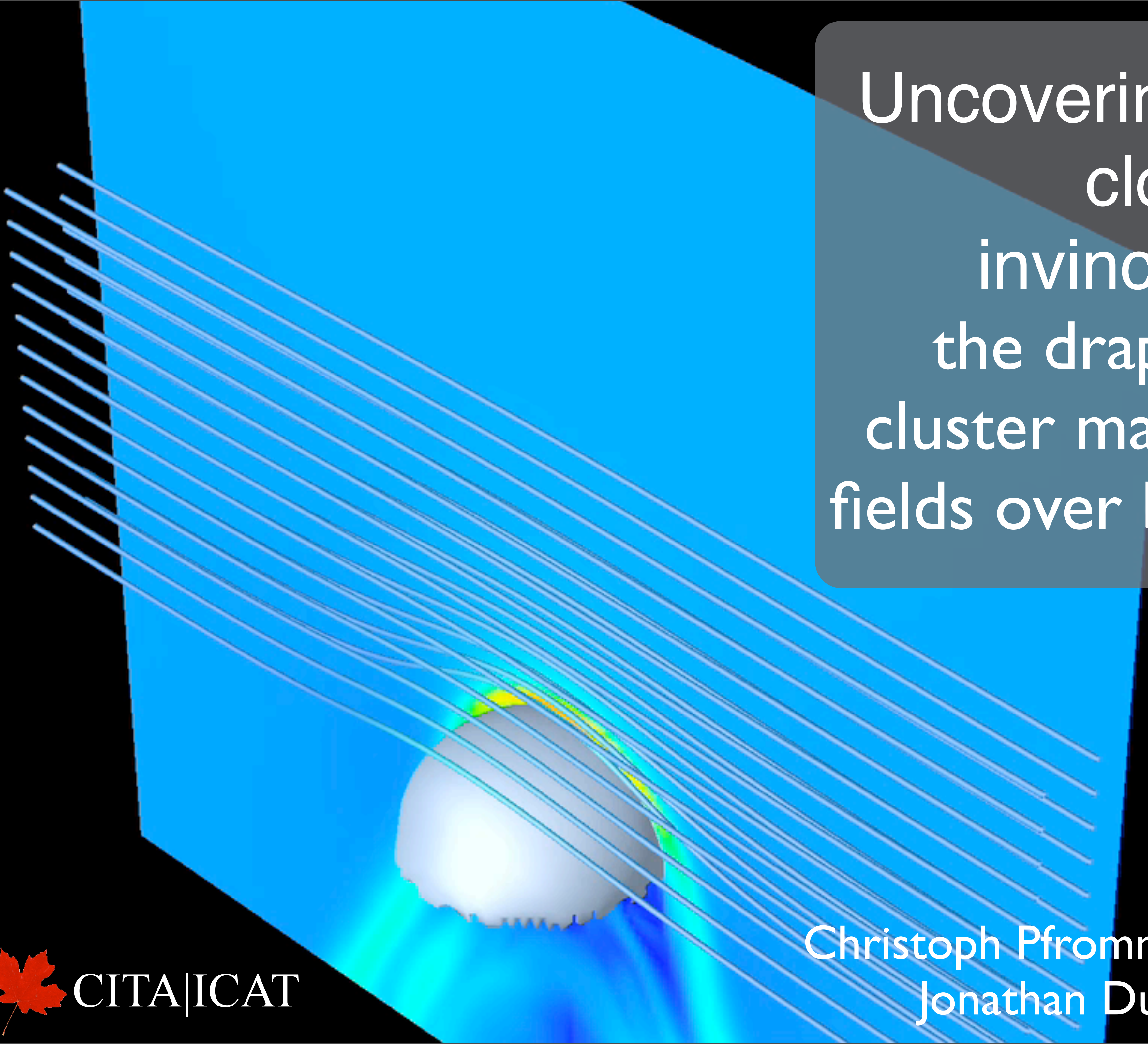
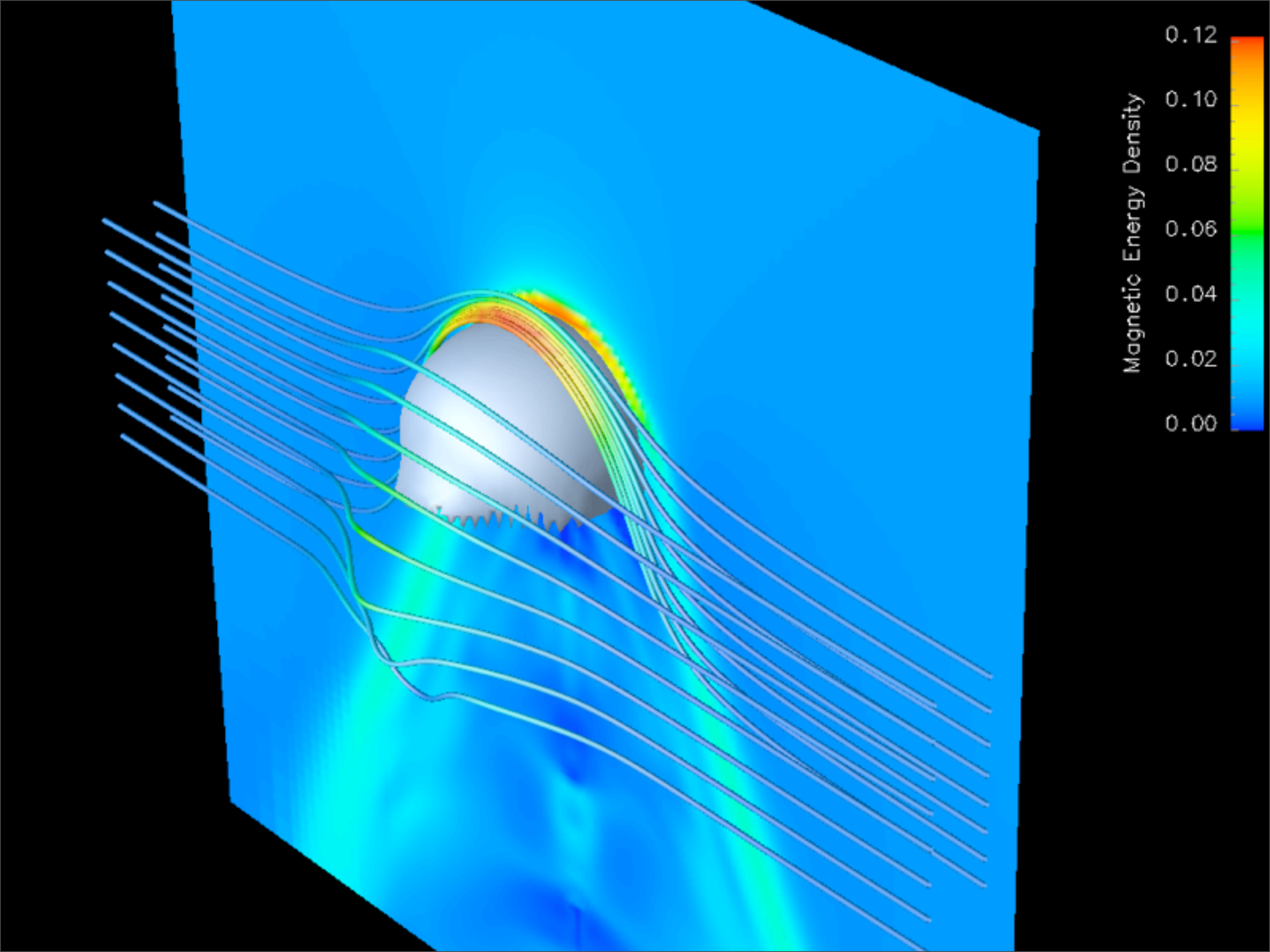
Uncovering the cloak of invincibility: the draping of cluster magnetic fields over bullets



Christoph Pfrommer, CITA
Jonathan Dursi, CITA

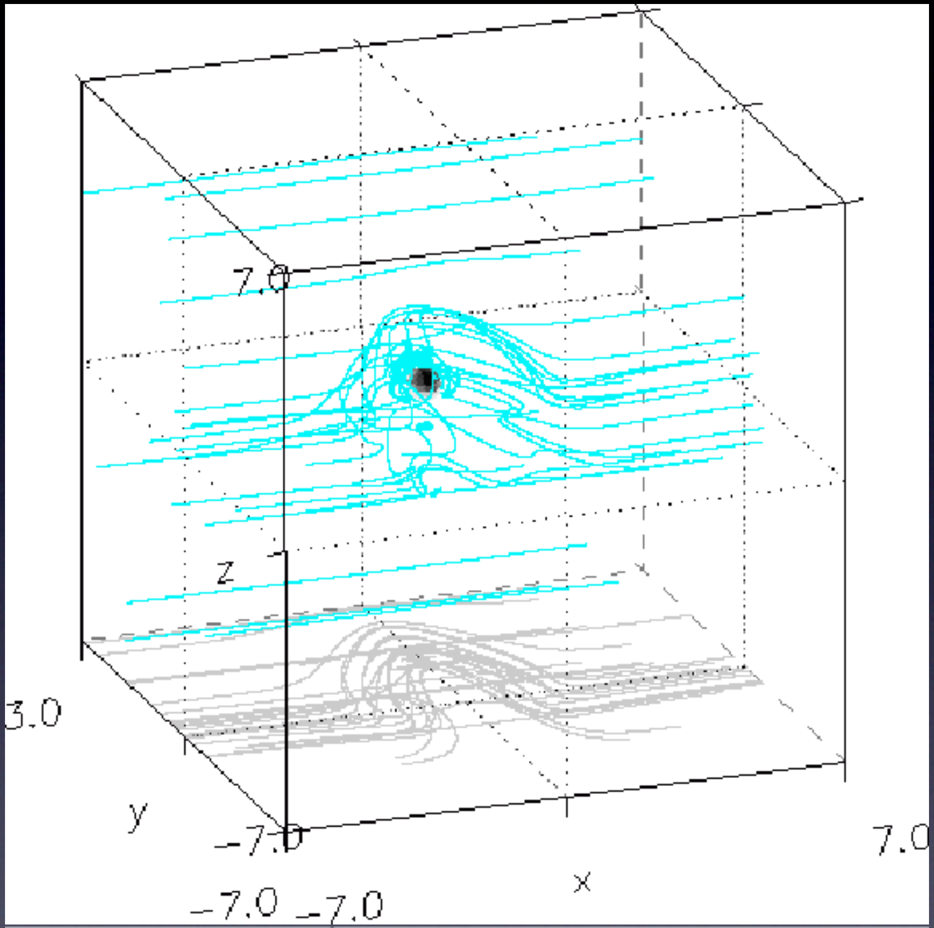
CITA|ICAT



Magnetic Energy Density
0.12
0.10
0.08
0.06
0.04
0.02
0.00

Draping of solar wind field around Earth

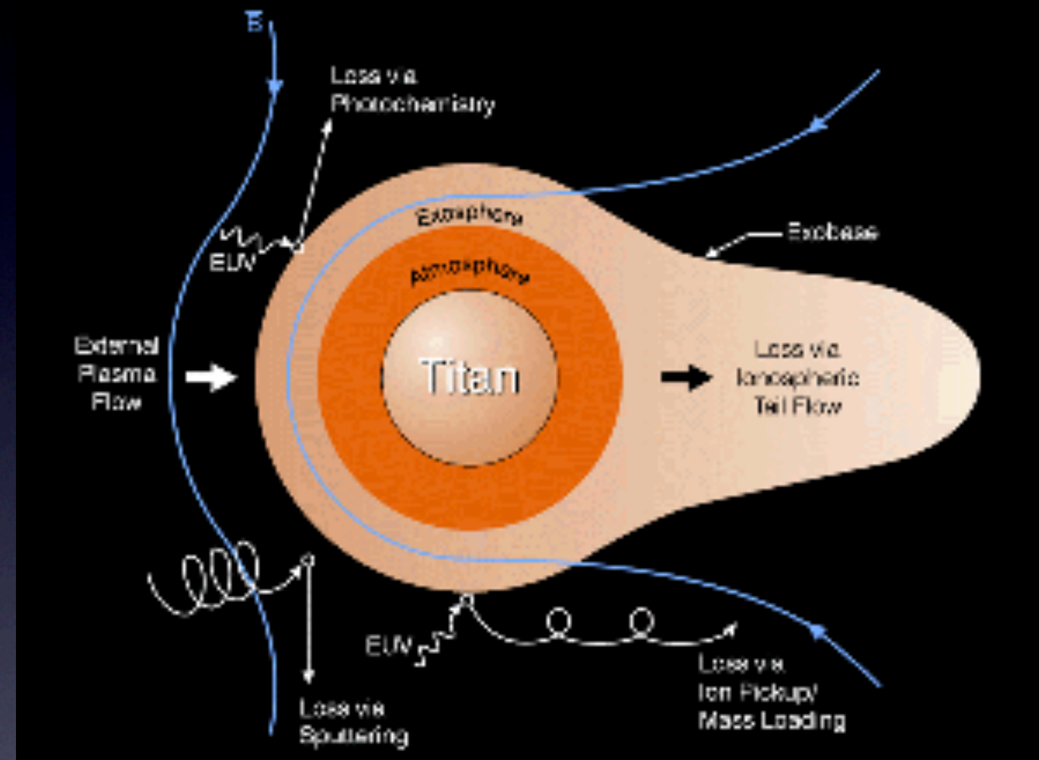
- Happens very quickly (figure to right -- 600 s)
- Can induce magnetic field in even a neutral atmosphere
- Earth Magnetic Field reversals may not be catastrophic to life



Birk et al (2004)

Draping of Saturn's field over Titan

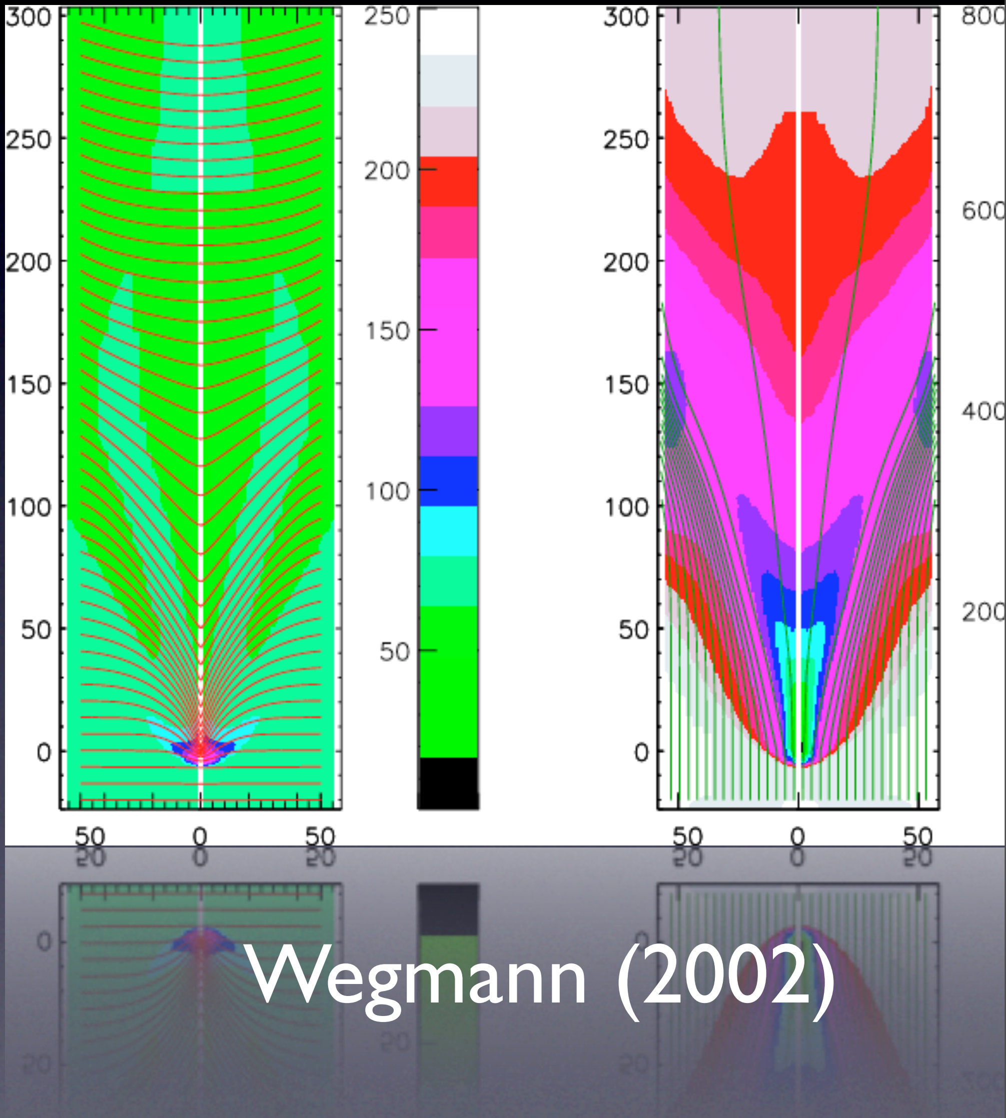
- Observable with Cassini
- Emission from draped field



S.A. Ledvina, UC Berkeley

Comets in solar wind

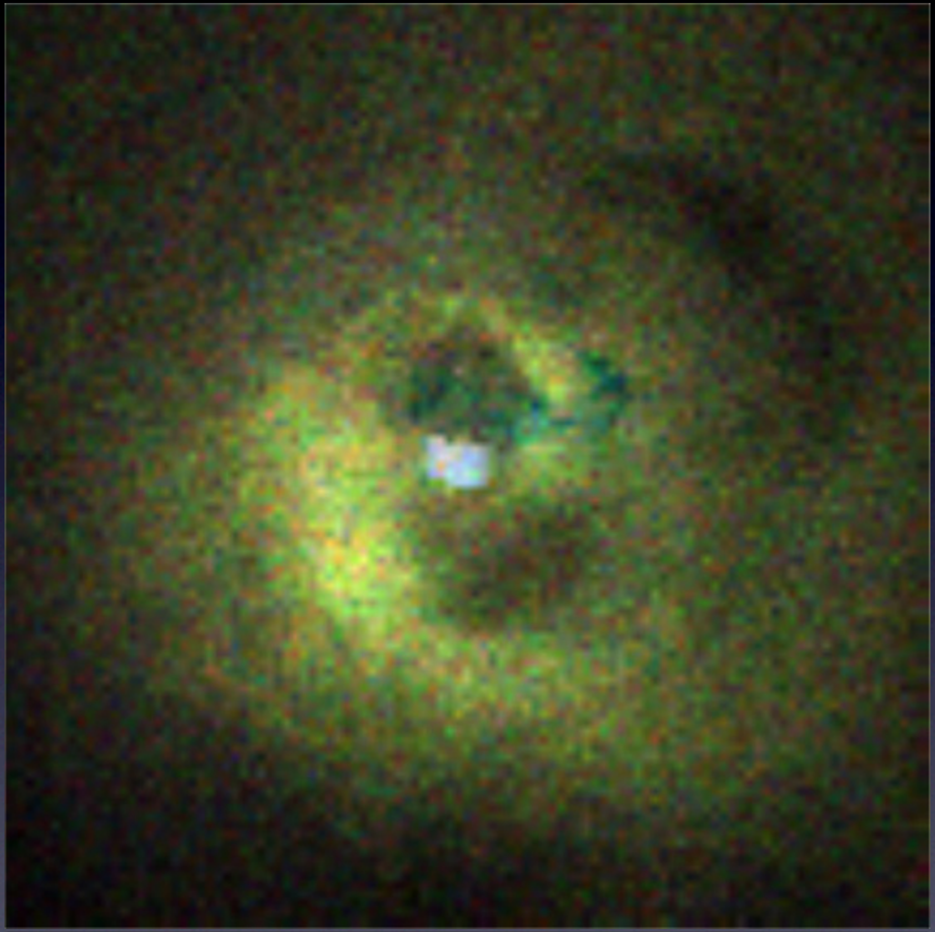
- Draping occurs and can distort velocity, magnetic fields in wind over significant distances



Wegmann (2002)

Applications to galaxy clusters

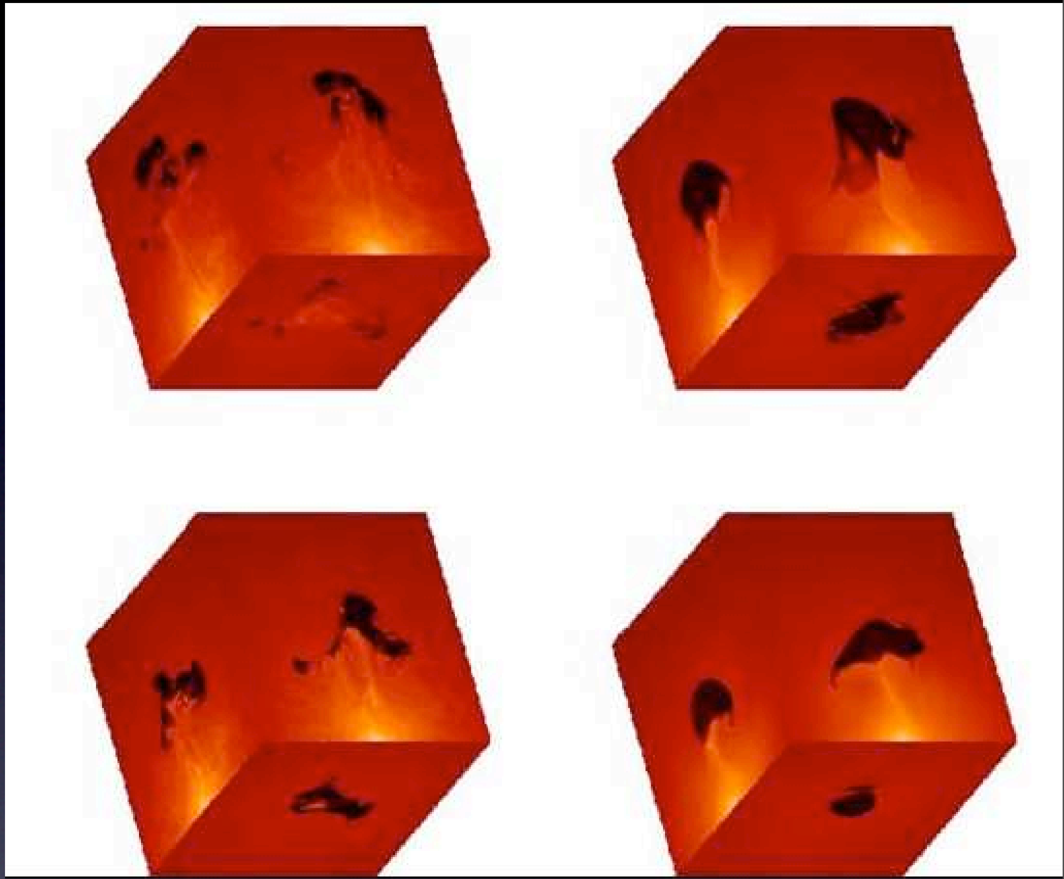
- Radio Bubbles, seen as voids in X-rays, are observed out to large distances and have very sharp interfaces
- Hydrodynamic instabilities should disrupt them, conduction should dissipate the interfaces in $\sim 10^8$ yrs
- Could bubble motions sweep up enough field to suppress instabilities and conduction?



NASA/IoA/A.Fabian et al.

Magnetic draping at work

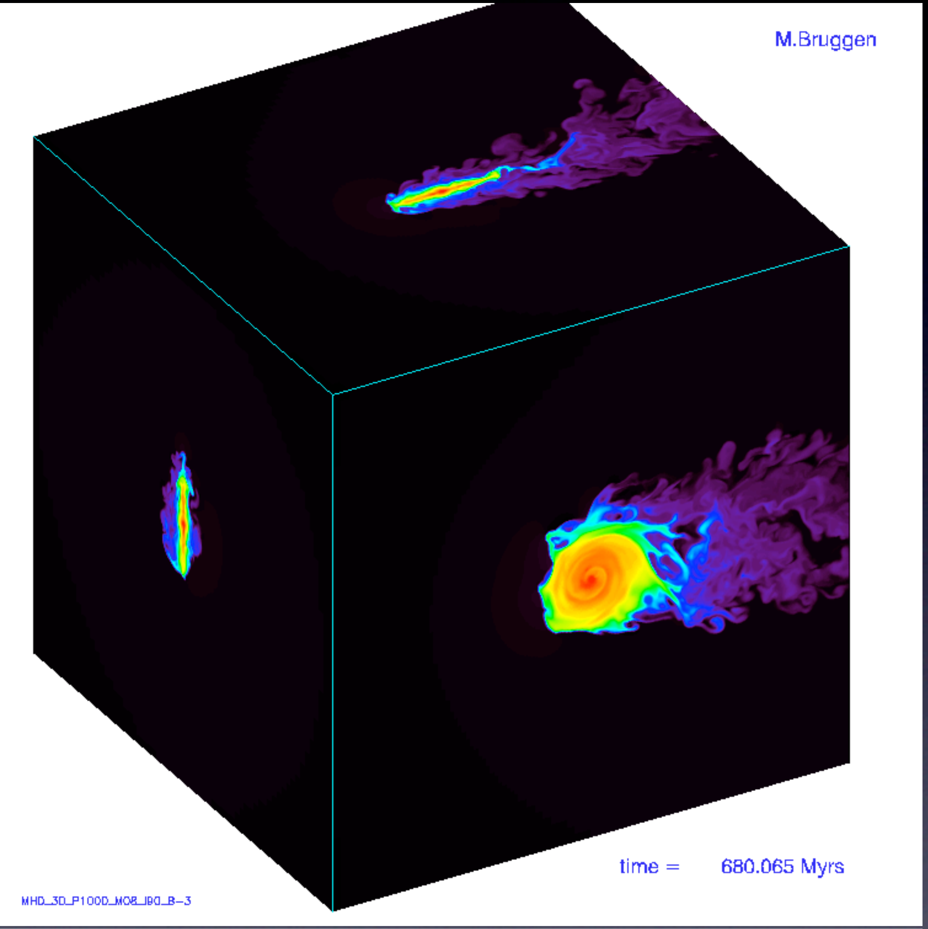
- Rising radio bubbles in a hot atmosphere
- shown is the log of the density for the non-draping versus draping case
- hydrodynamical instabilities are suppressed



Ruszkowski et al. 2007

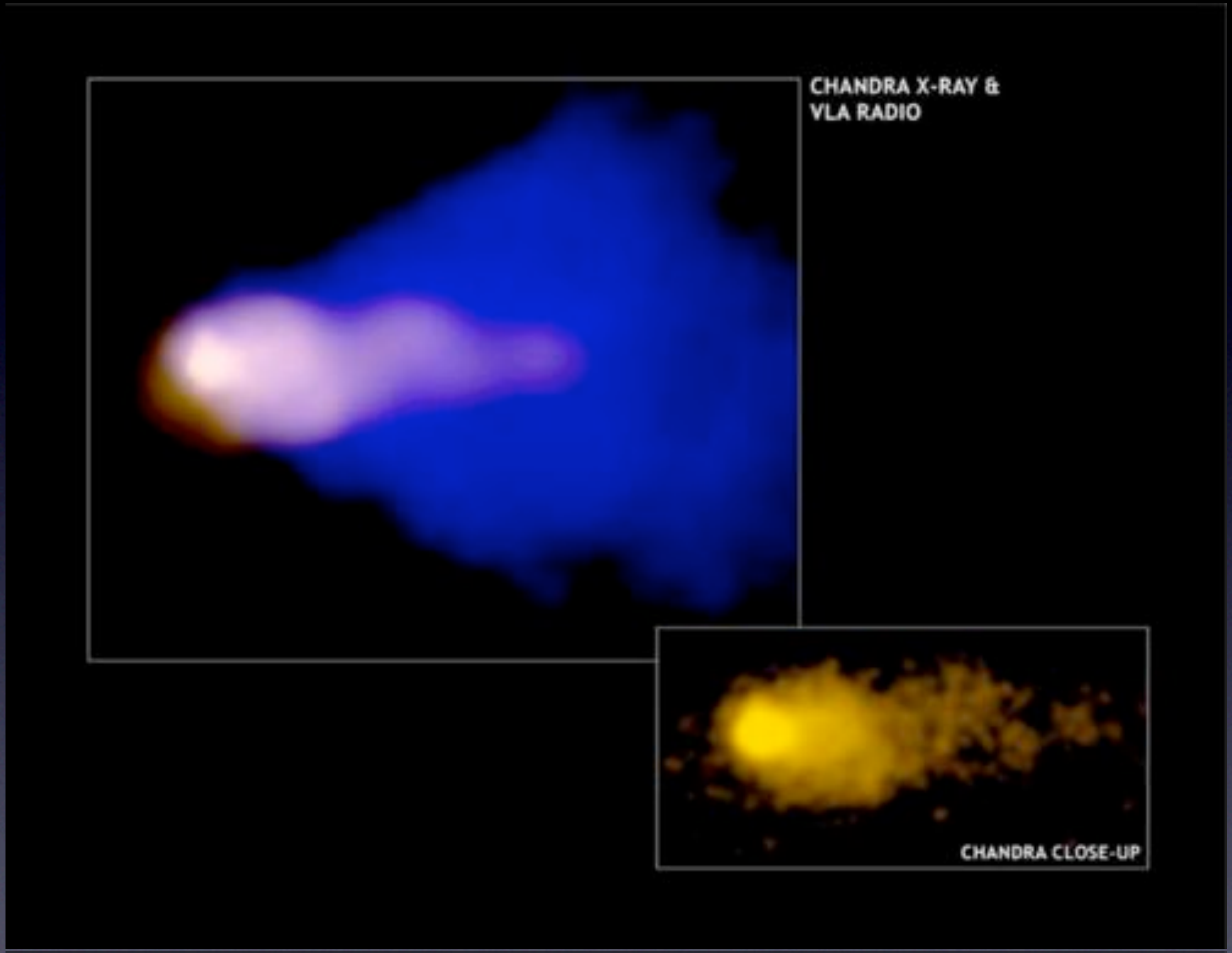
Mergers of galaxy clusters

- Minor mergers involve smaller cluster falling into more massive ones
- Stripping of small-mass ICM
- When/where does this occur?
- Consequences for enrichment, cold fronts, ...



M. Bruggen, Bremen

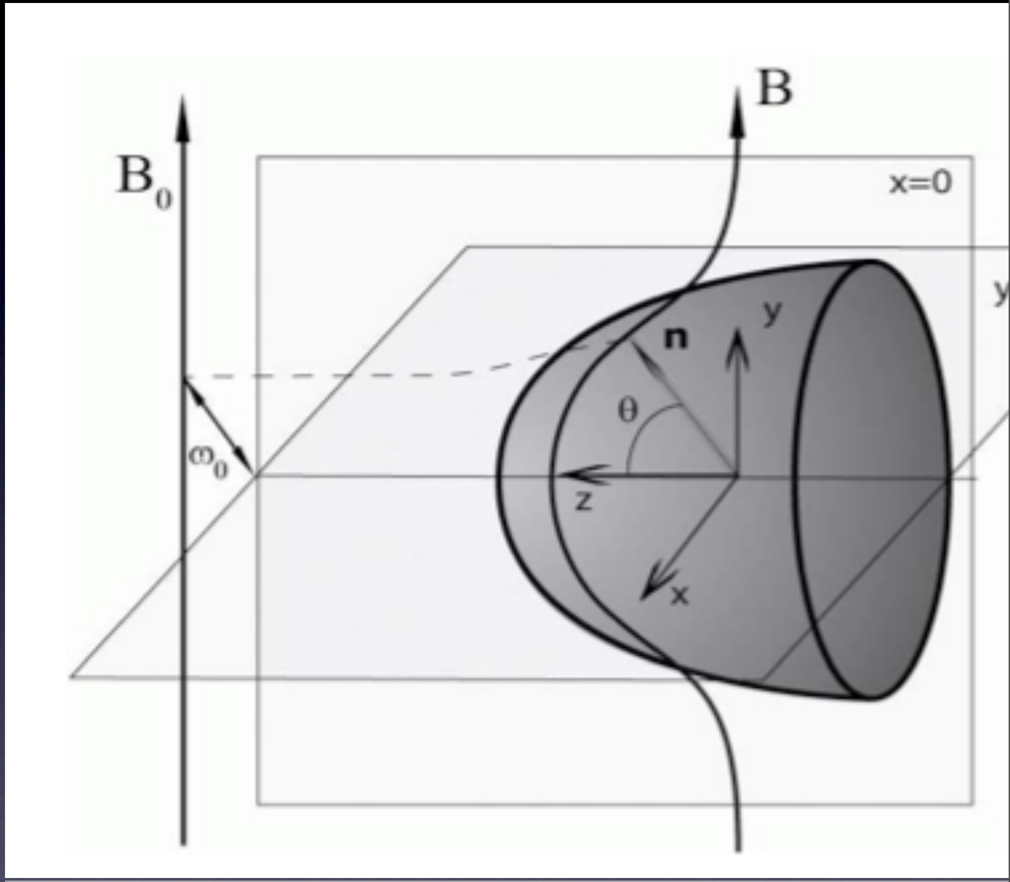
Pulsar wind nebulae?



Previous work: Lyutikov 2004

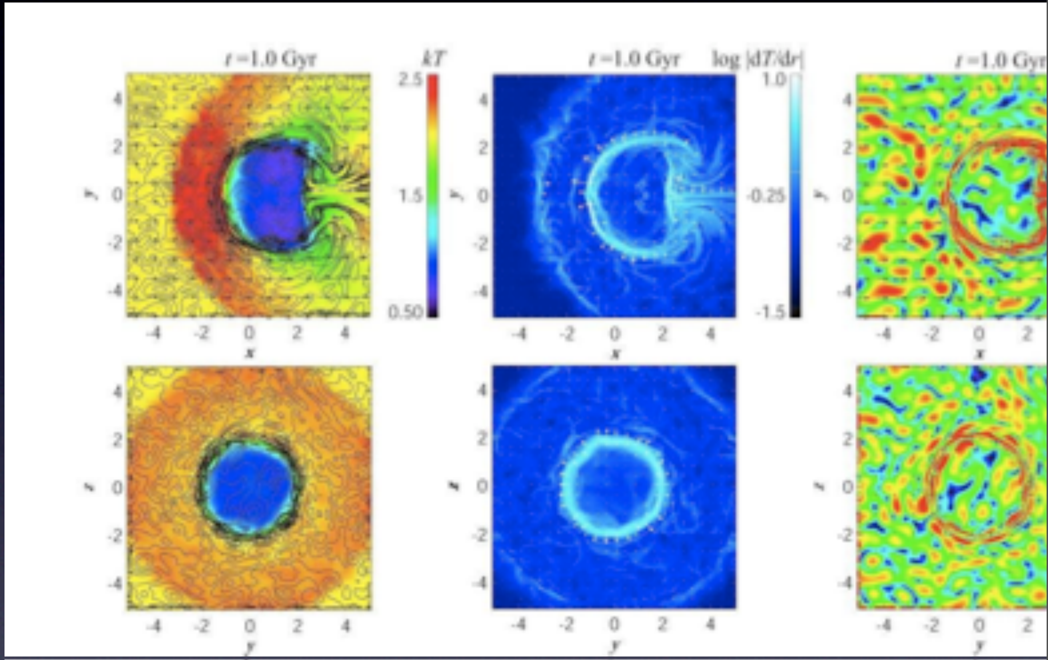
- Analytics
- Particularly along stagnation line

$$\frac{B}{\rho} = \frac{1}{\sqrt{1 - \frac{R_0^3}{r^3}}} \left(\frac{B}{\rho}\right)_0 ; \qquad l \approx \frac{1}{\mathcal{M}_A^2} R_0$$



Asai et al (2004,5,6..)

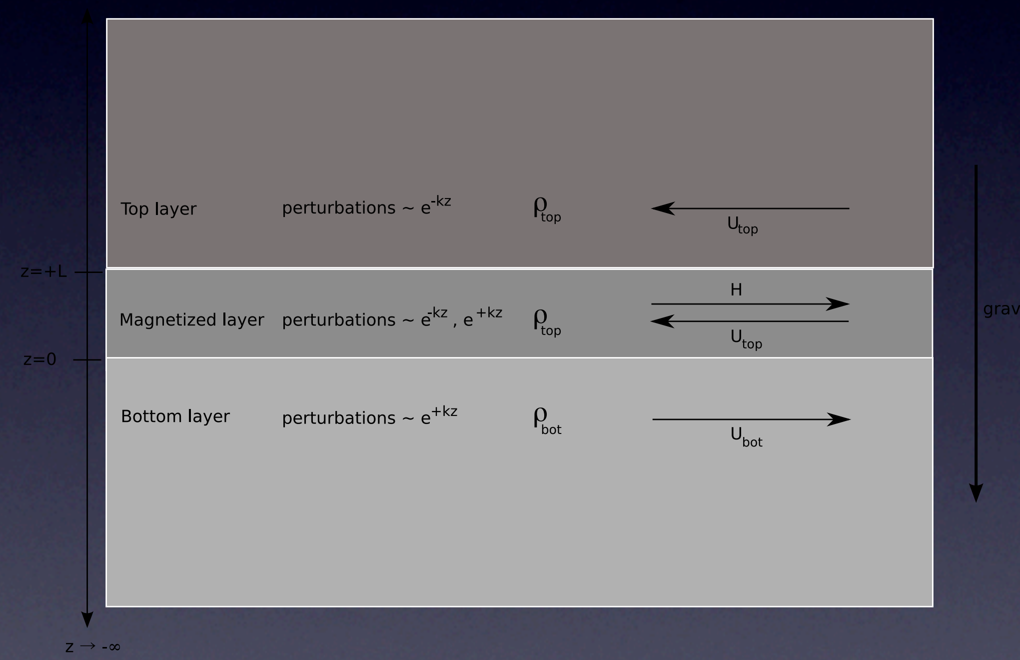
- Numerics
- 2d, 3d
- `Kitchen sink' - turbulent magnetic field, conduction,...
- Can draping effect conduction? Yes



Asai et al (2004...)

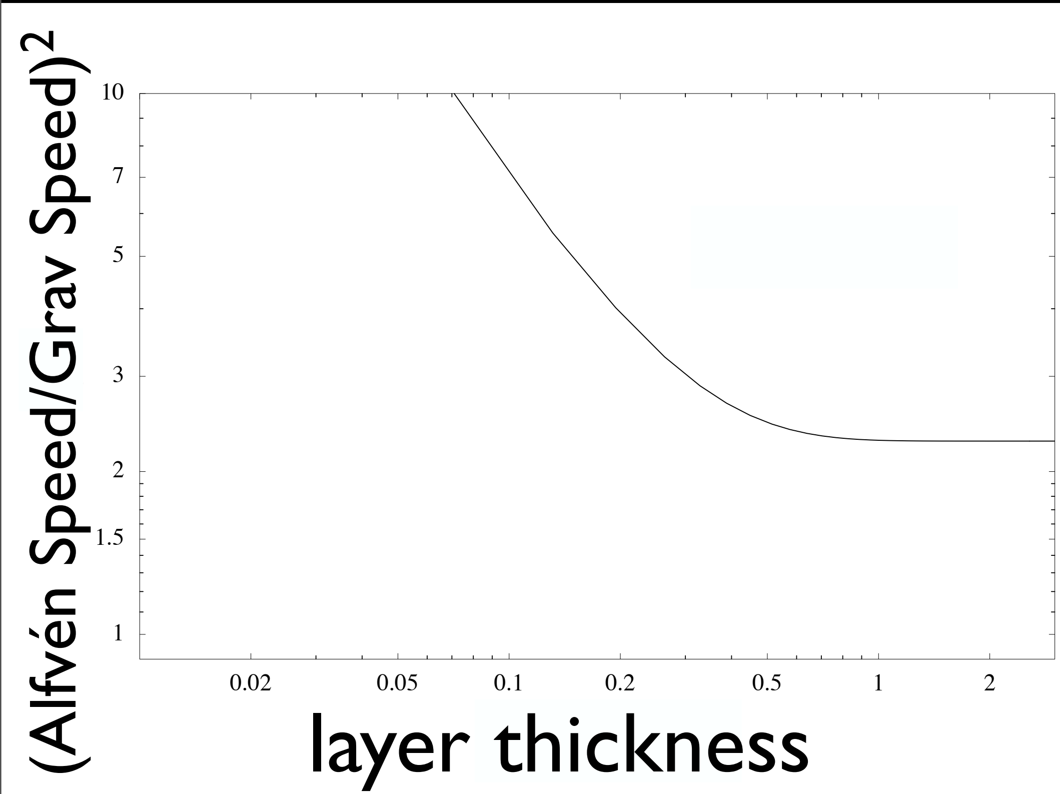
Linear theory analysis

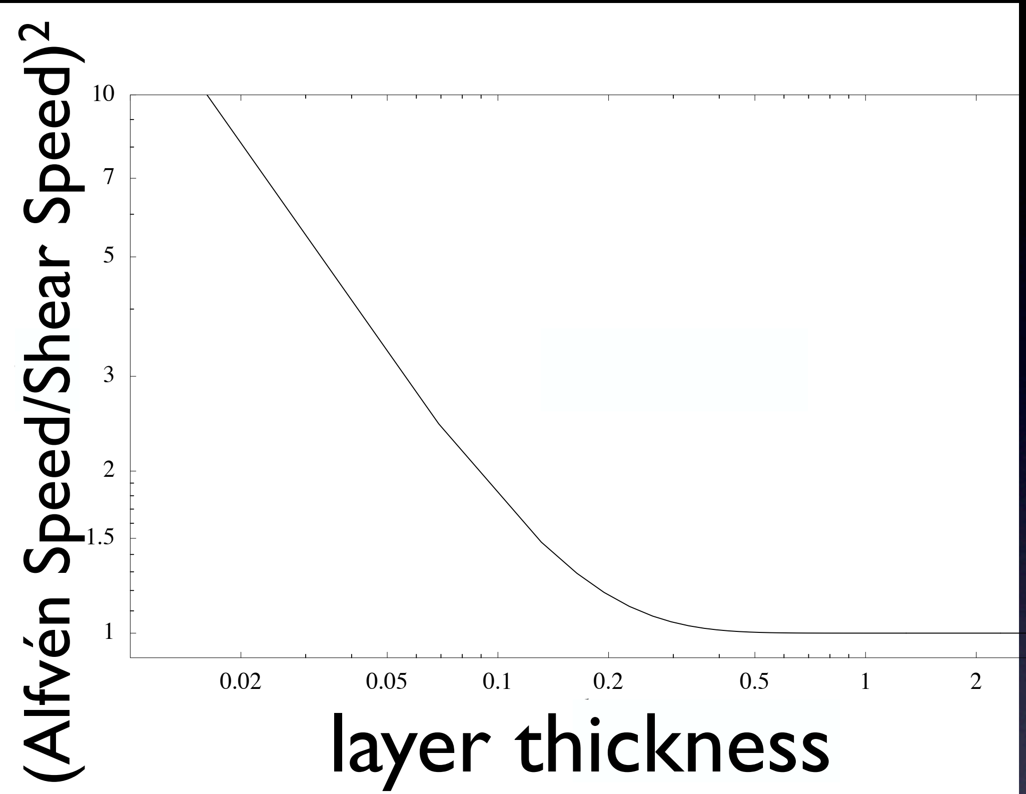
- Can such a thin layer have interesting dynamic effects?
- Three layers; velocity +/- U, magnetized layer of some thickness/ strength



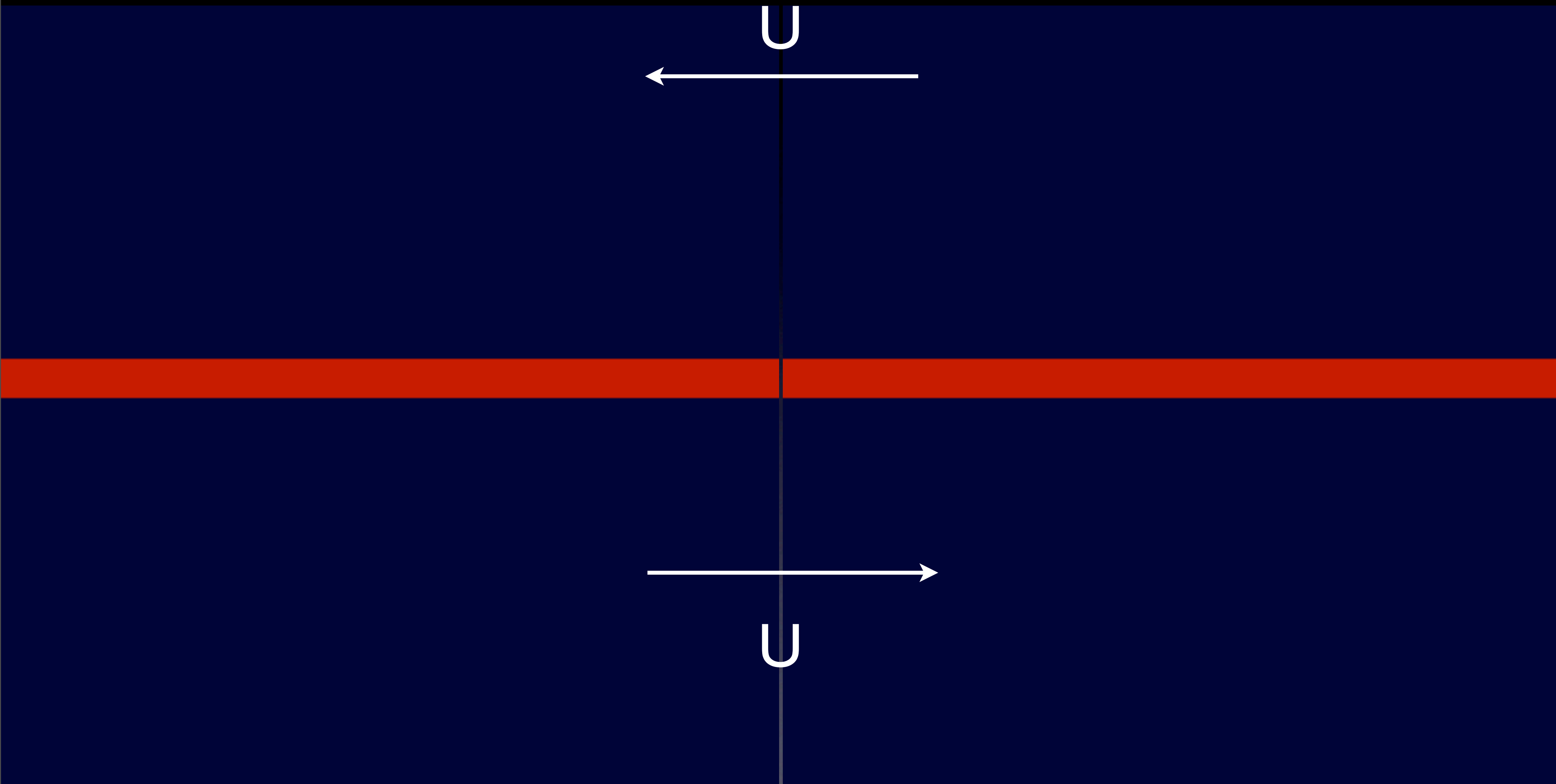
Dursi 2007

Rayleigh-Taylor | Kelvin-Helmholtz





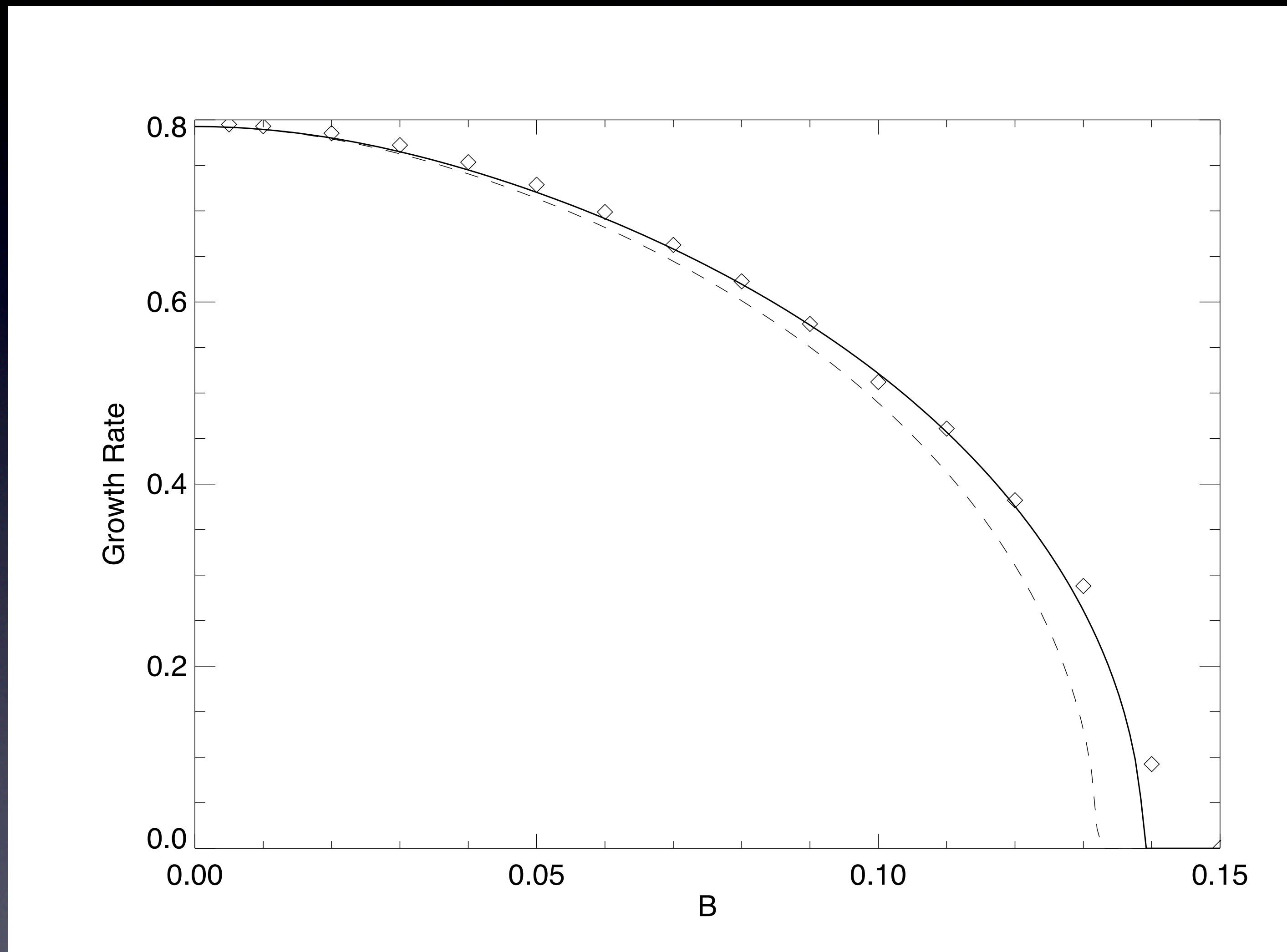
If V_A is a few times relevant velocity, can stabilize against wavelengths an order of magnitude longer than thickness of layer



$V_A = 0.2\ U$

$V_A = 1.25\ U$

Excellent agreement between theory and simulation!

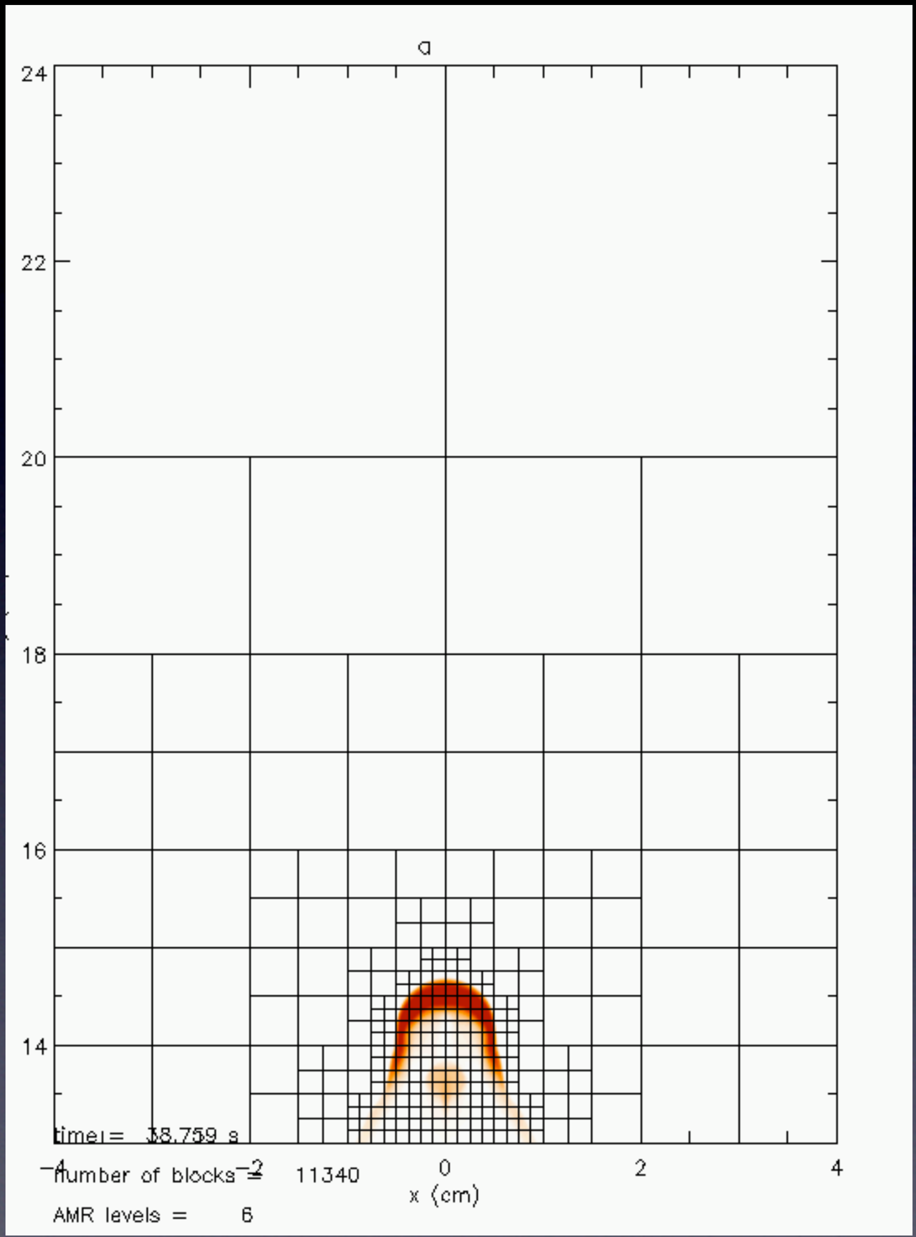


Our Contributions

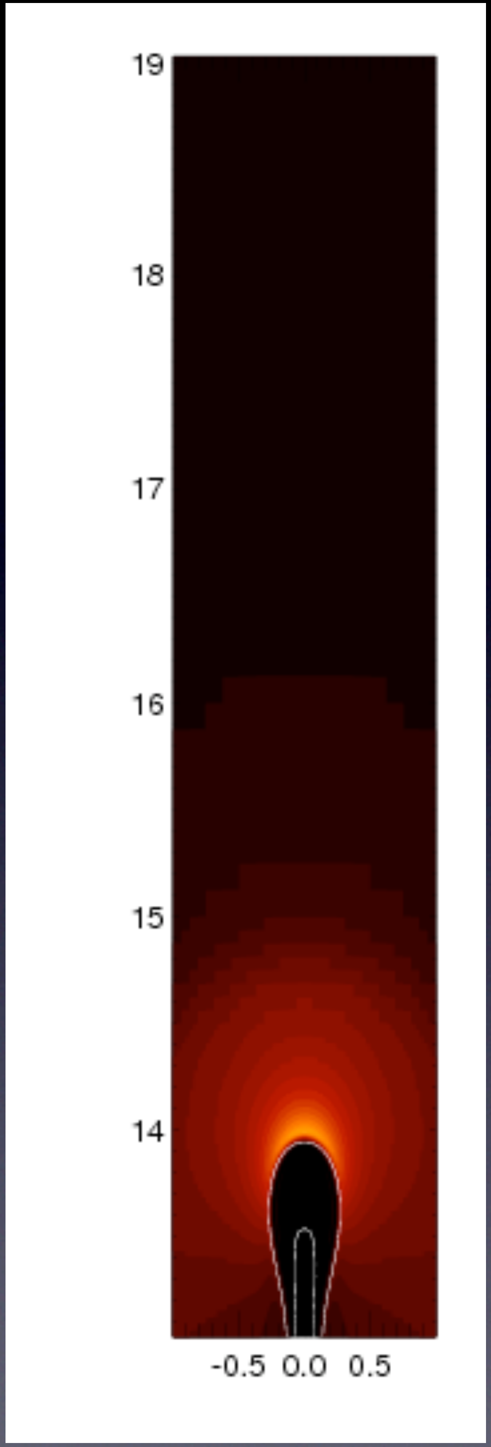
- 3D, AMR numerical experiments of draping of uniform magnetic fields in context relevant to galaxy clusters
- More careful analytic calculation in potential flow approximation to understand dynamics
- Analytic understanding of field strength in the draping layer, opening angle, deceleration due to magnetic tension, vorticity generation, instabilities in the perpendicular plane

3D simulations using FLASH

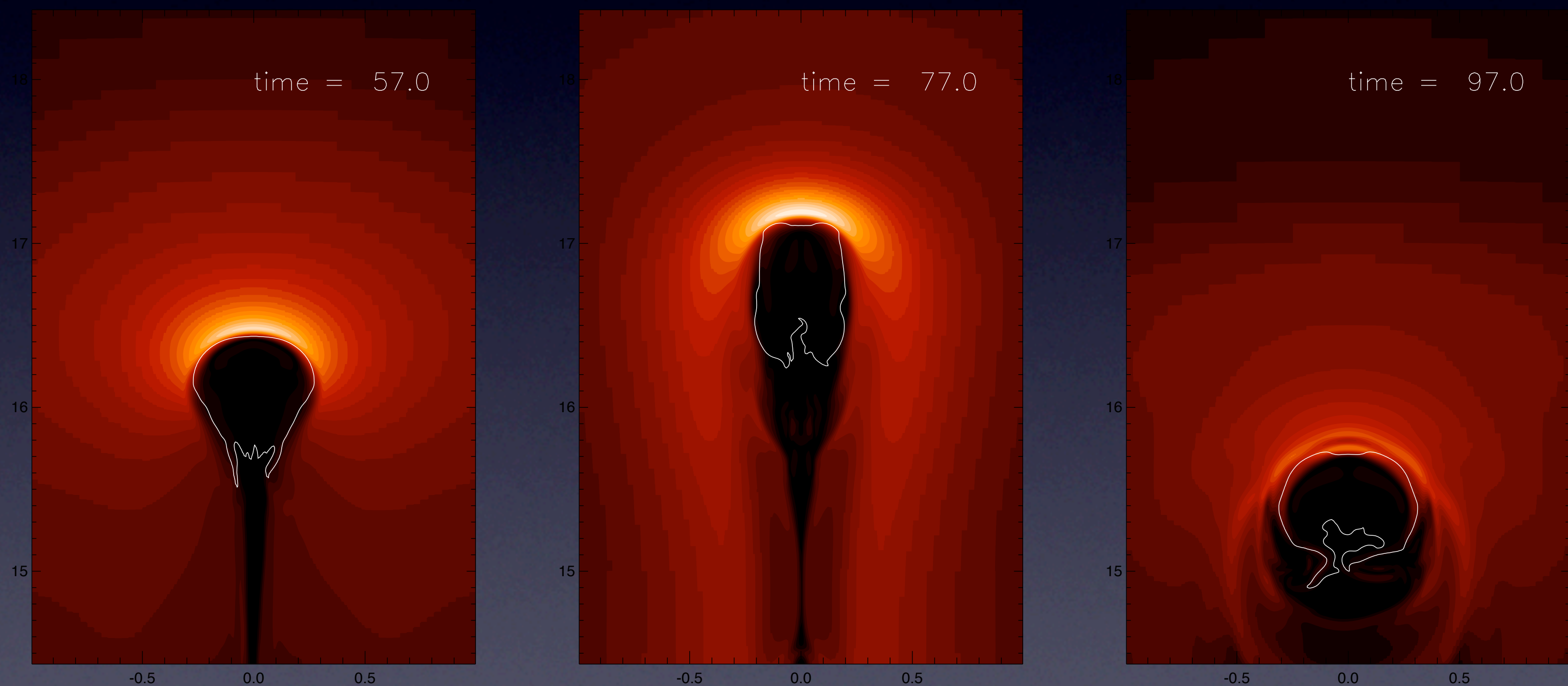
- AMR very useful for focusing resolution in near draped layer
- Large dynamic range between size of traversed region and thickness of layer
- Magnetic dynamics relatively straightforward



Sometimes,
2D just
isn't enough...



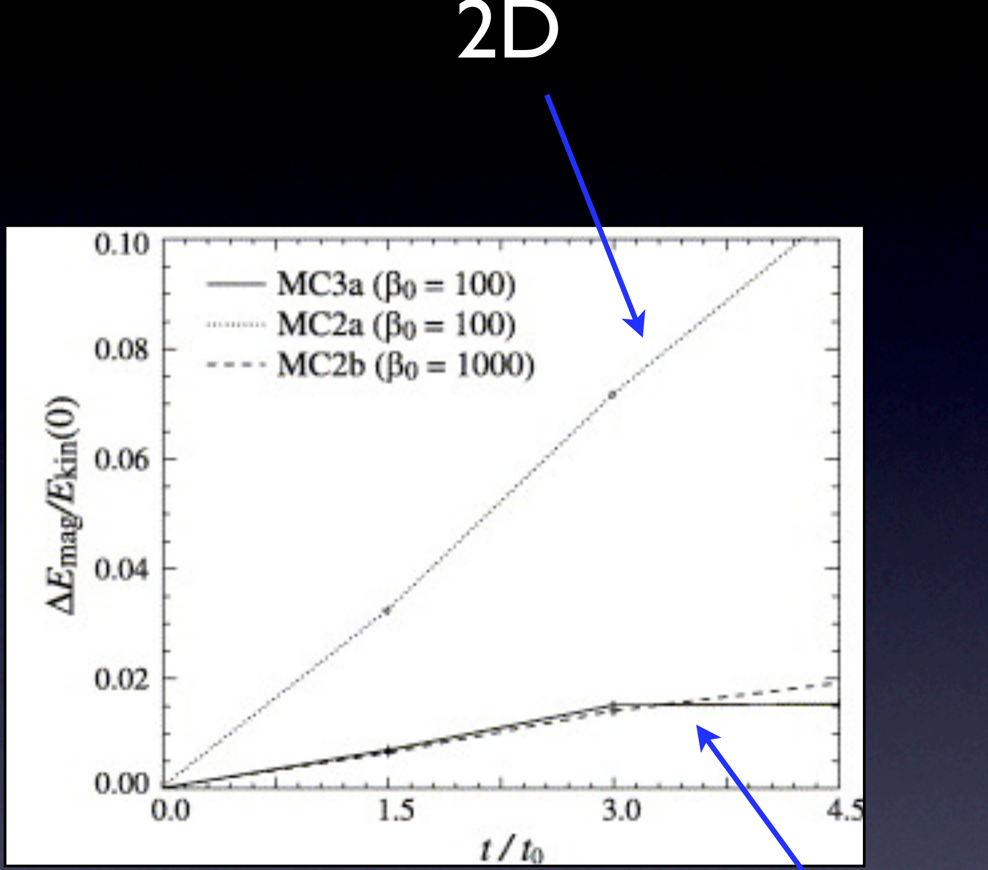
Magnetic energy density in 2D



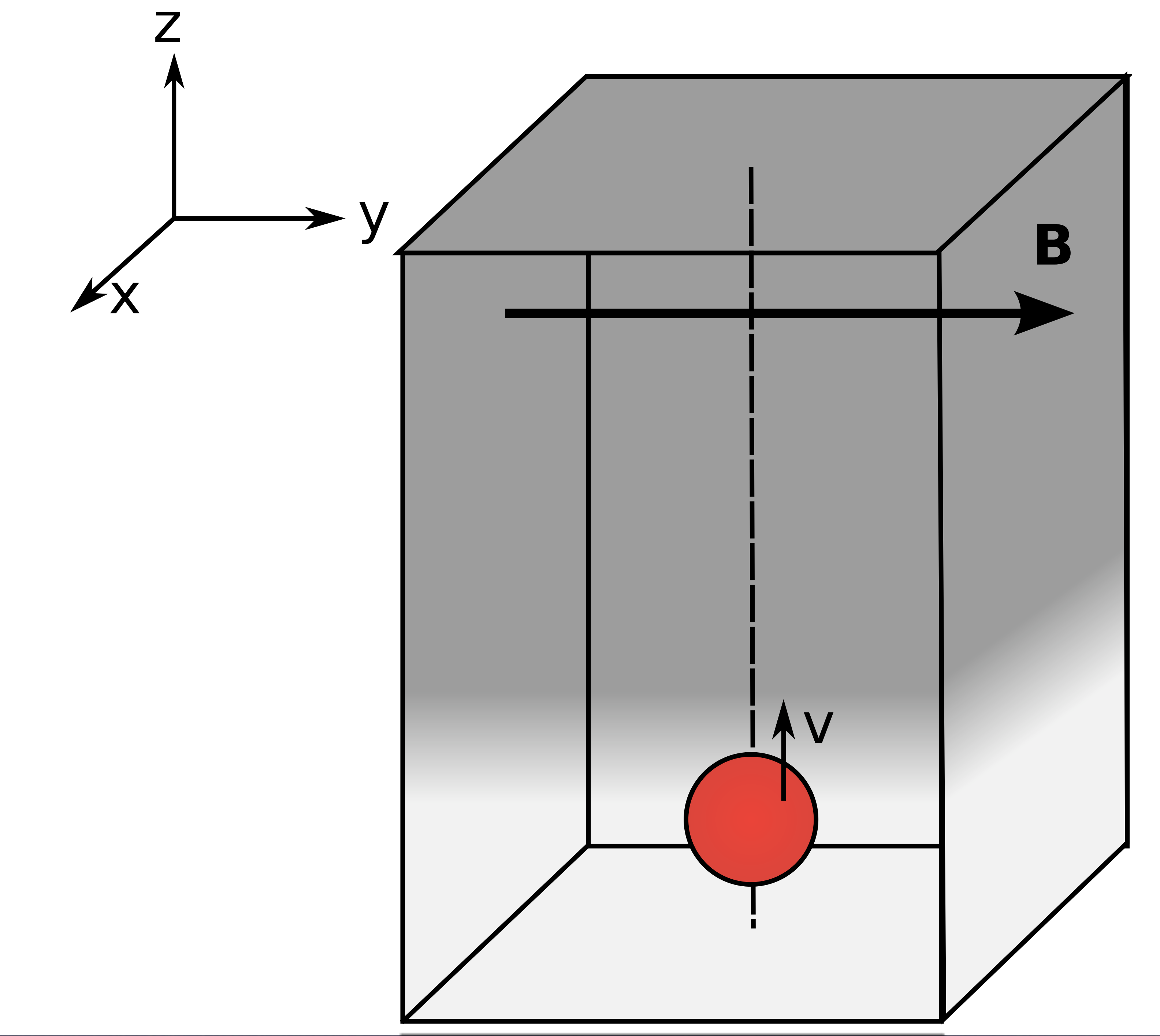
Not only slingshots back the bullet, but squishes it, too...

Foreshadowed earlier

- Asai et al 2004, 2005 saw strong growth of magnetic field in 2d, but only commented on it
- Simulation was not run long enough to see that there is no steady state



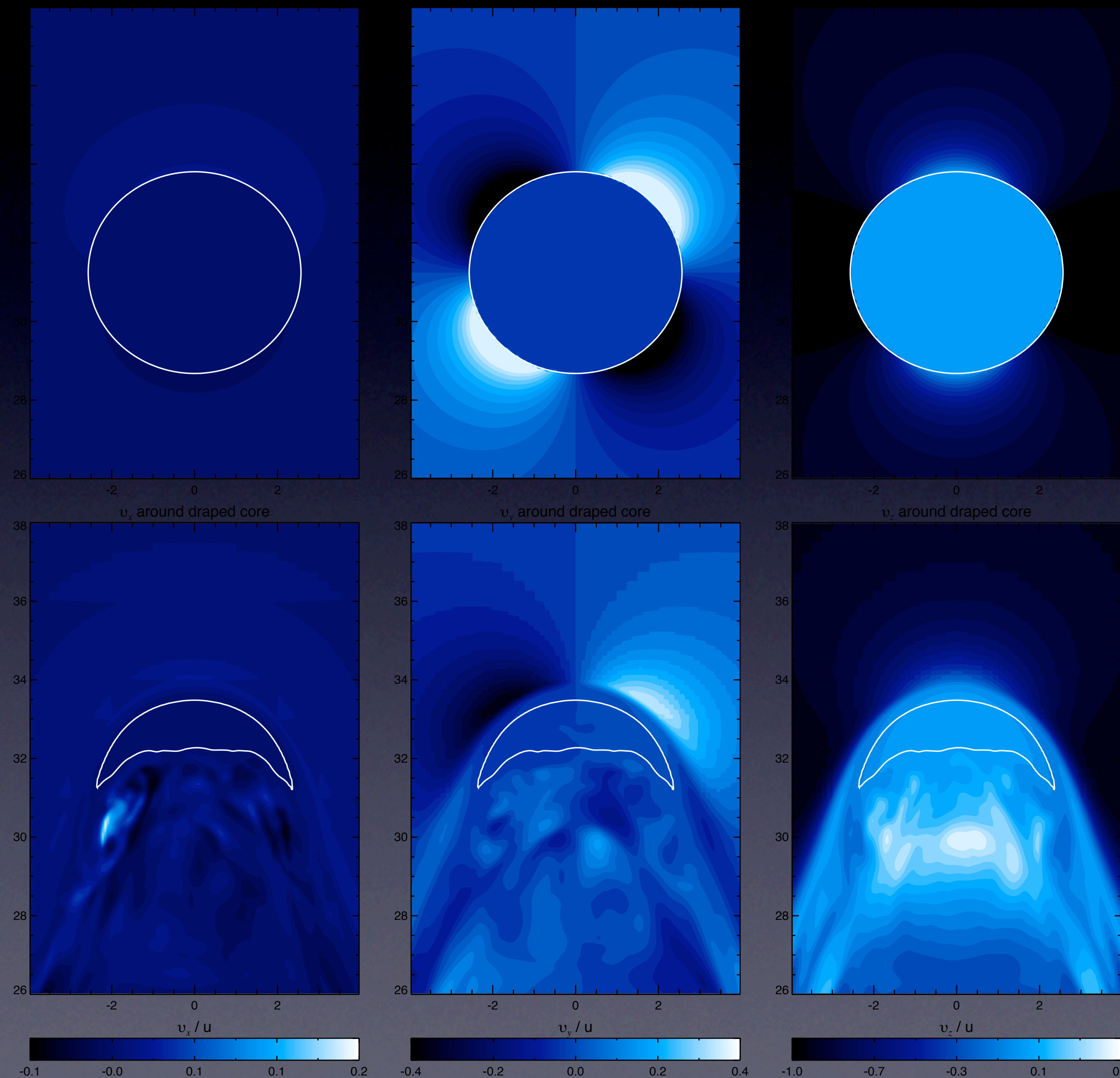
Asai et al (2005)


z
y
x
B
v

$$\mathbf{v} = \mathbf{e}_r \left(\frac{R^3}{r^3} - 1 \right) u \cos\theta + \mathbf{e}_\theta \left(\frac{R^3}{2r^3} + 1 \right) u \sin\theta$$

Potential flow around solid sphere

3d AMR results



Exact MHD solution: kinetic approximation

$$\mathrm{curl}(\mathbf{v} \times \mathbf{B}) = \mathbf{0} \qquad \mathrm{div}\,\mathbf{B} = 0$$

- given our potential flow solution it, looks straightforward to solve the usual Maxwell's equations for the B-field...

- but sometimes things only look simple

Exact MHD solution: kinetic approximation

$$\mathrm{curl}(\mathbf{v} \times \mathbf{B}) = \mathbf{0} \qquad \mathrm{div}\,\mathbf{B} = 0$$

$$B_\phi = \frac{B_0 \cos\phi}{\sqrt{1 - \frac{R^3}{r^3}}}$$

$$B_r = \frac{r^3 - R^3}{r^3} \cos\theta \left[C_1 \mp B_0 \sin\phi \int_\xi^r \frac{p(r,\theta)\, r'^4 \, \mathrm{d}r'}{(r'^3 - R^3 - p(r,\theta)^2 r')^{3/2} \sqrt{r'^3 - R^3}} \right.$$

$$B_\theta = \frac{2r^3 + R^3}{r^{5/2}\sqrt{r^3 - R^3}} \left| C_2 \pm 2B_0 \sin\phi \int_\xi^r \frac{r'^3\,(r'^3 + 2R^3)\sqrt{r'^3 - R^3}\,\mathrm{d}r'}{(2r'^3 + R^3)^2 \sqrt{r'^3 - R^3 - p(r,\theta)^2\, r'}} \right.$$

Approximate MHD solution near the sphere

$$\frac{\operatorname{curl}(\mathbf{v} \times \mathbf{B}) = \mathbf{0} \qquad \operatorname{div} \mathbf{B} = 0}{}$$

$$B_r = \tfrac{2}{3} B_0 \sin\phi \tfrac{\sin\theta}{1+\cos\theta} \sqrt{\tfrac{3s}{R}}$$

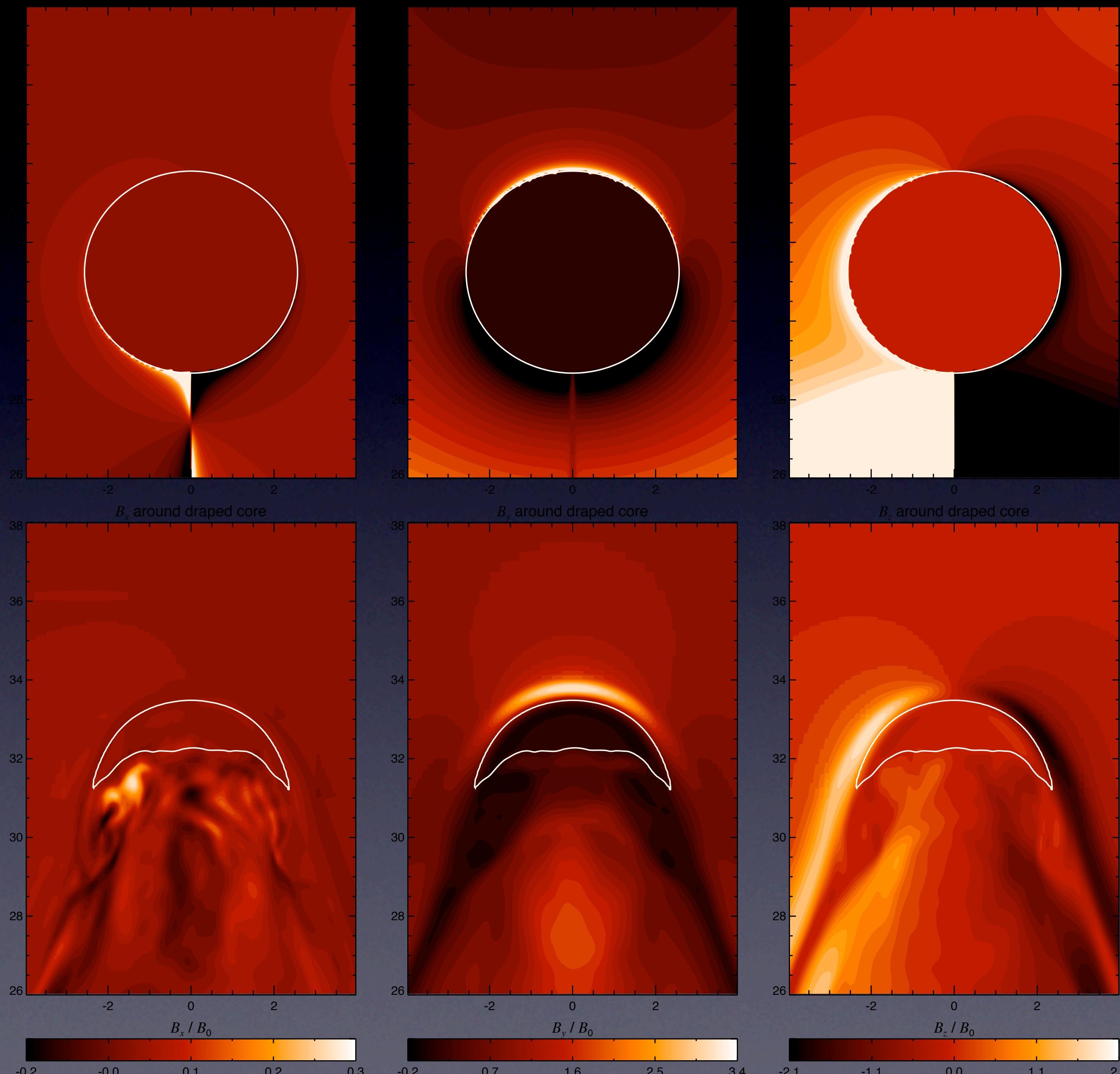
$$B_\theta = B_0 \sin\phi \sqrt{\frac{R}{3s}}$$

$$B_\phi = B_0 \cos\phi \sqrt{\frac{R}{3s}} \qquad \text{and } s = r - R$$

B_x, B_y, B_z in the plane of the initial B-field

Potential flow around solid sphere

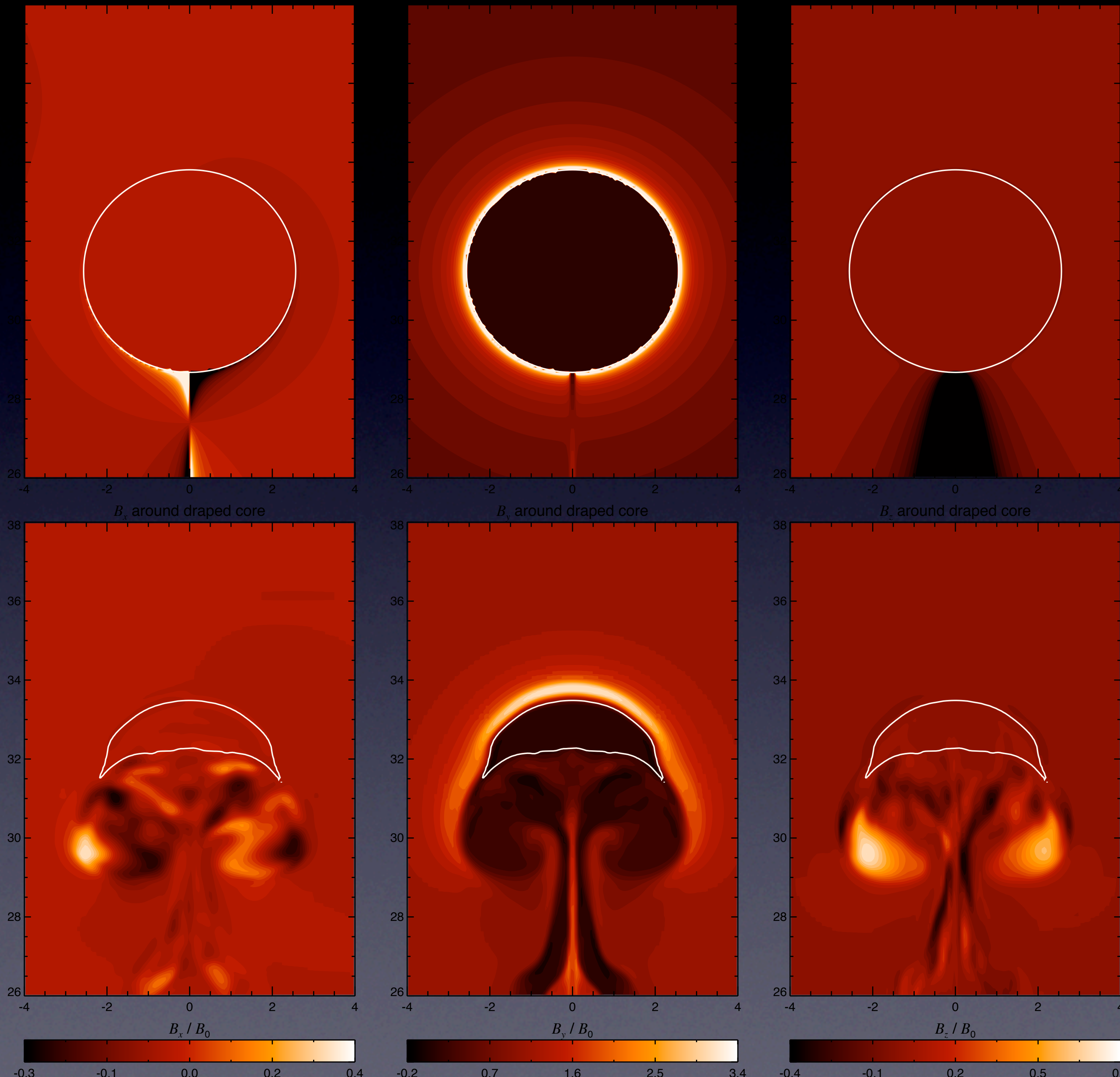
3d AMR results



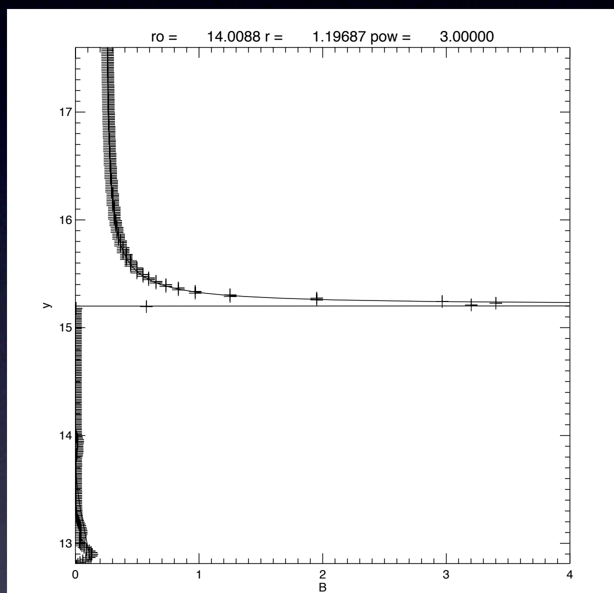
B_x, B_y, B_z in the plane transverse to the initial B-field

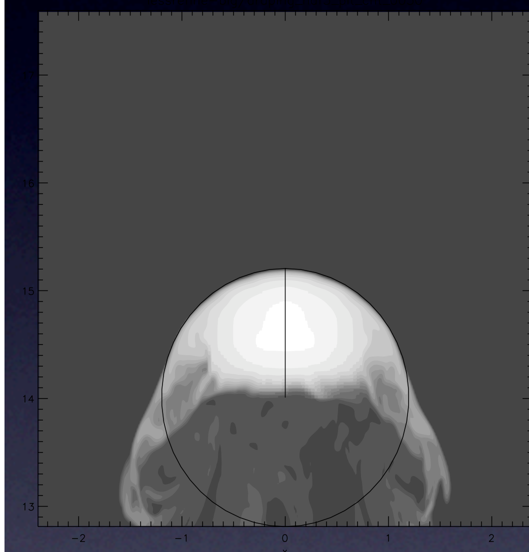
Potential flow around solid sphere

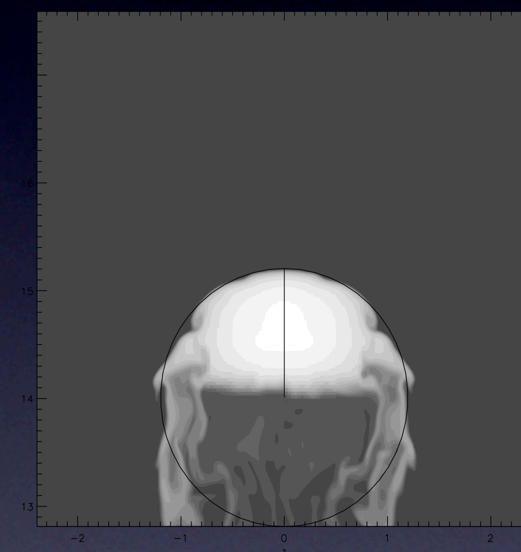
3d AMR results



Agreement with potential flow calculations

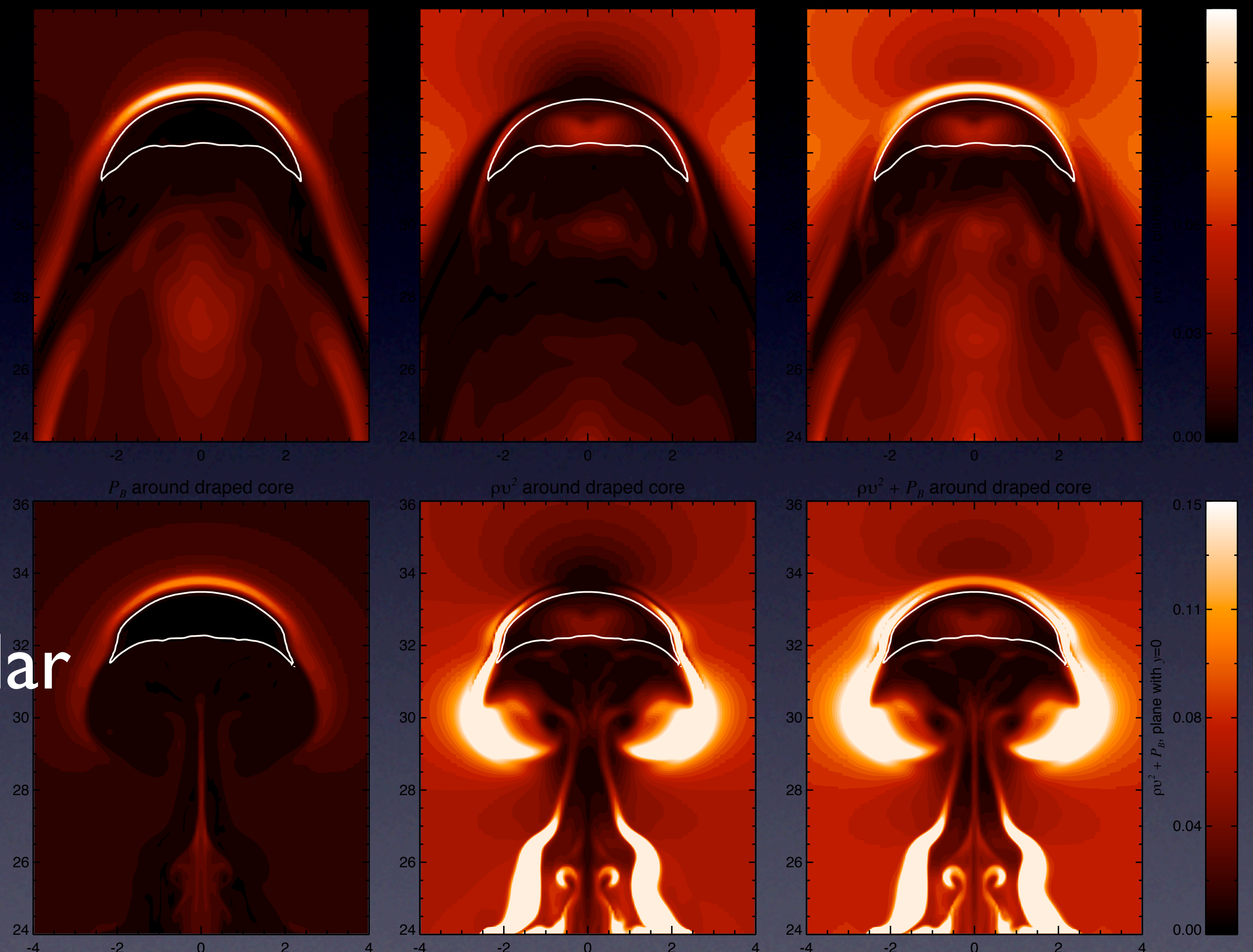






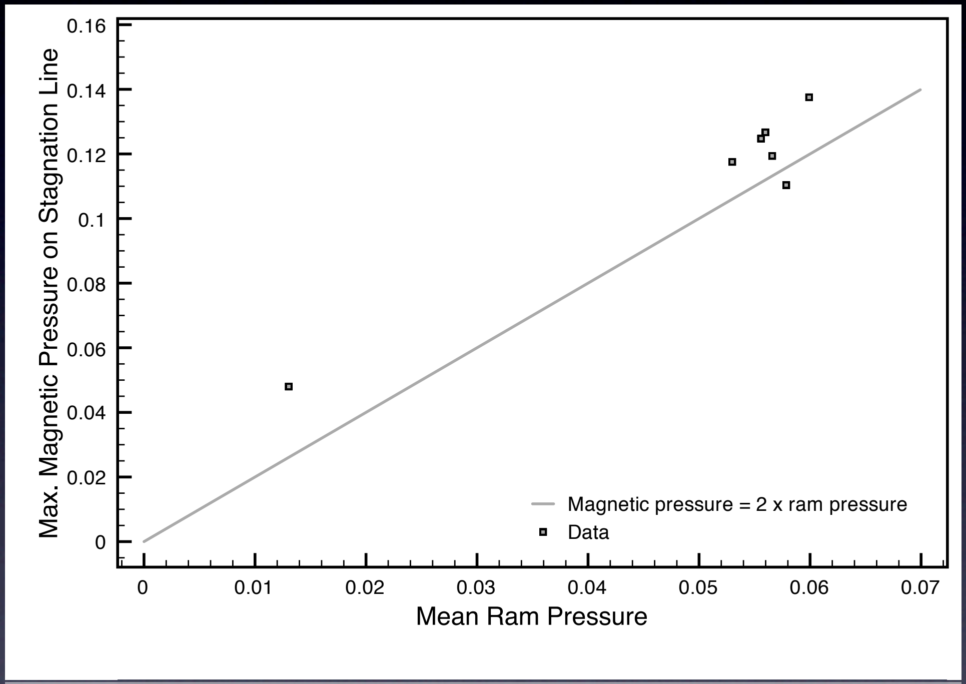
$$\frac{B}{\rho} = \frac{1}{\sqrt{1 - \frac{R_0^3}{r^3}}} \left(\frac{B}{\rho}\right)_0 ; \qquad l \approx \frac{1}{\mathcal{M}_A^2} R_0$$

	Magnetic pressure	Ram pressure	Total pressure
Magnetic field plane			
Perpendicular plane			



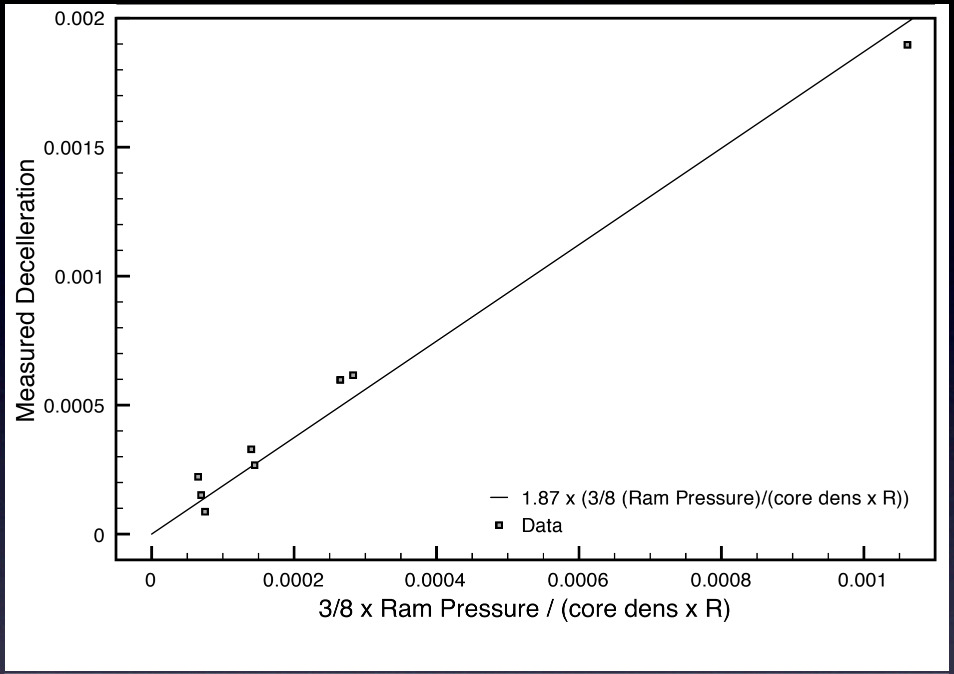
Magnetic field strength in draped layer

- To first order, depends only on ram pressure
- Maximum magnetic field strength ~ 2 x ram pressure



Deceleration due to mag. tension

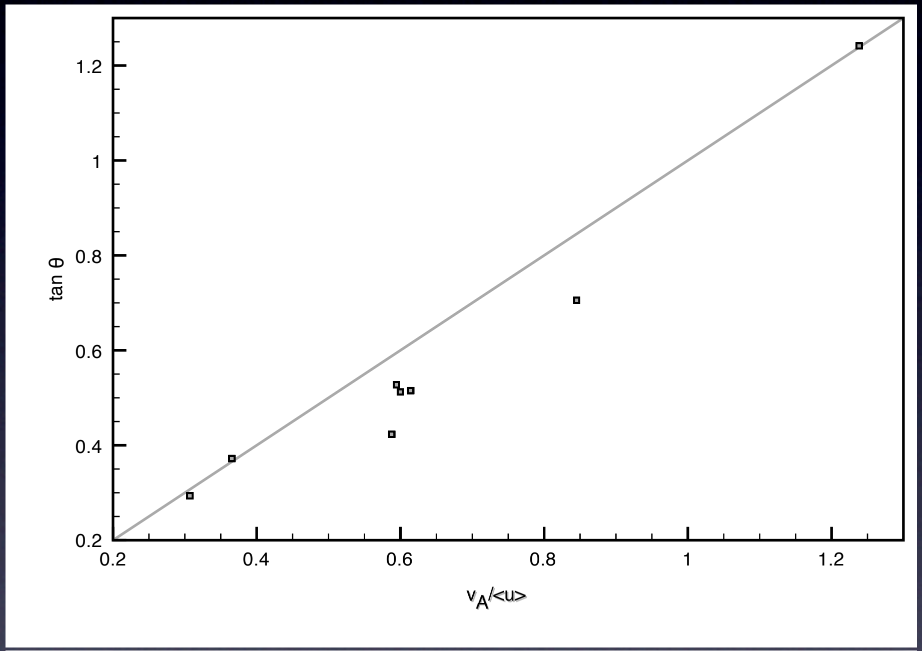
- Magnetic layer is strong enough (and curved enough) that it dominates deceleration
- Even in 3D case!
- ~ 4x stronger than viscous/ turbulent drag



$$\dot{u}_T = -\frac{3}{8}\frac{\rho u^2}{\langle \rho_c \rangle R} C_G$$

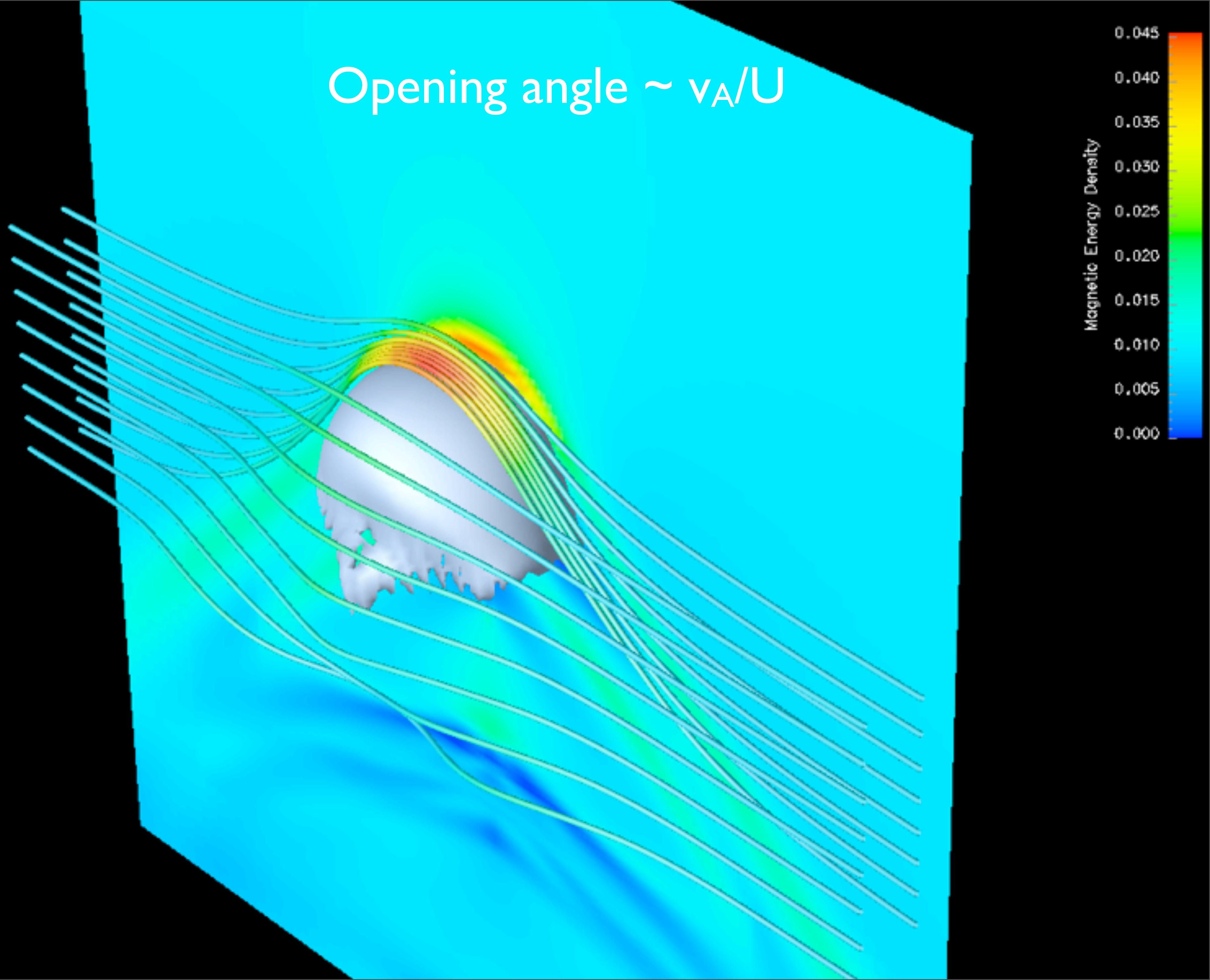
Opening angle of drape

- Comparison with 9 3D simulation
- Correlation a little rattier than other quantities --
- Largest scales in simulations, some effects of boundary conditions

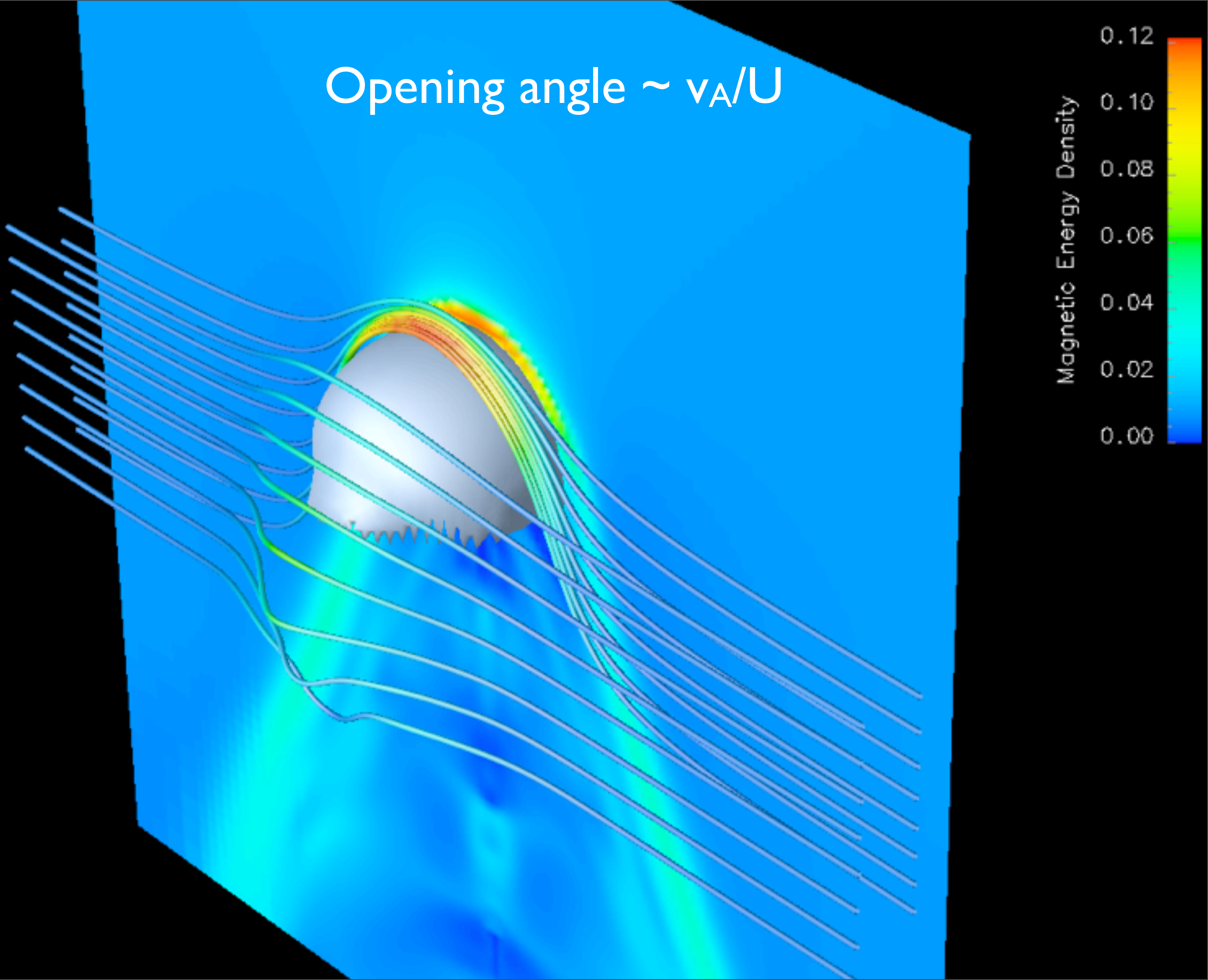


$$\tan\theta = v_{\mathrm{A}}/u$$

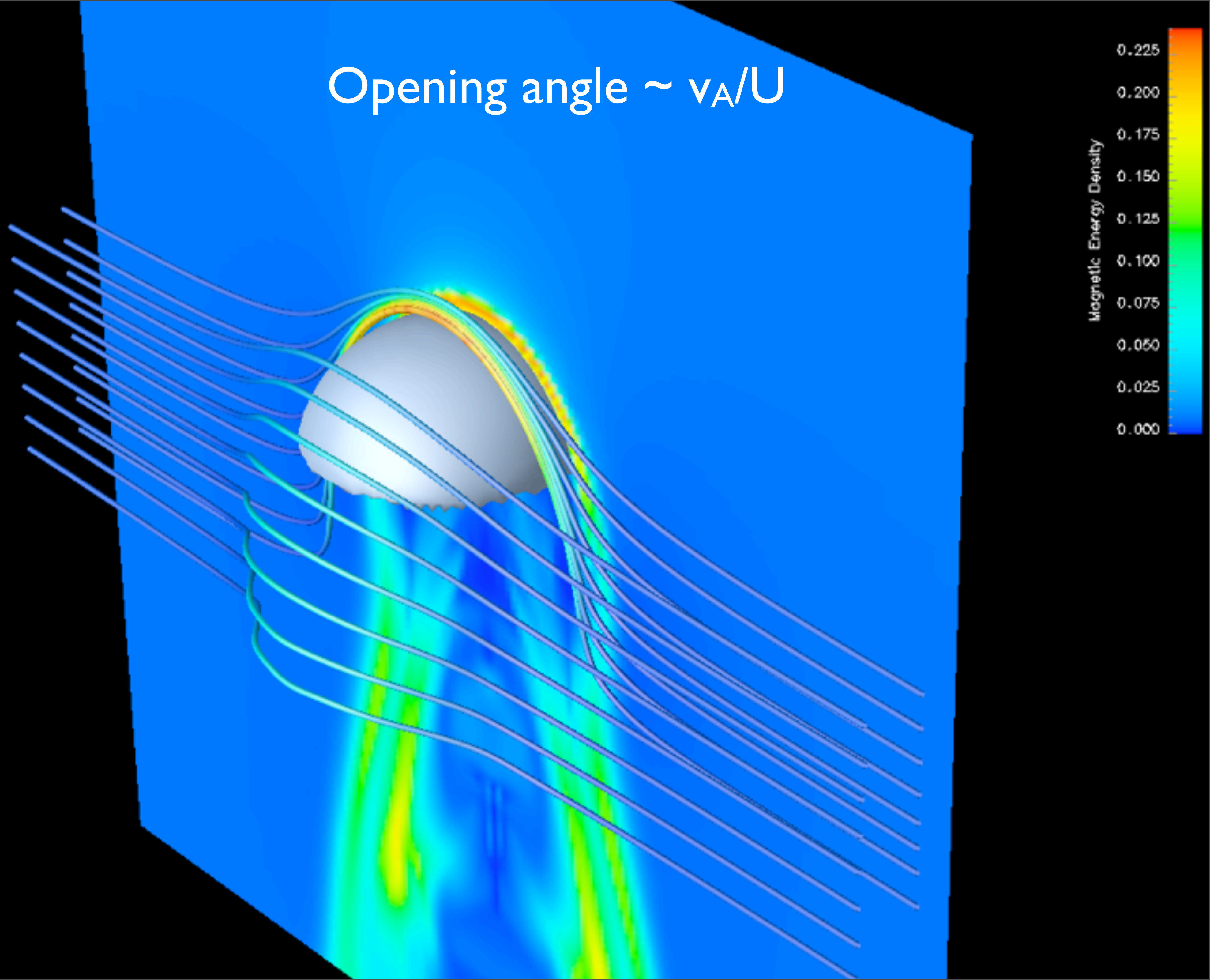
Opening angle ~ v_A/U



Opening angle ~ v_A/U



Opening angle ~ v_A/U



Vorticity generation

$$\rho \, \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla P + \mathbf{j} \times \mathbf{B} = -\nabla \left(P + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \, \mathbf{B}$$

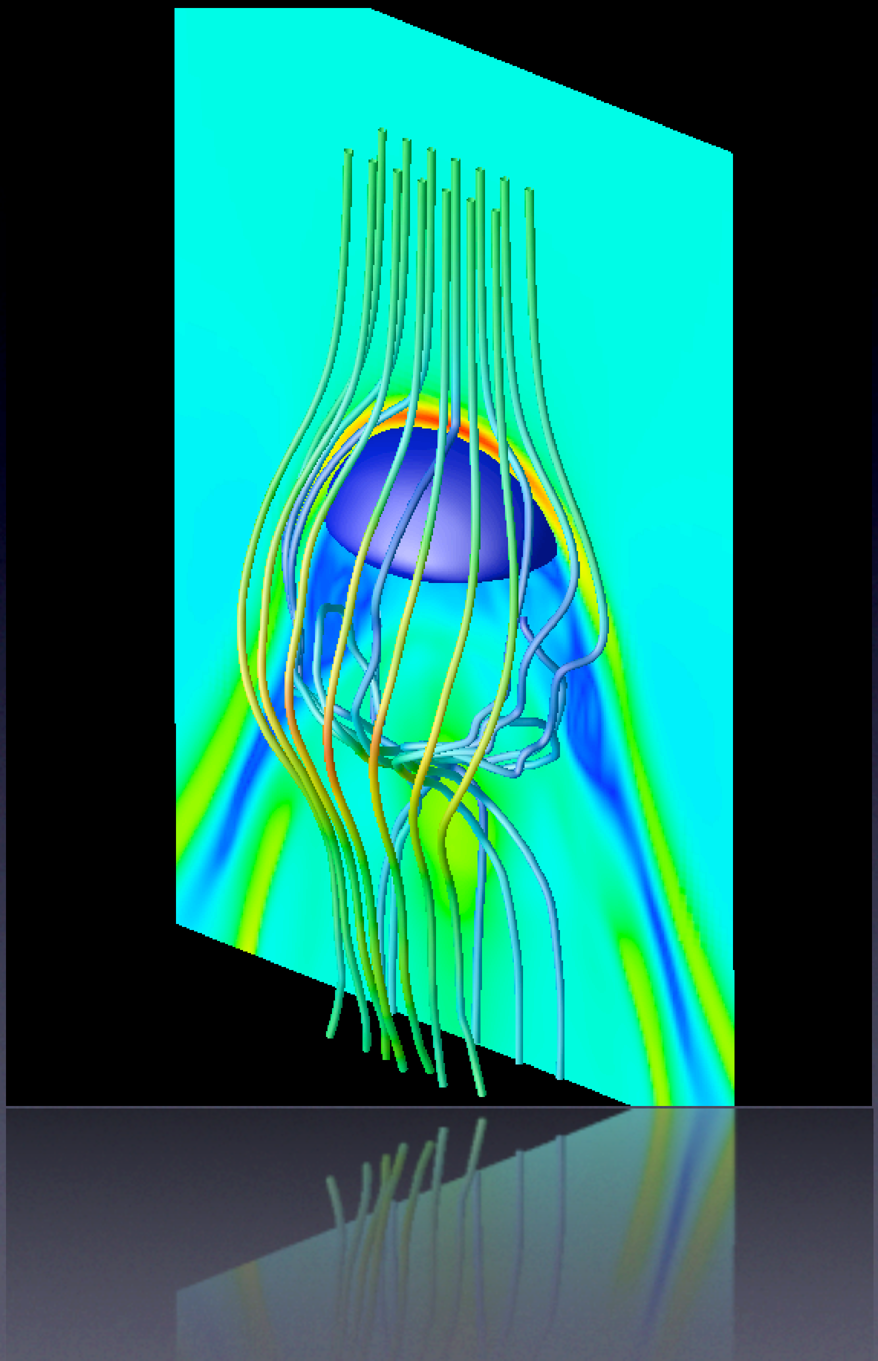
equilibrium: balance between magnetic tension and magnetic + thermal pressure

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\omega}{\rho} \right) \quad &= \quad \left(\frac{\omega}{\rho} \cdot \nabla \right) \mathbf{v} + \frac{1}{4\pi \, \rho^2} \nabla \times (\mathbf{B} \cdot \nabla) \, \mathbf{B} + \\ & \quad \frac{1}{\rho^3} \nabla \rho \times \left[\nabla \left(P + \frac{B^2}{8\pi} \right) - \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \, \mathbf{B} \right. \end{aligned}$$

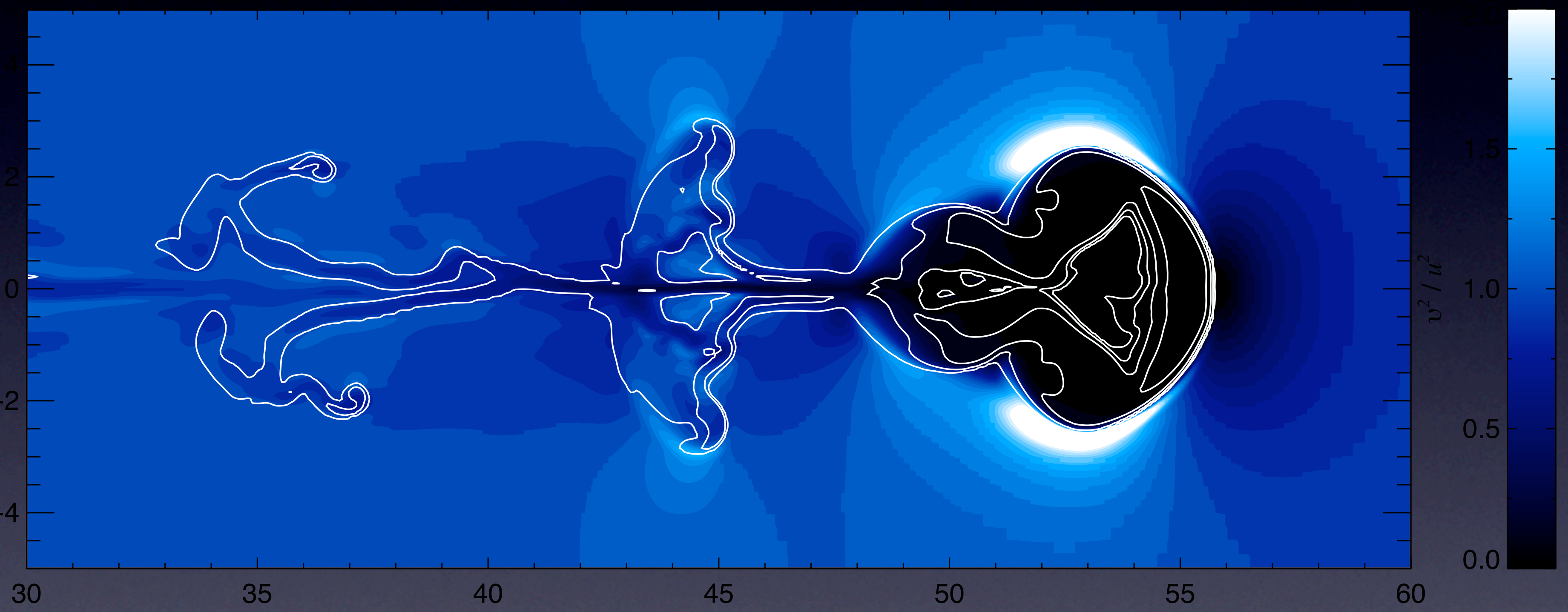
vorticity is frozen into the flow if source terms negligible

Generation of Vorticity

- Magnetic contact layer induces vorticity in fluid elements which cross it
- Operates primarily in plane along field lines
- Much less vorticity generation in other plane



Instabilities in transverse plane



- Material is stripped off the object,
- flow accelerated there due to Bernouli effect

Instabilities in transverse plane

$$\omega_{\mathrm{RT}}^2 = \frac{\langle\rho_{\mathrm{c}}\rangle - \rho_0}{\langle\rho_{\mathrm{c}}\rangle + \rho_0}\, \dot{u}_{\mathrm{T}}\, k \qquad \omega_{\mathrm{KH}} = \frac{\sqrt{\langle\rho_{\mathrm{c}}\rangle\, \rho_0}}{\langle\rho_{\mathrm{c}}\rangle + \rho_0}\, \Delta u\, k$$

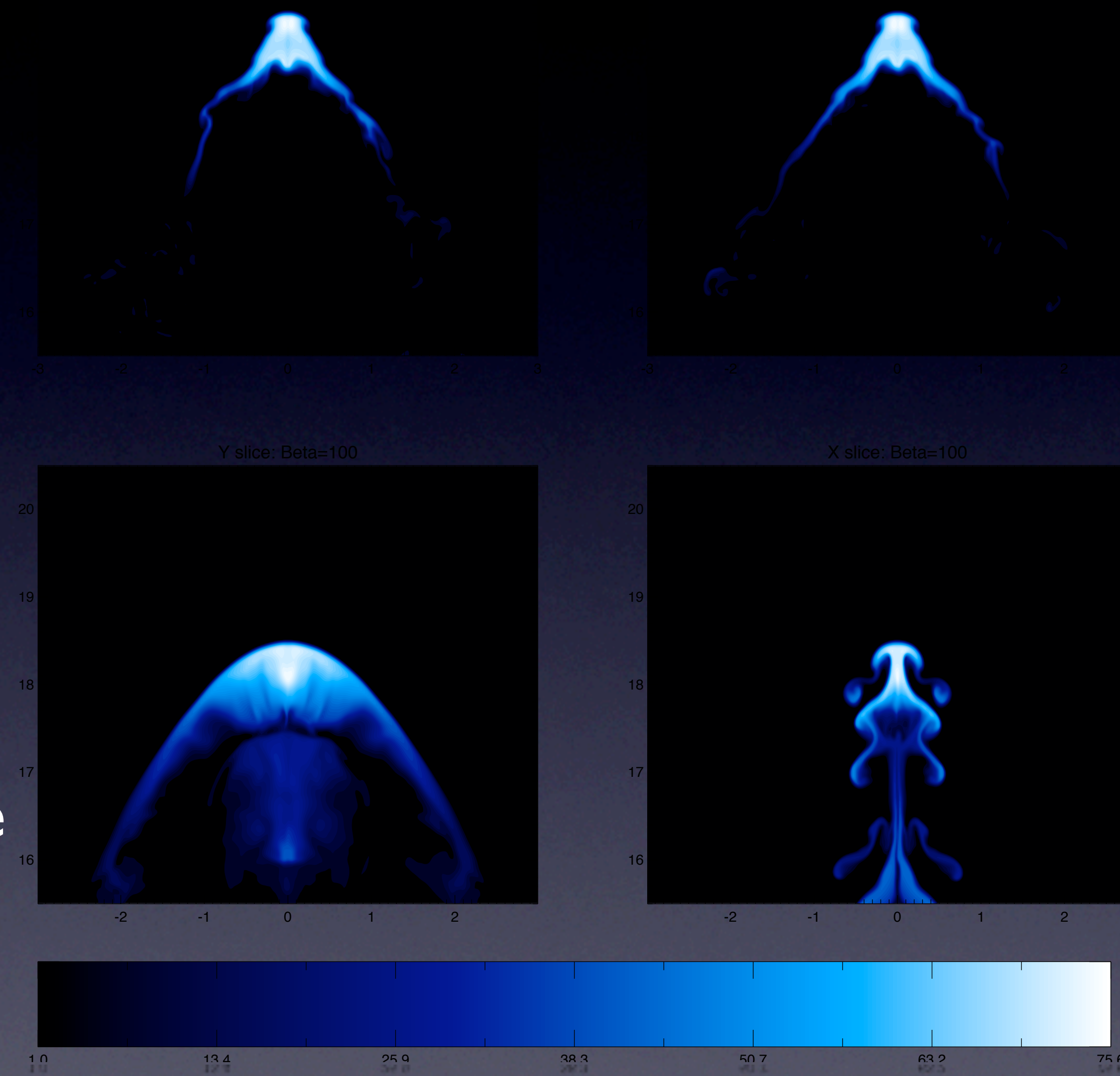
$$\frac{\omega_{\mathrm{KH}}^2}{\omega_{\mathrm{RT}}^2} \simeq \frac{12\pi}{C_{\mathrm{G}}}\, \frac{k}{k_0} \geq \frac{12\pi}{C_{\mathrm{G}}} \simeq 20$$

$$L_{\mathrm{KH}} = \frac{2\pi\, u}{\omega_{\mathrm{KH}}} \simeq \frac{2R}{3} \sqrt{\frac{\langle\rho_{\mathrm{c}}\rangle}{\rho_0}\, \frac{k_0}{k}} \leq 15$$

Kelvin-Helmholtz instability is responsible for disintegration of the core

Long-term behavior

- Evolution of core after it has swept past roughly its own mass
- Mixed material `fills up' drape
- Highly constrained in other plane!



Conclusions

- Very quickly drape strong magnetized layer
- Even thin layer can have interesting effects protecting object (bubble or bullet) against indignities of shearing into environment
- Pushing forward analytics, guided by simple numerical experiments
- Rich astrophysical applications