

Self-interacting dark matter

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in collaboration with

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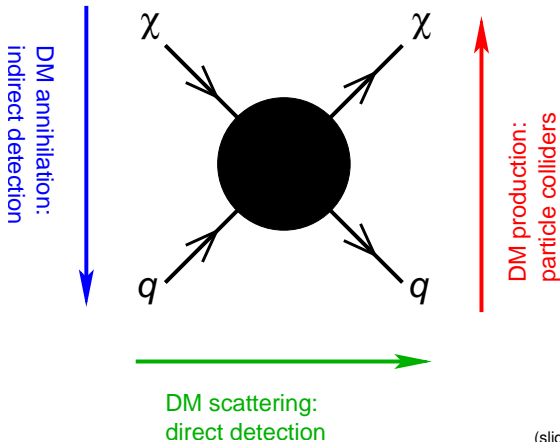
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Apr 22, 2015 / CITA Cosmology Seminar



Searching for dark matter (DM)

correct relic density \rightarrow DM annihilation in the Early Universe



(slide concept Feng)



Outline

- 1 Standard WIMPS
 - Chemical decoupling
 - Kinetic decoupling
 - Smallest protohalos
- 2 Self-interacting WIMPS
 - Sommerfeld effect
 - Small-scale problems
 - A solution to *all* Λ CDM problems
- 3 Cosmological simulations



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The WIMP “miracle”

- “freeze-out”: annihilation rate drops below expansion rate H
→ number density of Weakly Interacting Massive Particles:

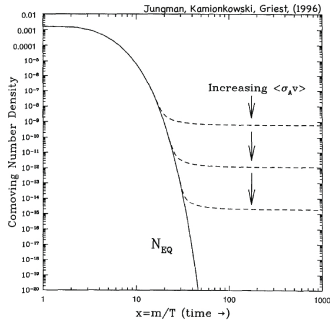
$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle (n_\chi^2 - n_{\chi,\text{eq}}^2), \quad \langle\sigma v\rangle : \chi\chi \rightarrow \text{SM SM}$$



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- assuming a particle χ , initially in thermal equilibrium, with a relic density

$$\Omega_\chi \sim \frac{1}{m_{\text{Pl}} T_0 \langle\sigma v\rangle} \sim \frac{m_\chi^2}{m_{\text{Pl}} T_0 g_\chi^4},$$

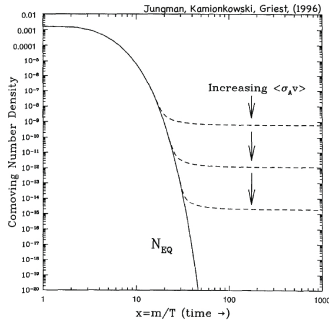
$$\left. \begin{array}{l} m_\chi \sim m_{\text{weak}} \sim 100 \text{ GeV} \\ g_\chi \sim g_{\text{weak}} \sim 0.6 \end{array} \right\} \Omega_\chi \sim 0.1$$



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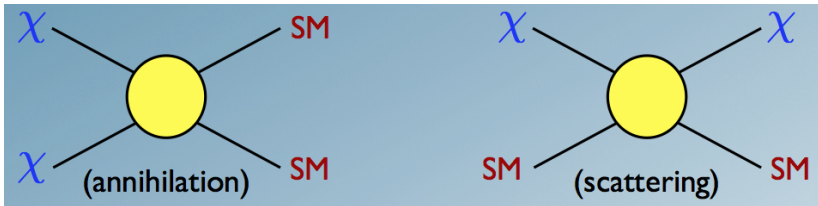
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- remarkable coincidence: particle physics independently predicts particles with the right density to be dark matter



Freeze-out \neq decoupling!

- WIMP interactions with **heat bath** of SM particles:



- Boltzmann suppression of n_χ :
 - scattering process more frequent
 - continue even after **chemical decoupling** (“freeze-out”) at $T_{\text{cd}} \sim m_\chi/25$
- kinetic decoupling** much later: $\tau(T_{\text{kd}}) \equiv N_{\text{coll}}/\Gamma_{\text{el}} \sim H^{-1}(T_{\text{kd}})$
 random walk in momentum space: $N_{\text{coll}} \sim m_\chi/T$ (Schmid+ 1999, Green+ 2005)



Kinetic decoupling

- evolution of phase space density f_{χ} given by the full Boltzmann equation in FRW space time:

$$E (\partial_t - H\mathbf{p} \cdot \nabla_{\mathbf{p}}) f_{\chi} = C[f_{\chi}]$$

- 1st moment ($\int d^3p$) recovers the familiar continuity equation:

$$\frac{dn_{\chi}}{dt} + 3Hn_{\chi} = -\langle\sigma v\rangle (n_{\chi}^2 - n_{\chi,\text{eq}}^2)$$

- consider the 2nd moment ($\int d^3p \mathbf{p}^2$) and introduce

$$T_{\chi} n_{\chi} \equiv \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^2}{3m_{\chi}} f_{\chi}(\mathbf{p})$$

→ analytic treatment possible without assumptions about $f_{\chi}(\mathbf{p})$

Bertschinger (2006), Bringmann & Hofmann (2007), Bringmann (2009)



Thermal history of WIMPs

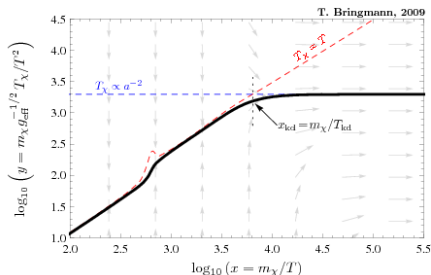
- resulting ODE for T_χ

$$\frac{dy}{dx} = 2 \frac{m_\chi c(T)}{H \tilde{g}^{-1/2}} \left(1 - \frac{T_\chi}{T} \right)$$

example:

$$m_\chi = 100 \text{ GeV}$$

$$|\mathcal{M}|^2 \sim g_Y^4 (E_\chi / m_\chi)^2$$



- fast transition allows definition of T_{kd} :

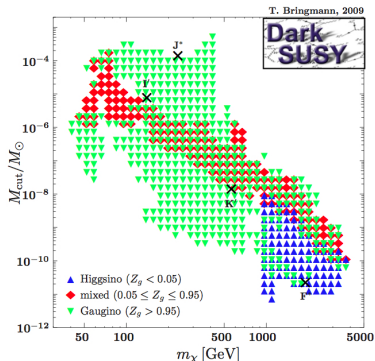
$$T_\chi = \begin{cases} T & \text{for } T \gtrsim T_{kd}, \\ T_{kd} (a_{kd}/a)^2 & \text{for } T \lesssim T_{kd} \end{cases}$$

Bringmann & Hofmann (2007), Bringmann (2009)



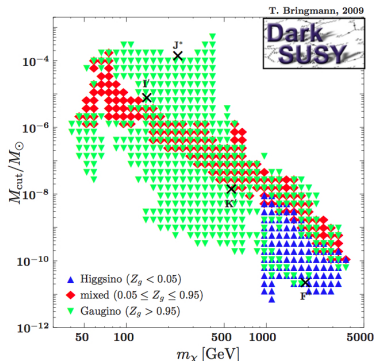
The smallest protohalos

- **free streaming** of WIMPS after t_{kd} at the thermal speed of decoupling erases small-scale fluctuations (Green+ 2005)
- initial coupling between WIMPS and the radiation field
 → **acoustic oscillations** in the power-spectrum at the horizon scale of kinematic decoupling (Loeb & Zaldarriaga 2005, Bertschinger 2006)



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- **cutoff** in the power spectrum corresponds to **smallest gravitationally bound objects** in the universe
- strong dependence on particle physics properties, no “typical” value of $M_{\text{cut}} \sim 10^{-6} M_{\odot}$ (Profumo+ 2006)



Consequences

- indirect detection experiments through WIMP annihilation:

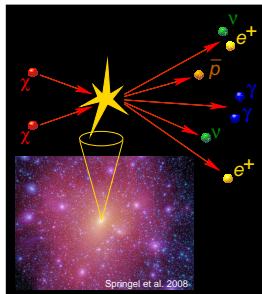
$$\Phi_{\text{SM}} \propto \langle \rho_{\chi}^2 \rangle = (1 + \text{BF}) \langle \rho_{\chi} \rangle^2,$$

$$\text{BF} \propto \log(M_{\text{halo}}/M_{\text{min}})$$

(Pinzke+ 2011, Gao+ 2012, Ludlow+ 2014)

- flux depends on **astrophysics**, **particle physics**, **detector properties**:

$$N_{\gamma} = \left[\int_{\text{LOS}} \rho_{\chi}^2 dl_{\chi} \right] \frac{\langle \sigma v \rangle}{2M_{\chi}^2} \left[\int_{E_{\text{th}}}^{M_{\chi}} \left(\frac{dN_{\gamma}}{dE} \right)_{\text{SUSY}} A_{\text{eff}}(E) dE \right] \frac{\Delta\Omega}{4\pi} \tau_{\text{exp}}$$



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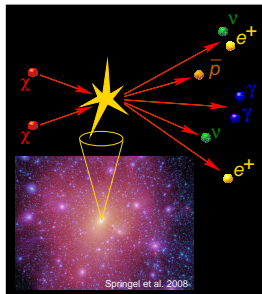
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- fluctuations in the event rate of **direct detection experiments**
- gravitational lensing of substructures → flux anomalies
- Lyman- α forest ...

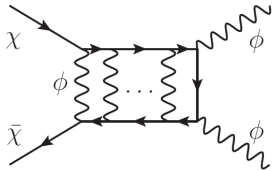


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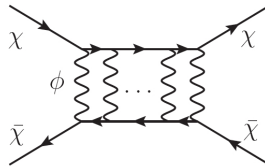
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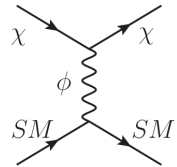
WIMPS with long-range forces



annihilation



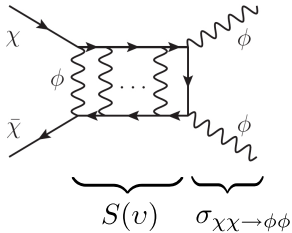
self-scattering



scattering



Sommerfeld effect



Arkani-Hamed+ (2009)

- **long range interaction:**
potential distorts wave function

$$\left(-\frac{\nabla^2}{m_\chi} + V\right)\psi(r) = m_\chi v^2 \psi(r)$$

$$\Rightarrow \sigma = \mathbf{S}(v)\sigma_{\chi\bar{\chi} \rightarrow \phi\phi},$$

- **kinematics:** non-relativistic DM particle χ interacts with **light force carrier ϕ** ($m_\phi \ll m_\chi$)
- **repeated exchange of ϕ :** each “rung” of ladder contributes at $\mathcal{O}(\alpha/v)$ \rightarrow resummation necessary
- **short-range interaction:** standard QFT result

$$\text{with } \mathbf{S}(v) = |\psi(0)|^2$$



Enhancement factor

- Coulomb potential: analytic solution

$$S(v) = \frac{\pi\alpha/v}{1 - \exp(-\pi\alpha/v)} \quad \xrightarrow{v \rightarrow 0} \quad \frac{\pi\alpha}{v}$$

- Yukawa potential: numerical solution
→ appearance of **resonances near bound states**



Enhancement factor

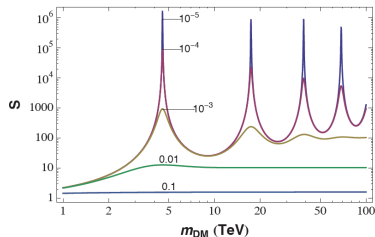
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→ appearance of **resonances near bound states**

- off resonance: $S \propto v^{-1}$
- on resonance: $S \propto v^{-2}$
- saturation for small v



Lattanzi, Silk (2009)



Enhancement factor

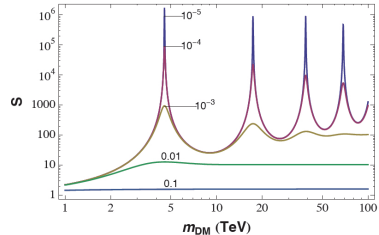
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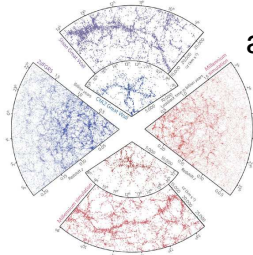
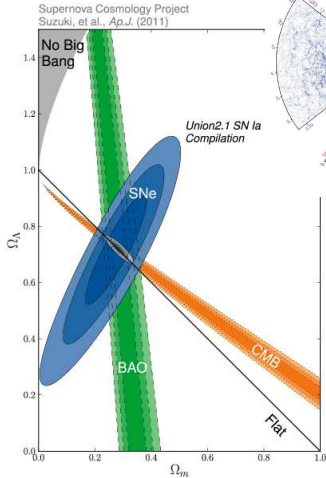
- off resonance: $S \propto v^{-1}$
 - on resonance: $S \propto v^{-2}$
 - saturation for small v
- for $m_\phi \lesssim 100$ MeV, ϕ can only decay into leptons (e, μ)
 → leptophilic DM



Lattanzi, Silk (2009)



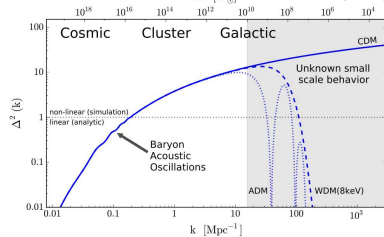
Λ CDM cosmology



a great success
story on **large**
scales

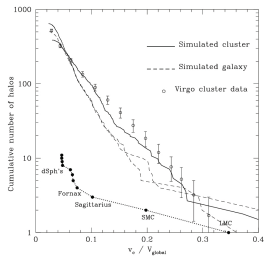
Springel+ (2006)

Kuhlen+ (2012)



Λ CDM small-scale problems

1. Missing satellites?



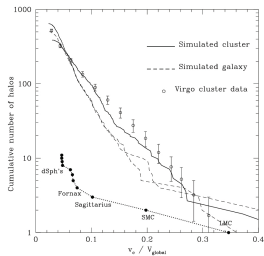
Moore+ (1999)

→ many more satellites
in simulations of MW-
sized galaxies than
observed



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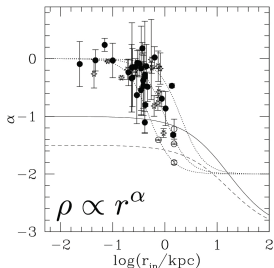
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2. Cusps or cores?



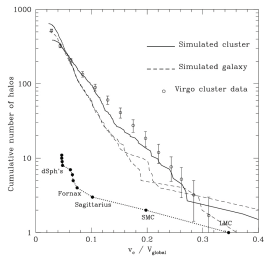
Blok+ (2001)

→ cuspy inner density
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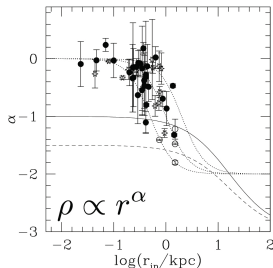
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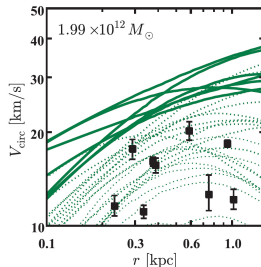
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3. Too big to fail?

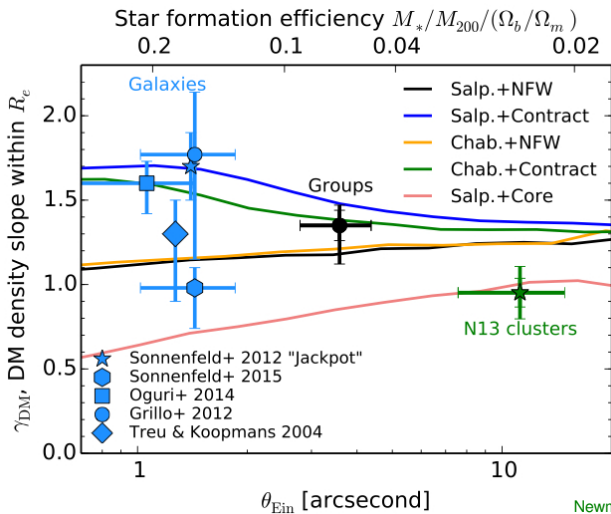


Boylan-Kolchin+ (2011)

→ most massive sub-
halos in simulations too
dense to host observed
brightest dwarf satellites



Inner DM profile in galaxy groups and clusters



Newman+ (2015)



Solutions?

many possibilities, no consensus reached yet:

- **astrophysical solutions:**
increased gas entropy, suppress cooling efficiency, SN feedback, large velocity anisotropy, other baryonic feedback, increased stochasticity of galaxy formation, small MW mass, ...
- **dark matter solutions:**
warm DM, interacting DM, DM from late decays, large annihilation rates, condensates, ...
- **all have shortcomings** and/or solve at most 2 problems at the time!



Solutions?

velocity-dependent self-interacting dark matter:

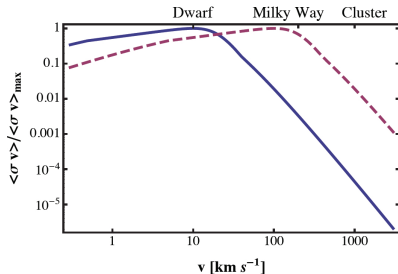
- scattering cross-section for **Yukawa potential** Khrapak+ (2003)
 $\sigma_{\chi\bar{\chi}} = \text{const.}$ unnatural from particle physics viewpoint!
- elastic DM self-scattering is completely analogous to screened Coulomb scattering in a plasma



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velocity-dependent self-interacting dark matter:

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- elastic DM self-scattering is completely analogous to screened Coulomb scattering in a plasma
- **cored profiles possible** without violating astrophysical constraints
Feng+ (2010), Loeb & Weiner (2011)
- N-body simulations: **“too big to fail” problem avoided**
Vogelsberger+ (2012)
- **what about missing satellites?**



Loeb & Weiner (2011)

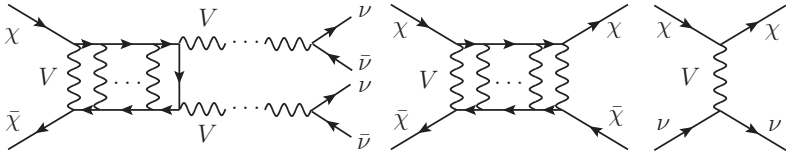


Our model

van den Aarsen, Bringmann, C.P. (2012)

- assume **light vector mediator** coupling to dark matter and neutrinos:

$$\mathcal{L}_{\text{int}} \supset -g_\chi \bar{\chi} \not{V} \chi - g_\nu \bar{\nu} \not{V} \nu$$



annihilation

- relic density
- indirect 4ν detection signal from galactic center(?)

self-scattering

- changes inner density and velocity profiles of dwarf galaxies

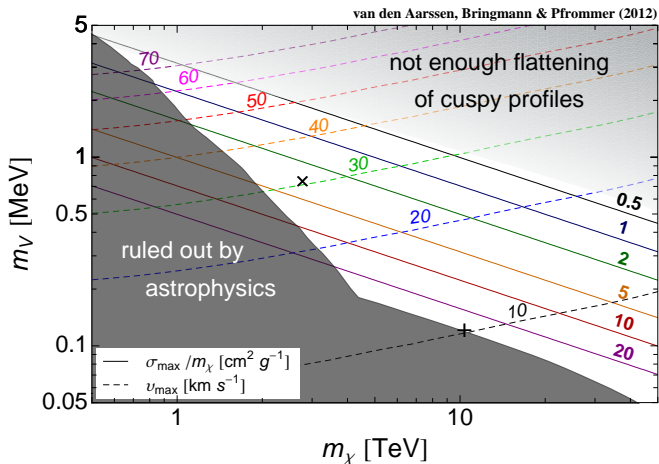
scattering

- large M_{min}



“Cusp vs. core” and “too big to fail” problems

- demand correct relic density
→ unique relation between $(v_{\max}, \sigma_{\max})$ and (m_χ, m_V)



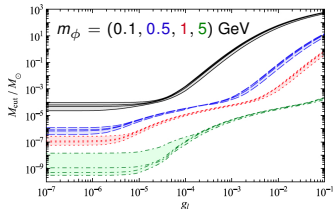
DM scattering off standard model particles

- free-streaming of WIMPs after **kinetic decoupling** creates cutoff in power spectrum
- acoustic oscillations leads to similar cutoff
- cutoff scale is set by size of horizon at KD: **late KD** \rightarrow **high M_{\min}**
- $M_{\min} = \max(M_{\text{fs}}, M_{\text{ao}})$: only objects with $M \geq M_{\min}$ form



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- $M_{\min} = \max(M_{\text{fs}}, M_{\text{ao}})$: only objects with $M \geq M_{\min}$ form
- **scalar mediator**:
 - scatters off ϕ, μ^\pm, e^\pm
 - saturation at $M_{\min} \sim 10^3 M_\odot$
 - ν 's negligible: $|\mathcal{M}_{\phi I \rightarrow \phi I}|^2 \propto m_I^2$
- **vector mediator**:
 - ν 's contribute:
 $|\mathcal{M}_{V\nu \rightarrow V\nu}|^2 \propto E_\nu^2$
 - M_{\min} increases to $\mathcal{O}(10^{11} M_\odot)$

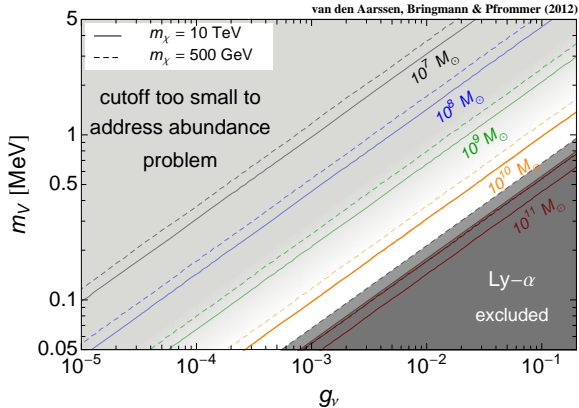


van den Aarsen+ (2012)



“Missing satellites” problem

- now compute M_{\min} from kinetic decoupling temperature ...



- in this simple phenomenological model, it is possible to **simultaneously solve all small-scale problems of Λ CDM!**



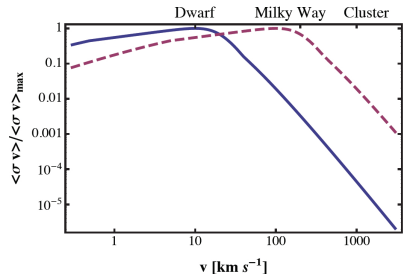
Cored central density profiles of clusters

- velocity-dependent DM self-scattering cores out central density slopes in clusters with rate

$$\Gamma \sim \frac{\rho}{m_\chi} \langle \sigma_{\chi\bar{\chi}} v \rangle \sim H$$

- ellipticals/clusters,
 $f_s = 10 - 100$:

$$\Gamma \sim \frac{f_s \rho}{m_\chi} \frac{\langle \sigma_{\chi\bar{\chi}} v \rangle |_{\max}}{f_s}$$



Loeb & Weiner (2011)



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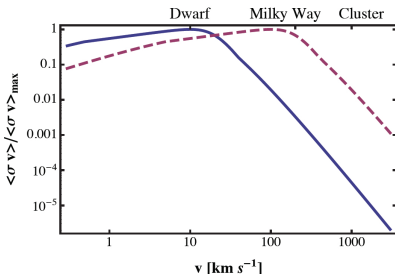
$$\Gamma \sim \frac{f_s \rho}{m_\chi} \frac{\langle \sigma_{\chi\bar{\chi}} v \rangle |_{\max}}{f_s}$$

- using $\rho \sim 1/r$ for $r \ll r_s$:

$$\left. \frac{r_{\text{core}}}{r_{200}} \right|_{\text{cluster}} \sim \frac{1}{f_s} \left. \frac{r_{\text{core}}}{r_{200}} \right|_{\text{dwarf}} \sim \frac{1}{f_s 10} \Rightarrow r_{\text{core}}(10^{15} M_\odot) \sim \mathcal{O}(1-10 \text{ kpc})$$

Loeb & Weiner (2011)

- need simulations to understand interplay of hierarchical evolution and determination of cluster- r_{core} : **merging history** \rightarrow **scatter**



Conclusions on small-scale problems of Λ CDM

small-scale problems of Λ CDM can be solved by a DM model with:

- velocity-dependent self-interactions mediated by (sub-)MeV vector:
 - transforms cusps to cores and solves “too big to fail” problem
- much later kinetic decoupling than in standard case follows naturally for vector mediator coupling to neutrinos:
 - potentially solves “missing satellites” problem
- predicts cores in clusters on scales $\mathcal{O}(1 - 10 \text{ kpc})$

→ need further model building and simulations to confirm

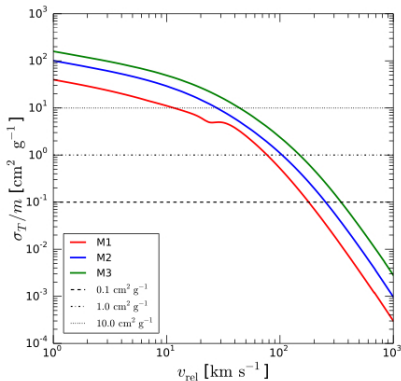
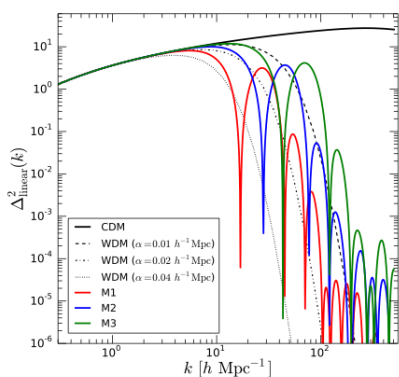


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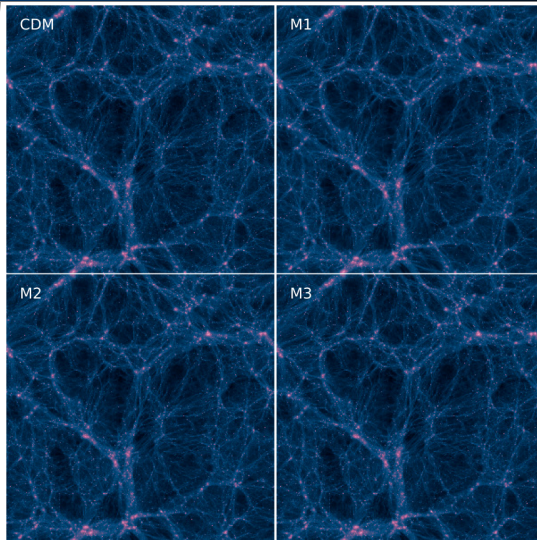
SIDM simulations: models



Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.



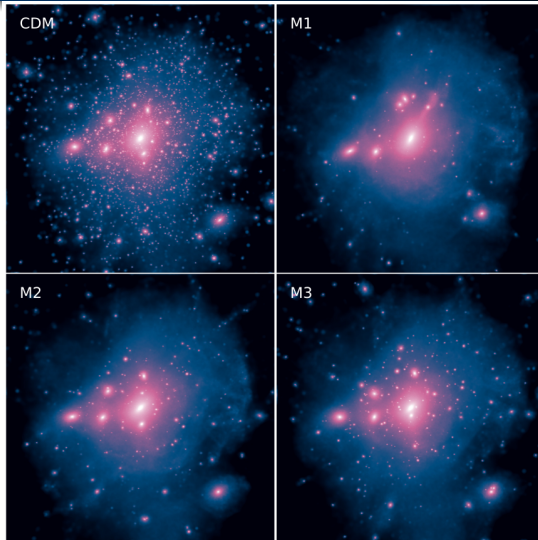
SIDM simulations: large-scale structure



Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.



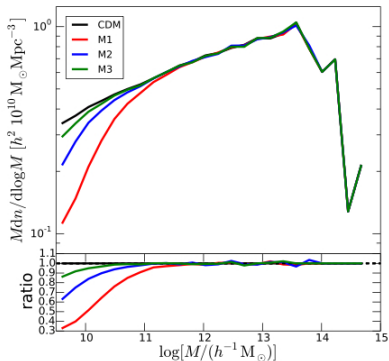
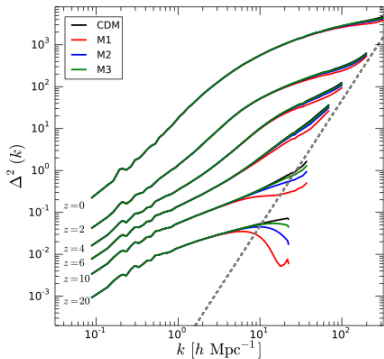
SIDM simulations: Milky Way-sized halos



Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.



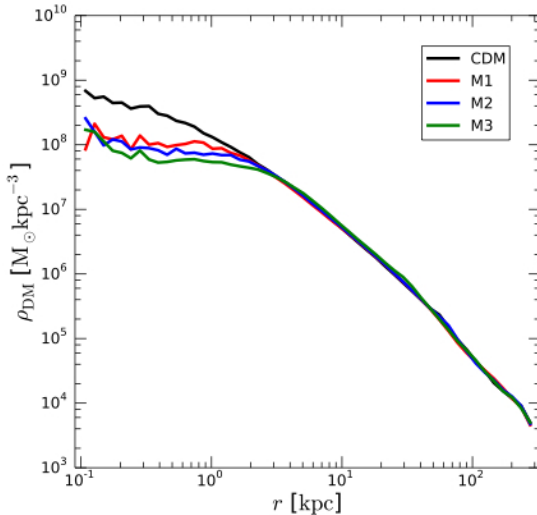
SIDM simulations: power spectrum and mass function



Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.



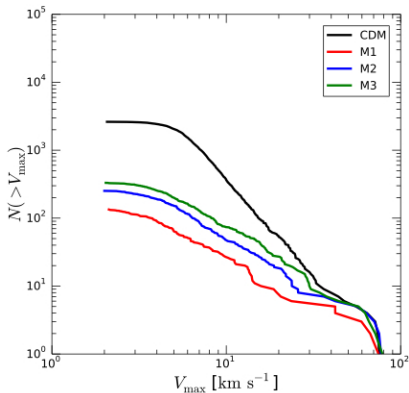
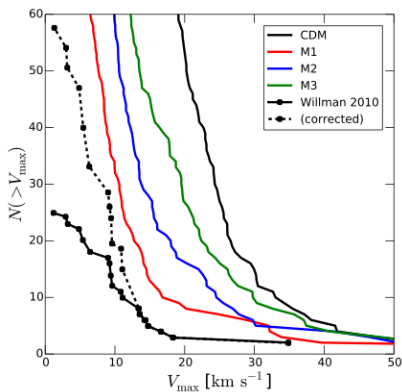
SIDM simulations: density profile of MW-sized halo



Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.



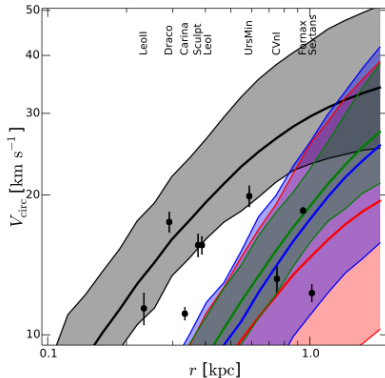
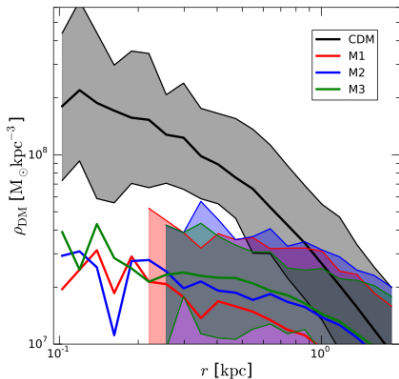
SIDM simulations: subhalo abundances



Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.



SIDM simulations: internal subhalo structure



Vogelsberger, Zavala, Cyr-Racine, Pfrommer, Bringmann, Sigurdson, in prep.



Conclusions

If DM searches (production, indirect, and direct experiments) continue to deliver null results, we need to search for alternative windows:

- **small-scale features of Λ CDM cosmology**: abundances, density profiles, ... in the most DM-dominated objects (dwarfs, clusters)
- **particle physics model building** that addresses anomalies (beam dump experiments, ...)
- **develop effective theory for structure formation** that connects particle physics properties to effective parameters of structure formation

