Lecture 7

Cosmic ray interactions – overview

1) CR electrons : Pointlike electromagnetic interactions of CR electrons include

- synchrotron radiation,
- inverse Compton scattering,
- non-thermal bremsstrahlung,

and are proportional to the Thomson cross section $\sigma_T = \frac{8\pi r_e^2}{3} = \frac{8\pi e^4}{3 m_e^2 c^4}$

2) CR protons : Pointlike EM interactions of CR nuclei are suppressed, since the Thomson cross section for nuclei is given by

$$\sigma_N = \frac{8\pi r_N^2}{3} = \frac{Z^4}{A^2} \left(\frac{m_e}{m_p}\right)^2 \sigma_T \quad , \quad r_N = \frac{(Ze)^2}{A m_p c^2} = \frac{Z^2}{A} \frac{m_e}{m_p} r_e \; .$$

Comparing CR electrons and protons with the same Lorentz factor, we can neglect pointlike EM of CR nuclei!

The remaining inelastic interaction processes can be divided into 2 groups:

i) interactions w/ photons

ii) interactions w/ matter

2.1) CR interactions with photons

1) pair production in the field of the nucleus:

$$A + \gamma \rightarrow A + e^+ + e^-$$

2) photoproduction of hadrons (mostly pions):

$$A + \gamma \rightarrow A + n_0 \pi^0 + n_+ \pi^+ + n_- \pi^-,$$

n_0, n_+, n_- are the numbers of produced pions.

3) photodisintegration of the nucleus:

$$A + \gamma \rightarrow (A-1) + N, \quad N = p, n$$

Notes: •) Considering CMB-photons, processes 1) and 2) occur only for UHECRs.

•) When process 2) becomes important, only protons need to be considered, since due to process 3) nuclei don't survive under these conditions.

Process 1): • In center of momentum system (CMS), the energy threshold is

$$E'_{th, e^+e^-} = 2 m_e c^2 \left(1 + \frac{m_e}{A m_p}\right) \simeq 2 m_e c^2 \simeq 1 \mathrm{MeV}$$

• If target photons are distributed isotropically in space, have mean energy $\langle E \rangle$ in the observers frame, minimum Lorentz factor is given

$$\gamma_{min} = \frac{\Delta m c^2}{2 \langle E \rangle} \left[1 + \frac{\Delta m}{2 m_p A} \right], \quad \Delta m \text{ is mass difference between incoming and outgoing particles}$$

$$\simeq \frac{m_e c^2}{\langle E \rangle} = 7 \cdot 10^8, \quad \langle E \rangle = 7 \cdot 10^{-4} \mathrm{eV}$$

Process 2): $\rightarrow$ GZK cutoff : ($\rightarrow$ homework)

2.2. CR interactions with matter

1) Hadronic p-p interaction: (details on pg 4 ff)

$$\mathrm{CR}\,p + p \begin{cases} \nearrow \pi^{\pm} \longrightarrow \overset{(-)}{\nu}_{\mu} + \mu^{\pm} \longrightarrow e^{\pm} + \overset{(-)}{\nu}_e + \overset{(-)}{\nu}_{\mu} \\ \searrow \pi^{0} \longrightarrow 2\gamma \end{cases}$$

2) Excitation of nuclei

$$\mathrm{CR}\,p + A \rightarrow \underset{+p}{A^{*}} \longrightarrow A + \gamma\text{-ray}$$

3) Coulomb and ionizing interactions of CR nuclei with mass $M = A m_p$:

- fully ionized plasma with species of respective mass m_s, charge $Z_s e$, n_s, T_s:

$$-\left(\frac{dE}{dt}\right)_{\mathrm{Coul}} = \frac{3 c \sigma_T m_e c^2 Z^2}{2\beta} \sum_s \frac{m_e}{m_s} Z_s^2 n_s W_s\left(\frac{\beta}{\beta_s}\right)$$

$$\beta_s \equiv \sqrt{\frac{2kT_s}{m_s c^2}}$$

$$W_s(x) = \frac{2}{\sqrt{\pi}} \left[\int_0^{\pi} dy\, e^{-y^2} - \left(1 - \frac{m_s}{M}\right) x\, e^{-x^2} \right]$$

$\rightsquigarrow$ Coulomb collisions are dominated by scattering off the thermal electrons

since $\frac{m_e}{m_p} \sim \frac{1}{1836}$

- Interactions in neutral matter: account also for ionization potential

4) Catastrophic losses from fragmentation and radioactive decay:

In inelastic p-nucleus and α-nucleus collisions with atoms and molecules of the ISM / IGM target gas, CR nuclei with charge greater than 1 can fragment in the reactions

$$A + \begin{pmatrix} p \\ \alpha \end{pmatrix} \rightarrow (A-k) + \text{anything}, \quad k = 1, 2, 3, \ldots (A-1)$$

3. Hadronic p-p interactions

3.1 Motivation

- produces secondary electrons that emit (1) radio synchrotron radiation (possibly responsible for galaxy cluster-wide radio emission; radio synchrotron emission of the Galaxy – in competition to radio emission produced by directly accelerated electrons)
  (2) inverse Compton (IC) scattering of CMB and starlight photons into the X-ray band
- produces γ-ray through neutral pion decay:
  -> dominates diffuse galactic γ-ray emission seen by Egret ($E > 100\,MeV$)
  -> expected to dominate clusters γ-ray emission
  -> possibly responsible for TeV γ-ray emission of individual SNR as seen by imaging air Cerenkov telescopes (HESS, Magic, Veritas, Cangaroo), model competes with IC emission from primary shock-accelerated CR electrons

- expected to be the primary reaction that should produce extragalactic neutrinos in starburst galaxies, clusters, AGN.
  -> ν-Astronomy with AMANDA, ICECUBE, ...

3.2. Threshold energy for $p+p \to 2p + \pi^0$

Gedankenexperiment: start with 2 slow protons, increase their relative energy until they have just enough energy to produce a pion at rest.

CMS:

E: $\quad 2\gamma m_p = 2m_p + m_\pi \qquad$ (notation: $\gamma_{CM} = \gamma$)

p: $\quad \beta\gamma m_p - \beta\gamma m_p = 0 \qquad$ exactly @ threshold for π-production!

$$\to \gamma = 1 + \frac{m_\pi}{2m_p} \quad \sim E_{th,CoM} = m_p c^2 \gamma = \underline{1.008\,\text{GeV}}$$

→ Choice of Lorentz transformation such, that one particle is at rest in the Lab frame:

$\underline{\gamma_{LT} = \gamma_{CM}} \qquad (\gamma' = \gamma_{Lab})$

$$\cancel{m_p}\begin{pmatrix} \gamma' \\ \beta'\gamma' \end{pmatrix} = \cancel{m_p}\begin{pmatrix} \gamma & \pm\beta\gamma \\ \pm\beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} \gamma \\ \beta\gamma \end{pmatrix} = \begin{pmatrix} \gamma^2 \pm \beta^2\gamma^2 \\ \pm\beta\gamma^2 + \beta\gamma^2 \end{pmatrix} = \begin{cases} \begin{pmatrix} \gamma^2(1+\beta^2) \\ 2\beta\gamma^2 \end{pmatrix} & \text{fast moving } p \\[2ex] \begin{pmatrix} \gamma^2 - \beta^2\gamma^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} & p \text{ @ rest} \end{cases}$$

using $\beta^2\gamma^2 = \dfrac{\frac{v^2}{c^2} - 1 + 1}{1 - \frac{v^2}{c^2}} = \gamma^2 - 1$

Lab:

$$\gamma'_{th} = \gamma^2 + \gamma^2\beta^2 = 2\gamma^2 - 1 = 2\left(1 + \frac{m_\pi}{2m_p}\right)^2 - 1$$

$$E'_{threshold} = m_p c^2 \gamma'_{th} = \underline{1.218\,\text{GeV}}$$

Mathematica: units$\left[\gamma' \cdot \dfrac{m_p c^2}{\text{GeV}}\right] = 1.218$

$$p'_{thr} \cdot c = \sqrt{E'^2_{thr} - m_p^2 c^4} = 0.7766\,\text{GeV} = 0.828 \cdot m_p c^2$$

3.3. Pion source function

the pion production spectrum $\quad s_\pi \equiv \dfrac{d^4N}{dt\, dV\, dp_\pi\, dp_p} \qquad p = \dfrac{P}{m_p c}$

$$s_\pi(p_\pi, p_p) = c\, n_N\, \xi(p_p)\, \sigma_{pp}^{\pi}(p_p)\, \delta_D(p_\pi - \langle p_\pi \rangle)\, \Theta(p_p - p_{th})$$

n_N ... target nucleon density,

σ_{pp}^{π} ... inelastic cross section for the pp interaction, $\bar{\sigma}_{pp} \simeq 32$ mbarn

$\langle p_\pi \rangle$ ... average energy of a single produced pion, $\langle p_\pi \rangle = \dfrac{m_p p_p}{2 m_\pi \xi}$,

$p_{th} = 0.78$ ... threshold energy for pion production,

$$s_\pi(p_\pi) = \int_{-\infty}^{\infty} dp_p\, f_p^{(1)}(p_p)\, s_\pi(p_\pi, p_p)$$

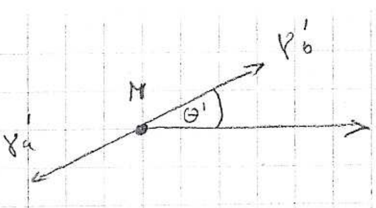
using $f_p = C_p\, p_p^{-\alpha}\, \Theta(p - q)$, assuming $q < p_{th}$

multiplicity $\xi \simeq 2$

$$s_\pi(p_\pi) = 2\, c\, n_N\, \sigma_{pp}\, C_p \int dp_p\, \underbrace{\delta\left(p_\pi - \frac{m_p p_p}{4 m_\pi}\right)}_{\frac{\delta\left(p_p - \frac{4 m_\pi p_\pi}{m_p}\right)}{\frac{m_p}{4 m_\pi}}}\, p_p^{-\alpha}$$

$$s_\pi(p_\pi) = 2 \left(\frac{4 m_\pi}{m_p}\right)^{1-\alpha} c\, n_N\, \sigma_{pp}\, C_p\, p_\pi^{-\alpha}$$

$$s_{\pi^\pm} = \frac{2}{3} s_\pi = 2\, s_{\pi^0}$$

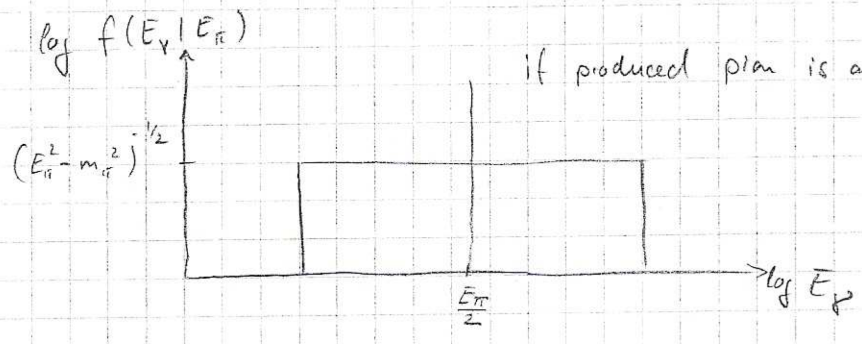


3.4. Pion decay induced γ-rays

$\pi^0 \rightarrow 2\gamma$, (prime denotes CMS quantities)

Homework: $E_\gamma(\theta') = \frac{1}{2}\gamma_\pi m_\pi c^2 (1+\beta_\pi \cos\theta') = \frac{E_\pi}{2}(1+\beta_\pi\cos\theta')$

$$f(E_\gamma|E_\pi) = \begin{cases} (E_\pi^2 - m_\pi^2 c^4)^{-1/2} & \text{for } \frac{E_\pi}{2}(1-\beta_\pi) \le E_\gamma \le \frac{E_\pi}{2}(1+\beta_\pi) \\ 0 & \text{otherwise} \end{cases}$$



if produced pion is at rest → γ-ray line at $E_\gamma = \frac{m_{\pi^0}c^2}{2}$

γ-ray source spectrum:

$$S_\gamma(E_\gamma) = \int_{E_{\pi,min}}^{E_{\pi max}} dE_\pi \; S_\pi(E_\pi)\, f(E_\gamma|E_\pi)\, \xi_{\pi^0\to 2\gamma}\, R_{\pi^0\to 2\gamma}$$

$\xi = 2$ (multiplicity) $R \simeq 1$ (branching ratio)

$E_{\pi,max}$: is the maximum pion energy that is able to produce a γ-ray with E_γ:

$$E_{\pi,max} \underset{\beta_{\pi,max}\simeq 1}{=} \frac{E_\gamma}{[\cos^2(\frac{\theta'}{2})]_{min}} \rightarrow \infty \quad \text{since} \quad [\cos^2(\tfrac{\theta'}{2})]_{min} = 0$$

$E_{\pi,min}$: Consider extreme case, where γ-rays are emitted in the direction of motion

$$E_{\gamma,min} = \frac{1}{2}E_\pi(1-\beta_\pi), \quad E_{\gamma,max} = \frac{1}{2}E_\pi(1+\beta_\pi)$$

$$\rightsquigarrow E_{\gamma_{min}} \cdot E_{\gamma_{max}} = \frac{1}{4} m_\pi^2 c^4 \gamma_\pi^2 (1-\beta_\pi^2) = \frac{m_\pi^2 c^4}{4}$$

$$\rightsquigarrow E_\pi = E_{\gamma,min} + E_{\gamma,max} = E_{\gamma,max} + \frac{m_\pi^2 c^4}{4 E_{\gamma max}}$$

reverse criterion to put lower limit on pion energy integration!

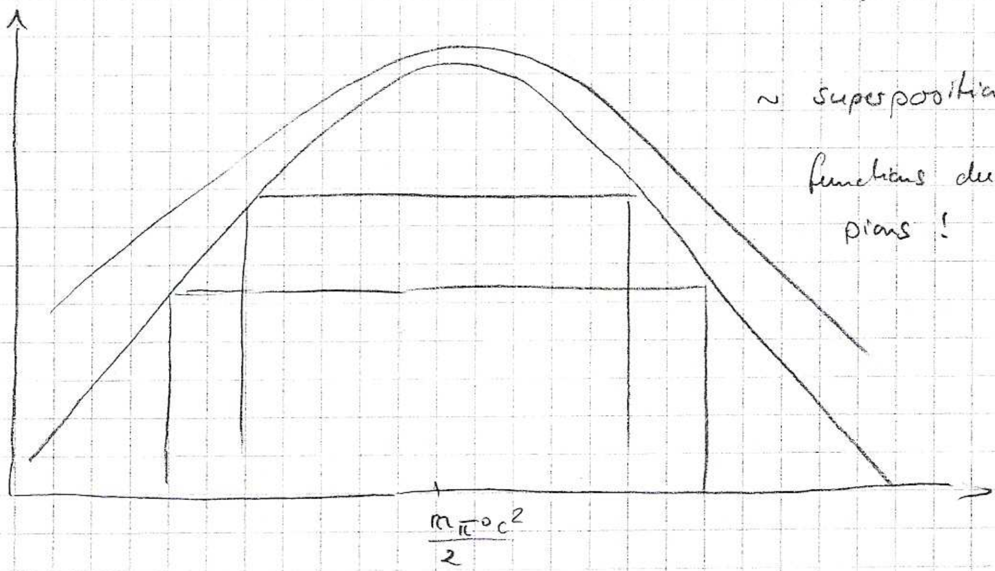
The omnidirectional (i.e. integrated over 4π solid angle) differential γ-ray source function:

$$s_\gamma(\vec{r}, E_\gamma) = 2 \int_{E_\gamma + \frac{m_\pi^2 c^4}{4 E_\gamma}}^{\infty} dE_\pi \frac{s_{\pi^0}(\vec{r}, E_\pi)}{\sqrt{E_\pi^2 - m_\pi^2 c^4}}$$

$$s_{\pi^0} = \frac{2}{3} \left(\frac{4 m_\pi}{m_p} \right)^{1-\alpha} c\, n_N \sigma_{pp} C_p\, p_\pi^{-\alpha}$$ for power-law proton distribution

If one works a little more careful and take into account detailed processes at the threshold of pion production (CR velocity distribution, kaon decay channels, momentum dependend σ_{pp}), on can derive a semi-analytic γ-ray source function:

$$s_\gamma(E_\gamma)\, dE_\gamma \simeq \frac{2^4 C_p}{3\alpha} \frac{\sigma_{pp} n_N}{m_p c} \left(\frac{m_p}{2 m_{\pi^0}} \right)^\alpha \cdot \frac{dE_\gamma}{\left[\left(\frac{2E_\gamma}{m_{\pi^0} c^2} \right)^\delta + \left(\frac{2E_\gamma}{m_{\pi^0} c^2} \right)^{-\delta} \right]^{\alpha/\delta}}$$



~ superposition of γ-ray source functions due to monoenergetic pions!

Electron source function

$$\pi^{\pm} \rightarrow e^{\pm} + 3\nu \quad \rightsquigarrow \quad \langle E_e \rangle = \frac{1}{4} \langle E_{\pi^\pm} \rangle$$

employing the transformation law for distribution function:

$$s_e(p_e)\, dp_e = s_{\pi^\pm}\left[p_\pi(p_e)\right] \frac{dp_\pi}{dp_e}\, dp_e = \frac{4 m_e}{m_\pi}\, s_{\pi^\pm}\left(\frac{4 m_e}{m_\pi} p_e\right) dp_e$$

$$s_e(p_e) = \frac{2}{3} s_\pi = \frac{8}{3} \frac{m_e}{m_\pi} \cdot 2 \left(\frac{4 m_\pi}{m_p}\right)^{1-\alpha} c\, n_N \sigma_{pp}\, C_p \left(\frac{4 m_e}{m_\pi}\right)^{-\alpha} p_e^{-\alpha}$$

$$= \frac{4}{3} \left(\frac{16\, m_e}{m_p}\right)^{1-\alpha} c\, n_N \sigma_{pp}\, C_p\, p_e^{-\alpha}$$

$\rightsquigarrow$ injection spectrum for secondary CR electrons:

$$f_{inj,pp}\, dp_e = C_{inj,pp}\, p_e^{-\alpha}\, dp_e$$

$$C_{inj,pp} = \frac{4}{3}\, 16^{1-\alpha}\, c\, \tau_{pp}\, n_N \sigma_{pp}\, C_p \left(\frac{m_e}{m_p}\right)^{1-\alpha}, \quad \tau_{pp} = \min\left[\frac{1}{c \sigma_{pp} n}, \tau_{Hubble}\right]$$

Steady state spectrum: balancing injection of secondaries and IC/synchrotron cooling of the CR electrons ($\dot{p} < 0$):

$$\frac{\partial}{\partial p}\left[\dot{p}(p)\, f_e(p)\right] = s_e(p)$$

$$\rightsquigarrow f_e(p) = \frac{1}{|\dot{p}|} \int_p^\infty dp'\, s_e(p') = C_e\, p^{-\alpha_e}$$

$$\dot{p}_{IC/syn} = \frac{4 \sigma_T c}{3 m_e c^2} \left(\varepsilon_{ph} + \varepsilon_B\right) \gamma^2 \beta^2$$

$$C_e = \frac{16^{2-\alpha_e}\, \sigma_{pp}\, n_N\, C_p\, m_e c^2}{(\alpha_e - 2)\, \sigma_T\, (\varepsilon_{ph} + \varepsilon_B)} \left(\frac{m_p}{m_e}\right)^{\alpha_e - 2}, \quad \alpha_e = \alpha_p + 1$$

$\Rightarrow$ Use f_e to compute resulting IC / sync emission!