Nonlinear Cosmic Ray Transport

$90°$ problem ?! No scattering at $90°$ in QLT $\Rightarrow l_{mfp} \to \infty$

Key assumption in QLT: unperturbed orbit.

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magnetic moment $\frac{v_\perp^2}{B}$ = const is an adiabatic invariant.

$$\frac{\Delta v_\parallel}{v_\perp} = \left(\frac{\langle (B-B_0)^2\rangle}{B_0^2}\right)^{\frac{1}{4}} \simeq \left[\frac{\langle \delta B_\parallel^2\rangle}{B_0^2} + o\left(\frac{\langle \delta B_\perp^2\rangle^2}{B_0^4}\right)\right]^{1/4}$$

Assuming the guiding center has a gaussian distribution along the field line,

$$f(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} e^{-\frac{(z-\langle z\rangle)^2}{2\sigma_z^2}}$$

Integrating over z, we get

$$\int_{-\infty}^{\infty} dz\, e^{ik_\parallel z} f(z) = e^{ik_\parallel \langle z\rangle} e^{-k_\parallel^2 \sigma_z^2/2}$$

$$\sigma_z = \langle \Delta v_\parallel^2\rangle t^2 = v_\perp^2 \left(\frac{\langle \delta B_\parallel^2\rangle}{B_0^2}\right)^{\frac{1}{2}} t^2$$

$$R_n(k_\parallel v_\parallel - \omega \pm n\Omega)$$

$$= Re \int_0^\infty dt\, e^{i(k_\parallel v_\parallel + n\Omega - \omega)t - \frac{1}{2}k_\parallel^2 v_\perp^2 t^2 \left(\frac{\langle \delta B_\parallel^2\rangle}{B_0^2}\right)^{1/2}}$$

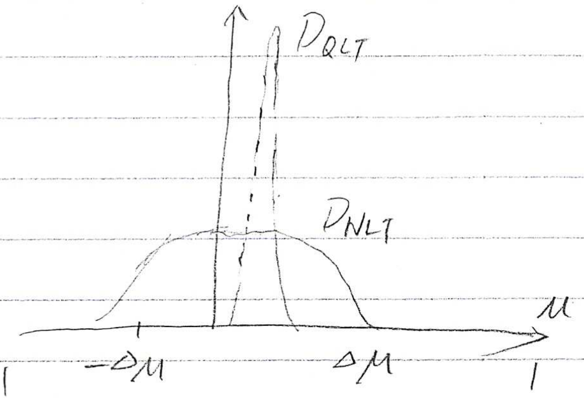
$$= \frac{\sqrt{\pi}}{|k_\parallel \Delta v_\parallel|} \exp\left[-\frac{(k_\parallel v_\parallel - \omega + n\Omega)^2}{k_\parallel^2 \Delta v_\parallel^2}\right] \qquad \Delta v_\parallel \simeq v_\perp M_A^{1/2}$$

(13)

$$D_{\mu\mu}^T = \frac{v\sqrt{\pi}(1-\mu^2)}{2LR^2}\int_1^{k_{max}L} dx \int_0^1 \frac{x^{-5/2}}{\Delta\mu_{\parallel}} J_1^2(w) \exp\left[-\frac{(\mu - v_A/v)^2}{\Delta\mu^2}\right].$$

$$W = \frac{k_\perp v_\perp}{\Omega} = x_\perp R\sqrt{1-\mu^2}, \quad R = \frac{v}{l\Omega}.$$

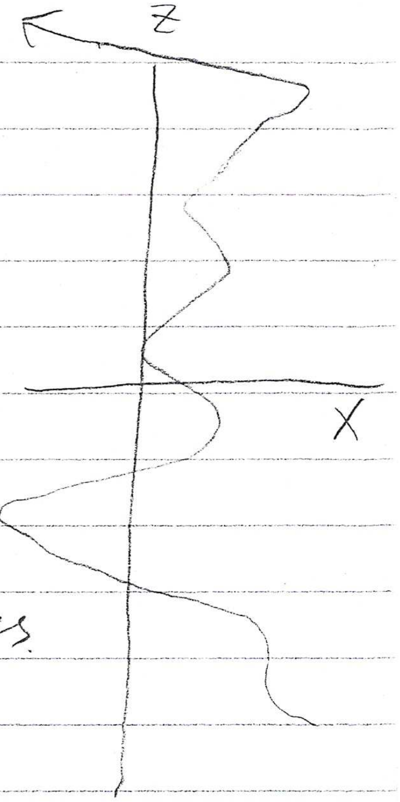
$$\Delta\mu = \frac{\Delta v_{\parallel}}{v} = \sqrt{1-\mu^2}\, M_A^{\frac{1}{2}}$$



CRs' mean free path:

$$\lambda_{\parallel}/L = \frac{3}{4}\int_0^1 d\mu \frac{v(1-\mu^2)^2}{(D_{\mu\mu}^T + D_{\mu\mu}^G)L}$$

Cross field transport



Is perpendicular transport subdiffusive?

$\delta z^2 = D_{\parallel} t$.

Random walk of field lines, $\delta x^2 = D_{spat}\, \delta z$ $\Rightarrow$
(Retracing)

$\delta x^2 = D_{spat} \cdot D_{\parallel}^{\frac{1}{2}} \cdot t^{1/2}$, only true in slab waves.

What happens in turbulence?

field line seperation Δx grows exponentially;

Once Δx reaches the size of minimum eddy, Δx grows monochromatically with δz, no retracing can happen in x direction any more. perpendicular motion becomes diffussive !!

1. perpendicular diffusion on large scale

1. $M_A > 1$, $\rho V_L^2 > B^2$ (e.g. cluster of galaxies)

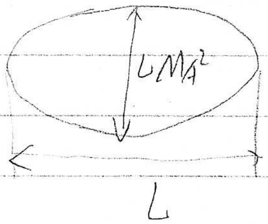
if a) $\lambda_{\parallel} \gg l_A$, $D_{\perp} = D_{\parallel} \simeq \frac{1}{3} l_A v$ $\quad l_A = L / M_A^3$

b) $\lambda_{\parallel} < l_A$, $D_{\perp} = D_{\parallel} \simeq \frac{1}{3} \lambda_{\parallel} v$.

No distinction between $\parallel$ and $\perp$ directions.

2. $M_A < 1$, eddies are anisotropic on large scale

$$L_\perp = L_\parallel \cdot M_A^2.$$



a) if $\lambda_\parallel > L$, the step size of random walk in $\perp$ direction is LM_A^2, to diffuse over a distance R, one needs $\left(\frac{R}{LM_A^2}\right)^2$ steps. Thus.

$$D_\perp = \frac{R^2}{\delta t} = \frac{R^2}{(R/LM_A^2)^2 L/v_\parallel} \simeq \frac{1}{3} L v M_A^4$$

b) if $\lambda_\parallel < L$, the time needed for individual step is $\frac{L^2}{D_\parallel}$.

$$D_\perp = \frac{R^2}{\delta t} = \frac{R^2}{(R/LM_A^2) \cdot \frac{L^2}{D_\parallel}} = D_\parallel M_A^4$$

II Perpendicular diffusion on small scales.

1. $M_A > 1$

$$\langle \delta x^2 \rangle^{1/2} = \frac{|\delta z|^{\frac{3}{2}}}{3^{3/2} l_A^{1/2}} = \frac{(|\delta z| M_A)^{3/2}}{3^{3/2} L^{1/2}}$$

$$D_\perp = \frac{\delta x^2}{\delta t} = \left(\frac{\delta x}{\delta z}\right)^2 D_\parallel = \frac{|\delta z| M_A^3}{3^3 L} D_\parallel \simeq D_\parallel (k_\parallel l_A)^{-1}$$

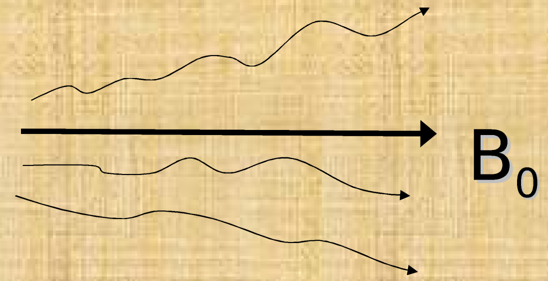
2. $M_A < 1$

$$\langle \delta x^2 \rangle^{1/2} = \frac{|\delta z|^{3/2}}{3^{3/2} L^{1/2}} M_A^2,$$

$$D_\perp = \left(\frac{\delta x}{\delta z}\right)^2 = \frac{D_\parallel \delta z}{3^3 L} M_A^4 \simeq D_\parallel (k_\parallel L)^{-1} M_A^4$$

Perpendicular transport

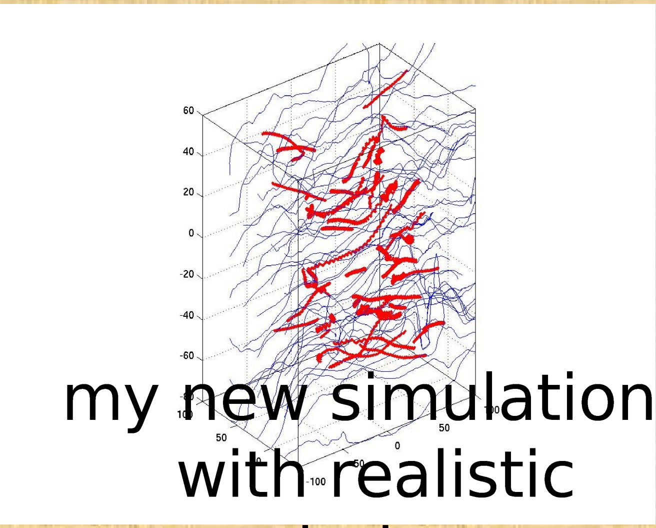
- Dominated by field line wandering.



FLRW model (Jokipii 1966)

Intensive studies:

e.g., Jokipii & Parker 1969, Forman 74, Urch 77, Bieber & Matthaeus 97, Giacolone & Jokipii 99, Matthaeus et al 03, Shalchi et al. 04



my new simulations with realistic turbulence

– Particle trajectory

— Magnetic field

What if we use the tested model of turbulence?

Is there subdiffusion ($\Delta x^2 \propto \Delta t^a$, $a<1$) ?

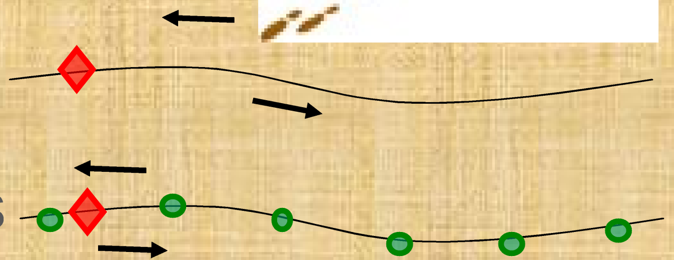
- Subdiffusion (or compound diffusion, Getmantsev 62, Lingenfelter et al 71, Fisk et al. 73, Webb et al 06) was observed in near-slab turbulence, which can occur on small scales, due to instability.

- What about large scale turbulence?

Example: diffusion of a dye on a rope

a) A rope allowing retracing, $\Delta t = l_{rope}^2 / D$

b) A rope limiting retracing within pieces l_{rope} / n, $\Delta t = l_{rope}^2 / nD$

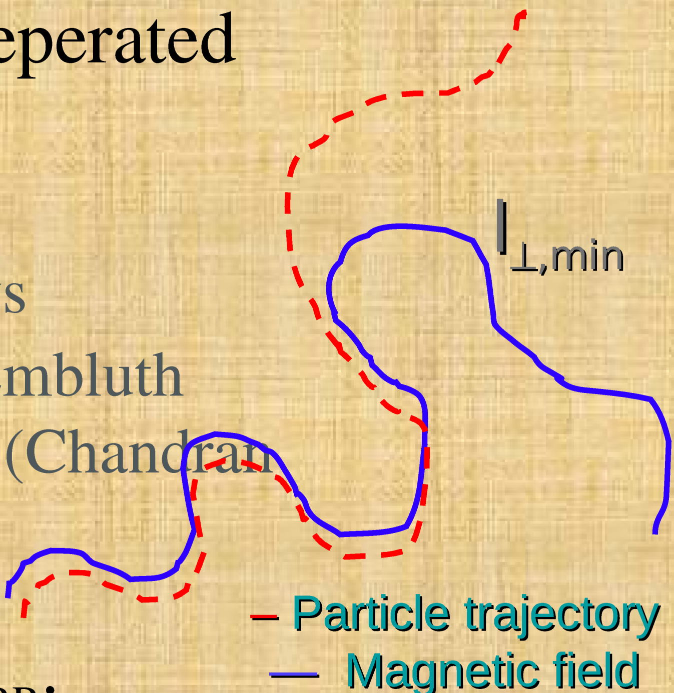


Diffusion is slow only if particles retrace their trajectories.

When does subdiffusion occur?

- In turbulence, CRs' trajectory become independent when field lines are seperated by the smallest eddy size , $l_{\perp,min}$.

The separation between field lines grows exponentially, provides Rechester-Rosembluth distance, $L_{RR} = l_{\|,min} \log(l_{\perp,min} / r_L)$ (Chandran & Cowley 98, Lazarian 06)



- Subdiffusion only occurs below L_{RR}. Beyond L_{RR}, normal diffusion applies and our calculations are correct.

Perpendicular transport ($\lambda_{||} > L$)

∀ $\lambda_{||} > L$, CR diffusion is controlled by field line wandering

- $M_A < 1$, CRs free stream over distance L, thus $\Delta t = (R/L\, M_A^2)^2\, L/v_{||}$, $D_\perp = R^2/\Delta t = L v_{||} M_A^4$ (differs from the FLRW result)
- $M_A > 1$, $D_\perp = L\, v_{||}$

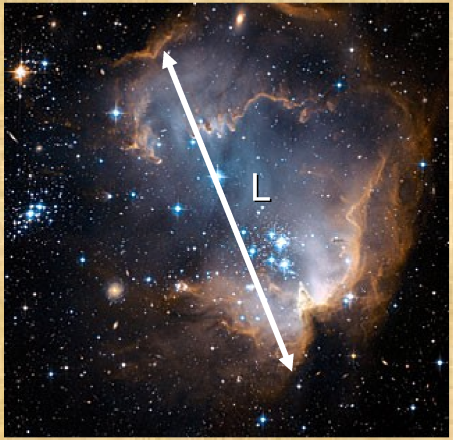
Whether and to what degree $\perp$ diffusion is suppressed depends on $\lambda_{||}$ and M_A.

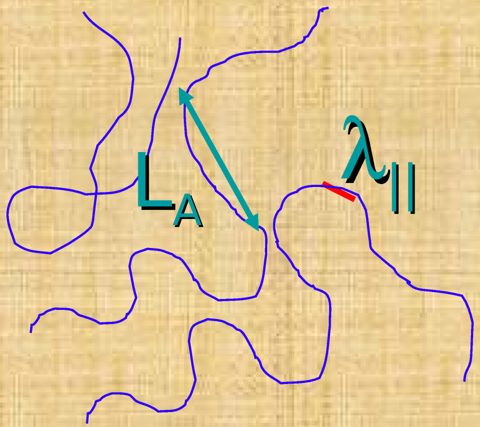
Perpendicular diffusion ($\lambda_{||} < L$)

- $M_A < 1$, on large scale CRs need to diffuse L in order to cover a distance LM_A^2 in ⊥ direction, thus $\Delta t = (R/LM_A^2)^2 L^2/D_{||}$ → $D_\perp = R^2/\Delta t = D_{||} M_A^4$

(Lazarian 06, Yan & Lazarian 07)

- $M_A > 1$, $D_\perp = D_{||}$; the stiffness of B field is negligible for $\lambda_{||} << L_A$





Perpendicular diffusion depends on $M_A = \delta B/B_0$.