Gravitational waves, black hole simulations, and the validity of Post-Newtonian theory

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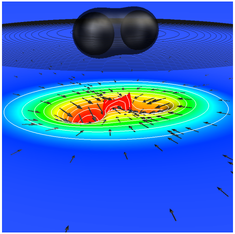
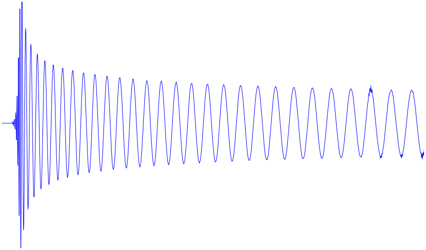




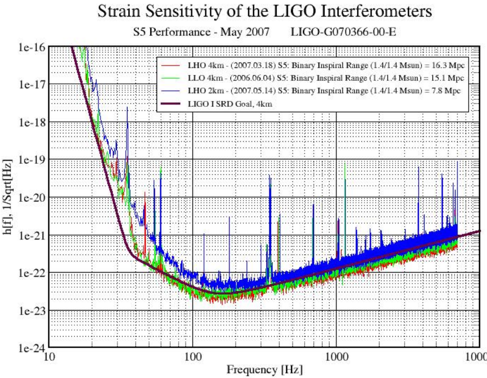
Gravitational Waves

- Einstein's equations admit wave-solutions

$$g_{ab} = \eta_{ab} + h_{ab} \qquad \Box \bar{h}_{ab} = 0$$

- Efforts are underway to detect these gravitational waves

LIGO Hanford



Gravitational wave detectors

LISA (planned)

LIGO (2 sites)

VIRGO

GEO 600

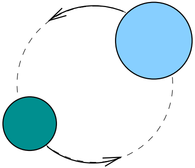
Gravitational Wave Sources

- Generated by changing quadrupole moments

$$h_{ij} = \frac{1}{r}\ddot{Q}_{ij}$$

- **Compact object binaries**

$$\Omega^2 r^3 = GM, \quad f_{\rm GW} = 2\frac{\Omega}{2\pi}$$



- Close to merger

$$r \sim 10Gm/c^2 \quad \Rightarrow \quad f_{\rm GW} \sim 2\text{kHz}\left(\frac{M}{M_\odot}\right)^{-1}$$

- $M = 1 \cdots 100 M_\odot \quad \Rightarrow$ LIGO
- $M = 10^4 \cdots 10^7 M_\odot \quad \Rightarrow$ LISA

Matched Filtering

- Tiny signal, $h = \frac{\Delta L}{L} \sim 10^{-21}$
- Detector output s, waveform template h_T

$$\mathrm{SNR} = \frac{< s, h_T >}{(< h_T, h_T >)^{1/2}},$$

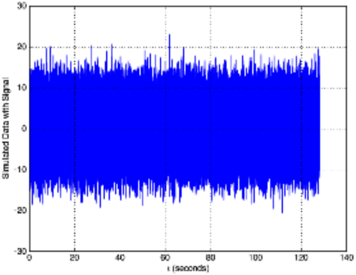
$$< a, b >= \int df\, \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_h(f)}$$

- **Phase of h_T crucial**

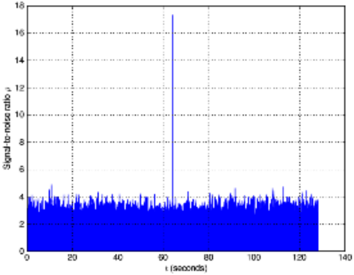
$$\delta\phi \lesssim 1/\mathrm{SNR}$$

$\mathrm{SNR} = 8 \ldots 50$ for advanced LIGO

Simulated detector output



SNR vs. coalescence time



(D. Brown)

Gravitational wave detection

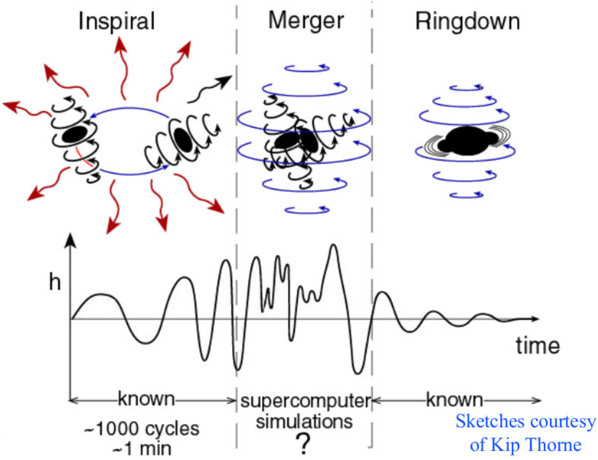
- Current LIGO data analysis pipeline developed before numerical waveforms were available

- Fractional loss in SNR due to “simple” templates
  (stationary phase, truncated at $f_{\rm ISCO}$)

mass	2PN	3.5PN
5+5	0.95	0.99
10+10	0.92	0.93
15+15	0.77	0.79
20+20	0.59	

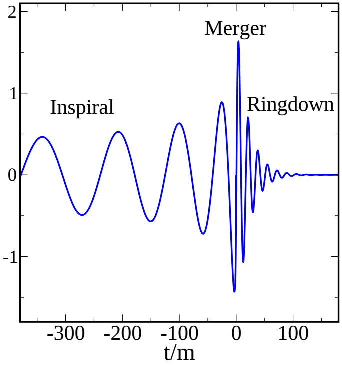
D. Brown; see also Pan, Buonanno et *al.*, 2007

Stages of binary black hole evolution



Tools for computing the waveform

- Inspiral
  - $v \ll c$: perturbative expansion in v/c (post-Newtonian expansion)
  - v/c large: Numerical relativity
- Merger
  - Numerical relativity
- Ringdown
  - BH perturbation theory
  - Numerical relativity



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  – v/c large: Numerical relativity
- Merger
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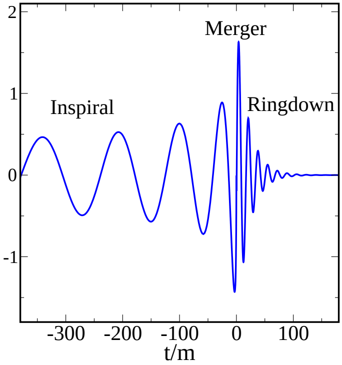
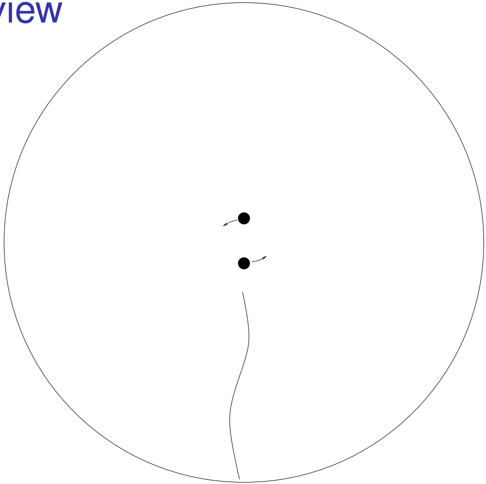
- Ringdown
  – BH perturbation theory
  – Numerical relativity
- Tasks for Numerical relativity:
  – simulate “late” inspiral and merger
  – determine what **“late”** means



BBH Simulations – Overview

Problem characteristics

- Multiple length scales
  - Size of BH's ~ 1
  - Separation ~ 10
  - Wavelength $\lambda \sim 100$
  - Wave extraction at several λ
- Gravitational wave flux small
  - $\dot{E}/E \sim 10^{-5}$
  - $\dot{E}$ drives inspiral
- High accuracy required
  - Absolute phase error $\delta\phi \ll 1$
- Solutions are smooth



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Computational approaches

- Finite difference AMR
  - Albert-Einstein Institut (Berlin), Goddard, Jena (Germany), LSU, Penn State, Princeton, Rochester
  - Impressive short inspirals with mergers (BH-kicks)
  - Accurate long inspirals difficult
- Multi-domain spectral methods
  - Cornell/Caltech collaboration
  - Impressive long inspirals
  - Merger difficult, but possible

Computational Framework I

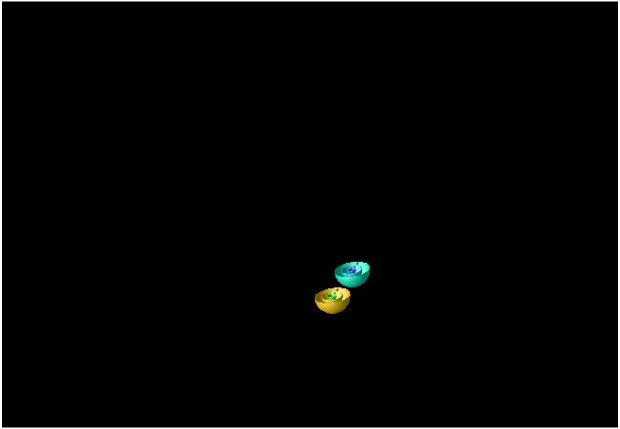
- Pseudo-spectral code

$$u(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi_k(x)$$

- Evaluate derivatives in spectral space,
  non-linear terms in physical space
- Elliptic problems
  Solve huge set of algebraic equations for $\tilde{u}_k$ (HP et *al.* 2003)
- Hyperbolic problems
  Evolve $\tilde{u}_k(t)$ with method of lines
- Code developed by Larry Kidder, HP, Mark Scheel; 250,000 lines

Computational Framework II

- Domain-decomposition



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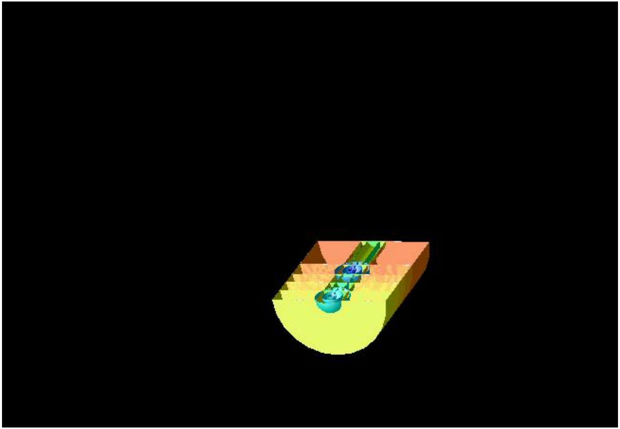

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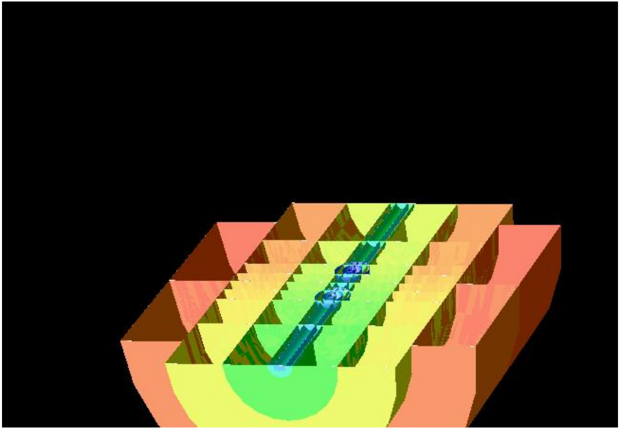

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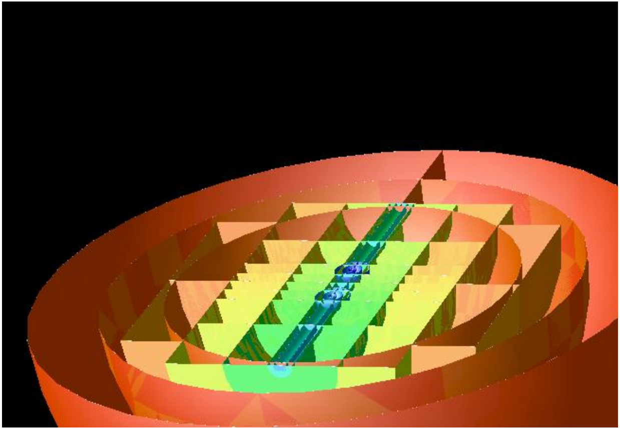

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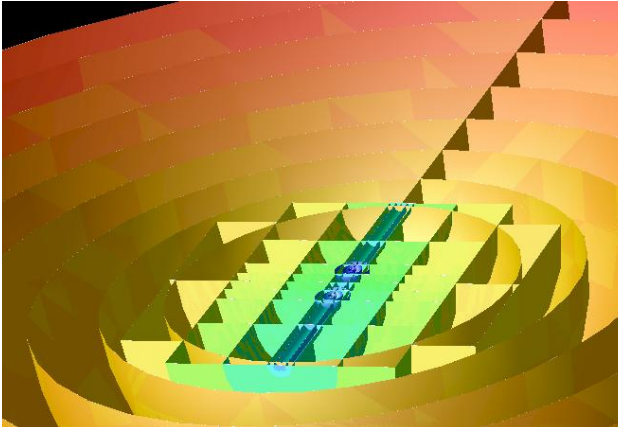


Evolution equations

- Einstein's equations

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms}, \qquad \Gamma_a = -g_{ab}\Box x^b.$$

- Generalized harmonic coordinates $g_{ab}\Box x^b \equiv H_a(x^a, g_{ab})$
(Friedrich 1985, Pretorius 2005; $H = 0$ used since 1920's)

$$\Box g_{ab} = \text{lower order terms.}$$

$\Rightarrow$ Constraint $C_a \equiv H_a - g_{ab}\Box x^b = 0$

- Constraint damping (Gundlach, et *al.*, Pretorius, 2005)

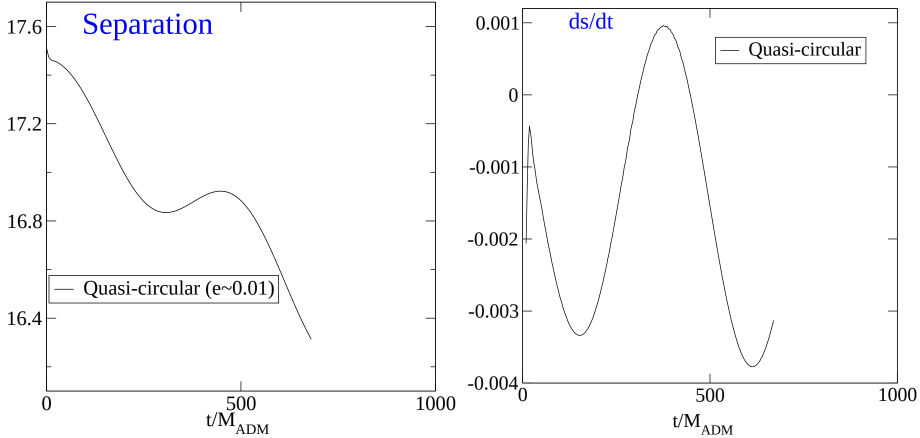
$$\Box g_{ab} = \gamma \left[t_{(a}C_{b)} - \frac{1}{2} g_{ab} t^c C_c \right] + \text{lower order terms}$$

$$\partial_t C_a \sim -\gamma C_a.$$

Boundary conditions

- **Black hole singularity excision**
  - Place artificial inner boundary just inside horizon
  - Causality $\Rightarrow$ pure outflow condition, no BC applied (Unruh, 80's)
  - Nasty details require dual coordinate frames (Scheel et *al.*, 2006)
- **Outer boundary**
  - Constraint preserving (Kidder et *al.* 05; Sarbach, Tiglio 05; Lindblom et al. 06)
  - Transparent to outgoing gravitational waves (Lindblom et *al.*, 2006)
  - No incoming gravitational waves (Lindblom et *al.*, 2006)
  - No reflections of gauge-modes (Rinne et *al.* 2007)

Iteratively control eccentricity (HP et *al.*, 2007)



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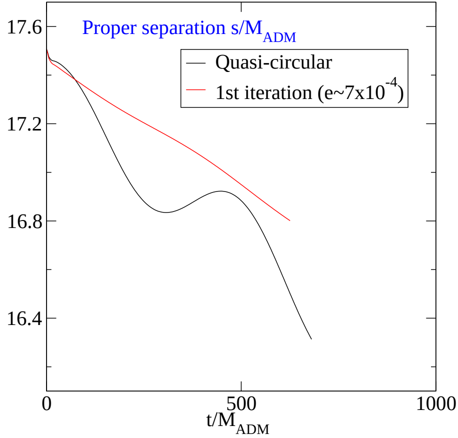

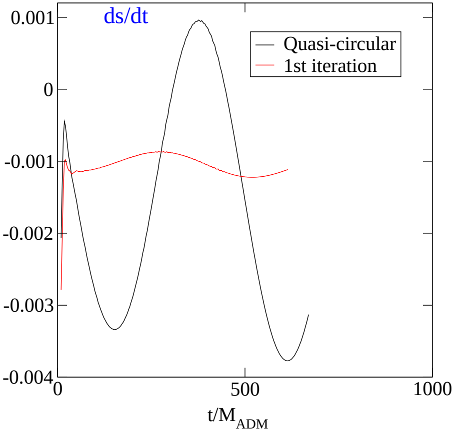

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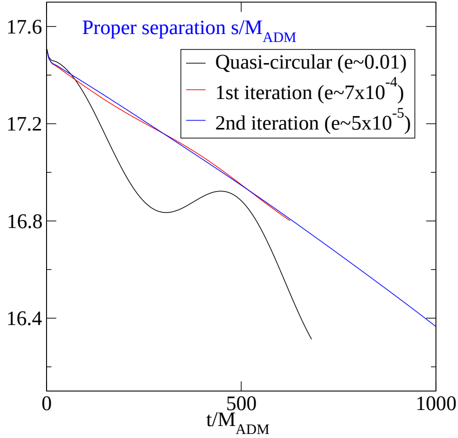
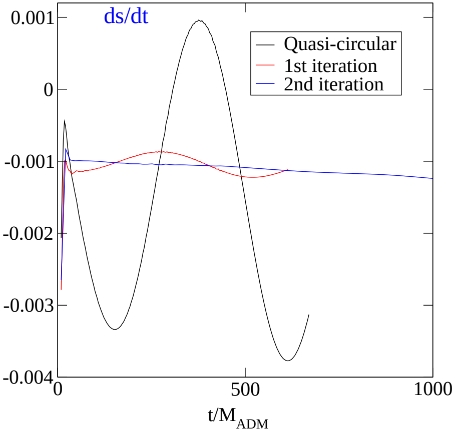
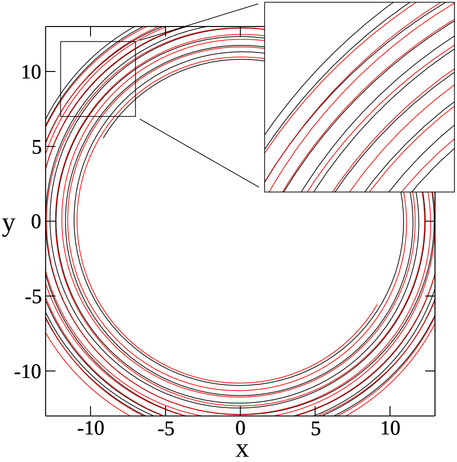
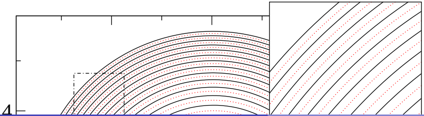




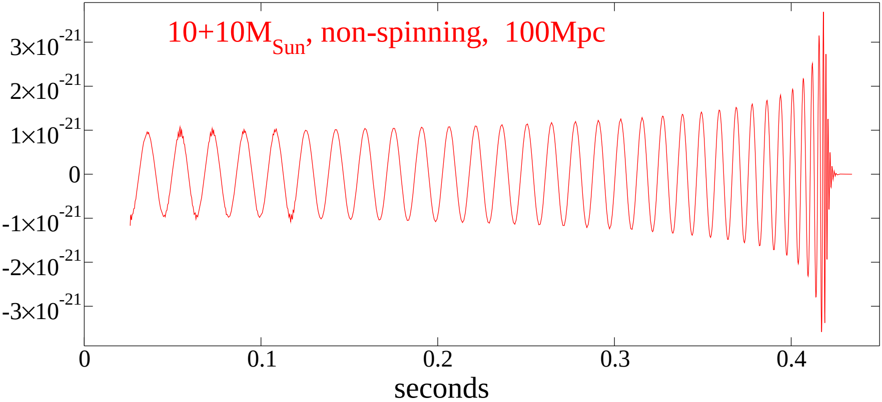
Orbital trajectory

$e = 0.01$ $\qquad$ $e = 5 \times 10^{-5}$

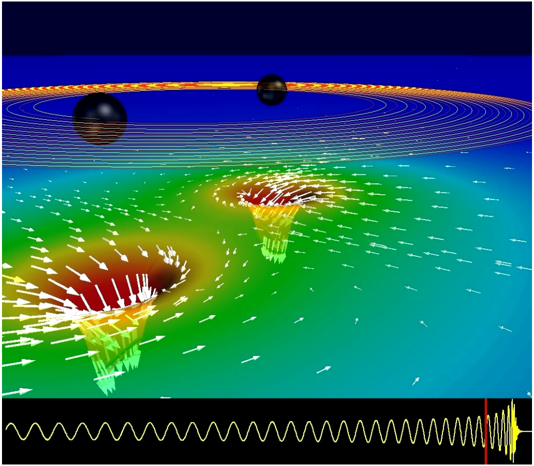




At last – a waveform!!

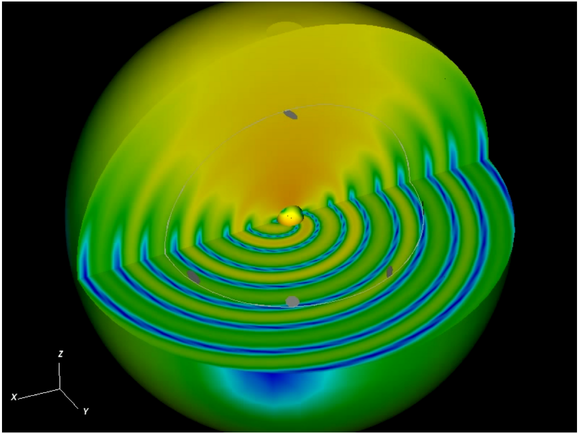


Movies I



www.black-holes.org/explore2.html

Movies II



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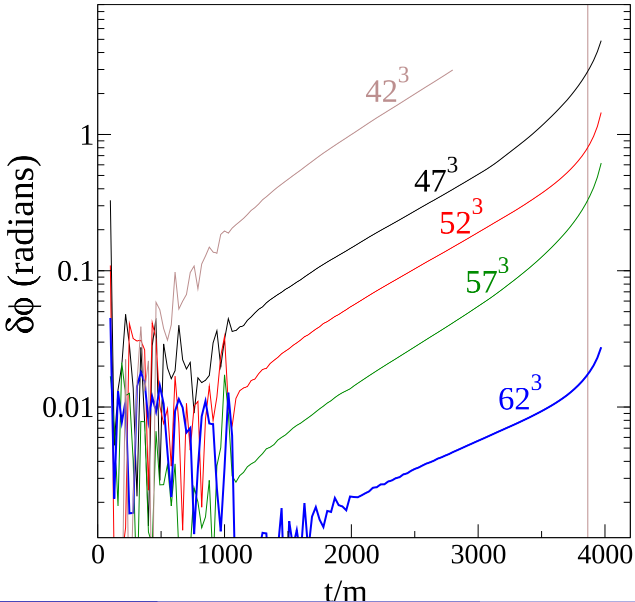
Phase-Accuracy (out of 200 radians)



Post-Newtonian theory

Blanchet, Damour, Iyer, Schäfer, Jaranowski, Faye; Will, Wiseman, Kidder, ...

- Expansion in velocity $v = v/c$
- For a binary in a circular orbit
  - Energy $$E(v) = -\frac{\mu}{2}v^2\left(1+\sum_{k=1}^{7} a_k v^k\right)$$
  - GW-Flux $$F(v) = \frac{32\nu}{5}\mu v^{10}\left(1+\sum_{k=1}^{7} b_k v^k\right)$$
- Energy-balance gives time-evolution:

$$\frac{dE}{dt} = -F \quad \Rightarrow \quad \frac{dv}{dt} = -\frac{F}{dE/dv}$$

- **Difficulty:** $v/c \sim 0.3$ during late inspiral
  - Slow convergence
  - Uncontrolled higher-order terms seizable!

PN-approximants

- Different treatment of uncontrolled higher-order terms, e.g.
  - Use of energy-balance equation (Damour, Iyer, Sathyaprakash, 01)

$$\frac{dv}{dt} = -\frac{F}{dE/dv} \qquad \text{TaylorT1}$$

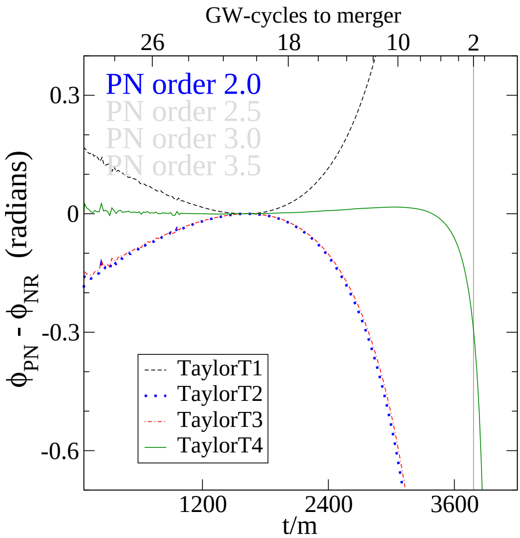
$$\frac{dv}{dt} = -\text{Series}\left[\frac{F}{dE/dv}, \text{v}\right] \qquad \text{TaylorT4}$$

$$\text{Series}\left[\frac{dE/dv}{F}, \text{v}\right]\frac{dv}{dt} = -1 \qquad \text{TaylorT2 \& T3}$$

  - Padé-resummation of F(v) (Damour et *al.* 98; Buonanno et *al.* 98)
  - Effective-One-Body formalism (Damour and collaborators)

Numerical relativity vs. post-Newtonian

Boyle et *al.*, 2007



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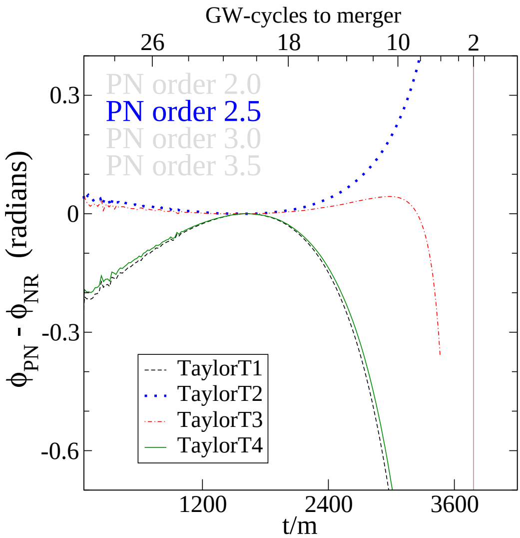

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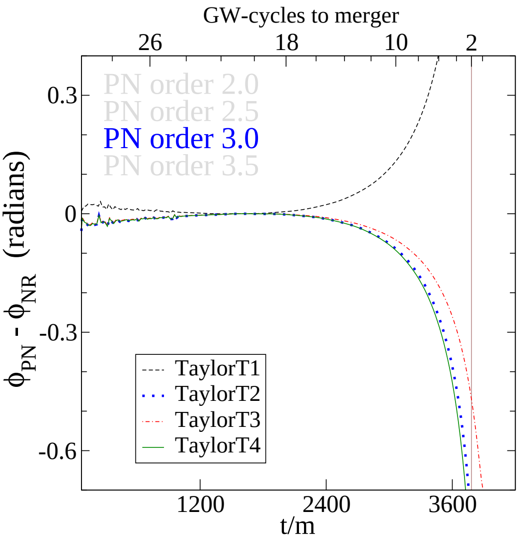

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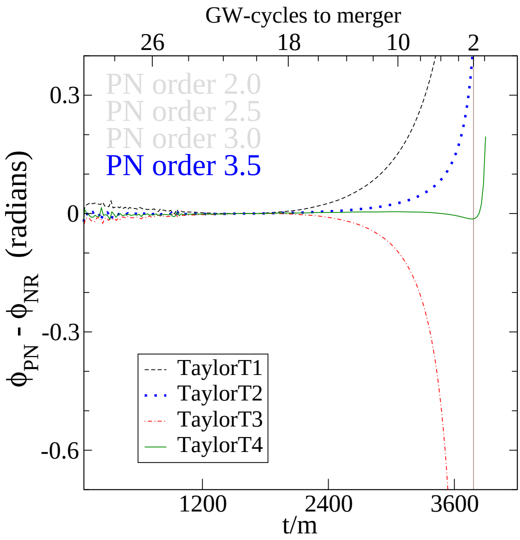
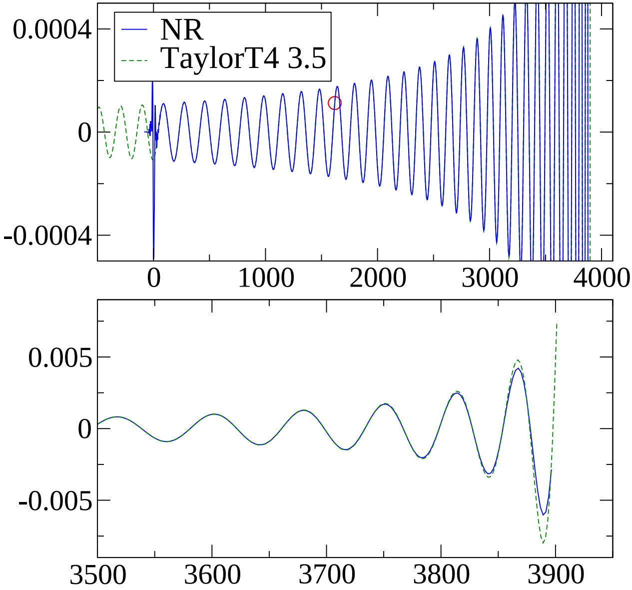


Comparing Waveforms



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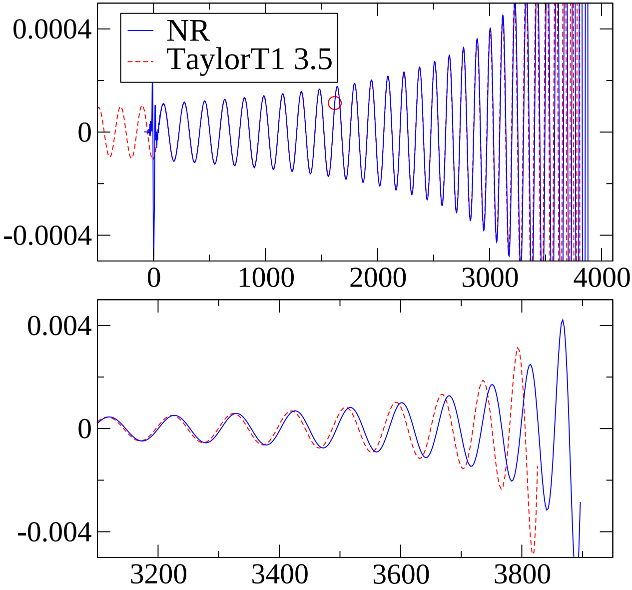
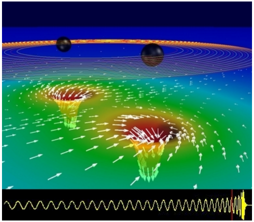


Summary

- GW-detectors require accurate **templates**



- **Spectral BBH evolution code**
  - 15 orbits & merger, $\delta\phi \lesssim \text{few} \times 10^{-2}$ radians
- **PN-NR comparison** (equal masses, non-spinning)
  - Agreement within PN-truncation error
  - Large PN-truncation error in last 20 GW-cycles
  - Only simulations can tell which PN approximants works
  - Must repeat for non-equal masses, spins, ...
- **Collaborators**: Mike Boyle, Lee Lindblom, Oliver Rinne, Mark Scheel (Caltech); Larry Kidder, Abdul Mroue, Saul Teukolsky (Cornell); Duncan Brown (Syracuse), Greg Cook (Wake Forest)