Binary Black Hole Simulations

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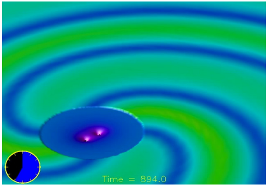
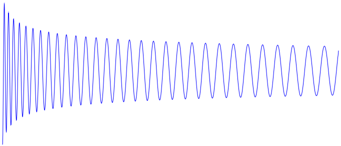
Larry Kidder, Mark Scheel, Saul Teukolsky





APS April Meeting, Jacksonville, Apr 17, 2007

Gravitational wave detectors

LISA (201x)

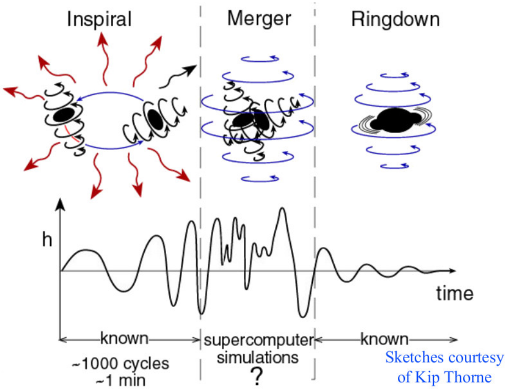
LIGO (2 sites)

VIRGO

GEO 600

- Among prime targets: Binary black hole systems

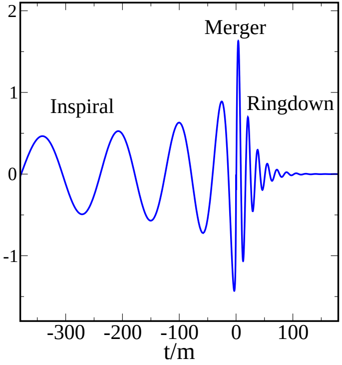
Stages of binary black hole evolution



- Knowledge of waveform allows to
  - Enhance detector sensitivity
  - Test general relativity
  - Extract information about source ($\rightarrow$ astrophysics)

Tools for computing the waveform

- Inspiral
  - $v \ll c$: perturbative expansion in v/c (post-Newtonian expansion)
  - v/c large: Numerical relativity
- Merger
  - Numerical relativity
- Ringdown
  - BH perturbation theory
  - Numerical relativity



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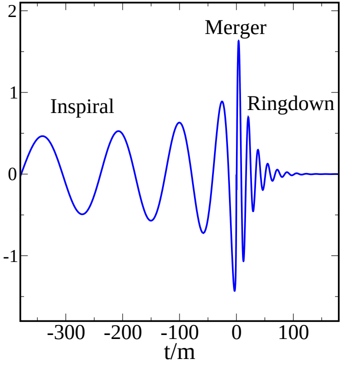
- Tasks for Numerical relativity:
  – simulate “late” inspiral and merger.
  – determine what “late” means.



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  - AMR and spectral infrastructures
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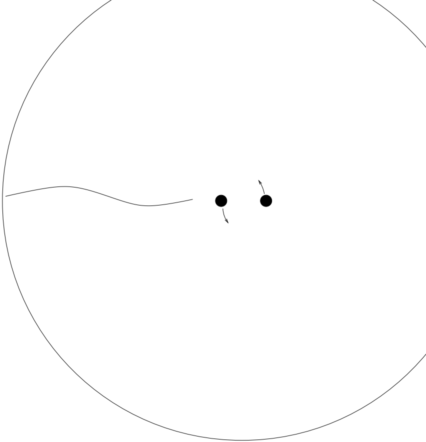
- Since 2005: The Golden Age of Numerical Relativity

Overview

Problem characteristics

- Multiple length scales
  - Size of BH's ~ 1
  - Separation ~ 10
  - Wavelength $\lambda \sim 100$
  - Wave extraction at several λ



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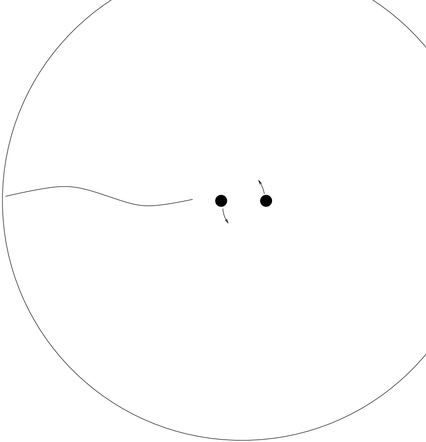
  - $\dot{E}$ drives inspiral



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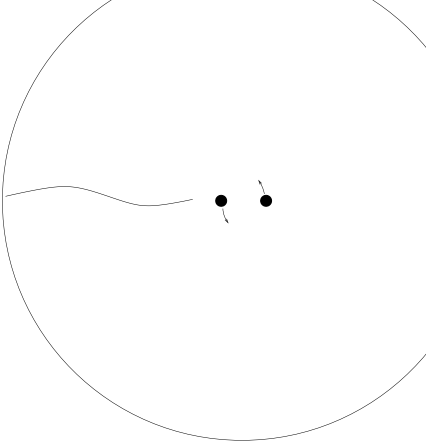

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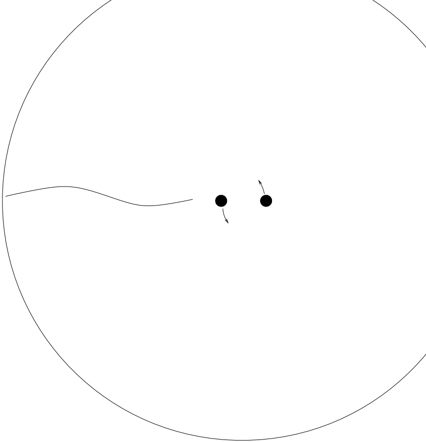

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- Multi-domain spectral methods
  - Cornell/Caltech
  - Impressive long inspiral simulations
  - Merger difficult

Solving Einstein's equations – basic idea

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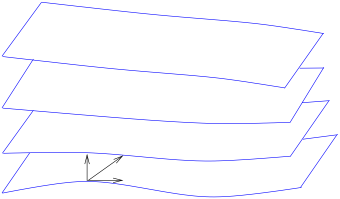

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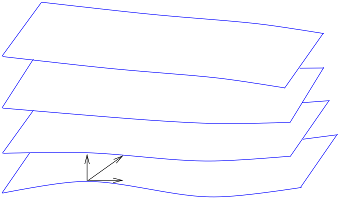

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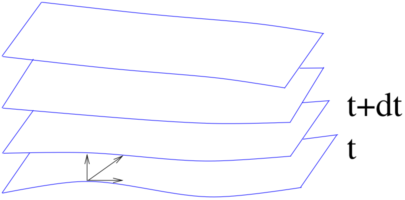

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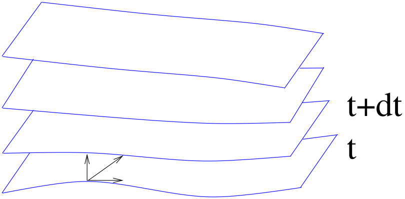

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cf. Maxwell equations

$$\partial_t \vec{E} = \nabla \times \vec{B}$$
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$$\nabla \cdot \vec{E} = 0$$
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- Einstein's equations:

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms} \qquad \Gamma_a = -g_{ab}\Box x^b.$$

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Constraint damping (Gundlach, et *al.*, Pretorius, 2005)

$$0 = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}C_{b)} + \gamma\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^c C_c\right] + \text{l. o.}$$

$$\partial_t C_a \sim -\gamma C_a.$$

Spectral Evolution code

- Rewrite as first order symmetric hyperbolic system (Lindblom et *al.* 2005)

$$\partial_t u + A(u)^k \partial_k u = F(u).$$

- Approximate solution by truncated series

$$u(x,t) \approx u^{(N)}(x,t) \equiv \sum_{k=0}^{N-1} \tilde{u}_k(t)\,\Phi_k(x),$$

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$$\frac{du^{(N)}(x)}{dx} = \sum_{k=0}^{N-1} \tilde{u}_k \frac{d\phi_k(x)}{dx}.$$
- Evolve $u(x_i)$ by method of lines
$$\partial_t u(x_i) = \Big[F - A(u)^k \partial_k u\Big]_{x=x_i}.$$

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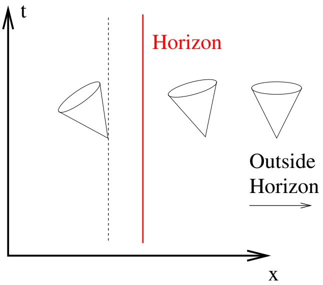

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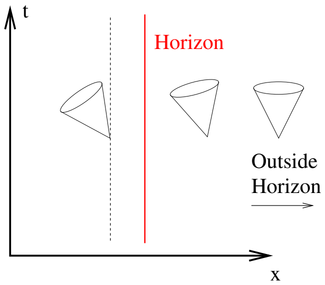
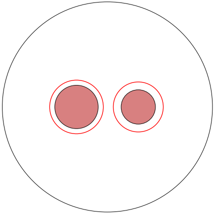
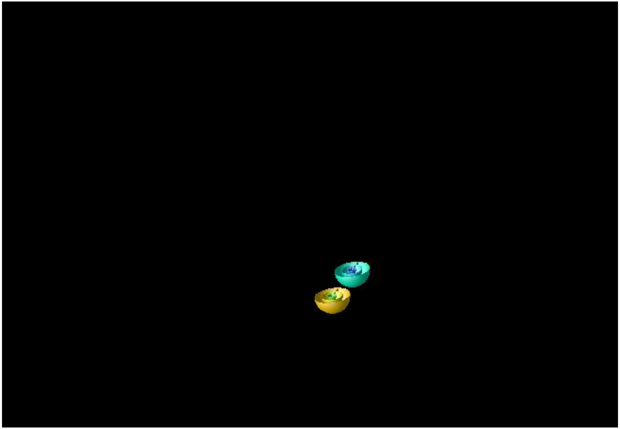




Domain-decomposition



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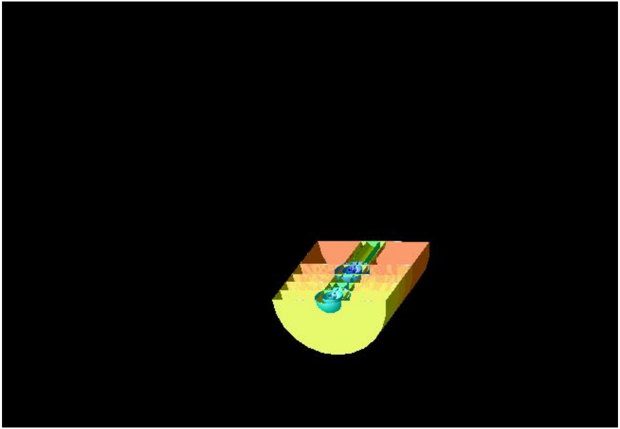

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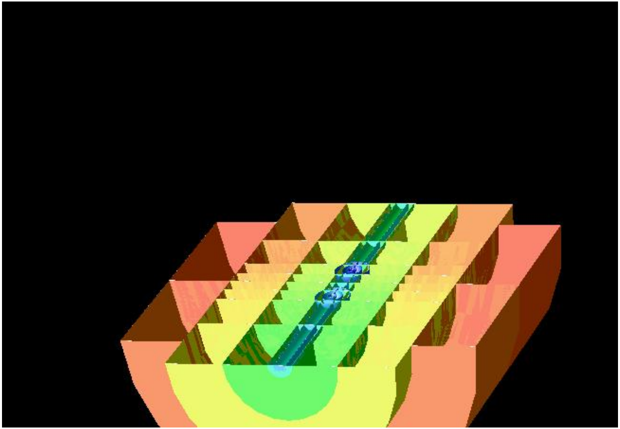

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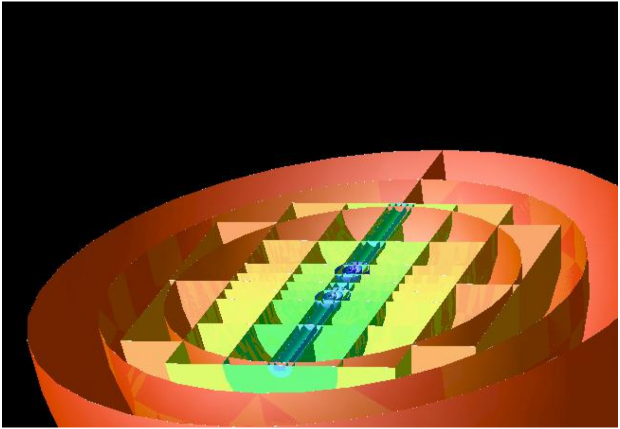

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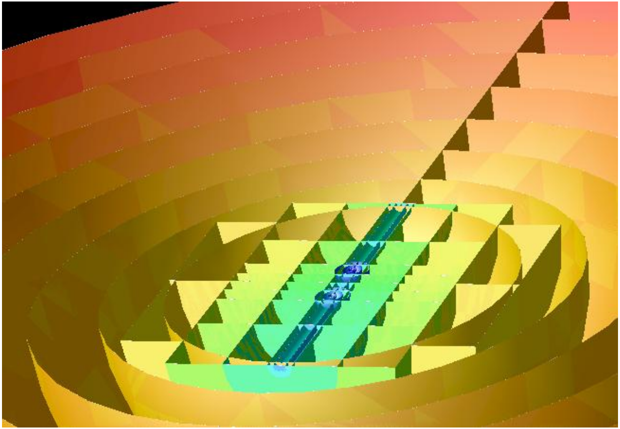
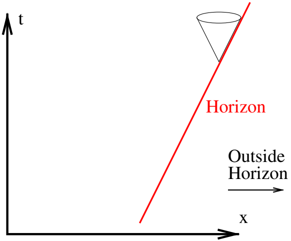


- Outer shells have fixed angular resolution. Cost increases linearly with radius of outer boundary.

Moving black holes – Dual frame method

Scheel et al., 2006



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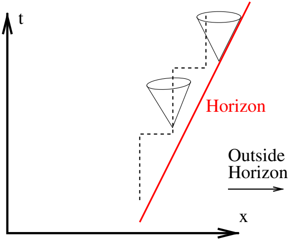

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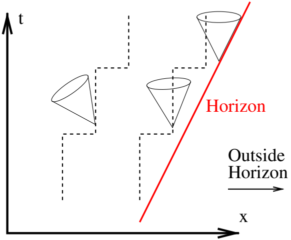

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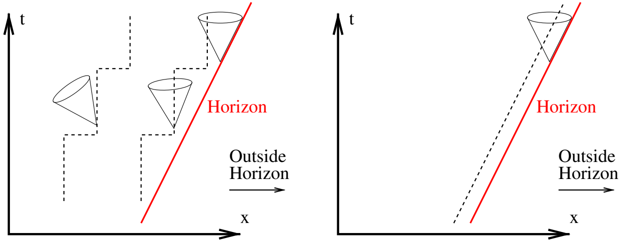

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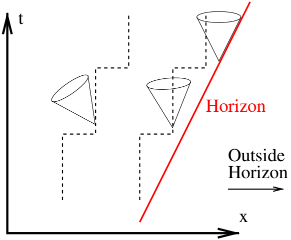

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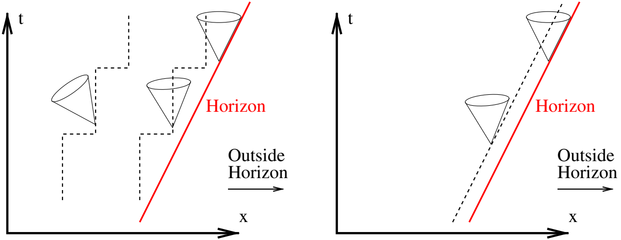
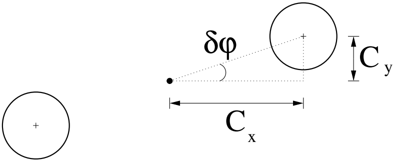


- Map between “moving” and “inertial” coordinates:

$$\vec{x}_{\text{inertial}} = a(t)R(t)\vec{x}_{\text{moving}}$$

$R(t)$ rotation, $a(t)$ radial scaling.

Dynamic feedback control

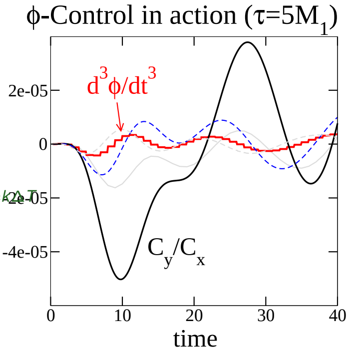


Example: Control of ϕ

- Measure BH location $C_x(t), C_y(t)$
- Update ϕ periodically s.t. $C_y \to 0$:

$$\frac{d^3\phi}{dt^3}(t) = \dddot{\phi}_k, \quad k\Delta T \le t < (k+1)\Delta T$$

$$\dddot{\phi}_k = -\left[\frac{1}{\tau^3}\frac{C_y}{C_x} + \frac{3}{\tau^2}\frac{d}{dt}\left(\frac{C_y}{C_x}\right) + \frac{3}{\tau}\frac{d^2}{dt^2}\left(\frac{C_y}{C_x}\right)\right]_{t=k\Delta T}$$



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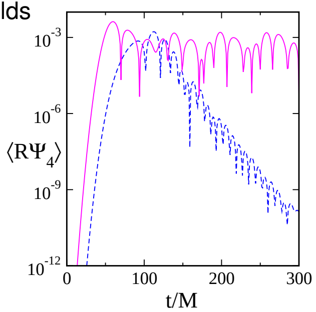

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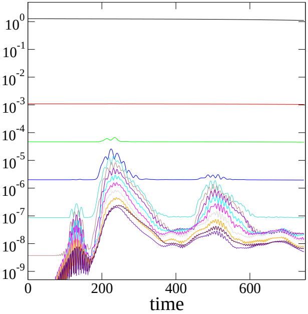

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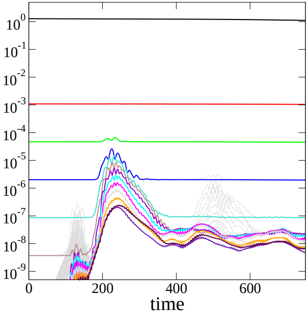
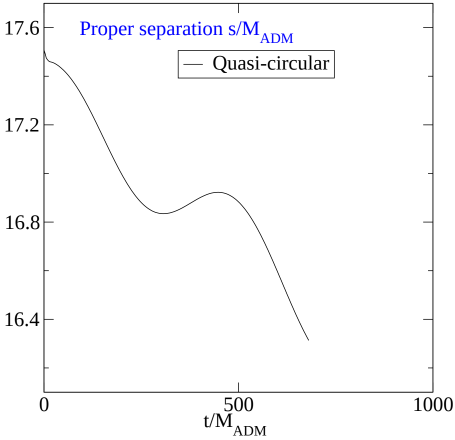
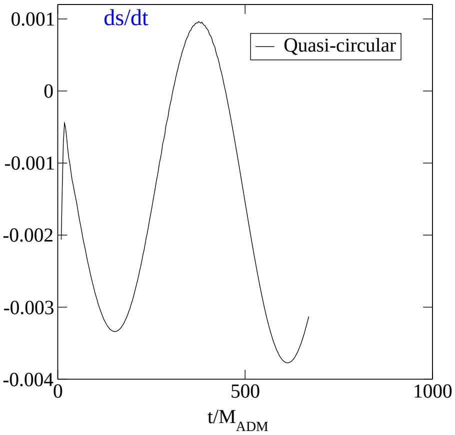


Improve initial data through evolutions

HP et *al.*, 2007

- Circular orbits $\Rightarrow$ eccentric inspiral.





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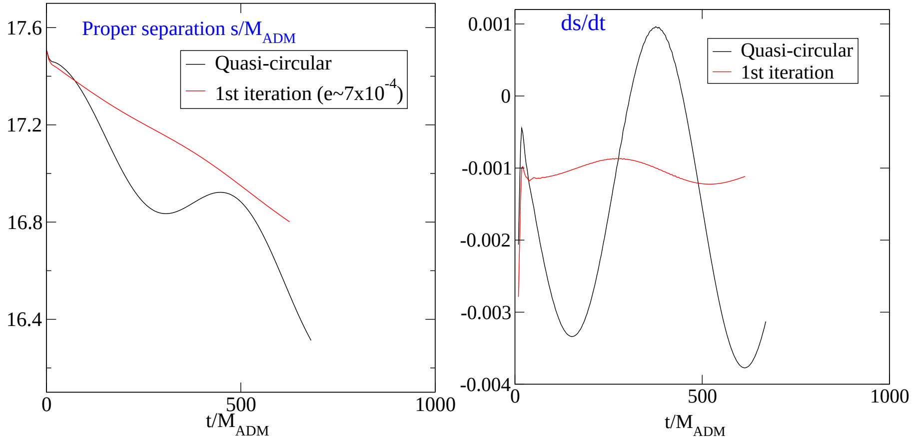
- Allow for nonzero initial radial velocities of BHs.
- Tune radial velocities (and Ω_0) to reduce orbital eccentricity.



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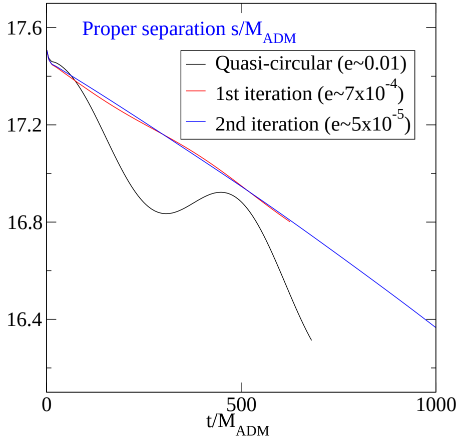
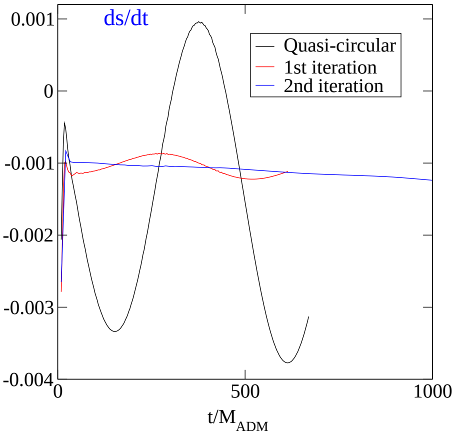
HP et *al.*, 2007

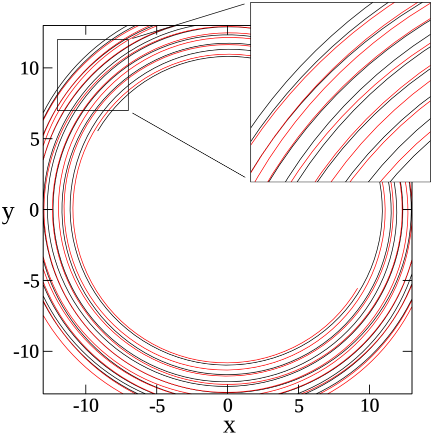
- Circular orbits $\Rightarrow$ eccentric inspiral.
- Allow for nonzero initial radial velocities of BHs.
- Tune radial velocities (and Ω_0) to reduce orbital eccentricity.





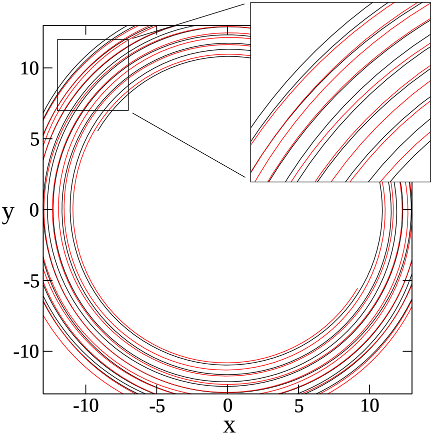
Orbital trajectory

Quasi-circular

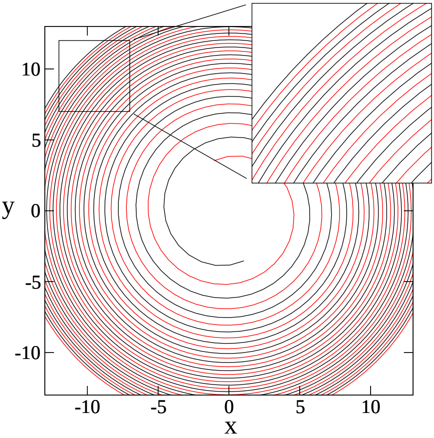


Orbital trajectory

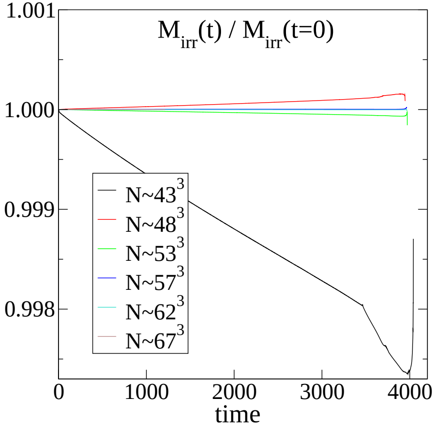
Quasi-circular



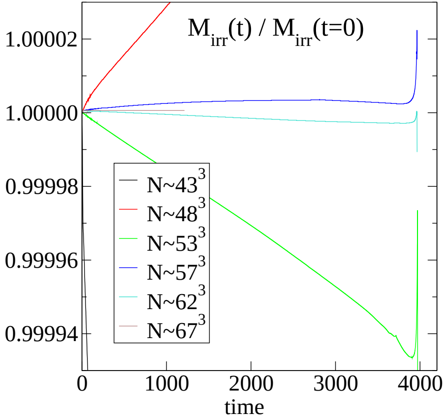
Eccentricity reduced



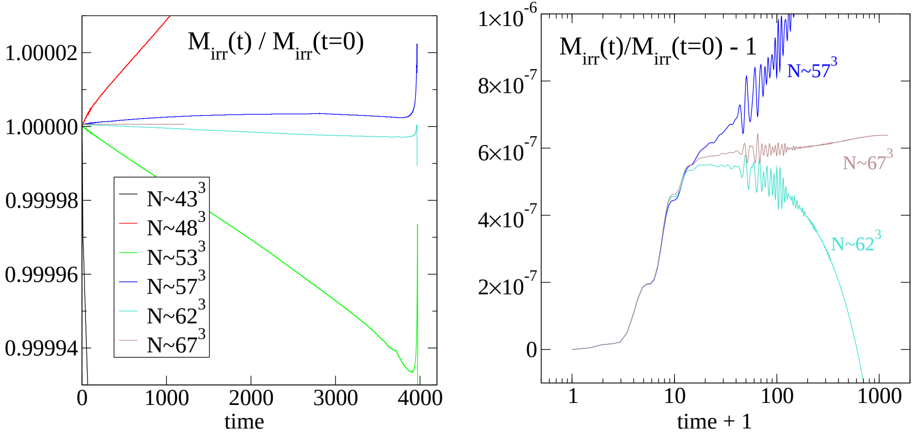
Irreducible Mass



Irreducible Mass



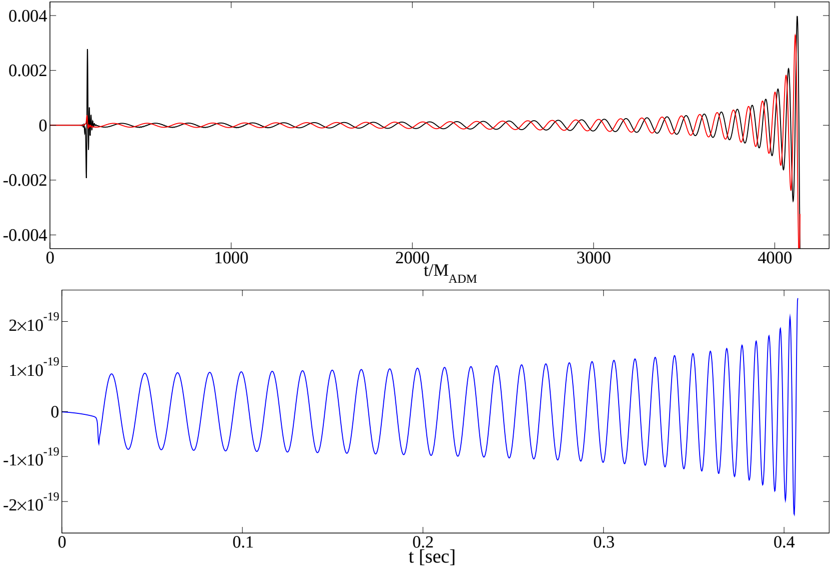
Irreducible Mass



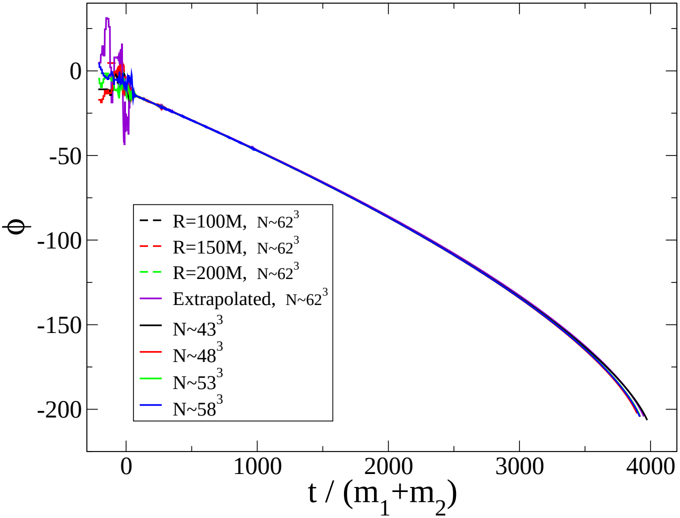
- $M_{\rm irr}$ initially increases by $6 \cdot 10^{-7}$ – "Junk" radiation falling into BH
- $|\delta M_{\rm irr}| < 10^{-7}$ in the next 3.5 orbits – Limit on tidal heating

Movie

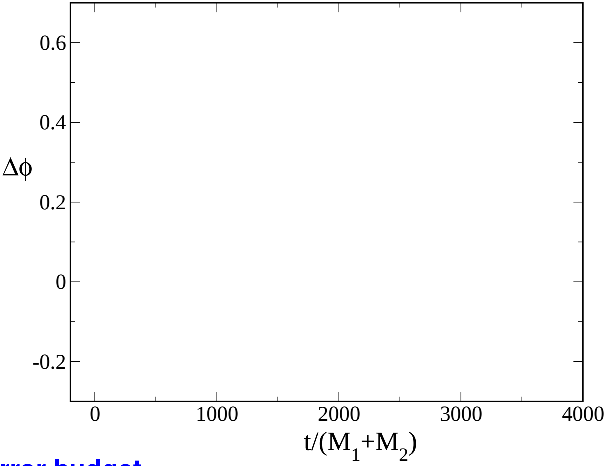
Waveforms


0.004
0.002
0
-0.002
-0.004
0
1000
2000
3000
4000
t/M_{ADM}
2×10^{-19}
1×10^{-19}
0
-1×10^{-19}
-2×10^{-19}
0
0.1
0.2
0.3
0.4
t [sec]

Gravitational wave phase

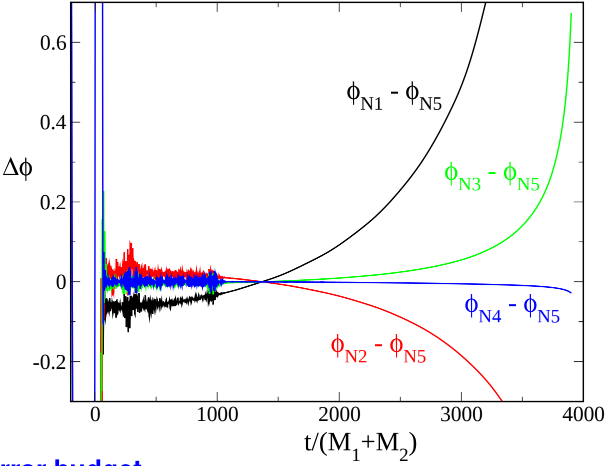


Comparison with post-Newtonian (GW-phase)



Numerical error budget

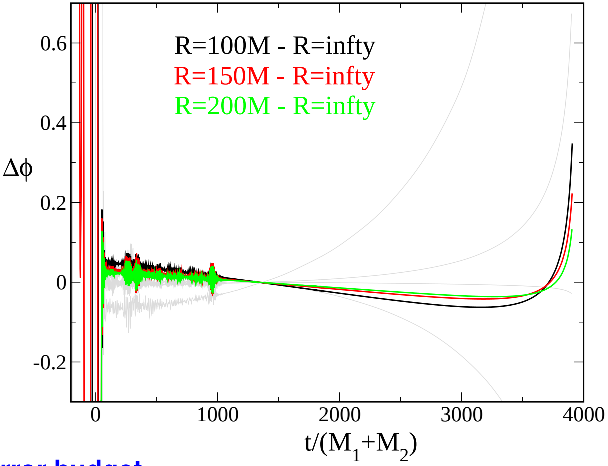
Comparison with post-Newtonian (GW-phase)



Numerical error budget

- Truncation error $\sim$ 0.03rad

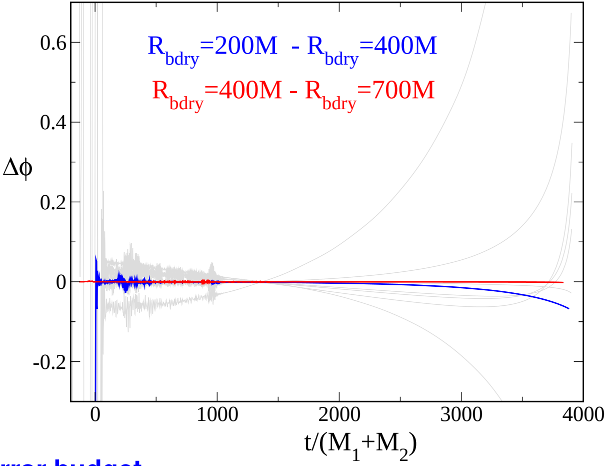
Comparison with post-Newtonian (GW-phase)



Numerical error budget

- Truncation error $\sim$ 0.03rad
- Extraction radius $\sim$ 0.1rad

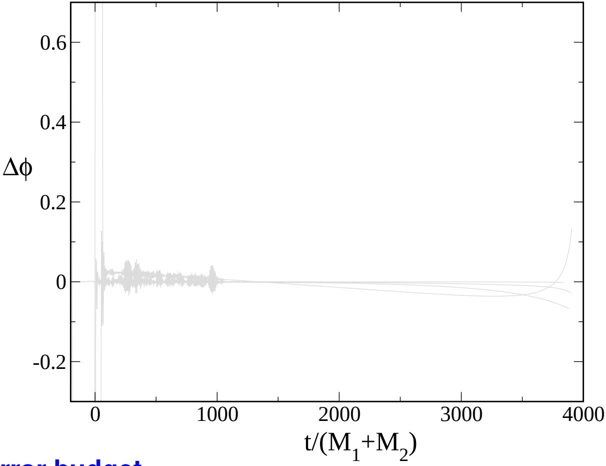
Comparison with post-Newtonian (GW-phase)



Numerical error budget

- Truncation error $\sim$ 0.03rad
- Extraction radius $\sim$ 0.1rad
- Outer boundary $\sim$ 0.07rad

Comparison with post-Newtonian (GW-phase)



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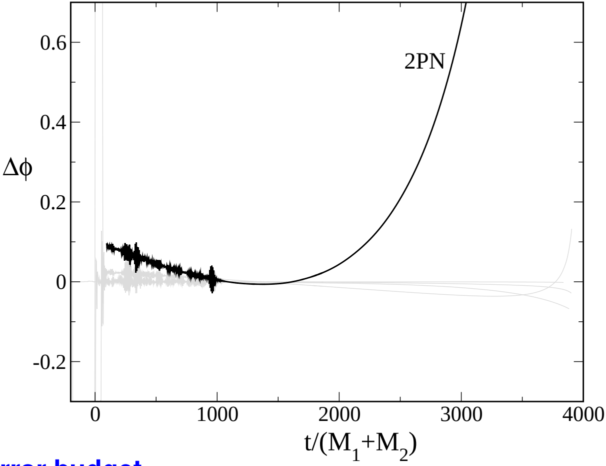

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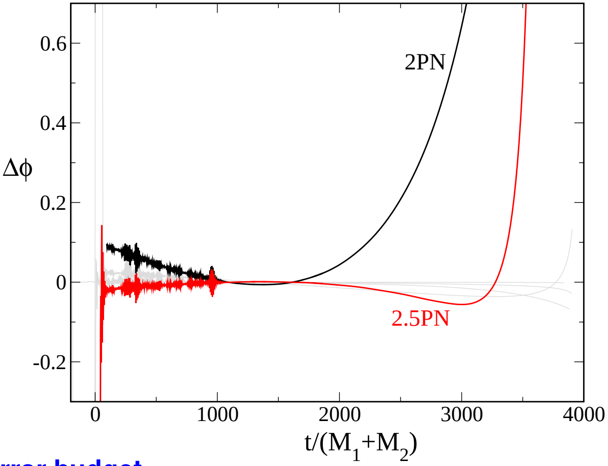

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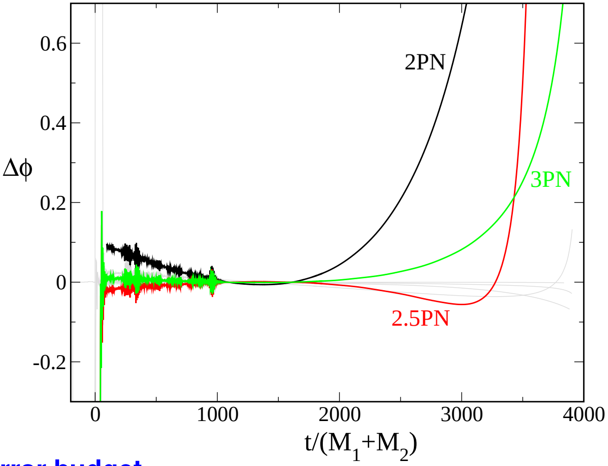

Numerical error budget

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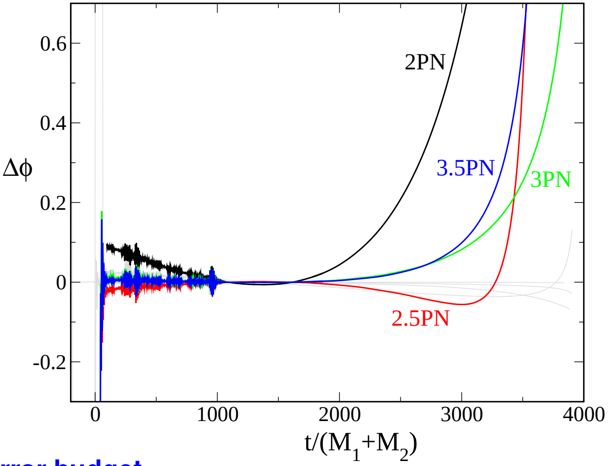
Comparison with post-Newtonian (GW-phase)



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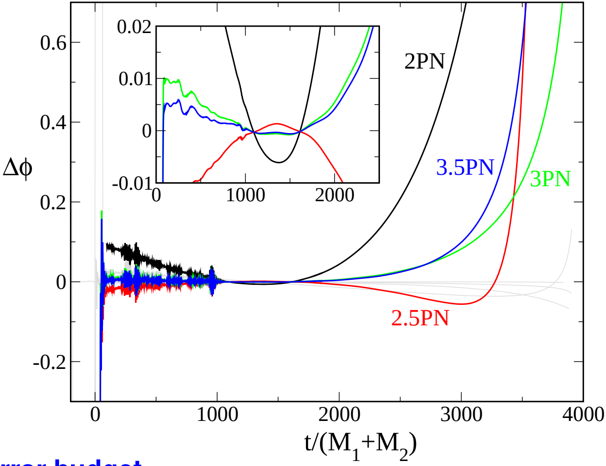
Comparison with post-Newtonian (GW-phase)



Numerical error budget

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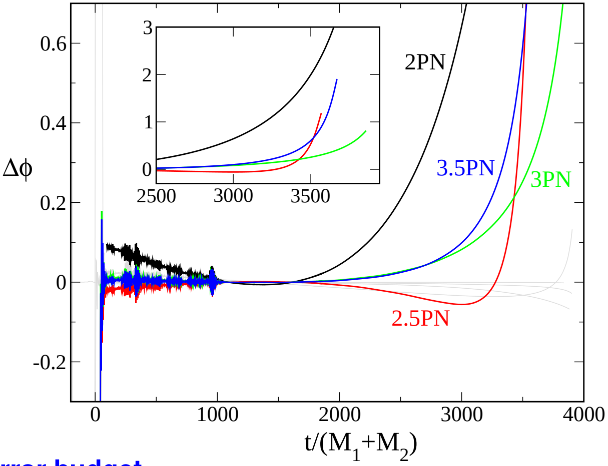
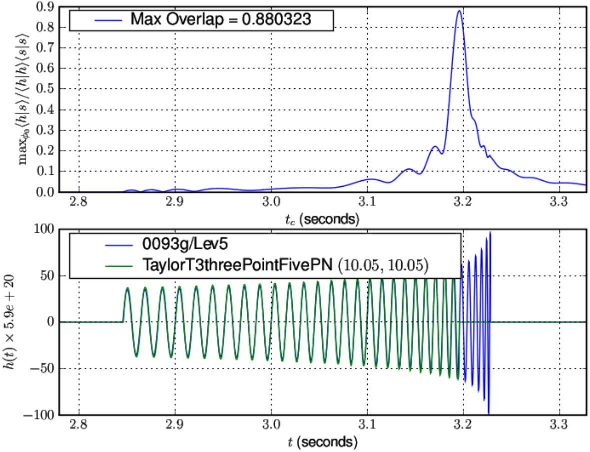
Comparison with post-Newtonian (GW-phase)



Numerical error budget

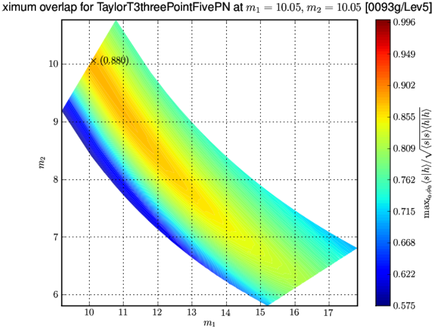
- Truncation error $\sim$ 0.03rad
- Extraction radius $\sim$ 0.1rad
- Outer boundary $\sim$ 0.07rad

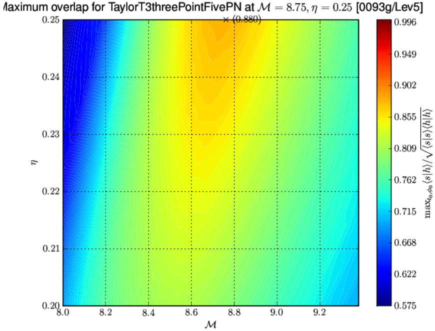
Testing LIGO detection templates



- In collaboration with D. Brown
- Good overlap!

Parameter estimation

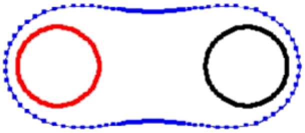




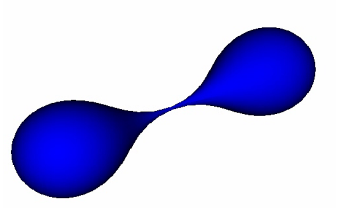
- Masses in simulation: 10.07Msun. Recovered 10.05Msun.

Head-on Merger

- Evolve to common horizon, regrid, continue.
- Apparent Horizons



Event horizon (M. Cohen, in prep.)



- Orbiting BHs have reached separation $2.2M$ (common horizon forms at $\sim 2M$.)

Summary

- Numerical relativity is in its golden age
- Spectral methods achieve stunning accuracy
- Comparison to PN in progress
  - Equal-mass non-spinning BHs
    – Full agreement NR - PN up to ~ 15 cycles before merger
    – Sufficient accuracy to identify deviations at all available PN-orders
- Future
  - Merger!
  - PN comparisons for spinning, non-equal mass binaries
    – Blanchet: PN converges exceptionally fast for q
    – For $q > 1$, $T_{\rm inspiral} \propto q$