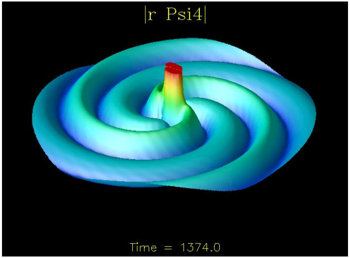
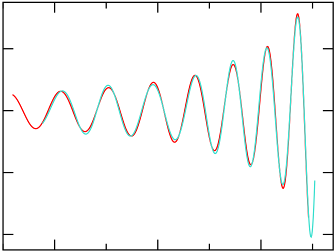
Binary BH simulations and gravitational waves

Harald Pfeiffer

California Institute of Technology





Theoretical Astrophysics Center, UC Berkeley, Oct 18, 2006

Outline & Bottom Line

1. Why to do black hole simulations
   – Templates for GW detectors
   – explore nonlinear gravity
   – solve two body problem

2. How to do black hole simulations
   Emphasis on the Caltech/Cornell spectral code
   – Really good for inspirals
   – No mergers yet

3. First results
   – Eccentricity of current inspiral simulations is small

Gravitational wave detectors

LISA (201x)

LIGO (Hanford)

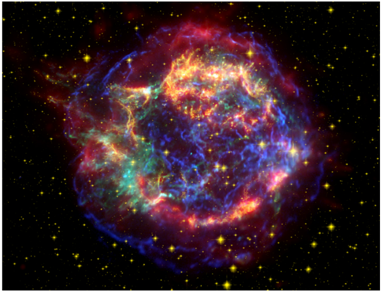
VIRGO

GEO 600

Gravitational Wave Sources

LIGO/GEO/TAMA/VIRGO

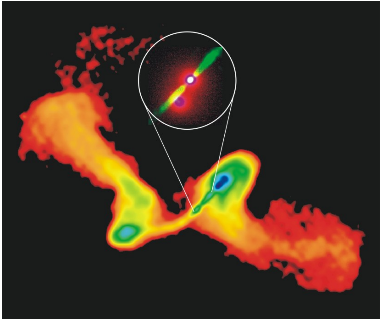
Compact Binary Inspiral
Pulsars, Supernovae, GRBs



Casiopeia A (Spitzer/HST/Chandra)

LISA

Supermassive BH mergers
Extreme mass ratio inspirals
White dwarf binaries

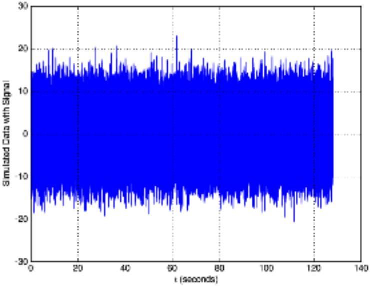


NGC 326 (NRAO/AUI/NSF)

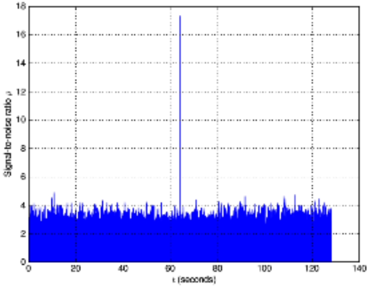
Signal Detection

- Signals extreme weak
- Detect via matched filtering against waveform templates

Instrument noise w/ signal

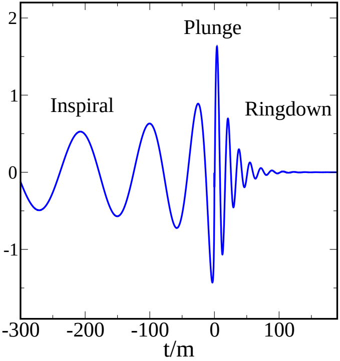


SNR vs. coalescence time



Waveform generation

- Inspiral
  – post-Newtonian expansions
- Late inspiral & merger
  – Numerical relativity
- Ringdown
  – BH perturbation theory



Small phase errors essential for matched filtering

Role of numerical relativity

- Essential for GW detectors
  - Supply waveform templates
  - Test general relativity
- Explore stong field behavior of general relativity
  - Toroidal black holes (Shaprio, Teukolsky)
  - Critical behavior in BH formation (Choptuik)
- Solve the two-body problem

Solving Einstein's equations – basic idea

- Task: Find space-time metric g_{ab} such that $R_{ab}[g_{ab}] = 0$

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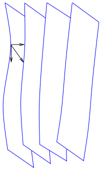

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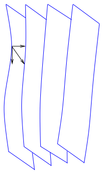

- Evolution equations

$$\partial_t g_{ij} = \ldots$$
$$\partial_t \ldots = \ldots$$

cf. Maxwell equations

$$\partial_t \vec{E} = \nabla \times \vec{B}$$
$$\partial_t \vec{B} = -\nabla \times \vec{E}$$
$$\nabla \cdot \vec{E} = 0$$
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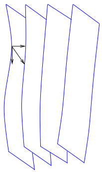

- Evolution equations

$$\partial_t g_{ij} = \ldots$$
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- Constraints

$$R[g_{ij}] + \ldots = 0$$
$$\ldots = 0$$

cf. Maxwell equations

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Generalized Harmonic evolution system

$$0 = R_{ab} = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms} \qquad \Gamma_a = -g_{ab}\Box x^b$$

- The gauge condition $g_{ab}\Box x^b \equiv H_a$ (with H_a) given removes nasty piece from principal terms, which become wave-equations.

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$$0 = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}C_{b)} + \gamma\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^c C_c\right] + \text{l. o.}$$

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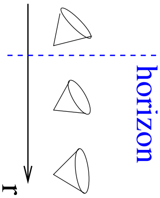
- Lower order terms are very complicated: 1000's of FLOPS per grid-point per timestep
- In practice, rewrite in first order from (Lindblom, et al 2005)

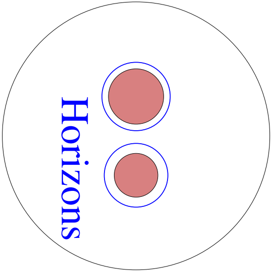
Boundary conditions & BH excision

- Generalized harmonic evolution system is symmetric hyperbolic

$$u^{\alpha} + A^{k\alpha}{}_{\beta}\partial_k u^{\beta} = F^{\beta}$$

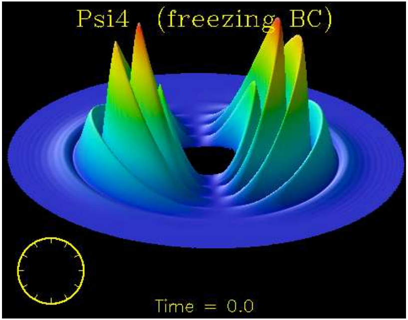
- Boundary conditions
  - Decompose into characteristic fields
  - Impose BCs on incoming fields
- All modes propagate inside light cone
  $\Rightarrow$ Excision boundaries inside horizon
  do not require any BC

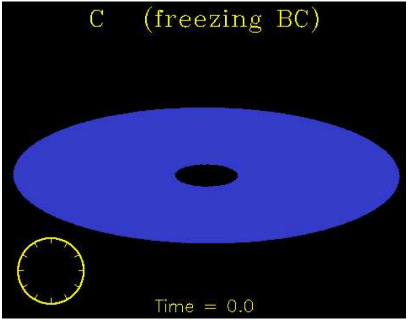




Outer boundary conditions

- Must prevent influx of constraint violations





Outer boundary conditions

- Must prevent influx of constraint violations
  - Derive constraint evolution system,
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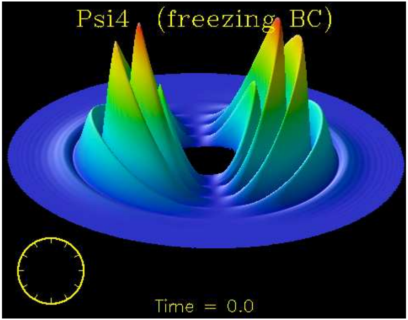
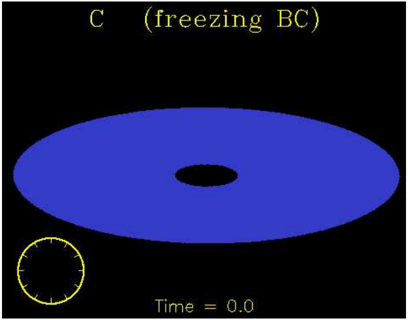
    set incoming fields to zero
    $\Rightarrow$ some BC on fundamental fields





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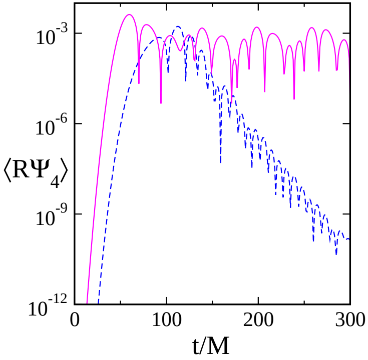
- Must allow gravitational waves to exit without reflection.
  - Consider Newman-Penrose scalars
    – Ψ_4 is represented by outgoing characteristic fields (good!)
    – $\Psi_0 \equiv 0$ implies conditions on some incoming char. fields



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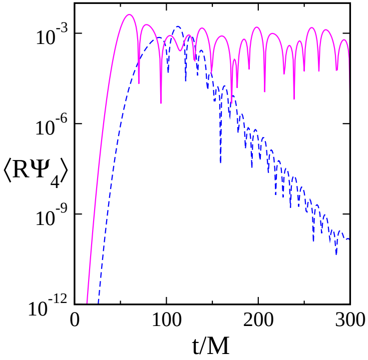
- Must keep coordinates well-behaved (work in progress)



Spectral Methods I

- Truncated series-expansion

$$u(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi_k(x)$$

  (Fourier series, Chebyshev series, spherical harmonics)

- Differentiation, integration, interpolation become analytic operations on the basis-functions

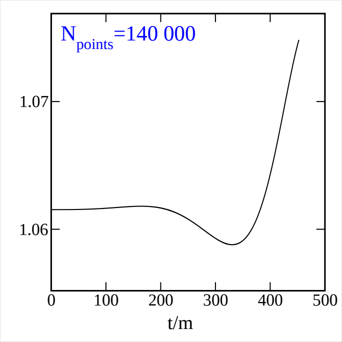
$$\int u(x,t)\,\mathrm{dx} = \sum_{k=1}^{N} \tilde{u}_k(t) \int \Phi_k(x)\mathrm{dx}$$

- Use method of lines to evolve $\{\tilde{u}_k(t)\}$

Exponential convergence for smooth solutions

Spectral Methods II: Exponential convergence

- Example: Irreducible mass of BH in BBH evolution



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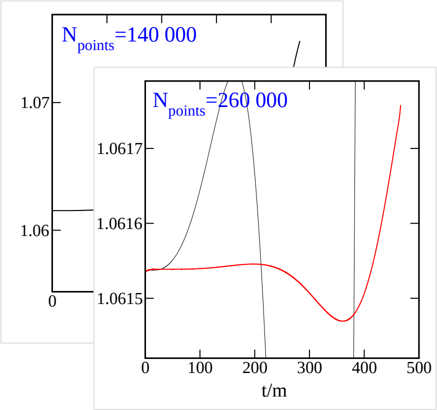

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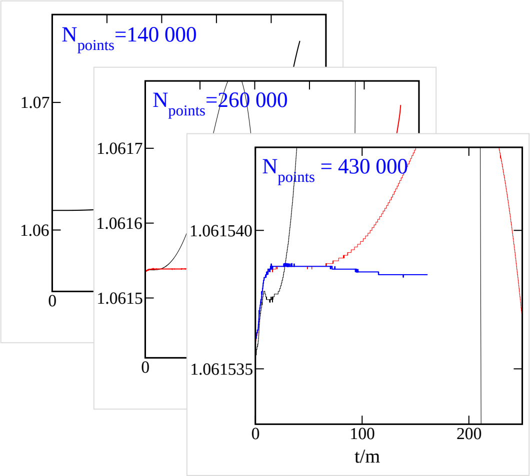
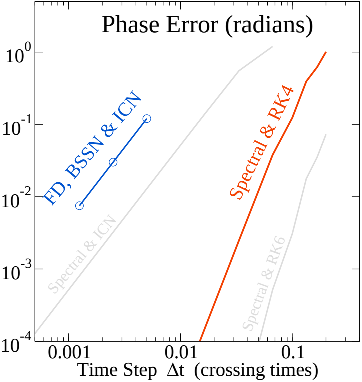
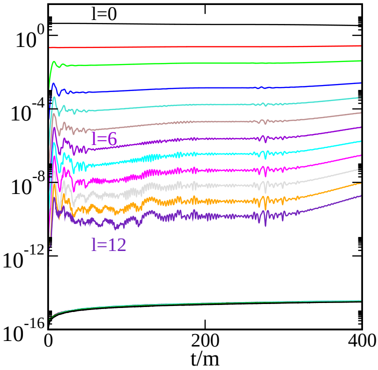


Spectral Methods III: Low phase errors, no viscosity

1D travelling wave
(courtesy Mike Boyle)



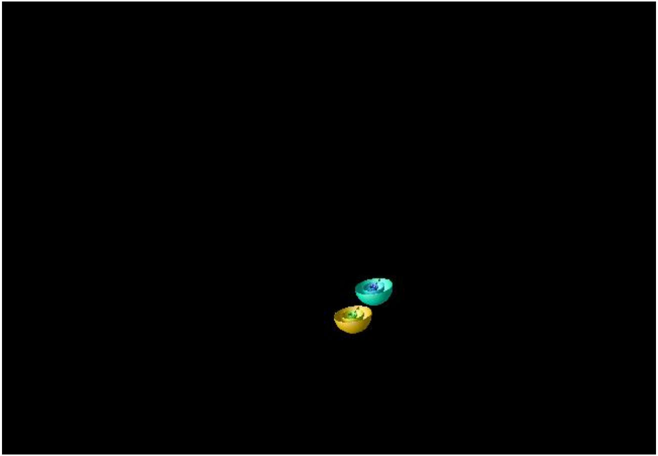
BBH Evolution: Ylm-coefficients
in sphere around BH



⇒ **expect small cummulative errors in long-term evolutions**

Multi domain-method

- Spectral methods work well for simple topologies: Blocks, shells, ...
- For BBH, must excise two spheres



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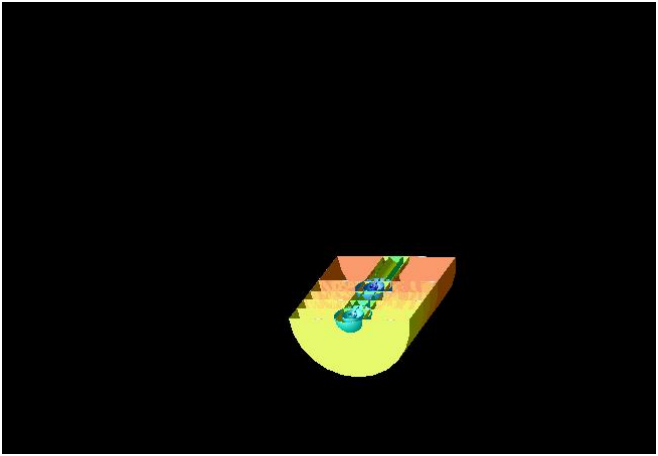

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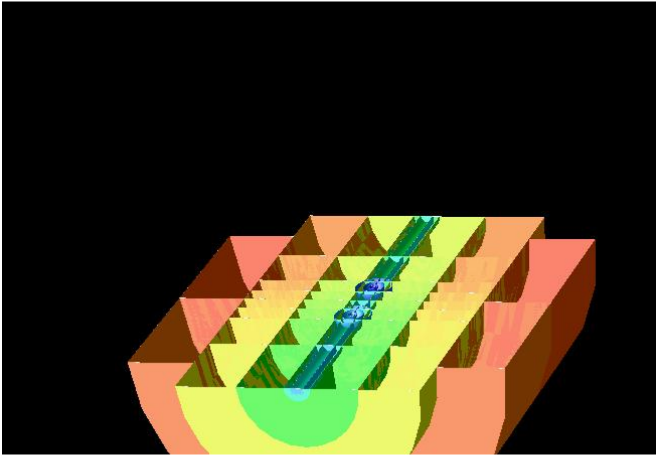

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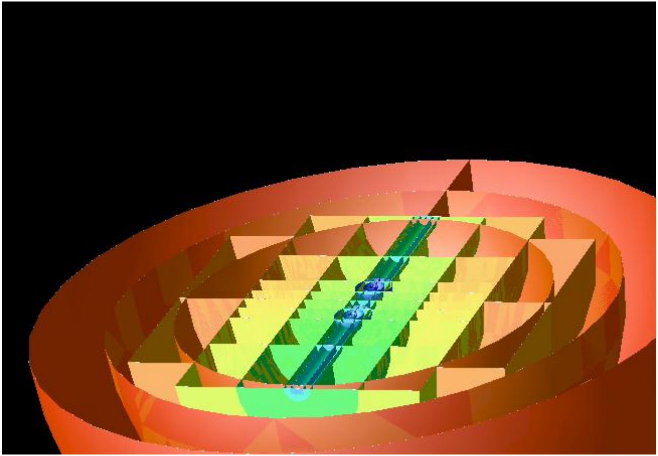

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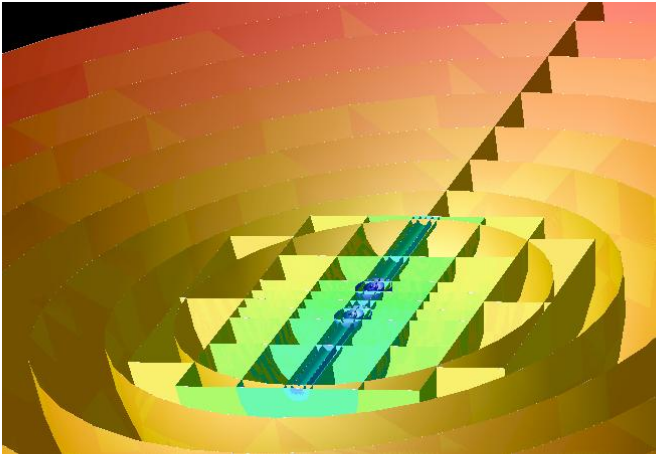

Comoving coordinates

- Changing domain-decomposition is difficult
  – *localize* horizons in coordinate space
  (Scheel, HP, etal, 2006):

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$$\vec{x}_{\rm inertial} = a(t)R(t)\vec{x}_{\rm computational}$$

$R(t)$ rotation matrix, $a(t)$ overall scale factor

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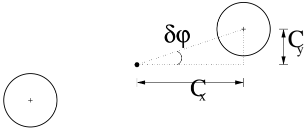
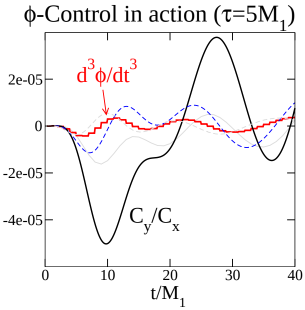
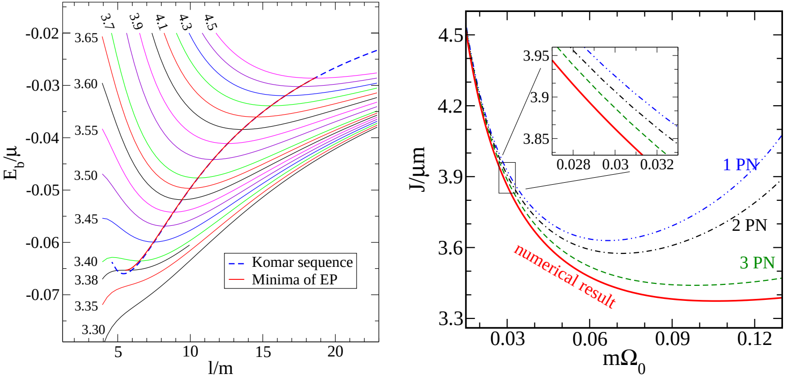
  3. R(t) and a(t) determined by dynamic control based on current AH location





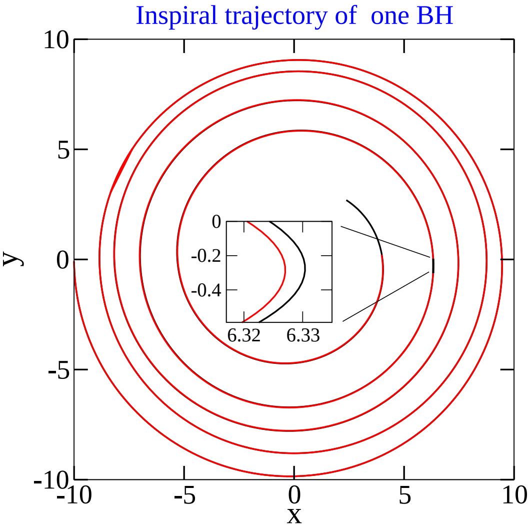
Initial data

- Quasi-equilibrium initial data (Cook, HP, 2002, 2004, 2006)
- Exploit that black holes are in *circular* orbit
- Construct sequences of circular orbits at different separation



Orbits, at last!

AH-MOVIE 2D



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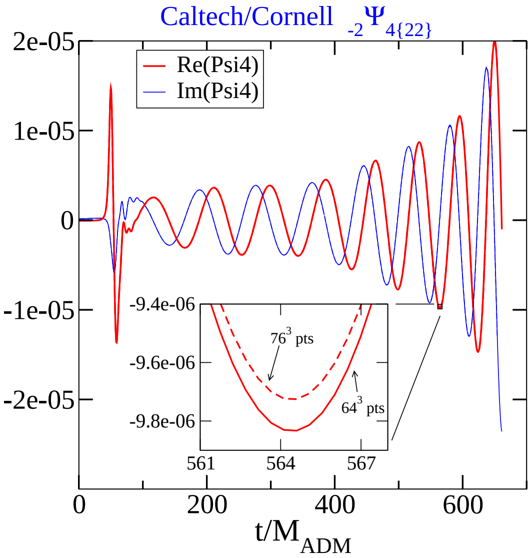

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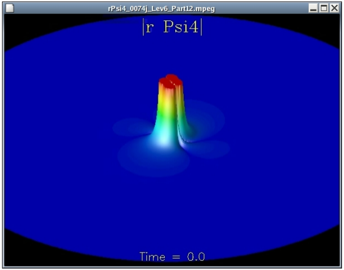
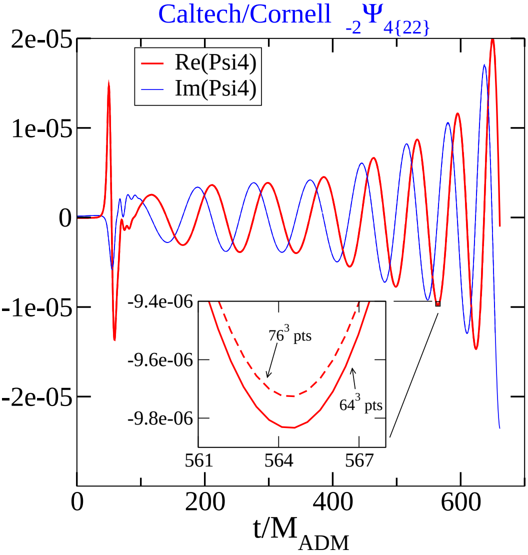
Computational requirements:

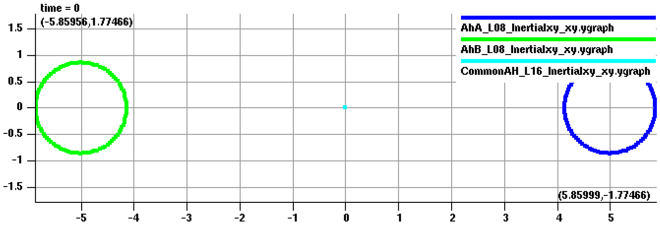
- 64^3 points: 6000 CPU-h (10 CPU-h/$M_{\rm ADM}$)
- 76^3 points: 18000 CPU-h (27 CPU-h/$M_{\rm ADM}$)





Mergers

- Our code does extremely well during inspiral
- Plan for coalescence:
  (a) Push BBH run to formation of common horizon
  (b) Regrid onto one set of concentric spherical shells
  (c) Continue
- No luck yet with orbiting binaries
- Practice with head-on collisions



Toward science – post-Newtonian expansions

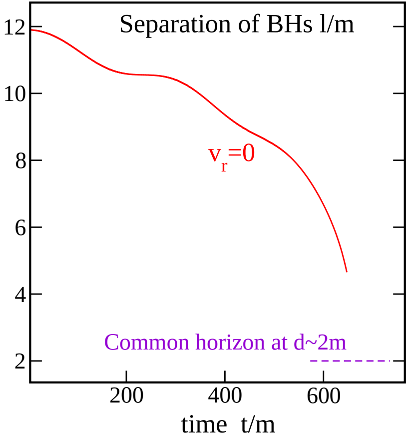
- Post-Newtonian theory generates inspiral waveforms

  When breaks PN down?
  Where must numerical relativity take over?

- Requires ...
  – long term, very accurate inspiral simulations $\Delta\phi \ll 1$ (ok!)
  – Realistic BBH initial data (??)

Eccentricity in BBH simulations

- $v_r = 0$ in initial data leads to oscillatory behavior.
  But BBH's will have circularized during inspiral.



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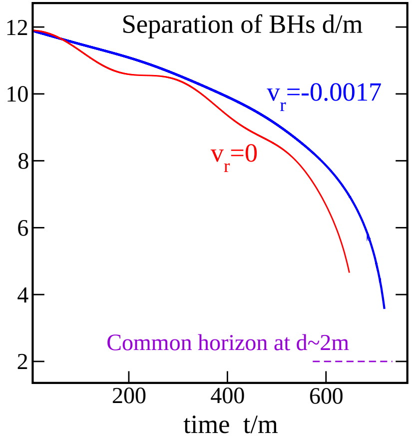

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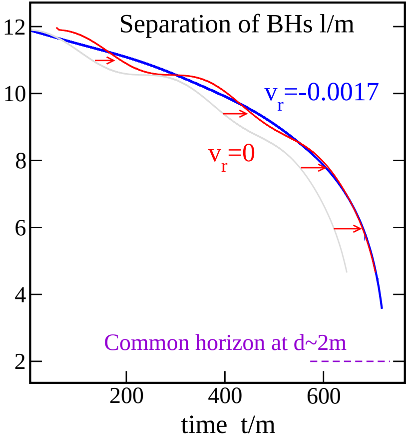

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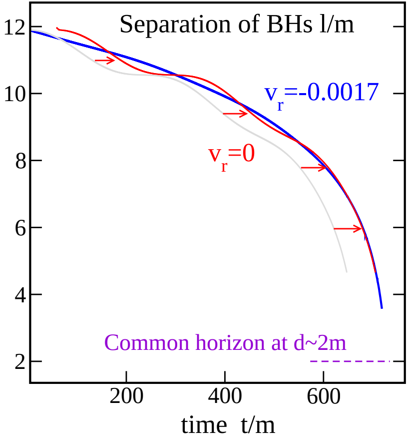
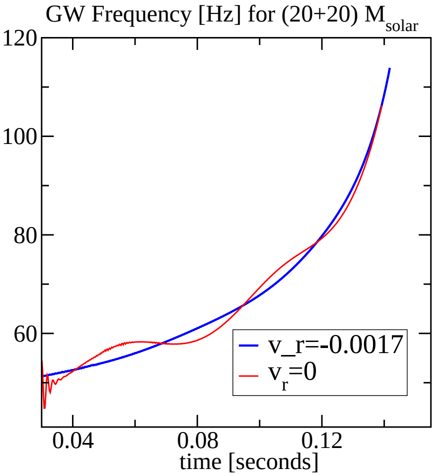
- Is this significant??



Significance of eccentricity



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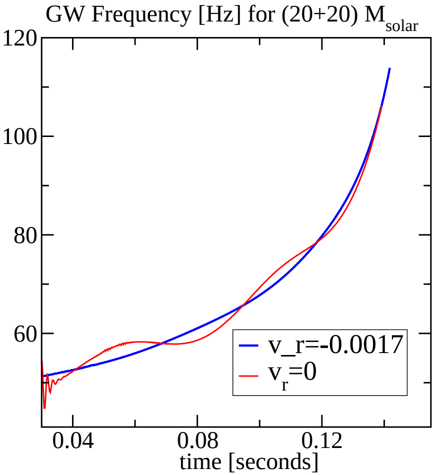

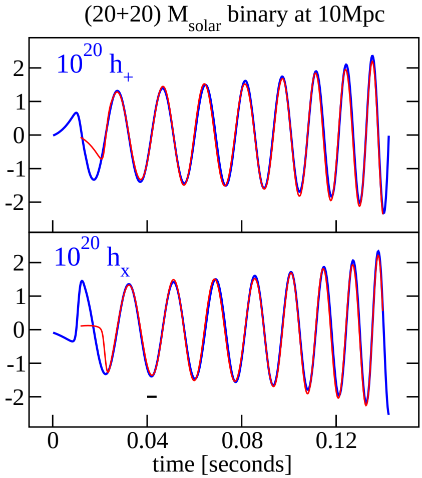

Overlap between waveforms 0.989

Quite good – good enough? Behavior for longer runs??

Overview of BBH simulations

who	when	system	N_{orbits}	notes
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Goddard Brownsville	Nov 2005 Nov 2005	BSSN BSSN	4.2 $2-3$	2nd/4th order FD, AMR 4th order FD uni-grid

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Penn State AEI / LSU FAU / U. Jena LSU	Jan 2006 early 2006 early 2006 –	BSSN BSSN BSSN $\sim$ GH	$\gtrsim 1$ $\gtrsim 1$ $\gtrsim 1$ –	FD FD FD multi-block 8th order FD

Overview of BBH simulations

who	when	system	N_{orbits}	notes
Caltech/Cornell	Apr 2006	GH	5.1	Spectral, excision
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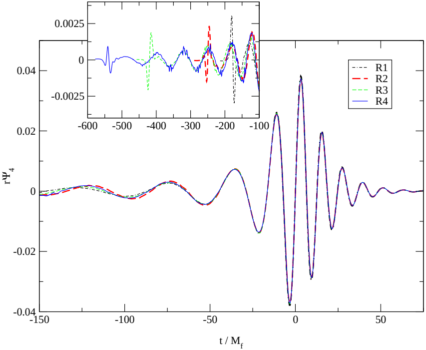
- Everybody can do mergers, except Caltech/Cornell
- Caltech/Cornell is at least 10x more accurate with 1/10-CPU cost
  – Important for inspiral simulations

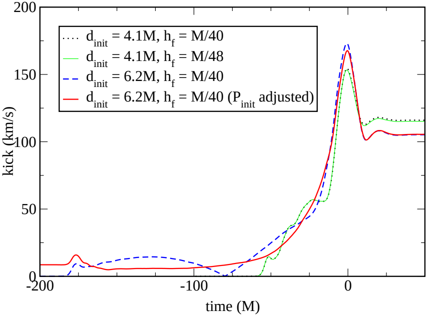
Goddard simulations

Merger waveform independent of early evolution



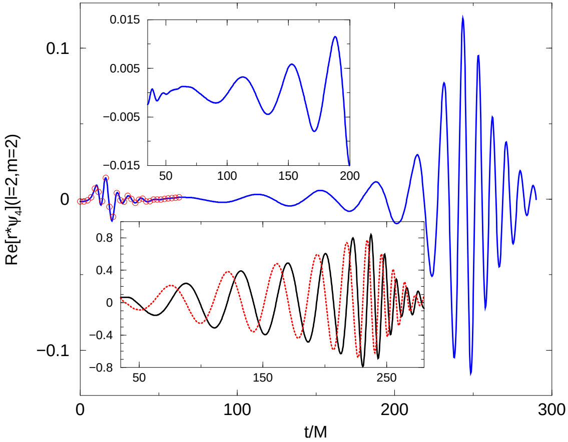
(Baker et al, 2006a, 2006b)

Black hole kicks for $M_1/M_2 = 1.5$



UTB Brownsville

Orbital hangup for corotating BHs $J_{\rm final} \approx 0.9 M_{\rm final}^2$



Campanelli et al 2006

Conclusions & Outlook

- Black hole evolution codes are finally stable!
- First science results are obtained
- Accuracy and efficiency will become increasingly important
  – Longer evolutions
  – Vast parameter space (masses, spins)
- Caltech/Cornell spectral code has bright future
  (once mergers are accomplished...)

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