Type III migration in a low viscosity disc

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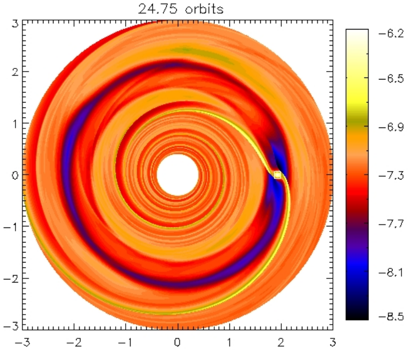
Outline

- Introduction: type III migration
- Numerical methods, first results and motivation
- Type III migration in an inviscid disc:
  - Formation of vortensity rings
  - Linear stability
  - Non-linear outcome and role in type III
- Summary

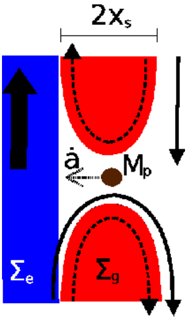
Introduction

- 405 exo-planets discovered (27 November 2009).
- First 'hot Jupiter' around 51 Pegasi, orbital period 4 days (Mayor & Queloz 1995). Fomalhaut b with semi-major axis 115AU.
- Formation difficult in situ, so invoke *migration*: interaction of planet with gaseous disc (Goldreich & Tremaine 1979; Lin & Papaloizou 1986).



Physics of type III migration

Consider a migrating Saturn-mass planets with partial gap, so there is flow of material across co-orbital radius.



- Fluid element orbital radius changes from $a - x_s \to a + x_s \Rightarrow$ torque on planet:

$$\Gamma_3 = 2\pi a^2 \dot{a} \Sigma_e \Omega x_s.$$

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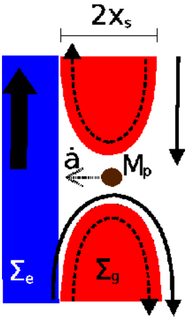

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$$\Gamma_3 = 2\pi a^2 \dot{a} \Sigma_e \Omega x_s.$$

- Migration rate for $M_p + M_r + M_h$:

$$\dot{a} = \frac{2\Gamma_\mathrm{L}}{\Omega a(\underbrace{M_p + M_\mathrm{r}}_{M_p'} - \delta m)} \tag{1}$$

where Γ_L is the total Lindblad torque and

$$\delta m = 4\pi\Sigma_e a x_s - M_h = 4\pi a x_s(\Sigma_e - \Sigma_g)$$

is the density-defined co-orbital mass deficit.

Key ideas in type III

- Co-orbital mass deficit:

  $\boxed{\text{larger } \delta m \Rightarrow \text{faster migration.}}$

- Horse-shoe width: $\boxed{x_s}$, separating co-orbital and circulating region. Take $x_s = 2.5r_h$ for result analysis ($r_h \equiv (M_p/3M_*)^{1/3}a$). Can show $x_s \lesssim 2.3r_h$ in particle dynamics limit.

- Vortensity: $\boxed{\eta \equiv \omega/\Sigma,}$ important for stability properties and η^{-1} also used to define δm (Masset & Papaloizou 2003).

- Modelling assumptions: steady, slow migration ($\boxed{\tau_{\mathrm{lib}}/\tau_{\mathrm{mig}} \ll 1}$), horse-shoe material moves with planet.

Numerics

Standard numerical setup for disc-planet interaction. 2D disc in polar co-ordinates centered on primary but non-rotating. Units $G = M_* = 1$.

- Hydrodynamic equations with local isothermal equation of state:

$$
\begin{aligned}
&\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{v}) = 0, \\
&\frac{\partial v_r}{\partial t} + \mathbf{v} \cdot \nabla v_r - \frac{v_\phi^2}{r} = -\frac{1}{\Sigma}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r} + \frac{f_r}{\Sigma}, \\
&\frac{\partial v_\phi}{\partial t} + \mathbf{v} \cdot \nabla v_\phi + \frac{v_\phi v_r}{r} = -\frac{1}{\Sigma r}\frac{\partial P}{\partial \phi} - \frac{1}{r}\frac{\partial \Phi}{\partial \phi} + \frac{f_\phi}{\Sigma}, \\
&P = c_s^2(r)\Sigma.
\end{aligned}
$$

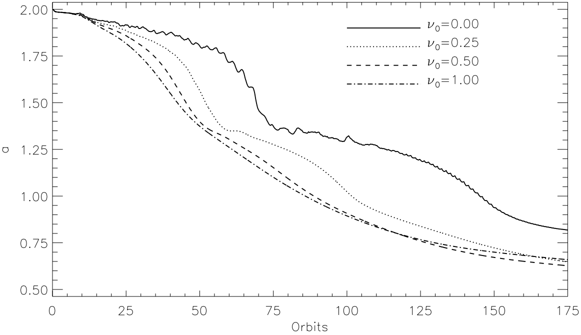
  Viscous forces $f \propto \nu = \nu_0 \times 10^{-5}$, temperature $c_s^2 = h^2/r$, $h = H/r$. Φ is total potential including primary, planet (softening $\epsilon = 0.6H$), indirect terms but no self-gravity.

- Method: FARGO code (Masset 2000), finite difference for hydrodynamics, RK5 for planet motion.

Type III migration as a function of visocisty

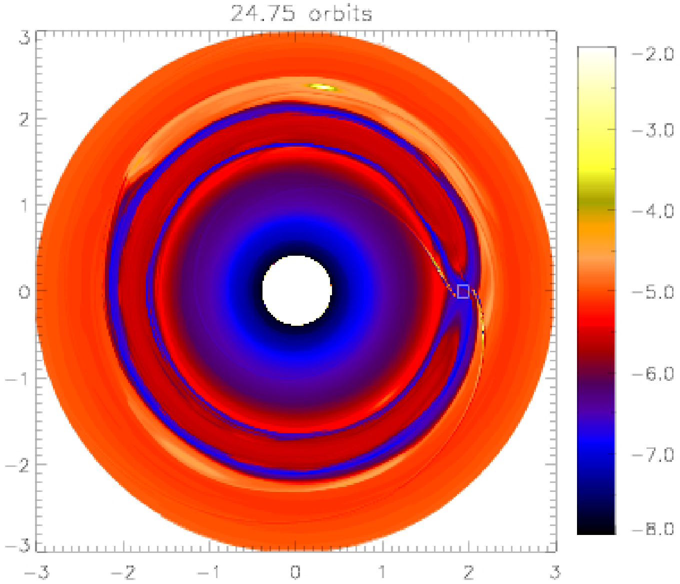
Discs: uniform density $\Sigma = 7 \times 10^{-4}$, aspect ratio $h = 0.05$ and different uniform kinematic viscosities. Note $\nu = 10^{-5}$ equivalent to $\alpha_{SS} = 4 \times 10^{-3}$ at $r = 1$.

Planet: Saturn mass $M_p = 2.8 \times 10^{-4}$ initially at $r = 2$.



What's going on at low viscosities?

Inviscid case: evolution of Σ/ω:



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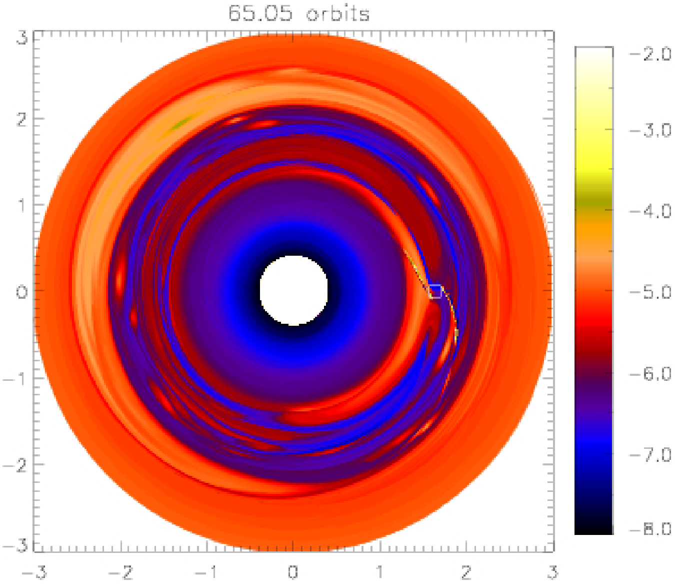
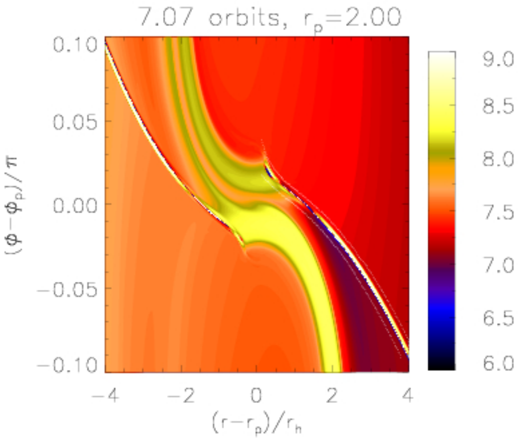


Vortensity rings: formation via shocks



Vortensity generated as fluid elements U-turn during its horse-shoe orbit.

Predicting the vortensity jump

We need:

- Vortensity jump across isothermal shock:

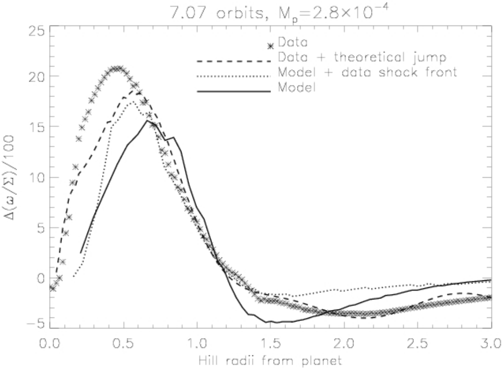
$$\left[\frac{\omega}{\Sigma}\right] = -\frac{(M^2-1)^2}{\Sigma M^4}\frac{\partial v_\perp}{\partial S} - \left(\frac{M^2-1}{\Sigma M^2 v_\perp}\right)\frac{\partial c_s^2}{\partial S}.$$

  RHS is pre-shock. $M = v_\perp/c_s$, S is distance along shock (increasing radius). Additional baroclinic term compared to Li et al. (2005) but has negligible effect ($c_s^2 \propto 1/r$).
- Flow field: shearing-box geometry, velocity field from zero-pressure momentum equations, density field from vortensity conservation following a particle.
- Shock location : generalised Papaloizou et al. (2004)

$$\frac{dy_s}{dx} = \frac{\hat{v}_y^2 - 1}{\hat{v}_x\hat{v}_y - \sqrt{\hat{v}_x^2 + \hat{v}_y^2 - 1}}.$$

$\hat{v} \equiv v/c_s$.

Theoretical jumps



- Vortensity generation near shock tip (horse-shoe orbits) , vortensity destruction further away (circulating region). Variation in flow properties on scales of $r_h \simeq H$.
- Variation in disc profiles on scale-heights enables shear instability $\Rightarrow$ vortices in non-linear stage (Lovelace et al. 1999, Li et al. 2001).

Ring stability

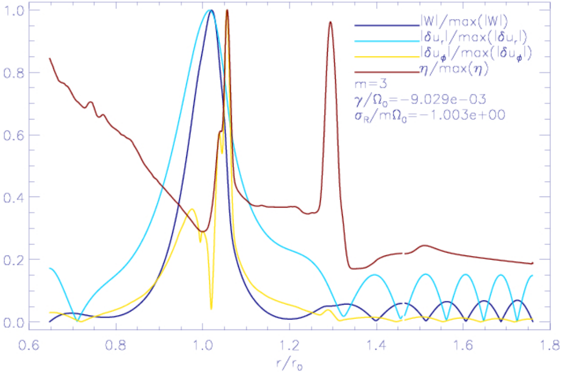
- Idea: linear stability analysis of inviscid disc but use simulation vortensity profile as basic state: axisymmetric, $v_r = 0$. Basic state checked explicitly.
- Governing equation for isothermal perturbations $\propto \exp i(\sigma t + m\phi)$:

$$\frac{d}{dr}\left(\frac{\Sigma}{\kappa^2 - \bar{\sigma}^2}\frac{dW}{dr}\right) + \left\{\frac{m}{\bar{\sigma}}\frac{d}{dr}\left[\frac{\kappa^2}{r\eta(\kappa^2 - \bar{\sigma}^2)}\right] - \frac{r\Sigma}{h^2} - \frac{m^2\Sigma}{r^2(\kappa^2 - \bar{\sigma}^2)}\right\} W = 0$$

$W = \delta\Sigma/\Sigma$; $\kappa^2 = 2\Sigma\eta\Omega$; $\bar{\sigma} = \sigma + m\Omega(r)$.

- Self-excited modes in inviscid disc with sharp vortensity profiles.

Example: $m = 3,\ h = 0.05$



- Disturbance focused around vortensity minimum (gap edge), exponential decays either side joined by vortensity term at co-rotation r_0. More extreme minimum $\Rightarrow$ more localised.
- Waves beyond the Lindblad resonances ($\kappa^2 - \bar{\sigma}^2 = 0$) but amplitude not large compared to co-rotation.

Link to type III migration

Recall co-orbital mass deficit

$$\delta m = 4\pi a x_s(\Sigma_e - \Sigma_g)$$

- Instability can increase δm by increasing Σ_e but not Σ_g (co-rotational modes are localised) $\Rightarrow$ favouring type III. When vortex flows across co-orbital region, Σ_g increases and migration *may* stall.

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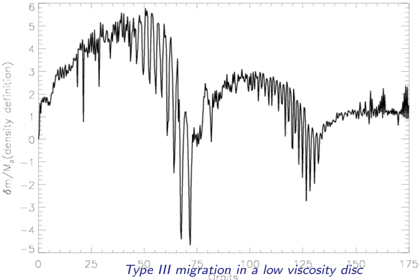
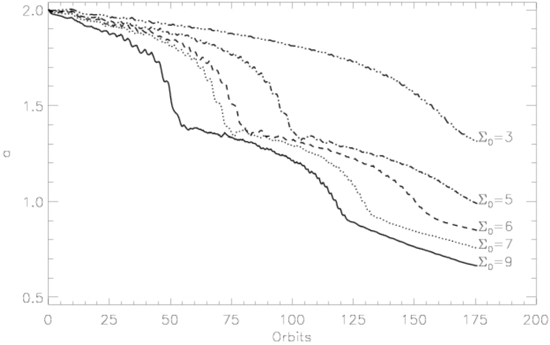
- Can expect interaction when δm of order planet mass.



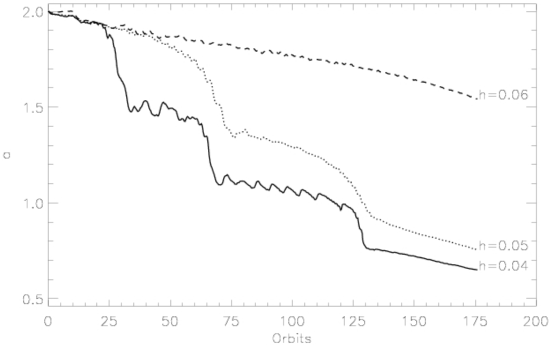
Vortex-driven migration in other discs

Growth rate indepdent of density scale. Higher density just means less time needed for vortex to grow sufficiently large for interaction.



Vortex-driven migration in other discs

$c_s^2 = T \propto h^2$. Lower temperature $\Rightarrow$ stronger shocks $\Rightarrow$ profile more unstable $\Rightarrow$ shorter time-scale to vortex-planet interaction.



Require disc profile to be sufficiently extreme and have enough mass to trigger vortex-planet interaction, but the extent of migration during one episode is the same.

Summary

- Migration in low viscosity/inviscid discs is non-smooth due to shear instabilities associated with gap edge (vortensity minima). Effects appear at $\nu = O(10^{-6})$ or $\alpha_{SS} = O(10^{-4})$.
- Provided an over-all picture of vortex-planet interaction: formation of unstable basic state via shocks, linear stability analysis and hydrodynamic simulations.
- Instability favours type III migration by increasing the co-orbital mass deficit. Vortex-planet interaction when $\delta m/M_p \sim 4$—5. Associated disruption of co-orbital vortensity structure, unlike previous notion of type III migration where flow-through does not affect co-orbital region.