Vortex instabilities in self-gravitating disc-planet interactions

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Self-gravitating protoplanetary discs

Inclusion of gravitational potential Φ of the disc via

$$\nabla^2\Phi = 4\pi G\rho$$

Why bother?

- Planet formation via gravitational instability
- Specific disc-planet interactions
- **Disc vortices are over-densities**

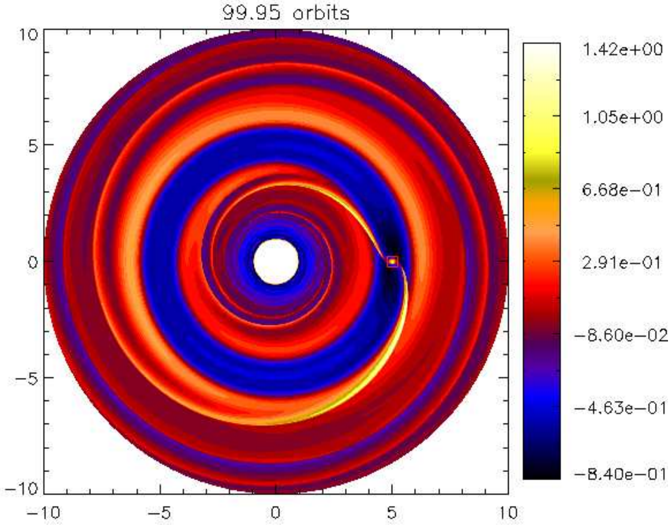
Previous works: Li et al. (2009), Lyra et al. (2009).

Plan

- Results
- Linear theory
- Linear calculations
- Hydrodynamic simulations I
- Hydrodynamic simulations II

Gaps in protoplanetary discs

Necessary condition for Rossby wave instability satisfied for gaps opened by a planet. Vortensity (potential vorticity) profile with extrema at gap edges.



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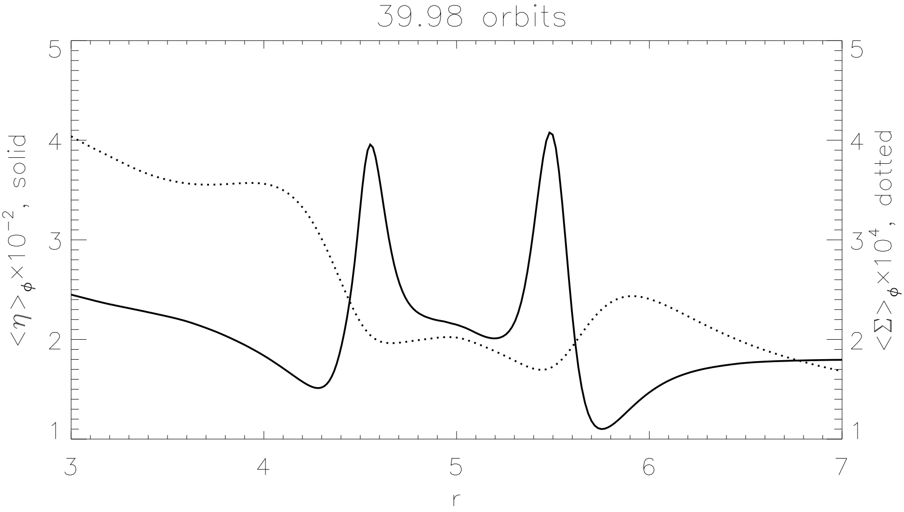
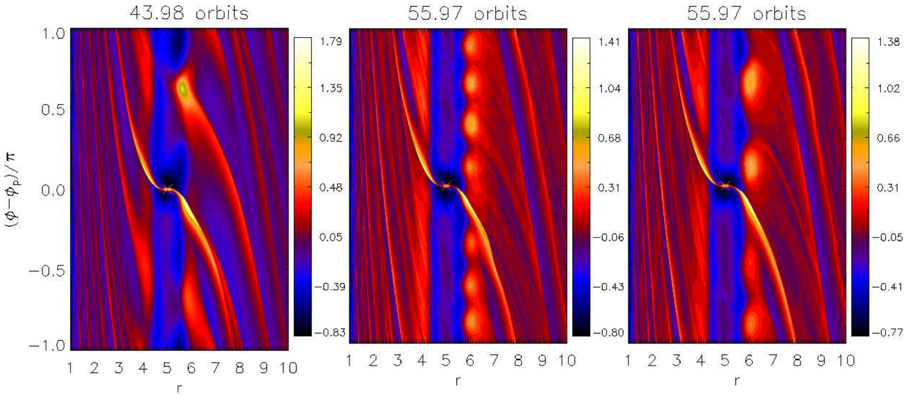


Effect of self-gravity

Consider a series of disc-planet simulations with increasing M_d:



$M_d = 0.063M_*$ $\quad$ $M_d = 0.032M_*$ $\quad$ $M_d = 0.011M_*$

Linear problem (barotropic discs)

- Perturb the system, e.g. $\Sigma \to \Sigma + \delta\Sigma(r)\exp i(\sigma t + m\phi)$.
- Linearize to get ODE for $S \equiv c_s^2\delta\Sigma/\Sigma + \delta\Phi$ (enthalpy plus gravitational potential).

$$\delta\Phi = -G\int K_m(r,\xi)\delta\Sigma(\xi)\xi d\xi,$$
$$L(S) = \delta\Sigma$$

- L is a linear operator

$$L(S) = \frac{mS}{r\bar{\sigma}(1-\bar{\nu}^2)}\frac{d}{dr}\left(\frac{1}{\eta}\right) + \cdots\ ,$$

$\bar{\sigma} = \sigma + m\Omega$, $\bar{\nu} = \bar{\sigma}/\kappa$ and

$\eta = \kappa^2/2\Omega\Sigma$ is the *vortensity*.

- When $\bar{\sigma}(r_c) = 0$, need $d\eta/dr = 0$ there.

Energy balance

$$\int rS^* L(S) dr = \int rS^* \delta\Sigma dr = \text{energy}.$$

- Association of disturbance with vortensity extremum: $\eta'(r_c) \simeq 0$ and $\bar{\sigma}(r_c) \simeq 0$.
- On LHS, only keep vortensity gradient term.

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$$\int \frac{m|S|^2}{\bar{\sigma}(1-\bar{\nu}^2)} \frac{d}{dr}\left(\frac{1}{\eta}\right) dr \sim \int rc_s^2 \frac{|\delta\Sigma|^2}{\Sigma} dr - G \int\int r\xi K_m(r,\xi)\delta\Sigma^*(r)\delta\Sigma(\xi)drd\xi$$

Energy balance

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- Sign of LHS depends on sign of $(1/\eta)''/\Omega'$ at r_c. Note $\Omega' < 0$.
- Weak SG: RHS > 0 so $\boxed{r_c \text{ is } \min(\eta)}$.
- Numerics $\rightarrow$ vortex instability only at vortensity minimum.
- Balance does not work if SG too strong (RHS < 0, gravitational disturbance).

Formal argument

- Consider neutral vortex mode with co-rotation at vortensity minimum: $\bar{\sigma}(r_c) = \eta'(r_c) = 0$.
- Tweak self-gravity via $G \to G + \delta G$ (only in linear response, $\delta\Phi$).
- Perturb eigensolution: $\sigma \to \sigma + \delta\sigma$ with $\delta\sigma = \delta\sigma_R + i\gamma$; $S \to S + \delta S$; $\delta\Sigma \to \delta\Sigma + \delta\Sigma_1$. γ is assumed small negative (unstable).

Can show:

$$\boxed{\gamma = \beta \left.\frac{d^2\eta}{dr^2}\right|_{r_c} \times \delta G,}$$

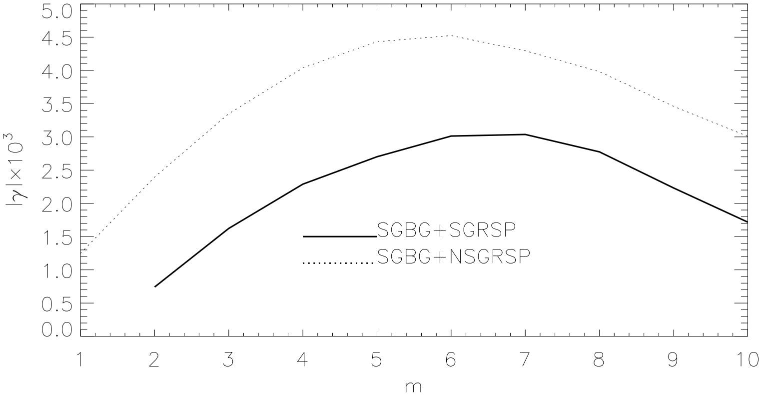
with $\beta > 0$ for Ω decreasing outwards. Vortex modes have $\eta''(r_c) > 0$. Need $\delta G < 0$ to destabilize them, i.e. increasing SG stabilizes them.

Note: Papaloizou-Pringle instability also stabilized by self-gravity.

Stabilization of vortex modes

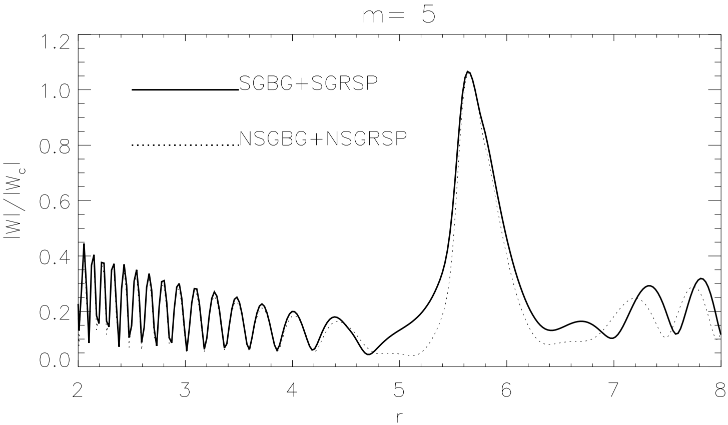
- Numerical solution to linear problem with fixed temperature profile.
- Also solved with $\delta\Phi = 0$.

Growth rate $|\gamma|$ as a function of azimuthal wave-number *m*:



Solid: with SG in response. Dotted: no SG in response.

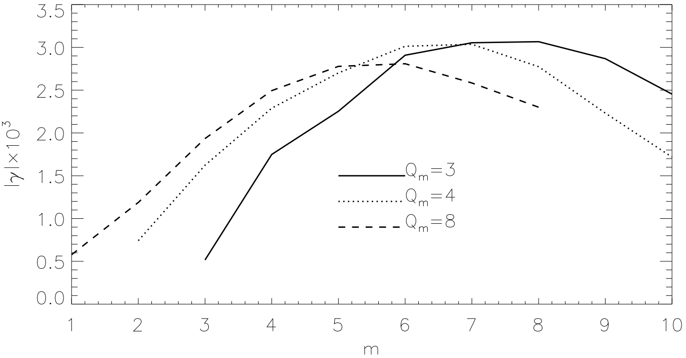
Eigenfunctions



Solid: completely with SG. Dotted: completely without SG.

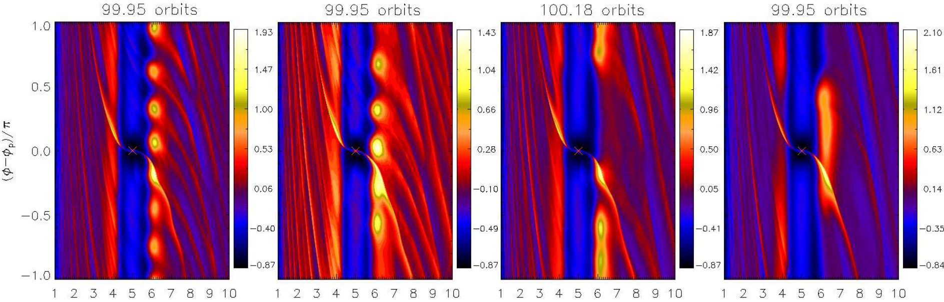
Getting more vortices

Now include SG all the way in linear problem. Parameter $Q_m \propto 1/M_d$.



- Higher m preferred with increasing SG. Get more vortices.
- Loss of low m (stabilization by SG in response).
- Enable higher m (effect of SG via basic state).

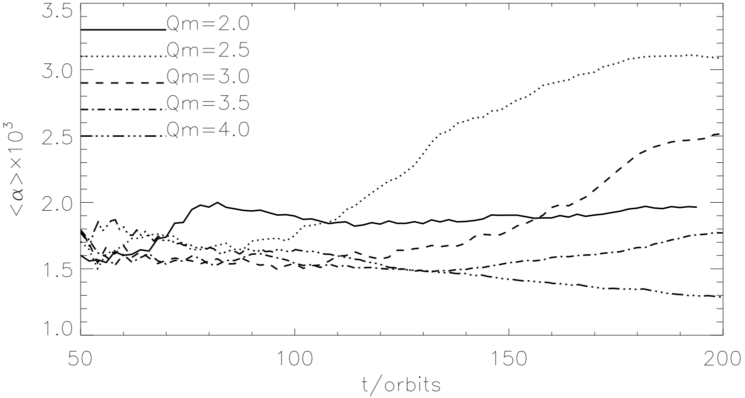
Resisted vortex merging



$M_d = 0.032M_*$ $M_d = 0.027M_*$ $M_d = 0.024M_*$ $M_d = 0.012M_*$

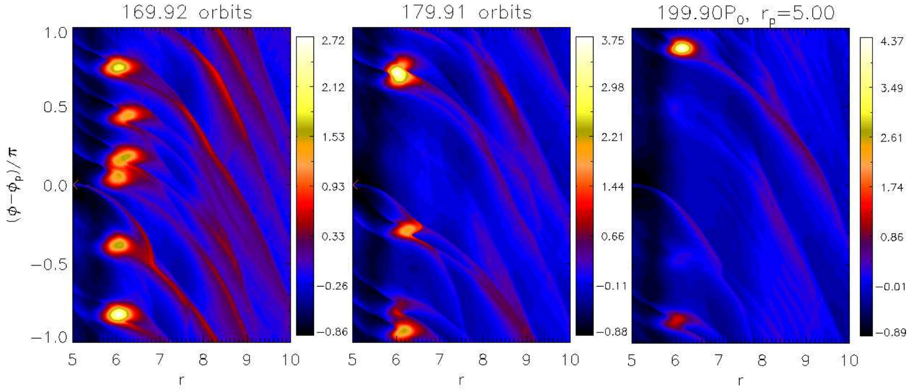
- Multi-vortex configuration is sustained longer with increasing SG.

Resisted vortex merging

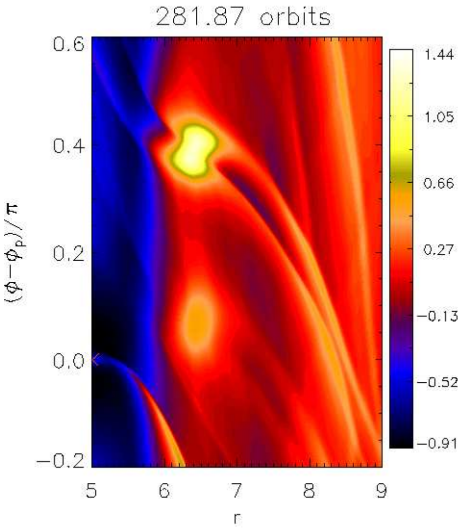


- Get α growth for intermediate range of disc mass.

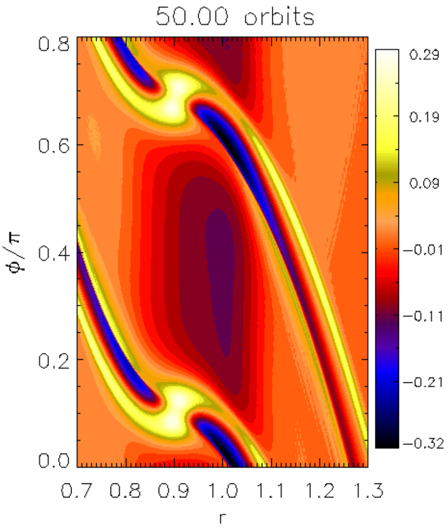
Resisted vortex merging



Vortices as co-orbital planets

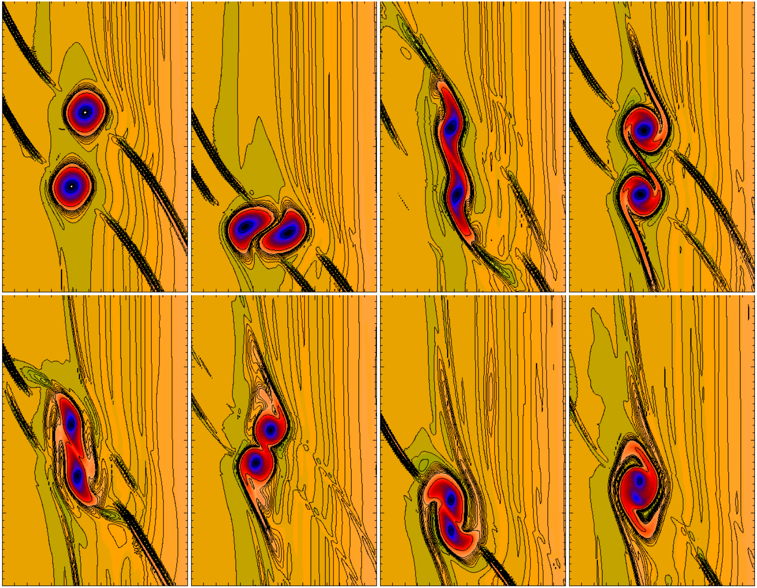


Gap edge vortex-pair



Kida vortex pair

Vortices as co-orbital planets



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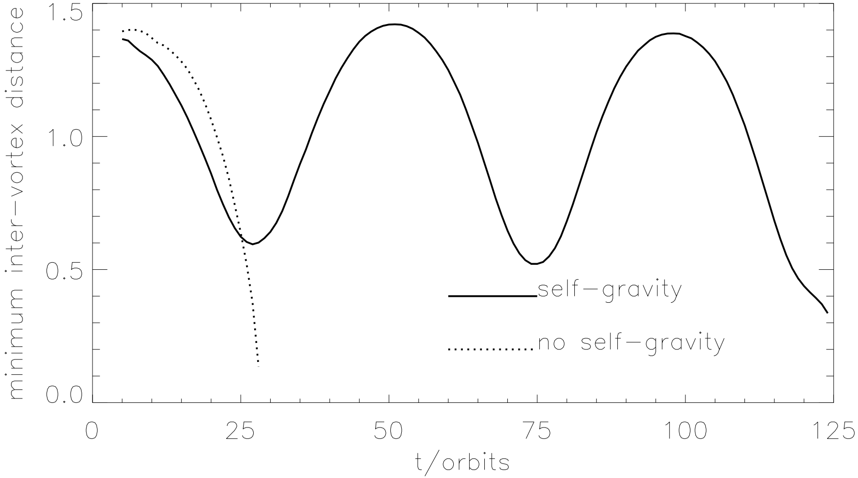

- SG imply minimum inter-vortex distance. If still larger than critical → no merging.

Summary & future work

- Self-gravity stabilizes low m Rossby wave instability through linear response.
- More vortices at planetary gaps as self-gravity increased.
- Mutual gravitational interaction resists vortex merging.

See Lin & Papaloizou (2011a) for vortex modes. See Lin & Papaloizou (2011b) for vortensity maximum modes.

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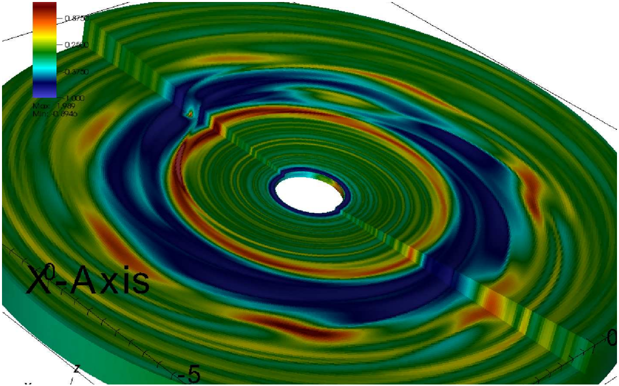
In preparation: 3D self-gravitating disc-planet simulations.



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