VORTICES IN PLANETARY MIGRATION

Min-Kai Lin and John C.B. Papaloizou

UNIVERSITY OF CAMBRIDGE

Department of Applied Mathematics and Theoretical Physics

INTRODUCTION

Vortices at gap edges of Saturn-mass planets originate from instability of vortensity minima and influence type III migration through their effect on the co-orbital mass deficit. The instability is enhanced by lowering viscosity. In the limit of zero viscosity, type III migration occur as a sequence of slow/fast phases, unlike smooth migration in viscous discs. Rapid migration corresponds to flow of vortex material across the co-orbital region, and migration can stall when the gap is partially filled.

MODEL

We consider a planet orbiting a central star in an inviscid disc and units such that the gravitational constant $G = 1$ and primary mass $M_* = 1$. The planet has fixed mass $M_p = 2.8\times10^{-4}$, corresponding to Saturn if M_* is the Solar mass. The disc evolves under the two-dimensional hydrodynamic equations:

$$\frac{\partial\Sigma}{\partial t} + \nabla\cdot(\Sigma\mathbf{v}) = 0, \qquad (1)$$

$$\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla P/\Sigma - \nabla\Phi. \qquad (2)$$

Σ is the surface density and initially $\Sigma_0\times10^{-4}$ where Σ_0 =constant. $P = c_s^2\Sigma$ is the vertically integrated pressure with $c_s^2 = h^2/r$, where $h = H/r$ is the constant aspect ratio, H being the disc semi-thickness . Φ is the total potential including primary, planet (with softening length $\varepsilon = 0.6H$) and indirect. Disc self-gravity is neglected. We use the FARGO code (Masset 2000a,b) to evolve the disc with damping boundary conditions. The planet motion is integrated with a 5th order Runge-Kutta method under disc, primary and indirect potentials.

VORTENSITY RINGS: ORIGIN AND STABILITY

The vortensity $\eta = \hat{\mathbf{z}}\cdot\nabla\times\mathbf{v}/\Sigma$ is central to the stability of inviscid discs; and the *inverse* vortensity η^{-1} can be used to define the co-orbital mass deficit describing type III migration (Masset & Papaloizou 2003). In the disc-planet case, Fig. 1 shows vortensity generation across shocks as fluid elements execute a U-turn. This leads to a global profile with double vortensity rings near separatrices.

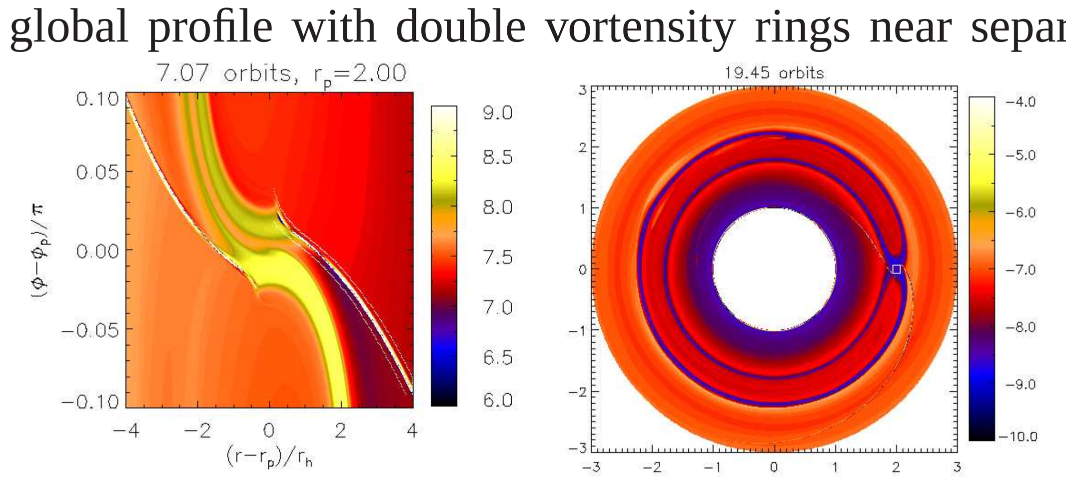

Fig. 1: *Left: vortensity generation across spiral shocks. Right: global inverse vortensity distribution. Fixed-orbit simulation with $\Sigma_0 = 1$, $h = 0.05$.*

Shocks can extend well inside the co-orbital region provided the planet is sufficiently massive. A simple flow model in supersonic regions is particle dynamics (neglecting pressure) in a shearing box centred on the planet. To estimate the shock location, we generalised the method described by Papaloizou et al. (2004), by including non-zero radial velocity, to obtain the equation of the characteristic separating supersonic and subsonic flow

$$\frac{dy_s}{dx} = \frac{\hat{v}_y^2 - 1}{\hat{v}_x\hat{v}_y - \sqrt{\hat{v}_x^2 + \hat{v}_y^2 - 1}}, \qquad (3)$$

where $(\hat{v}_x, \hat{v}_y)$ are shearing box velocities scaled by c_s. Fig. 2 shows results from this model. Keplerian flow, applicable for low mass planets, cannot shock within a distance $2H/3$ from the planet since flow is subsonic. This is resolved by inducing radial velocity due to large planet mass. Vortensity rings then become co-orbital features.

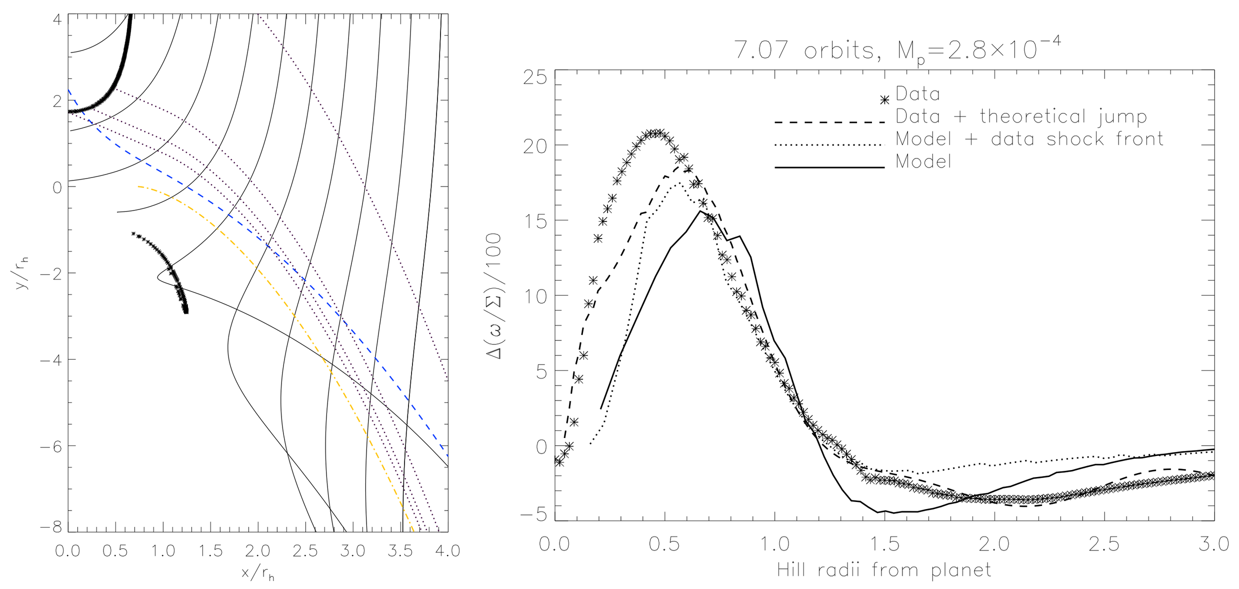

Fig. 2: *Left: particle paths (thin black), sonic points (thick black), solutions to Eq. 3 (purple), solution for Keplerian flow (yellow) and simulation shock front (blue). Right: vortensity jump across the outer shock. Here, $H \simeq r_h$, the Hill radius.*

The ring structure is reached within ~ 10 orbits and is the relevant profile for migration. Steep gradients associated with vortensity rings are dynamically unstable. Isothermal perturbations of the form $f(r)\exp i(\sigma t + m\phi)$ obey the governing equation

$$\frac{d}{dr}\left(\frac{\Sigma}{\kappa^2-\bar{\sigma}^2}\frac{dW}{dr}\right) + \left\{\frac{m}{\bar{\sigma}}\frac{d}{dr}\left[\frac{\kappa^2}{r\eta(\kappa^2-\bar{\sigma}^2)}\right] - \frac{r\Sigma}{h^2} - \frac{m^2\Sigma}{r^2(\kappa^2-\bar{\sigma}^2)}\right\}W = 0, \quad (4)$$

$W \equiv \delta\Sigma/\Sigma$, $\bar{\sigma} \equiv \sigma + m\Omega$ and $\kappa^2 = 2\Omega\Sigma\eta$. Σ and Ω were calculated by imposing hydrostatic equilibrium given η from simulations. In principle, gap structure may be calculated indirectly from modelling vortensity generation across shocks. Solutions for $m = 3$ are shown in Fig. 3, which shows vortensity minima are unstable. In the non-linear regime they develop into vortices and interact with the planet. Table 1 shows growth rates as a function of h.

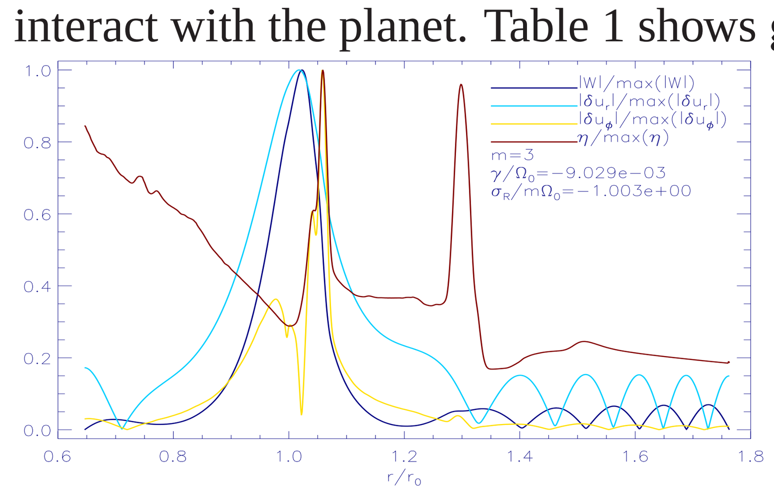

Fig. 3: *Eigenmodes for $m = 3$, $h = 0.05$ with $W = 0$ at boundaries. r_0 is radius of the vortensity minimum.*

h	$\gamma_{\rm in}/\Omega_{\rm in}\times10^3$	$\gamma_{\rm out}/\Omega_{\rm out}\times10^3$
0.03	-20.82	-22.33
0.04	-18.20	-35.01
0.05	-9.029	-31.95
0.06	-2.33×10^{-3}	-5.741

Table 1: *Growth rates of inner/outer gap edges as a function of h, for $m = 3$ with zero boundary condition.*

IMPLICATIONS ON MIGRATION

Vortices at gap edges influence type III migration. The type III torque increases with co-orbital mass deficit

$$\delta m = 2\pi x_s r_p^{-1/2}\left(\langle\Sigma/\omega\rangle_{\rm edge} - \langle\Sigma/\omega\rangle_{\rm gap}\right) \qquad (5)$$

with $x_s = 2.5r_h$. Eq. 5 is a simplified version of the definition by Masset & Papaloizou (2003). δm measures gap depth as a difference between η^{-1} inside the gap to that just inside the inner gap edge. $\langle\Sigma/\omega\rangle_{\rm edge}$ grows because of the instability. In our disc model, we find vortex-planet interaction when $\delta m \simeq 4-5M_p$, and vortex material flows across the co-orbital region (Fig. 4, $t = 65$) and there is rapid migration (Fig. 5). Migration stalls when $\delta m < 0$ as some vortex material becomes co-orbital, or 'gap filling'. Such non-smooth migration is not observed in viscous discs.

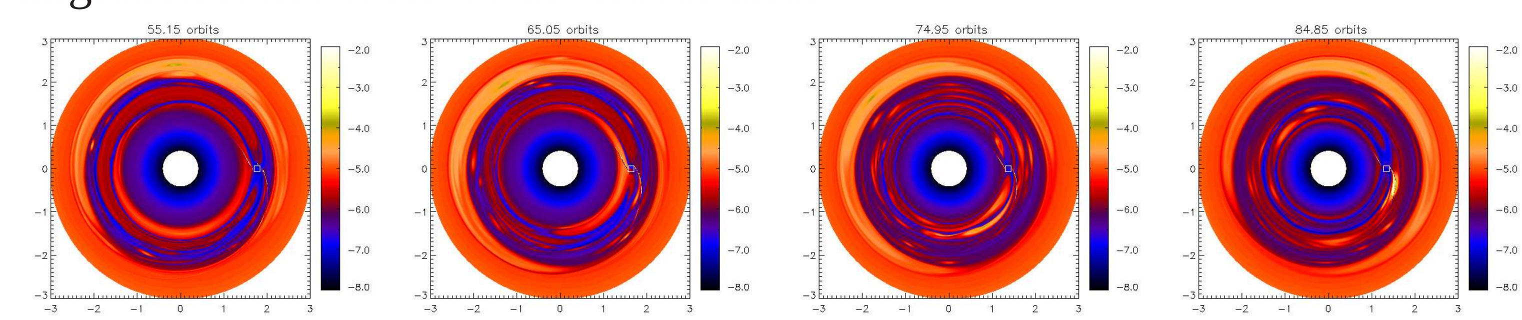

Fig. 4: $\ln(\Sigma/\omega)$ *in type III migration in the inviscid disc with $\Sigma_0 = 7$, $h = 0.05$.*

Fig. 5 shows migration with different Σ_0. Since growth rates are independent of density scaling, increasing the disc mass only shortens the time needed for vortex production to become critical to induce fast migration, but the extent of migration during one episode is unchanged. However, if there is insufficient mass in the annulus where vortices develop, rapid migration cannot occur.

Linear theory show decreasing h, equivalent to lower temperatures, makes the gap edge more unstable. This is due to stronger shocks, hence more extreme profiles. The co-rotational modes in Fig. 3 can only exist if disc profile is sufficiently extreme. For $h = 0.06$ we did not find such modes in linear theory, consistent with Fig. 5 where rapid migration was not observed for $h = 0.06$.

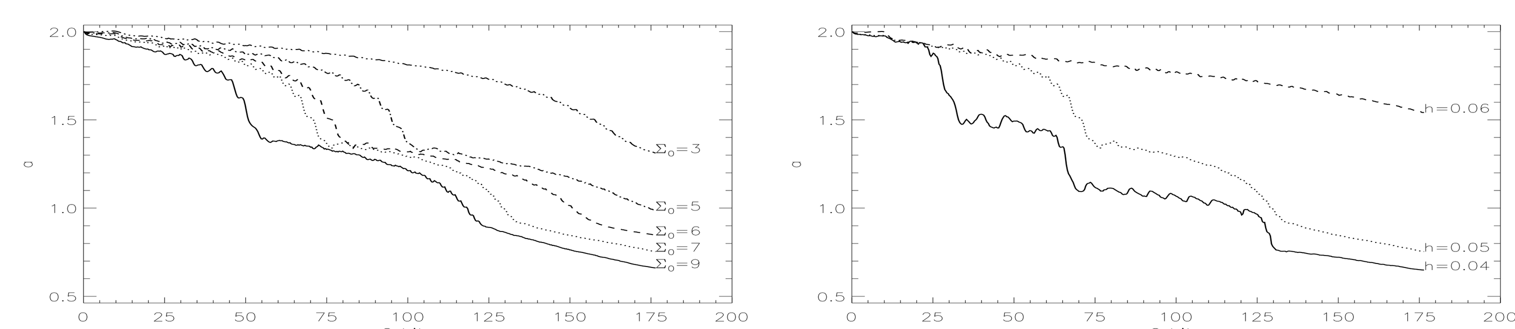

Fig. 5: *Vortex-induced migration in discs of different masses (fixed $h = 0.05$) and temperatures (fixed $\Sigma_0 = 7$).*

CONCLUSIONS

Vortensity rings are natural structures in protoplanetary discs and are unstable. The instability plays a role type III migration due to their formation at gap edges. When δm is sufficiently large, an interaction of vortices generated by the instability with the planet enables rapid migration, which may repeatedly stall and restart, depending on the surface density profile.

Email: *mkl23@cam.ac.uk*